Midterm Examination

FOR UNDERGRADUATE STUDENTS ONLY

Instructions: This is a 75 minute examination worth 100 total points. All questions are worth 33 points. Choose THREE of following FOUR questions. DO NOT ANSWER ALL OF THE QUESTIONS. If you do, your grade will be based on the LOWEST of the questions.

In order to get full credit, you must give a clear, concise, and correct answer, including all necessary calculations. Notes and books will not be permitted. Explain your answers clearly and use graphs when helpful.
1. Consider the simplified real business cycle model studied directly as a planner’s problem. Households have preferences over consumption:

\[\sum_{t=0}^{\infty} \beta^t \log C_t.\]

Output is produced with capital as the only input:

\[Y_t = z_t K_t,\]

where \(z_t\) is the level of total factor productivity. Suppose that capital depreciates fully each period (\(\delta = 1\)), so that the aggregate feasibility (or goods market clearing) condition is:

\[Y_t = C_t + K_{t+1}.\]

The planner maximizes the household utility subject to the feasibility condition.

(a) Find the Euler equation characterizing the optimal consumption allocation.

(b) Show that the optimal decision rule is to save a constant fraction of output \(K_{t+1} = sY_t = sz_t K_t\) and consume a constant fraction \(C_t = (1-s)z_t K_t\) and find an expression for the constant \(s\).

(c) How do increases in productivity at date \(t\) affect output at date \(t + 1\)?

2. Suppose that the economy is initially in equilibrium and then a short-lived war breaks out which requires a temporary increase in government spending. However rather than increase taxes to fund the expenditure, the government decides to simply print more money in the current period. Answer the following using the Lucas model where prices are flexible but agents may not be able to distinguish changes in the price level from changes in productivity.

(a) If the public understands this policy and knows that the money supply is increased, how do these changes affect the equilibrium levels of output, interest rates, employment, real wages, and prices?

(b) Now suppose that the public cannot directly observe the money supply or the general price level and so tries to make inference about these via observing the nominal wage. Suppose that households were expecting the increase in spending to be met by changes in (lump sum) taxes, so that the money supply increase was unanticipated. Now what happens to the equilibrium levels of output, interest rates, employment, real wages, and prices?
3. Suppose that a household has separable utility over consumption and next period’s real balances \( m' = M'/P' \) which are given by:

\[
U(C, C', m') = \frac{C^{1-\gamma}}{1-\gamma} + \beta \left( \frac{(C')^{1-\gamma}}{1-\gamma} + \frac{(m')^{1-\gamma}}{1-\gamma} \right)
\]

The household gets constant income \( Y \) in each period and faces the real intertemporal budget constraint:

\[
C + \frac{1}{1+r} C' + \frac{R}{1+r} m' = Y + \frac{Y}{1+r}
\]

Suppose that this is a representative agent endowment economy, so the goods market equilibrium conditions are \( C = Y \) and \( C' = Y \).

(a) From the household optimality conditions and the goods market equilibrium, find an aggregate money demand relationship of the form \( M'/P' = L(Y, R) \), recalling the relationship \( R = (1 + r)(1 + \pi) - 1 \).

(b) Implicitly here the government is raising seignorage revenue and rebating it to the consumers lump sum. Find an expression for the level of inflation which maximizes seignorage revenue.

4. This problem uses a Keynesian model with sticky prices and efficiency wages to compare the effects of a recession in Europe and the US.

(a) In both Europe and the US housing prices have fallen, reducing the wealth of consumers. Describe the effects of this fall on consumption, output, interest rates, and unemployment in the short run.

(b) Both economies are in a recession. In the US, the equilibrium real interest rate is likely negative, and inflation expectations are near zero (a liquidity trap). In Europe, equilibrium real interest rates are low but positive, and inflation expectations are low. How effective will monetary policy be in these economies to restoring full employment by changing interest rates (or increasing the money supply)? Illustrate and explain your results.