More on the New Keynesian Model

Noah Williams

University of Wisconsin-Madison

The New Keynesian Phillips Curve

- Recall that the inflation adjustment equation can be written:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t \tag{1}$$

  where $\kappa = (\eta + \sigma)\,\tilde{\kappa} = (\eta + \sigma)\,(1 - \omega)\,[1 - \beta\omega]\,/\omega$ and $x_t \equiv \hat{y}_t - \hat{y}_t^f$ is the gap between actual output and the flexible-price equilibrium output.
- This inflation adjustment or forward-looking Phillips curve relates output, in the form of the deviation around the level of output that would occur in the absence of nominal price rigidity, to inflation.

The demand side of the model

- Start with Euler condition for optimal consumption choice

$$C_t^{-\sigma} = \beta R_t \mathrm{E}_t \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}$$

- Linearize around steady-state:

$$-\sigma \hat{c}_t = (\hat{\imath}_t - \mathrm{E}_t p_{t+1} + p_t) - \sigma \mathrm{E}_t \hat{c}_{t+1}$$

or

$$\hat{c}_t = \mathrm{E}_t \hat{c}_{t+1} - \left(\frac{1}{\sigma} \right) (\hat{\imath}_t - \mathrm{E}_t p_{t+1} + p_t) .$$

- Goods market equilibrium (no capital)

$$Y_t = C_t .$$

The demand side of the model

Linearization

- Euler condition becomes

$$\hat{y}_t = \mathrm{E}_t \hat{y}_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{\imath}_t - \mathrm{E}_t p_{t+1} + p_t\right).$$

- This is often called an "expectational IS curve", to make the comparisons with old-style Keynesian models clear.

Demand and the output gap

- Express in terms of the output gap $x_t = \hat{y}_t - \hat{y}_t^f$:

$$\hat{y}_t - \hat{y}_t^f = \mathrm{E}_t\left(\hat{y}_{t+1} - \hat{y}_{t+1}^f\right) - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - \mathrm{E}_t p_{t+1} + p_t) + \left(\mathrm{E}_t \hat{y}_{t+1}^f - \hat{y}_t^f\right)$$

or

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(r_t - r_t^n),$$

where $r_t = \hat{\imath}_t - E_t p_{t+1} + p_t$ and

$$r_t^n \equiv \sigma\left(\mathrm{E}_t \hat{y}_{t+1}^f - \hat{y}_t^f\right).$$

- Notice that the nominal interest rate affects output through the interest rate gap $r_t - r_t^n$.

The general equilibrium model

- Two equation system

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t$$

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - \mathrm{E}_t \pi_{t+1} - r_t^n)$$

The general equilibrium model

- Consistent with
  - optimizing behavior by households and firms
  - budget constraints
  - market equilibrium
- Two equations but three unknowns: x_t, π_t, and i_t – need to specify monetary policy

Solving the model for the rational expectations equilibrium

- Suppose i is exogenous.
- Write system as

$$\begin{bmatrix} \beta & 0 \\ \frac{1}{\sigma} & 1 \end{bmatrix} \begin{bmatrix} \mathrm{E}_t \pi_{t+1} \\ \mathrm{E}_t x_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -\kappa \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\sigma} \end{bmatrix} (i_t - r_t^n)$$

- or

$$\begin{bmatrix} \mathrm{E}_t \pi_{t+1} \\ \mathrm{E}_t x_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ -\frac{1}{\sigma\beta} & 1 + \frac{\kappa}{\sigma\beta} \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\sigma} \end{bmatrix} (i_t - r_t^n)$$

- or

$$\mathrm{E}_t Z_{t+1} = M Z_t + N(i_t - r_t^n)$$

Solving the model for the rational expectations equilibrium

- There exists a unique, stationary rational expectations equilibrium if and only if the number of eigenvalues of M outside the unit circle is equal to the number of forward-looking variables (two).
- Condition is not satisfied!
- So a policy that just sets $i_t = r_t^n$ exogenously does not result in a unique rational expectations equilibrium.
- Self-fulfilling increase in expected inflation is possible.

Solving the model for the rational expectations equilibrium

- Suppose $i_t = r_t^n + \delta \pi_t$.
- Write system as

$$\begin{bmatrix} \beta & 0 \\ \frac{1}{\sigma} & 1 \end{bmatrix} \begin{bmatrix} \mathrm{E}_t \pi_{t+1} \\ \mathrm{E}_t x_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -\kappa \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\sigma} \end{bmatrix} \delta \pi_t$$

- or

$$\begin{bmatrix} \mathrm{E}_t \pi_{t+1} \\ \mathrm{E}_t x_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ \frac{\beta\delta - 1}{\sigma\beta} & 1 + \frac{\kappa}{\sigma\beta} \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}$$

- Two eigenvalues outside the unit circle if and only if

$$\delta > 1$$

The Taylor Principle

- Policy must respond sufficiently strongly to inflation.

Definition

The condition that the nominal interest rate respond more than one-for-one to inflation is called the Taylor Principle.

Lessons

- Policy based on responding to exogenous disturbances does not ensure a unique equilibrium.
- Policy must respond to endogenous variables.
- In particular, the Taylor Principle needs to be satisfied.
  - If policy also responds to the output gap, then Bullard and Mitra show condition becomes

$$\kappa(\delta_\pi - 1) + (1 - \beta)\delta_x > 0.$$

The Wicksellian interest rate

- Basic model:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t$$

- The impact of monetary policy on output and inflation operates through the real rate of interest;
- Wicksellian interest rate gap $i_t - E_t \pi_{t+1} - r_t^n$ summaries impact of monetary policy.

The Wicksellian interest rate

Definition

Woodford (2003) has labelled r_t^n as the Wicksellian real interest rate. It is the interest rate consistent with output equal to the flexible-price equilibrium level. r^n is also called the natural rate of interest.

The Wicksellian interest rate

- Output is affected by expected current and future one-period real interest rates. The presence of expected future output implies that the future path of the one-period real rate matters for current demand.
  - To see that this is the case, let $r_t \equiv i_t - E_t \pi_{t+1}$ be the one-period real interest rate and then recursively solve the Euler condition forward to yield (assume $C = Y$)

$$x_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} \mathrm{E}_t \left(r_{t+i} - r_{t+i}^n \right).$$

  - Changes in the one-period rate that are persistent, so that they also influence expectations of future interest rates, will have stronger effects on x_t then more temporary changes in r.

Policy analysis

Key issues

- What are the objectives of optimal policy
- Is the policy environment one of commitment or discretion?
- What instrument rule implements the optimal policy?
- What are the properties of the resulting equilibrium?

Welfare

- Given the specification of the economic environment, what are the appropriate objectives of the central bank?
- Standard to assume central bank is concerned with minimizing a quadratic loss function that depended on output and inflation – plausible, but ultimately *ad hoc*. Common in the Barro-Gordon tradition
- Woodford (2003) has provided the most detailed analysis of the link between a welfare criteria derived as a log-linear approximation to the utility of the representative agent and the type of quadratic loss functions so common in the literature.

Welfare

Quadratic approximation

- Woodford demonstrates that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda \left(x_{t+i} - x^* \right)^2 \right]. \tag{2}$$

- x_t is the gap between output and the output level that would arise under flexible prices, and x^* is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the steady-state level of output.

Comparison to a standard loss function

- This looks a lot like the standard quadratic loss function. There are, however, two critical differences.
  1. The output gap is measured relative to the rate of output under flexible prices.
  2. Inflation variability enters because, with price rigidity, higher inflation results in an inefficient dispersion of output among the individual producers.
     - ★ Because prices are sticky, higher inflation results in an increase in overall price dispersion.

Policy weights

- Theory says something about the weights in the loss function:

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda \left(x_{t+i} - x^* \right)^2 \right],$$

where

$$\Omega = \frac{1}{2} \bar{Y} U_c \left[\frac{\omega}{(1-\omega)(1-\omega\beta)} \right] \left(\theta^{-1} + \eta \right) \theta^2$$

and

$$\lambda = \left[\frac{(1-\omega)(1-\omega\beta)}{\omega} \right] \frac{(\sigma + \eta)}{(1 + \eta\theta)\,\theta}.$$

- Greater nominal rigidity (larger ω) reduces λ.
- Loss function endogenous.
- Calvo specification implies λ is small – Taylor specification leads to larger weight on output gap.

Instrument Rules

- A common approach to "optimal" policy is in terms of simple rules.
- The most famous of such instrument rules is the Taylor Rule (Taylor 1993):

$$i_t = \pi_t + 0.5x_t + 0.5\left(\pi_t - \pi^T\right) + r^*,$$

  where π^T was the target level of average inflation (Taylor assumed it to be 2%) and r^* was the equilibrium real rate of interest (Taylor assumed this too was equal to 2%).
- The Taylor Rule for general coefficients is

$$i_t = r^* + \pi^T + \delta_x x_t + \delta_\pi\left(\pi_t - \pi^T\right). \tag{3}$$

Taylor rules

- A larger literature has now developed that has estimated the Taylor Rule, or similar simple rules, for a variety of countries and time periods.
  - For example, Clarida, Galí, and Gertler (2000) do so for the Federal Reserve, the Bundesbank, and the Bank of Japan.
  - Estimates for the United States under different Federal Reserve Chairman are reported by Judd and Rudebusch (2000).
  - In general, the basic Taylor Rule, when supplemented by the addition of the lagged nominal interest rate, does quite well in matching the actual behavior of the policy interest rate.
- The argument for simple rules relies not on their optimality but on their simplicity; they may serve as a useful benchmark for policy or aid in promoting policy transparency.

Policy Implication of forward-looking models

- The basic new Keynesian inflation adjustment equation took the form

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

- That is, there is no additional disturbance term.

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t \Rightarrow \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i}$$

- The absence of a stochastic disturbance implies there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero.
- Just set $x_{t+i} = 0$ for all i; keeps inflation equal to zero.

Optimal policy in forward-looking models

- Thus, the key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.
- No policy conflicts.
- When prices are sticky but wages are flexible, the nominal wage can adjust to ensure labor market equilibrium is maintained in the face of productivity shocks. Optimal policy should then aim to keep the price level stable.

Policy implications of price stickiness

- Models that combine optimizing agents and sticky prices have very strong policy implications.
- When the price level fluctuates, and not all firms are able to adjust, price dispersion results. This causes the relative prices of the different goods to vary. If the price level rises, for example, two things happen.
  1. The relative price of firms who have not set their prices for a while falls. They experience in increase in demand and raise output, while firms who have just reset their prices reduce output. This production dispersion is inefficient.
  2. Consumers increase their consumption of the goods whose relative price has fallen and reduce consumption of those goods whose relative price has risen. This dispersion in consumption reduces welfare.

Optimal policy

- The solution is to prevent price dispersion by stabilizing the price level.
- What is critical for this result is that nominal wages are assumed to be completely flexible.
- But the same argument would apply if wages are sticky and prices flexible. With sticky wages and flexible prices, monetary policy should stabilizes the nominal wage.

Woodford versus Friedman

- The basic new Keynesian model suggests price stability (i.e., zero inflation) is optimal.
  - Zero inflation eliminates inefficient price dispersion.
- Friedman rule: zero nominal rate of interest is optimal.
  - Zero nominal rate eliminates inefficiency in money holdings.
  - Optimal inflation is negative (deflation) at rate equal to real rate of interest.
- Khan, King, and Wolman (2000) analysis model with both distortions.
- The conclude optimal inflation is closer to zero than to the Friedman rule.

Cost shocks

- Assume

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

  where e represents an inflation or cost shock.
- Then

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i E_t e_{t+i}$$

- Cannot keep both x and π equal to zero.
- Trade-offs must be made.

Implications of cost shocks

- Stochastic wedge between marginal rate of substitution and real wage.
- Wedge between flexible-price output and efficient level of output.
- Other sources: sticky nominal wages.

Other channels of monetary transmission

The role of money

- So far, monetary policy only works via the Wicksellian interest rate gap.
- No direct role for money.
- Direct effects of the quantity of money: if utility is not separable, then changes in the real quantity of money would alter the marginal utility of consumption. The absence of money constitutes a special case.
  - The real money stock would appear in the household's Euler condition.
  - To replace real marginal cost with a measure of the output gap in the inflation equation, the real wage was equated to the marginal rate of substitution between leisure and consumption, and this will involve real money balances.

Other channels of monetary transmission

The role of money

- However, McCallum and Nelson (2000) and Woodford (2003) have both argued that the effects arising with nonseparable utility are quite small, so that little is lost by assuming separability.
- Empirical evidence: Ireland (2001).

Adding lagged inflation

- To capture the inflation persistence found in the data, it is common to augment the basic forward-looking inflation adjustment equation with the addition of lagged inflation:

$$\pi_t = (1-\phi)\beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + \phi \pi_{t-1} + \varepsilon_t. \tag{4}$$

- In this formulation, the parameter ϕ is often described as a measure of the degree of backward-looking price setting behavior.
  - Fuhrer (1997) finds little role for future inflation once lagged inflation is added to the inflation adjustment equation.
  - Rudebusch (2000) estimates (2) using U. S. data and argues that ϕ is on the order of 0.7, suggesting that inflation is predominantly backward-looking.

Indexation

- Christiano, Eichenbaum, and Evans (2001) make a distinction between firms that reoptimize it setting their price and those that do not:
  - each period a fraction $1 - \omega$ of all firms optimally set their price;
  - the remaining firms either simply adjust their price based on the average rate of inflation, so that $p_{jt} = \bar{\pi} p_{jt-1}$ where $\bar{\pi}$ is the average inflation rate, or they adjust based on the most recently observed rate of inflation, so that $p_{jt} = \pi_{t-1} p_{jt-1}$.
- Costly to optimize

Indexation and decision lag

- This specification results in an inflation adjustment equation of the form
$$\pi_t = \left(\frac{\beta}{1+\beta}\right) \mathrm{E}_t \pi_{t+1} + \left(\frac{1}{1+\beta}\right) \pi_{t-1} + \tilde{\kappa}\hat{\varphi}_t.$$
The presence of lagged inflation in this equation introduces inertia into the inflation process.
- CEE also assume prices set before time t information is available:
$$\pi_t = \left(\frac{\beta}{1+\beta}\right) \mathrm{E}_{t-1} \pi_{t+1} + \left(\frac{1}{1+\beta}\right) \pi_{t-1} + \tilde{\kappa}\mathrm{E}_{t-1}\hat{\varphi}_t.$$

Structural estimates of the frequency of adjusting

- Estimates of new Keynesian Phillips curve yield values of ω that too high.
- Estimates range from 0.758 to 0.911 (Dennis 2006)
- Value of 0.8 implies prices adjusted on average every $(1 - 0.8)^{-1} = 5$ quarters.
- Micro evidence for U.S. suggests duration between price changes closer to 2 quarters, implying $\omega = 0.5$.

The sensitivity of marginal cost to output

- Empirically, inflation does not seem to respond strongly to the output gap: κ is small.
- In basic theory,

$$\kappa = (\eta + \sigma)\frac{(1-\omega)\left[1-\beta\omega\right]}{\omega}$$

where $1-\omega$ is the fraction of adjusting firms, σ is the coefficient of relative risk aversion, and η is the (inverse) of the wage elasticity of labor supply.

The sensitivity of marginal cost to output

So κ small if

- ω large – high degree of price rigidity (estimates often imply unrealistic values around 0.8)
- σ small – very little risk aversion
- η is small – high degree of labor supply elasticity.

The sensitivity of marginal cost to output

- Researchers have extended basic model to make marginal cost less sensitive to output.
- Christiano, Eichenbaum, and Evans (2001) – variable capital utilization
- Basic idea:
  - In standard model, increase in demand can only increase production if real wage rises to induce an increase in labor supply. If wage elasticity of labor supply is small, the real wage has to rise a lot. This boosts real marginal cost and inflation.
  - If output can increase by utilizing capital more intensely, wages and marginal cost will rise less.

The sensitivity of marginal cost to output

Firm-specific capital

- Generates decreasing returns to labor;
- Marginal cost varies across firms.;
- Marginal cost is increasing in firm's output;
- Elasticity of marginal cost to output depends on short-run returns to scale in variable factor.

Firm-specific capital

- Intuition
  - if aggregate real wages rise, firms that adjust price raise their price;
  - but rise in price lowers output at firm;
  - with fixed capital, marginal cost falls as output declines;
  - this dampens amount firm will raise price.
- If capital at firm is costly to adjust, firm only slowly adjusts its capital.
- Eichenbaum and Fisher find they get estimates of ω around one half – more plausible.

Demand persistence

The trouble with Euler conditions

- Euler condition is purely forward looking – same problems arise as with inflation equation.
- Output is discounted value of future interest rate gaps:

$$x_t = -\left(\frac{1}{\sigma}\right) E_t \sum_{i=0}^{\infty} \left(r_{t+i} - r_{t+i}^n\right).$$

Habit persistence

- To match the hump shaped response of output seen in the data, habit persistence has become a standard component of new Keynesian models (Fuhrer 2000, Christiano, Eichenbaum, and Evans 2001).
- External Habit persistence: Marginal utility of current consumption depends on past aggregate consumption.
- Internal Habit Persistence: Marginal utility of current consumption depends on household's past consumption.

General equilibrium, estimated models

- Christiano, Eichenbaum, and Evans (2001)
- Smets and Wouters (2003)
- Levin, Onatski, Williams, and Williams (2005)
  - Components:
    - ★ Habit persistence
    - ★ Variable capital utilization
    - ★ Investment with 2nd-order adjustment costs
    - ★ Price adjustment at start of period (based on expectations – information delay)
    - ★ Wage and price stickiness

Conclusions

- Basic model fairs poorly when faced with data – too forward-looking;
- Habit persistence, variable capital utilization, firm specific capital, sticky wages all help.
- Models fit data, but decomposition into flexible-price and gap may miss major historical episodes.