Lecture 1: Dynamics and the Role of Expectations

Economics 712, Fall 2014

Class focuses on DSGE models: dynamic, stochastic general equilibrium. We will build up to each of these elements.

Start without optimization, just focus on dynamics and add role of expectations.

1 The Muth (1961) Model

Demand:

$$x_t^d = \alpha - \beta p_t$$

Supply:

$$x_t^s = \delta + \gamma p_t^e + u_t$$

Equilibrium:

$$p_t = \frac{\alpha - \delta}{\beta} - \frac{\gamma}{\beta} p_t^e - \frac{1}{\beta} u_t$$

What determines p_t^e?

1.1 Adaptive expectations

$$p_t^e = p_{t-1}^e + \eta(p_{t-1} - p_{t-1}^e)$$

Solve backward:

$$p_t^e = \eta \sum_{j=0}^{\infty} (1-\eta)^j p_{t-1-j}$$

Shocks propagated over time, any forecast error is permanent

1.2 Adaptive learning

Suppose agents believe prices are i.i.d.:

$$p_t = \mu + \varepsilon_t$$

with $E_{t-1}\varepsilon_t = 0$. Then $p_t^e = \mu$, which implies:

$$p_t = \frac{\alpha - \delta}{\beta} - \frac{\gamma}{\beta}\mu - \frac{1}{\beta}u_t$$

So there is a **temporary equilibrium** where beliefs μ are mapped into outcomes consistent with the same from of beliefs (i.i.d. prices) but with different mean $T(\mu)$:

$$T(\mu) = \frac{\alpha - \delta}{\beta} - \frac{\gamma}{\beta}\mu$$

At a fixed point of the T beliefs are consistent with outcomes:

$$\mu^* = T(\mu^*) = \frac{\alpha - \delta}{\beta} - \frac{\gamma}{\beta}\mu^* = \frac{\alpha - \delta}{\beta + \gamma}$$

Models of **adaptive learning** assume that agents have subjective models with unknown parameters, update those parameters with observations. Estimate of μ based on data through $t-1$ is μ_t. Update as:

$$\begin{aligned} \mu_{t+1} &= \mu_t + \eta(p_t - \mu_t) \\ &= \mu_t + \eta(T(\mu_t) - \mu_t) - \eta\frac{1}{\beta}u_t \end{aligned}$$

Adjust beliefs toward $T(\mu)$. Learning literature studies when does $\mu_t \to \mu^*$.

1.3 Rational expectations

Subjective expectation is mathematical expectation conditioned on all all available information:

$$p_t^e = E[p_t | p_{t-1}, p_{t-2}, \ldots] \equiv E_{t-1} p_t$$

Implication:

$$E_{t-1} p_t = \frac{\alpha - \delta}{\beta + \gamma}$$

p_t is i.i.d.

2 Dynamics of Linear Difference Equations

2.1 Scalar example: Linear Solow Model

Output $Y_t = Y_0 + AK_t$

Consumption: $C_t = (1 - s)Y_t$

Capital accumulation: $K_{t+1} = (1 - \delta)K_t + Y_t - C_t = (1 - \delta + sA)K_t + sY_0 \equiv aK_t + b$

What happens to K_t (and C_t, Y_t) over time?

Recursive substitution:

$$K_{t+1} = aK_t + b = a^2 K_{t-1} + ab + b = \ldots = a^t K_0 + \sum_{i=0}^{t} a^i b$$

So if $|a| < 1$ we have a **steady state**:

$$\lim_{t \to \infty} K_t = \frac{b}{1 - a}$$

Phase diagram: plot K_{t+1} vs. K_t.

2.2 Vector example: Dornbusch (1976) exchange rate model

Aggregate demand: $y_t = \delta(e_t + p^* - p_t) - \sigma(r_t - p^e_{t+1} + p_t)$

Phillips curve: $p_{t+1} - p_t = \alpha(y_t - \bar{y})$

Money demand: $m - p_t = \phi\bar{y} - \lambda r_t$

Interest parity: $r_t = r^* + e_{t+1} - e_t$

Combine equations, impose **perfect foresight** $p^e_{t+1} = p_{t+1}$. Gives system:

$$
\begin{aligned}
\lambda(e_{t+1} - e_t) &= \phi\bar{y} - \lambda r^* + p_t - m \\
(1 - \alpha\sigma)(p_{t+1} - p_t) &= \alpha\left[\delta(e_t + p^* - p_t) - \bar{y} + \frac{\sigma}{\lambda}(\phi\bar{y} - m + p_t)\right]
\end{aligned}
$$

Steady state: $e_t = \bar{e}$, $p_t = \bar{p}$:

$$
\begin{aligned}
\bar{p} &= \lambda r^* + m - \phi y \\
\bar{e} &= \bar{p} - p^* + \frac{1}{\delta}(\bar{y} + \sigma r^*)
\end{aligned}
$$

Then re-write dynamics as deviations from steady state:

$$
\begin{aligned}
e_{t+1} - \bar{e} &= e_t - \bar{e} + \frac{1}{\lambda}(p_t - \bar{p}) \\
p_{t+1} - \bar{p} &= \frac{\delta\alpha}{1 - \alpha\sigma}(e_t - \bar{e}) + \left[1 - \frac{\alpha}{1 - \alpha\sigma}(\delta + \sigma/\lambda)(p_t - \bar{p})\right]
\end{aligned}
$$

Define vector $X_t = [e_t - \bar{e}, p_t - \bar{p}]'$, so can write this as a vector difference equation $X_{t+1} = AX_t$. What happens to X_t over time?

Qualitatively, can construct phase diagram.

$$
\begin{aligned}
\Delta e_t &= e_{t+1} - e_t = (p_t - \bar{p})\lambda \\
\Delta p_t &= \frac{\alpha}{1 - \alpha\sigma}\left[\delta(e_t - \bar{e}) - (\delta + \sigma/\lambda)(p_t - \bar{p})\right]
\end{aligned}
$$

Analyze regions where $\Delta e_t > 0, \Delta e_t = 0, \Delta e_t < 0$, same for p_t.

Given p_t, there is a unique value of e_t such that the economy is stable. This mapping $e_t(p_t)$ defines the **saddle path**.

3 Forward Looking Model: Cagan Model with Rational Expectations

$$M_t^d = \exp\left(-\alpha \frac{E_t p_{t+1} - p_t}{p_t}\right)$$

Take logs, rewrite:

$$p_t = aE_t p_{t+1} + cm_t$$

What happens to prices over time? Solve forward if $|a| < 1$:

$$p_t = c\sum_{j=0}^{\infty} a^j E_t m_{t+j} + \lim_{T\to\infty} a^T E_t p_{t+T}$$

Example:

$$m_{t+1} = \rho m_t + w_{t+1}$$

Then:

$$p_t = \frac{c}{1 - a\rho} m_t$$

Lucas critique and cross-equation restrictions