Midterm Examination Solutions\(^1\)

FOR UNDERGRADUATE STUDENTS ONLY

**Instructions:** This is a 75 minute examination worth 100 total points. Question 1 is worth 40 points, all other questions are worth 30 points. **ANSWER QUESTION 1** then choose **TWO** of following **THREE** questions. **DO NOT ANSWER ALL OF THE QUESTIONS.** If you do, your grade will be based on the **LOWEST** of the questions.

In order to get full credit, you must give a clear, concise, and correct answer, including all necessary calculations. Notes and books will not be permitted. Explain your answers clearly and use graphs when helpful.

**ANSWER THIS QUESTION (40 points)**

1. Suppose you run across the following argument in the newspaper. “The GDP of Neverland has been growing at a rate more than double that of the US. In ten years, Neverland will likely be much richer (in per capita GDP terms) than the US.”

   (a) What facts would you want to collect to address the validity of this claim?

   **Solution:** According to the growth accounting, if \( Y = AK^\alpha N^{1-\alpha} \), suppose the growth rate of \( A, K, N \) are \( g_A, g_K, g_N \) respectively, then the growth rate of \( Y \), \( g_Y = g_A + \alpha g_K + (1 - \alpha)g_N \). We need to figure out how much of the growth is due to growth in inputs versus growth in productivity. If it’s mainly growth in inputs it won’t be sustainable, while if it’s growth in productivity it may be. So we should collect the data of the growth rate of \( g_A, g_K, g_N \), capital share \( \alpha = \frac{r(t)K(t)}{Y(t)} \) and labor share \( 1 - \alpha = \frac{w(t)N(t)}{Y(t)} \).

   (b) Suppose that the population growth rates in the US and Neverland were the same, but Neverland had a much higher savings rate than the US. (Suppose that both of these savings rates are constant.) How would this affect your conclusions? Use the Solow model to guide your answer.

   **Solution:** If they are both in the balanced growth path, the growth rate of output per capita equals the technology growth rate \( g \), and the growth rate of total output equals the sum of population growth rate and the technology growth rate \( n + g \). While per capita GDP in the balanced growth path is \( y(t) = A(t) \left( \frac{s}{n+g+\delta} \right)^{\frac{n}{\delta}} \). Since \( s \) in Neverland is much higher than the US, and \( n \) is the same, it’s possible that Neverland will be much richer (in per capita GDP terms) than the US.

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(c) Suppose you find out that the population growth rate in Neverland is much higher than in the US, and these rates won’t change over time. In addition, suppose the claim was that the size of the overall economy (levels of GDP, not per-capita) of Neverland would surpass the US. How would this affect your conclusions? Use the Solow model to guide your answer.

Solution: If they are both in the balanced growth path, total GDP in the balanced growth path is

\[ Y(t) = A(t)N(t) \left( \frac{s}{n+g+\delta} \right)^{\frac{1}{1-\alpha}}. \]

Suppose \(s\), \(\delta\), \(\alpha\) are the same in both countries, \(n\) and \(g\) are higher in Neverland than in the US, then \(\tilde{y} = \left( \frac{s}{n+g+\delta} \right)^{\frac{1}{1-\alpha}}\) is smaller in Neverland than in the US, but \(A(t)\) and \(N(t)\) in Neverland could be much larger that \(Y(t)\) of Neverland would likely to surpass the US.

ANSWER TWO OF THE FOLLOWING THREE QUESTIONS (30 points each)

2. Consider the optimal growth model with inelastic labor supply, and for simplicity assume that there is no population or productivity growth. Household preferences are:

\[ \sum_{t=0}^{\infty} \beta^t u(C_t) \]

The capital evolution equation is:

\[ K_{t+1} = (1 - \delta) K_t + F(K_t) - C_t. \]

Suppose the economy is initially in the steady state and then there is an unexpected and permanent increase in household patience, so \(\beta\) increases (or \(\theta = \frac{1}{\beta} - 1\) falls).

(a) What are the long-run effects of this change on consumption?

Solution: The Lagrangian function is

\[ \mathcal{L} = \sum_{t=0}^{\infty} \left( \beta^t u(C_t) + \lambda_t \left[ (1 - \delta) K_t + F(K_t) - C_t - K_{t+1} \right] \right) \]

By solving the FOCs with respect to \(C_t\) and \(K_{t+1}\), we can get the Euler Equation:

\[ u'(C_t) = \beta u'(C_{t+1})(1 + F'(K_{t+1}) - \delta) \]

where \(\beta = \frac{1}{1+\theta}\).

In Steady State, \(C_t = C_{t+1}\), from the Euler Equation, we get the \(\Delta C = 0\) curve

\[ F'(K_{t+1}) = \delta + \theta \]

In Steady State, \(K_t = K_{t+1}\), from the capital evolution equation, we get the \(\Delta K = 0\) curve
\[ C = F(K) - \delta K \]

When \( \theta = 1/\beta - 1 \) falls, the \( \Delta K = 0 \) curve is unaffected, while the \( \Delta C = 0 \) curve shift to the right. Thus the \( C^* \) increases in the long run.

We can draw the graph as follows:

(b) What are the long-run effects of this change on capital?

**Solution:** From the analysis above, when \( \theta = 1/\beta - 1 \) falls, the \( \Delta K = 0 \) curve is unaffected, while the \( \Delta C = 0 \) curve shift to the right. Thus the \( K^* \) increases in the long run.

3. Consider a two period problem where a consumer has preferences over consumption in the two periods given by:
\[
\log c + \beta \log c' .
\]

She has no initial assets and has income \( y \) in the first period \( y' \) in the second, pays taxes (net of benefits) \( T \) in the first and \( T' \) in the second, and can borrow and lend at interest rate \( r \), thus giving the present value budget constraint:
\[
c + \frac{c'}{1 + r} = y - T + \frac{y' - T'}{1 + r} .
\]

The government finances spending through taxes and borrowing:
\[
G = T + B, \quad G' + (1 + r^G) B = T',
\]

where the government borrows at a lower rate than households: \( r^G < r \).
(a) Solve for the agent’s optimal consumption choices \(c\) and \(c'\).

**Solution:** The present value budget constraint is

\[
c + \frac{c'}{1 + r} = y - T + \frac{y' - T'}{1 + r} \equiv y^{PV}
\]

The Lagrangian function is

\[
\mathcal{L} = \log c + \beta \log c' + \lambda \left( y^{PV} - c - \frac{c'}{1 + r} \right)
\]

FOCs:

\[
\frac{1}{c} = \lambda \\
\frac{\beta}{c'} = \frac{\lambda}{1 + r}
\]

We can get the consumption Euler equation

\[
\frac{1}{c} = \beta (1 + r) \frac{1}{c'}
\]

Combined with the present value life time budget constraint, we have

\[
c = \frac{1}{1 + \beta} \left( y - T + \frac{y' - T'}{1 + r} \right)
\]

\[
c' = \frac{\beta (1 + r)}{1 + \beta} \left( y - T + \frac{y' - T'}{1 + r} \right)
\]

(b) Now suppose that the government cuts taxes in the current period, so \(T\) falls by some amount \(\Delta\), but government spending is unchanged. Thus future taxes must rise to pay back the principal and interest on the deficit this policy creates. How does this affect the consumer’s optimal choices?

**Solution:** The government present value budget constraint is

\[
T + \frac{T'}{1 + r^G} = G + \frac{G'}{1 + r^G}
\]

The new tax is \(\tilde{T} = T - \Delta\), while government spending is unchanged. So

\[
\tilde{T}' = T' + (1 + r^G) \Delta
\]

The new life time income for the consumer is

\[
y^{PV} = y - \tilde{T} + \frac{y' - \tilde{T}'}{1 + r}
\]

\[
= y - T + \frac{y' - T'}{1 + r} + \Delta - \frac{1 + r^G}{1 + r} \Delta
\]

\[
= y^{PV} + \frac{r - r^G}{1 + r} \Delta
\]
Since \( r^G < r \), we have \( \bar{y}^{PV} > y^{PV} \). Thus the consumer will increase consumption in both periods.

4. Consider the two-period dynamic general equilibrium model, which we can depict graphically as in class with equilibrium in the labor market (labor supply and demand) and the goods market (output supply and demand). Suppose the economy is initially in equilibrium, and then a new government program is announced. This program will make public infrastructure investments in the current period that will be funded by lump sum tax revenue and will increase future productivity. That is, the program combines an increase in \( G \) today (only, not \( G' \) as well) with an increase in \( z' \) in the future. As in class, assume that the response of labor supply to interest rates is small.

(a) What effect will the program have on consumption and investment demand, and thus on output demand?

Solution: The program combines an increase in \( G \) today with an increase in \( z' \) in the future. **Distinguish between shift of the demand, supply curve and change of the equilibrium result.**

If we just consider an increase in \( G \) today, then increase in current or future taxes reduces household wealth, thus Consumption demand \( C^d \) shifts to the left, but by less than the amount \( G \) shifts to the right, and \( Y^d(r) = C^d(r) + I^d(r) + G \) shifts to the right.

If we just consider an increase in \( z' \) in the future, since \( z'F_K(K, N) \) increases, \( I^d(r) \) shifts to the right, \( Y^d(r) = C^d(r) + I^d(r) + G \) shifts to the right.

In sum, \( C^d \) shifts to the left, but by less than the amount \( G \) shifts to the right, \( I^d(r) \) shifts to the right, and \( Y^d(r) = C^d(r) + I^d(r) + G \) shifts to the right.

(b) What effect will this program have on labor supply and labor demand? How will the program affect the output supply curve?

Solution:

If we just consider an increase in \( G \) today, then increase in current or future taxes reduces household wealth, thus leisure falls and so labor supply \( N^s \) increases (shifts to the right), and output supply \( Y^s(r) \) increases (shifts to the right). There is no effect on labor demand curve \( N^d \).

If we just consider an increase in \( z' \) in the future, there is no (direct) effect on labor market (only through change in \( r \), if \( r \) increases, \( N^s \) shifts to the right), hence no (direct) effect on \( Y^s(r) \). There is no effect on labor demand curve \( N^d \).

In sum, \( N^s \) shifts to the right, there is no effect on labor demand curve \( N^d \), \( Y^s(r) \) shifts to the right.

(c) What will be the net equilibrium effects on output, interest rates, employment, and wages?

Solution: We can draw the equilibrium effects in the two graphs below. The equilibrium output increases, interest rate increases, employment increases, wages decrease.