Problem Set 4
Due on May 9, either in class or in my mailbox by 4:30 PM.

1. Consider a simple IS-LM model given by the following equations:

\[
IS: \quad Y = a_0 - a_1 r + u \\
LM: \quad \frac{M}{P} = b_0 + b_1 Y - b_2 r + v
\]

where all \(a_i\) and \(b_i\) coefficients are positive and \(u\) and \(v\) are i.i.d. shocks. Suppose the price level is fixed in the current period to \(P = 1\), and that there is a constant full employment level of output given by \(\bar{Y}\). The monetary authority has the option to either control the real interest rate \(r\) or the money supply \(M\).

(a) First suppose that there are no shocks \((u, v)\). Find the values of \(r\) and \(M\) (call them \(r^*\) and \(M^*\)) that lead to the full employment level of output. Does it matter which policy instrument is chosen?

(b) Now suppose that there are shocks \(u\) to the IS equation and shocks to money demand \(v\). Policymakers must set policy before the shocks are realized, and the goal of policy is to minimize a loss function that depends on fluctuations of output from the full employment level:

\[
\min(Y - \bar{Y})^2
\]

i. Find the values of the loss \((Y - \bar{Y})^2\) which results (after the shocks are realized) when the policymakers set \(r = r^*\) and when they set \(M = M^*\).

ii. Consider the special case when \(a_1 = 1, b_1 = 3, b_2 = 1\). When is it optimal for policymakers to choose to set the interest rate instead of the money supply?

2. Let \(\pi_t\) denote inflation, \(\hat{\pi}_t\) the public’s expected inflation, and \(u_t\) the unemployment rate. Then the expectations-augmented Phillips curve is given by:

\[
u_t = \bar{u} - \theta(\pi_t - \hat{\pi}_t)\]

where \(\bar{u} > 0\) is the natural unemployment rate and \(\theta > 0\) is the slope of the Phillips curve. Suppose policymakers want to minimize the losses from inflation and unemployment, with the loss function given by:

\[
\pi_t^2 + \lambda u_t^2,
\]

where \(\lambda > 0\). Suppose the policymakers directly choose \(\pi_t\).
(a) Find the policymakers’ optimal choice of \( \pi_t \), taking expected inflation \( \hat{\pi}_t \) as given. Then supposing that the public is rational, so \( \hat{\pi}_t = \pi_t \), solve for \( \pi_t \).

(b) Now suppose that policymakers can commit to an inflation policy, meaning that they take into account how their actions affect expectations. (That is, they know that \( \hat{\pi}_t = \pi_t \).) Solve for the optimal \( \pi_t \) now.

(c) How do the gains from commitment depend on the preferences of central bankers? In particular, in the absence of commitment (as in part (a)) is there a case for appointing conservative central bankers (with lower \( \lambda \))?

3. This problem considers a simplified New Keynesian model, given by the IS equation:

\[ x_t = -\phi(R_t - E_t \pi_{t+1}) + g_t \]

and the New Keynesian Phillips curve:

\[ \pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \]

Suppose the cost shock is given by:

\[ u_t = \rho_u u_{t-1} + \epsilon_t^u \]

while the demand shock \( g_t \) is i.i.d. Suppose that monetary policy is conducted by the policy rule:

\[ R_t = au_t + (1/\phi)g_t \]

where \( a > 0 \) is a constant and \( \phi \) is the same constant as in the IS equation.

(a) Solve for the equilibrium levels of inflation \( \pi_t \) and the output gap \( x_t \) as a function of the shocks \( u_t \) and \( g_t \). To do so, use the policy rule in the IS equation and solve for \( x_t \) in terms of the other variables. Substitute your answer into the Phillips curve, and guess that \( \pi_t = ku_t \) for some constant \( k \). Then solve for \( k \) and determine \( \pi_t \) and \( x_t \).

(b) If policymakers want to minimize expected losses, with a loss function:

\[ L_t = \frac{1}{2} (\omega x_t^2 + \pi_t^2) \]

what is the optimal value of the policy rule coefficient \( a \)?
4. Consider a search model of the labor market in which workers have linear utility
\[ U(c) = c, \] all jobs are the same and pay a wage of \( w \) units of consumption, un-
employed workers get \( b < w \) in unemployment benefits, the separation rate is 50%
\( (s = 1/2) \), the job finding rate is 50% \( (p = 1/2) \), and agents discount the future
with factor \( \beta = 1/2 \).

(a) The equilibrium unemployment rate occurs when flows into and out of unem-
ployment are equal. Find the equilibrium unemployment rate.

(b) Find the values (i.e. the total expected discounted utility) of an employed
worker and an unemployed worker.

(c) Instead of the scenario outlined above, suppose that the job finding rate was
\( p = 1 \), but that 1/2 of jobs paid a wage \( w_1 < b \) while 1/2 of jobs paid a wage
\( w_2 > b \). How would this change your results?