Lecture 13: Asset Pricing

Economics 714, Fall 2014

1 Asset Pricing

1.1 Pricing General Claims

Say transition function has density $f(s, s')$:

$$Q(s, s') = \int_{-\infty}^{s'} f(s, u)du$$

Pricing kernel $q(s', s)$: equilibrium price P^A of unit of period $t + 1$ when $s_{t+1} \in A$ conditional on $s_t = s$ is

$$P^A(s) = \int_{s' \in A} q(s, s')ds'$$

Here we have:

$$q(s, s') = \beta \frac{u'(s')}{u'(s)} f(s, s')$$

Price of any contingent claim $g(s')$ one-period ahead:

$$\begin{aligned} p^g(s) &= \int q(s, s')g(s')ds' \\ &= \int \beta \frac{u'(s')}{u'(s)} g(s')f(s, s')ds' \\ &= E\left[\beta \frac{u'(s')}{u'(s)} g(s') \middle| s\right] \end{aligned}$$

Component $m = \beta \frac{u'(s')}{u'(s)}$ is called stochastic discount factor

For multi-period claims, can chain together one-step claims:

$$q^j(s, s^j) = \int q(s, s')q^{j-1}(s', s^j)ds'$$

Ownership of tree is claim to entire $\{s_t\}$ so can define price as:

$$p(s) = \beta \int \frac{u'(s')}{u'(s)} s' f(s,s') ds' + \beta^2 \int \frac{u'(s'')}{u'(s)} s'' f^2(s,s'') ds'' + \dots$$

Or in sequence notation:

$$p_t = E_t \left[\sum_{j=1}^{\infty} \beta^j \frac{u'(s_{t+j})}{u'(s_t)} s_{t+j} \right]$$

Infinite horizon equilibrium rules out bubbles.

1.2 Risk Neutrality

With linear utility $u'(c_t)$ constant, so risk free rate:

$$1 = E_t(\beta R) \quad \Rightarrow R = \frac{1}{\beta}$$

So then

$$p_t = E_t \left[\sum_{j=1}^{\infty} \beta^j s_{t+j} \right] = E_t \left[\sum_{j=1}^{\infty} \frac{s_{t+j}}{R^j} \right]$$

1.3 Risk Corrections

Risk free rate:

$$1 = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} R \right] \quad \Rightarrow R = \frac{1}{E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} \right]}$$

or $R = 1/E_t m_{t+1}$.

For general payoff x_{t+1},

$$\begin{aligned} p_t &= E_t(m_{t+1} x_{t+1}) \\ &= E_t m_{t+1} E_t x_{t+1} + cov_t(m_{t+1}, x_{t+1}) \\ &= \frac{E_t x_{t+1}}{R} + cov_t(m_{t+1}, x_{t+1}) \end{aligned}$$

1.4 Power Utility and Risk-Free Rates

Now assume $u(c) = c^{1-\gamma}/(1-\gamma)$

Risk-free rate when c_{t+1} known:

$$R = \frac{1}{E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right]} = \frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^{\gamma}$$

Now assume c_{t+1}/c_t conditionally lognormal, $\beta = e^{-\delta}$:

$$\begin{aligned} R &= \frac{1}{E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right]} \\ &= \frac{1}{e^{-\delta-\gamma E_t(\Delta \log c_{t+1})+\frac{\gamma^2}{2}\sigma_t^2(\Delta \log c_{t+1})}} \\ \log R &= \delta + \gamma E_t(\Delta \log c_{t+1}) - \frac{\gamma^2}{2}\sigma_t^2(\Delta \log c_{t+1}) \end{aligned}$$

2 Equity Premium

2.1 Characterization

Define $r^f = R - 1$, $\beta = \frac{1}{1+\delta}$, then (net) stock return r_{t+1} satisfies:

$$1 = E_t\left[\frac{1}{1+\delta}(1+\Delta c_{t+1})^{-\gamma}(1+r_{t+1})\right]$$

Take 2nd order Taylor approximation of right side, unconditional expectations:

$$E(r) = \delta + \gamma E(\Delta c_t) + \gamma cov(r_t, \Delta c_t) - \frac{1}{2}\gamma(\gamma+1)\sigma^2(\Delta c_t)$$

Which can be expressed as:

$$\frac{E(r_t) - r^f}{\sigma(r)} = \gamma\sigma(\Delta c_t) corr(\Delta c_t, r_t)$$

Left side known as Sharpe ratio

2.2 Attempted Resolutions

- Change preferences: recursive preferences, robustness, habit persistence
- Change constraints: Limited participation, transaction costs, incomplete markets
- Change shocks: disaster models, long-run risk, learning