We have a household that decides how much to work, $N$, and how much to consume, $c$ to maximize utility. It takes as given the wage, $w$, and the interest rate, $r$.

We have a firm that decides how much to produce $Y$ and how much capital, $K$, and labor, $N$ to hire. It takes as given the wage, $w$, and the interest rate, $r$.

We have a government that raises taxes $T$, and spends $G$. We will switch for now to lump sum taxes.

We are in a static world: we will assume $K = \bar{K}$ constant.
We will now put these together. Why?

1. Consistency: we are sure that everyone is doing things that are compatible.
2. To derive positive predictions from the model.

We will take as exogenous some objects: $\bar{K}$, $G$, and $z$. As well as specifications of $u$ and $F$.

Will derive endogenous objects: $w$, $r$, $N$, $c$, $T$. These imply $Y$, $\pi$, $l$. 
A Competitive Equilibrium is an allocation \( \{ Y, N, K, c \} \), a price system \( \{ w, r \} \) and a government policy \( \{ T, G \} \) such that:

1. Given the price system and the government policy, households choose \( N^s \) and \( c \) to maximize their utility.
2. Given the price system and the government policy, firms maximize profits by choice of \( N^d, K \).
3. The government budget is balanced: \( G = T \).
4. Markets clear:

   Capital: \( K = \bar{K} \)
   Labor: \( N^d = N^s = N \)
   Goods: \( Y \equiv zF(K, N) = c + G \)
Note that if we impose the other conditions, the goods market clearing condition automatically holds.

Budget constraint: \( c = wN^s + \pi + r\bar{K} - T \)

Profits: \( \pi = zF(K, N^d) - wN^d - rK \)

Substitute \( \pi \) into BC, use \( K = \bar{K}, N^d = N^s = N \):

\[
c = wN + (zF(\bar{K}, N) - wN - r\bar{K}) + r\bar{K} - G
\]

\[
c + G = zF(\bar{K}, N)
\]

This is an implication of Walras law: if all markets but one clear, the other must as well.
Solving for an Equilibrium

- Key here is find $w$ to clear labor market.
- From consumer utility maximization:
  \[ MRS = \frac{u_l(c, l)}{u_c(c, l)} = w \]
- From firm profit maximization:
  \[ MPN = zF_N(K, h - l) = w \]
- Equate and impose goods market clearing:
  \[ \frac{u_l(zF(\bar{K}, h - l) - G, l)}{u_c(zF(\bar{K}, h - l) - G, l)} = zF_N(K, h - l) \]
- Solve for $l$. Note $MRS$ decreasing in $l$, $MPN$ increasing in $l$. 
Figure 5.2 The Production Function and the Production Possibilities Frontier
Figure 5.3  Competitive Equilibrium
An allocation is **Pareto Optimal** if there is no way to rearrange production or reallocate goods so that someone is made better off without making someone else worse off. (Limited notion.)

Let us imagine we have a powerful dictator, the Social Planner, that can decide how much the households consume and work and how much the firms produce.

The Social Planner does not follow prices. But he understands opportunity cost.

The Social Planner is benevolent. He searches for the best possible allocation, which will be Pareto optimal.
Maximizes utility of household given $G, K$.

$$\max_{c,l} u(c, l)$$

subject to: $c + G = zF(K, h - l)$

Note: we do not have prices in the budget constraint.

Again, either Lagrangian or impose constraint. Here impose:

$$\max_l u(zF(K, h - l) - G, l)$$
\[
\max_l u(zF(K, h - l) - G, l)
\]

- First Order Condition with respect to \( l \):

\[
-u_c zF_N + u_l = 0
\]

\[
\frac{u_l}{u_c} = zF_N
\]

- Same as the competitive equilibrium.
- A simple example of the first welfare theorem.
First Fundamental Welfare Theorem: under certain conditions (made clear later), a competitive equilibrium is Pareto optimal.

We also have the converse.

Second Fundamental Welfare Theorem: under certain conditions, a Pareto optimal allocation can be decentralized as a competitive equilibrium.

To decentralize an optimal allocation, may need a (lump sum) redistribution of wealth.
Some consequences

- First Fundamental Welfare Theorem states that, under certain conditions, an allocation achieved by a market economy is Pareto optimal.
- Formalization of Adam Smith’s “invisible hand” idea.
- Strong theoretical point in favor of decentralized allocation mechanisms: prices direct agents to do what is needed to get a Pareto optimum.
- Second Fundamental Welfare Theorem states gives the best way to change allocations: redistribute income. Do not change prices.
How robust is the First Welfare theorem?

- Key is the phrase “under certain conditions”. Plenty of reasons to deviate from a Pareto optimum:
  1. Distorting (non lump-sum) taxes, as before.
  2. Externalities.
  3. Imperfect Competition.
  4. Asymmetric Information.
  5. Market Incompleteness.

- Example: With proportional taxes we saw that household optimality implied.

\[
\frac{u_l}{u_c} = (1 - \tau) w
\]

Opens a wedge between \( MRS \) and \( MPN \).
Can we take the planner’s problem literally?

- How do we allocate resources in society?
- Could a social planner do as well, or better? Our basic model suggests so.
- Why is this important? a little bit of history
- Could Central Planning work? Mises, Hayek in the 20’s: NO.
- Experience is rather clear that it did not, but maybe they just did not apply the recipe properly.
“The problem of rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exist in concentrated or integrated form, but solely as the dispersed bits of incomplete knowledge which all the separate individuals possess...

The problem is thus in no way solved if we can show that all the facts, if they were known to a single mind (as we hypothetically assume them to be given to the observing economist) would uniquely determine the solution; instead we must show how a solution is produced by the interactions of people, each of whom possesses only partial knowledge.”

–F.A. Hayek, “The Use of Knowledge in Society” (1945)
Using the General Equilibrium Model

- We can now analyze the equilibrium response of the economy to exogenous changes.
- Example: suppose that there is an increase in TFP $z$.
- Increase in $z$ shifts out $MPN$, rotates out production possibility frontier. Can produce more with the same amount of labor, giving more scope for consumption.
- Will increase $C$ unambiguously. Effect on $N$ will depend on income and substitution effects.
- Wage likely to increases, due to shift in $MPN$ for any given $N$. Possible that $N$ falls slightly, but not enough to offset increase in $z$ (which is what leads to the change in $N$).
Figure 5.9  Competitive Equilibrium Effects of an Increase in Total Factor Productivity
Figure 5.10  Income and Substitution Effects of an Increase in Total Factor Productivity
During WWII government spending to finance the war effort increased to levels unseen previously in the US.

What are the predictions of the model for this increase in spending?

The assumption that government spending is a pure loss of output arguably makes sense here. Pure spending/diversion of resources in short run. Positive effects more long-run and harder to measure.
Figure 1.01  Output of the U.S. economy, 1869-1996
Figure 1.06 U.S. Federal government spending and tax collections, 1869-1999
Analysis of the Change in $G$

- We’ll work out a parametric example with preferences $u(c, l) = \log c + \gamma l$ and Cobb-Douglas technology.

- Production possibilities (goods market):

$$c = Y - G = zK^\alpha(h - l)^{1-\alpha} - G$$

To simplify: write $g = G/Y$. So:

$$c = (1 - g)zK^\alpha(h - l)^{1-\alpha}.$$  

- Firm profit maximization:

$$MPN = zF_N(K, h - l) = z(1 - \alpha)K^\alpha(h - l)^{-\alpha} = w$$
- Household utility maximization:

\[ MRS = \frac{u_l(c, l)}{u_c(c, l)} = \frac{\gamma}{1/c} = w \]

- Equate and impose goods market clearing:

\[
\frac{u_l(zF(\bar{K}, h - l) - G, l)}{u_c(zF(\bar{K}, h - l) - G, l)} = zF_N(\bar{K}, h - l)
\]

\[ \Rightarrow \gamma(1 - g)z\bar{K}^\alpha(h - l)^{1-\alpha} = z(1 - \alpha)\bar{K}^\alpha(h - l)^{-\alpha} \]

\[ \Rightarrow N = h - l = \frac{1 - \alpha}{\gamma(1 - g)}. \]

- Government spending has a pure income effect here (since financed by lump sum taxes). Increases labor supply.
Solve for rest of allocation:

\[ Y = z\bar{K}^\alpha \left[ \frac{1 - \alpha}{\gamma(1 - g)} \right]^{1-\alpha} \]

\[ c = (1 - g) Y = z(1 - g)^\alpha \bar{K}^\alpha \left[ \frac{1 - \alpha}{\gamma} \right]^{1-\alpha} \]

Output increases with \( g \), consumption decreases.

Solve for wages and interest rates:

\[ w = z(1 - \alpha)\bar{K}^\alpha \left[ \frac{1 - \alpha}{\gamma(1 - g)} \right]^{-\alpha} \]

\[ r = z\alpha\bar{K}^{\alpha - 1} \left[ \frac{1 - \alpha}{\gamma(1 - g)} \right]^{1-\alpha} \]

So wages decrease with \( g \), interest rates increase.
Summing Up:

- Following increase in $g = G/Y$, the model predicts an increase in $(Y, N, r)$, decrease in $(c, w)$.
- Private consumption spending is “crowded out” by increased government spending.
- Output increases but loss of welfare as both $c, l$ fall.
- These predictions match US experience of WWII.
Figure 5.6 Equilibrium Effects of an Increase in Government Spending
Figure 5.7  GDP, Consumption, and Government Expenditures
Government debt. A large fraction of the wartime spending was financed by government debt. Deficit/GDP ratio hit 24% by 1944.

Debt allows for intertemporal substitution of resources and smoothing burden of taxation. If needed to increase (distortionary) taxes to finance full war spending, production would have been less.

Increased productivity. Wartime mobilization of production increased labor productivity dramatically.

Led to larger increase in production than our model suggests.
Figure 15.04  Deficits and primary deficits: Federal, state, and local, 1940-1998
Figure 1.02  Average labor productivity in the United States, 1900-1998
Can define the **multiplier** for government spending as the percentage by which output increases for a given increase in government spending:

\[
\text{multiplier} = \frac{\Delta Y}{\Delta G} = \frac{Y' - Y}{G' - G}
\]

In our model, using \( G = gY \), \( G' = g'Y' \):

\[
\frac{Y' - Y}{g'Y' - gY} = \frac{z\bar{K}^\alpha \left[ \frac{1-\alpha}{\gamma} \right]^{1-\alpha} ((1 - g')^{\alpha-1} - (1 - g)^{\alpha-1})}{z\bar{K}^\alpha \left[ \frac{1-\alpha}{\gamma} \right]^{1-\alpha} (g'(1 - g')^{\alpha-1} - g(1 - g)^{\alpha-1})}
\]

\[
= \frac{(1 - g')^{\alpha-1} - (1 - g)^{\alpha-1}}{g'(1 - g')^{\alpha-1} - g(1 - g)^{\alpha-1}}
\]
In current discussions of fiscal stimulus, the size of the multiplier a source of some controversy. Obama administration suggested $\approx 1.5$, Barro suggested $\approx 0$.

In our model with $g = 0.2$, $\alpha = 0.3$, $g' = 0.25$ the multiplier is about 0.75.

This isn’t the best framework for current issues, as there’s no unemployment or idle resources. These are the main rationale for the fiscal stimulus.

In the model here, increase in $G$ always bad for welfare even if output increases.
Large literature documenting increase in income and wealth inequality in the US. Started in the 1970s and continues today.

At same time, large increase in the returns to education.

Average wages of college graduates from increased by 60% for males and 90% for females from 1963 to 2002.

Average wages of high school graduates only increased by 20% for males and 50% for females over same period.

**Main explanation:** Skill-biased technical change. Skilled and unskilled labor are effectively different labor markets. Productivity changes have increased the relative demand for skilled labor.
Figure 4  Men's Earnings by Quantiles

Index of Real Wages of Full-Time Full-Year Men Ages 22–65 by Specific Percentiles

Index, 1961=100
Figure 5  Women's Earnings by Quantiles

Index of Real Wages of Full-Time Full-Year Women Ages 22–65 by Specific Percentiles

Index, 1981=100
Figure 6  Men's Earnings by Education

Index of Mean of Real Wages of Full-Time Full-Year Men Ages 22–65 by Education Group

Index, 1963=100

![Chart showing the index of mean real wages for full-time full-year men ages 22–65 by education group, with data from 1963 to 2000.](chart_image)
Figure 7  Women's Earnings by Education

Index of Mean of Real Wages of Full-Time Full-Year Women Ages 22–65 by Education Group

Index, 1963=100
Extend the previous to two sectors: skilled and unskilled.

Households: Assume both skilled and unskilled workers have same preferences given by:

\[ u(c, l) = c - \frac{(h - l)^2}{2} \]

Assume skilled workers own a share \( \beta \) of the capital stock, unskilled a share \((1 - \beta)\).

Wages \( w_s \) for skilled \( w_u \) for unskilled.
Household Problem

- Skilled household problem (unskilled parallel):
  \[
  \max_{N_s} \{(N_s w_s + \beta rK) - (N_s)^2 / 2\}
  \]

  Optimality conditions:
  \[
  \frac{u_l}{u_c} = N_s = w_s
  \]

- So we get:
  \[
  \begin{align*}
  N_s &= w_s \\
  N_u &= w_u \\
  c_s &= w_s^2 + \beta rK \\
  c_u &= w_u^2 + (1 - \beta) rK
  \end{align*}
  \]
Firms: Assume representative firm hires both skilled and unskilled labor. Each has different productivity \((z_s, z_u)\).

Firm substitutes between skilled and unskilled for total labor input.

\[
N = (z_s N_s^\rho + z_u N_u^\rho)^{1/\rho}
\]

where \(0 < \rho < 1\). Thus production is:

\[
Y = K^\alpha N^{1-\alpha} = K^\alpha (z_s N_s^\rho + z_u N_u^\rho)^{(1-\alpha)/\rho}
\]
Firms maximize profits:

\[ K^\alpha \left(z_s N_s^\rho + z_u N_u^\rho\right)^{\frac{1-\alpha}{\rho}} - rK - w_s N_s - w_u N_u \]

FOC’s – each type paid its marginal product:

\[
(1 - \alpha) K^\alpha \left(z_s N_s^\rho + z_u N_u^\rho\right)^{\frac{1-\alpha}{\rho} - 1} z_s N_s^{\rho - 1} = w_s \\
(1 - \alpha) K^\alpha \left(z_s N_s^\rho + z_u N_u^\rho\right)^{\frac{1-\alpha}{\rho} - 1} z_u N_u^{\rho - 1} = w_u
\]

Divide firm FOC’s:

\[
\frac{z_s N_s^{\rho - 1}}{z_u N_u^{\rho - 1}} = \frac{w_s}{w_u}
\]
Equate to relative demands to labor supplies:

\[
\frac{z_s N_s \rho^{-1}}{z_u N_u \rho^{-1}} = \frac{N_s}{N_u}
\]

\[
\Rightarrow \frac{N_s}{N_u} = \frac{w_s}{w_u} = \left(\frac{z_s}{z_u}\right)^{1 \over 2-\rho}
\]

Skill-biased technical change: \(z_s\) increases faster than \(z_u\).
Implies \(\frac{w_s}{w_u}\) increases \(\Rightarrow\) inequality.
Also implies expansion in skill sector: \(\frac{N_s}{N_u}\) increases.
Figure 3.14  The effects of skill-biased technical change on wage inequality

(a) Skilled workers

1. Skill-biased technical change occurs
2. Skilled wages rise
2. Skilled employment rises

(b) Unskilled workers

1. Skill-biased technical change occurs
2. Unskilled wages fall
2. Unskilled employment falls


- Captures broad aggregate facts. Misses on some dimensions.

- **Consumption Inequality.** Evidence that inequality in consumption was less than inequality in income (Kreuger & Perri, 2003). Here we have greater consumption inequality (since $\beta \approx 1$):

  \[
  \frac{c_s}{c_u} = \frac{w_s^2 + \beta rK}{w_u^2 + (1 - \beta) rK} \approx \left(\frac{w_s}{w_u}\right)^2 + \frac{rK}{w_u^2}
  \]

- **Changes by Gender.** Most dramatic effects have been increase in female labor supply, especially in skilled labor. Hard to argue this was all from skill-biased technical change. Composition effects may be more important. (Eckstein & Nagypal, 2004)