Lecture 1: Incomplete Markets Models

Stationary Distributions

Economics 714, Spring 2015

1 Incomplete Markets Models

So far have studied complete markets models where agents are fully insured.

Now study (exogenous) incomplete markets, where not all possible asset markets exist.

Agents bear idiosyncratic risk.

Model will imply heterogeneity: income and wealth distributions

Aggregate impact on quantities (capital stock), prices (rates of return)

1.1 Bewley-Aiyagari-Huggett Models

Put consumption-savings model with borrowing constraint into general equilibrium

Only asset is risk free bond, agents face idiosyncratic (labor) income risk

No aggregate uncertainty. Agent's problem, as before:

$$\max_{\{c_t, a_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

with a_0, l_0 given and subject to:

$$c_t + a_{t+1} = Ra_t + wl_t$$

Huggett: endowment economy, $wl_t = y_t$, specify income directly.

Aiyagari: production economy with representative firm, so w endogenous.

Constraints: $c_t \geq 0, \quad a_t \geq \underline{\mathrm{a}}$. Debt limit.

l follows Markov process w/transition function Q on L

We've seen that optimization gives policy function $a' = a'(a, l)$, Euler inequality: $u'(c_t) \geq \beta R E_t u'(c_{t+1})$, with equality if $a_{t+1} > \underline{\mathrm{a}}$

Also have seen that $a_t \to \infty$ if $\beta R \geq 1$.

We add a representative firm with constant returns to scale production $F(K, N)$.

Aggregate capital follows $K_{t+1} = (1 - \delta)K_t + I_t$.

Firm optimality conditions: $w = F_N(K, N)$, $r = F_K(K, N) - \delta$.

1.2 General Equilibrium

Each agent i has endowment l_t^i with transition Q but independent across agents

Assume Q has stationary distribution μ_l^*.

If $l_0^i \sim \mu_l^*$, aggregate labor supply is constant:

$$\int l_t^i d\mu_l^* = N$$

Assumes continuum law of large numbers

Definition: A (recursive, competitive) stationary equilibrium is a policy function $a'(a, l)$, a probability distribution $\mu^*(a, l)$ and positive real numbers (K, r, w) such that:

(i) Firms optimize: $w = F_N(K, N)$, $r = F_K(K, N) - \delta$.

(ii) Households optimize: $a'(a, l)$ solves the household problem

(iii) Stationarity: $\mu^*(a, l)$ is the stationary distribution implied by household behavior:

$$\mu^*(a', l') = \int_L \int_{\{a:a'=a'(a,l)\}} Q(l, l')\mu^*(da, dl)$$

(iv) Individual and aggregate behavior are consistent:

$$K = \int_A \int_L a'(a,l) d\mu^*(a,l)$$

Note: build in labor market clearing by assuming $N = \int l_t^i d\mu_l^*$

Interpretation of stationary distribution: ergodic characterization of household assets also cross-section characterization of population assets.

No ex-ante heterogeneity, all driven by realizations. Eventually any agent will become richest or poorest in the economy.

2 Stationary Distributions

2.1 Background

We want to establish under what conditions a stationary distribution of assets exists

For general Markov transition P, measure μ on measurable space $(S, \mathcal{S})$, define operator:

$$T^*\mu(\hat{s}) = \int P(s,\hat{s})\mu(ds) \quad \forall \hat{s} \in S$$

Invariant or stationary distribution μ^* satisfies $T^*\mu^* = \mu^*$

Let $(S, \mathcal{S})$ be a measurable space, $C(S)$ bounded, continuous functions on S, $\{\mu_n\}$ and μ probability measures on $(S, \mathcal{S})$. We focus on convergence in the weak sense.

Definition: A sequence of probability measures $\{\mu_n\}$ converges weakly to μ, written $\mu_n \Rightarrow \mu$ if

$$\lim_{n\to\infty} \int f d\mu_n = \int f d\mu \quad \forall f \in C(S)$$

We need to define the joint transition over assets a and labor l:

$$
\begin{aligned}
P((a,l),\hat{A}\times\hat{L}) &= Q(l,\hat{L}) \;\text{ if } a'(a,l)\in\hat{A}, \\
&= 0 \;\text{ if } a'(a,l)\notin\hat{A}
\end{aligned}
$$

The basic existence result is:

Theorem: If $S\subset\mathbb{R}^n$ is compact, and P has the Feller property, then there exists an invariant measure μ^*.

Does not rule out multiple stationary distributions, or establish stability

Definition: For $S\subset\mathbb{R}^n$, P is monotone if for $f:S\to\mathbb{R}$ which are bounded and nondecreasing, $Tf(s)=\int f(s')P(s,ds')$ is also nondecreasing.

This ensures convergence to a stationary distribution

Theorem: If $S=[a,b]\subset\mathbb{R}^n$, and P is monotone and has the Feller property, let δ_a and δ_b be Dirac mass at a and b. Then $(T^*)^n\delta_a\to\mu^*a$ with $T^*\mu_a^*=\mu_a^*$ and $(T^*)^n\delta_b\to\mu^*b$ with $T^*\mu_b^*=\mu_b^*$.

To rule out multiple stationary distributions, need a mixing condition.

Assumption: $\exists c\in S$, $\varepsilon>0$, $N\geq 1$ such that $P^N(a,[c,b])\geq\varepsilon$ and $P^N(b,[a,c])\geq\varepsilon$.

This gives us:

Theorem: If $S=[a,b]\subset\mathbb{R}^n$, and P is monotone, has the Feller property, and satisfies the mixing condition, then there exists a unique invariant distribution μ^* and $(T^*)^n\mu_0\to\mu^*$ for all probability measures μ_0 on $(S,\mathcal{S})$.