Problem Set 3
Due in class on March 29.

1. This problem considers the qualitative effects of different types of shocks as potential sources of business cycle fluctuations. For each part use the two period dynamic general equilibrium model (with output supply and demand, labor supply and demand) to analyze the effects on output, interest rates, wages, and employment. In each case discuss whether the equilibrium effects are consistent with observed business cycles.

   (a) Suppose there is a temporary increase in government spending.

   (b) Suppose that there is a shift in consumer preferences, so that given the market real interest rate the representative agent now prefers less leisure and more consumption goods.

   (c) Suppose that there is an exogenous reduction in investment demand by firms (bad “animal spirits”).

2. Suppose that a household does not face a cash in advance constraint, but instead money is the only asset for transferring income over time. That is, suppose that household preferences are given by:

   \[ \log C + \beta \log C'. \]

   The household has real unearned income \( Y \) in period 1, and uses money \( M \) to transfer assets to the future, thus facing the budget constraints:

   \[ PC + M = PY \]
   \[ P'C' = M \]

   (a) Find the household’s money demand function and show that it is decreasing in the inflation rate.

   (b) Relate your results to the quantity theory of money \( MV = PY \). What is the velocity of money in this problem?

3. Consider a real business cycle model where preferences are given by:

   \[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \]

   where \( c_t \) is consumption and \( \gamma > 0 \) is a constant. The household thus does not value leisure. The sole input in production is capital \( k_t \), which is owned by households and
rented to firms at a rental rate \( r_t \). That is output is produced via \( y_t = z_t k_t \) where \( z_t \) is the (possibly stochastic) level of productivity. Capital depreciates at rate \( \delta \), and investment is given by \( I_t \) so the law of motion for capital is:

\[
k_{t+1} = (1 - \delta)k_t + I_t
\]

The household budget constraint each period is thus:

\[
c_t + I_t = r_t k_t
\]

(a) Write down the firms profit maximization problem, and derive an expression for the real interest rate at any date \( t \).

(b) Write down the household utility maximization problem, and considering the choice of consumption at two arbitrary dates \( t \) and \( t + 1 \) find the Euler equation.

(c) Suppose that \( z_t \) is constant. Under what conditions is equilibrium consumption increasing? Interpret your answer.

(d) Suppose that \( z_t \) is constant for all time. Under what conditions is equilibrium consumption increasing? Interpret your answer.

(e) Now suppose that \( z_{s} = z \) had been constant for \( s < t \) then there is a permanent, unanticipated increase to \( z_t = z' > z \). What happens to output, consumption, and interest rates?

4. Consider a money-in-the-utility function endowment economy model. Suppose that preferences are given by:

\[
\sum_{t=0}^{\infty} \beta^t (\log(C_t) + \gamma \log(M_t/P_t))
\]

where \( \gamma > 0 \) and \( 0 < \beta < 1 \). The household budget constraint is:

\[
Y + \tau_t + \frac{(1 + i_{t-1})B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t},
\]

where \( Y \) is the exogenously given endowment (which we assume is constant each period), \( \tau_t \) lump sum transfers, \( B_t \) nominal bond holdings, \( i_t \) the nominal inflation rate, \( M_t \) nominal money holdings, \( C_t \) consumption and \( P_t \) the price level. The government primary deficit \( D_t \) can be defined as:

\[
D_t = \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - (1 + i_{t-1})B_{t-1}}{P_t},
\]

capturing the net revenue from money creation and bond issuance.

(a) Find the optimality conditions characterizing an equilibrium.
(b) Suppose there is no government spending, that the real value of bonds $B_t/P_t$ is constant, and that the nominal money supply grows at a constant rate. Solve for a steady state of the economy.

(c) What is the maximal steady state primary deficit? Can it be attained, and if so, how?