1. Suppose that there is a temporary change in consumer sentiment, so that households cut back on consumption spending, but this has no other direct effects on the economy. Consider the Keynesian model with sticky prices, and discuss the short and long run effects on output and interest rates of the following policy options.

**Solution:** The shock causes a leftward shift in the IS curve. Recall that the SRAS is horizontal and LRAS is vertical. Also, in the long run $Y$ always returns to its original level, and in the short run, $P$ does not change (sticky prices), so I won’t mention these explicitly below. The question doesn’t mention efficiency wages, so we’ll assume that the FE curve is our usual output supply curve, and so is not vertical. However answers with a vertical FE curve are also acceptable.

(a) The money supply is adjusted to return the economy to full employment.

**Solution:** If the money supply is adjusted, the LM curve shifts to the right so that IS, LM and FE intersects at the same point, $Y$ falls slightly (due to the slope of the FE curve), $r$ falls, and $P$ is of course unchanged. This holds in the LR too, as the policy adjusted things immediately. (If the FE curve were vertical, then there would be no change in output.)

(b) Government spending is adjusted to return the economy to full employment.

**Solution:** If $G$ is expanded, this shifts the IS curve back to its original level. No change in $Y, r, P$ in either the short or the long run compared to the equilibrium before the shock.

2. Suppose that a household does not face a cash in advance constraint, but instead money is the only asset for transferring income over time. That is, suppose that household preferences are given by:

$$\log C + \beta \log C'.$$

The household has real unearned income $Y$ in period 1, and uses money $M$ to transfer assets to the future, thus facing the budget constraints:

$$PC + M = PY$$

$$P'C' = M$$

(a) Find the household’s money demand function and show that it is decreasing in the inflation rate.
Solution: We first eliminate $M$ and solve for the present value budget constraint, which can be written:

$$ PC + P'C' = PY $$

or: $$ C + (1 + \pi)C' = Y $$

To solve the household problem we can then form a Lagrangian with multiplier $\lambda > 0$.

$$ \mathcal{L} = \log(C) + \beta \log(C') + \lambda (Y - C - C'(1 + \pi)) $$

FOC: $$ \frac{1}{C} = \lambda $$

$$ \frac{\beta}{C'} = \lambda(1 + \pi) $$

Eliminating $\lambda$ between these, then using the budget constraint gives:

$$ C = \frac{Y}{1 + \beta} $$

$$ C' = \frac{\beta}{(1 + \pi)(1 + \beta)} Y $$

$$ M = P'C' = P' \frac{\beta}{(1 + \pi)(1 + \beta)} Y $$

Since $\pi$ is in the denominator of this last expression, we clearly see that money demand is decreasing in $\pi$.

(b) Relate your results to the quantity theory of money $MV = PY$. What is the velocity of money in this problem?

Based on our previous expressions, we get:

$$ V = \frac{PY}{M} = \frac{PY}{P' \frac{\beta}{(1 + \pi)(1 + \beta)} Y} = \frac{P(1 + \pi)(1 + \beta)}{P' \beta} = \frac{1 + \beta}{\beta} $$

Thus velocity is constant here, so the quantity theory of money always holds.

3. Suppose that instead of being on a fiat money system, where it is essentially costless to change the money supply, an economy runs on a commodity money system. In particular, to print more money the government must mine more gold and this costs real resources, which the government finances via lump sum taxes. As usual, assume that increases in money supply are distributed in a lump sum fashion to the households, so that the government retains no seignorage revenue. Assuming that all prices and wages are flexible, answer the following.

(a) What are the effects of a one-time increase in the money supply in this commodity money system? Consider the effects on output, real interest rates, employment, real wages, and the price level.
Solution: Here the effect is a combination of an increase in the money supply and a temporary tax increase, as real resources are used up in mining (recall that all taxes and transfers are lump-sum, so we can just net them out, and the result is a tax).

The increased taxes lead to an increase in the labor supply, shifting aggregate supply to the right too. This results in lower $w$ and $r$, and higher $N$ and $Y$ (wealth effect). Also, output demand shifts to the right, increasing $Y$ further, and raising $r$ (the net result is a rise in $r$). The higher $Y$ would also reduce prices (increased money demand), but because of the increased money supply, the effect on $P$ is ambiguous.

(b) Now suppose that there is a new government mine discovered, and the government increases the growth rate of the money supply requiring ongoing taxes to pay for the extraction of the gold. What are the effects of this on output, real interest rates, employment, real wages, and the price level?

Solution: Here a continuing increase in money supply (and hence inflation) is combined with a permanent tax increase.

The permanent increase in taxes will be matched by a permanent decrease in consumption, hence $Y^d$ does not shift, only $N^s$ and $Y^s$ shift to the right, resulting in lower $w$ and $r$, and higher $N$ and $Y$ (wealth effect). Higher $Y$ would increase, while higher inflation would decrease money demand, which would make the initial effect on the price level ambiguous. In the long run, however, the price level will keep increasing.

4. During the recession, the federal government implemented a temporary expansion of government purchases in an effort to help stimulate the economy. Suppose that current taxes remained unchanged. Compare and contrast the effects of this change in our basic real intertemporal model (with Ricardian equivalence) and the Keynesian model (with efficiency wages). In particular, what will be the effects on current output, employment, the real interest rate, and the real wage? How could you distinguish between the models?

Solution: In the basic real intertemporal model (with Ricardian equivalence), increasing government spending means that people should pay more tax in the future. So, there is a negative wealth effect. People consume less, invest less, and work more. The direct effect of the increase in government spending is an increase in output demand, with the labor supply effect leading to a (smaller) increase in output supply. So employment, output, and the real interest rate increase. The real wage falls.

In the Keynesian model (with efficiency wages), real wage and employment do not change, and FE is vertical. Increase in government spending shifts the IS curve to the right, and thus in the short run, output $Y$ increases, and the real interest rate $r$ increases. In the long run, price changes so that the LM curve shifts to the left, new general equilibrium has higher $r$, same $Y$. 

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They can be distinguished mainly from the labor market behavior.

5. In class we worked with the Lucas model in a labor market version, but this problem derives a product market version closer to Lucas’s original model. Suppose consumer \( i \) owns his own firm and produces output using his own labor: \( Y_i = N_i \), and his budget constraint in nominal terms is \( PC_i = P_i Y_i \), where \( P_i \) is the price of the consumer’s own good and \( P \) is the aggregate price (and the price of consumption goods). The consumer has preferences:

\[
U(C_i, N_i) = C_i - \frac{N_i^{1+\phi}}{1 + \phi}
\]

(a) Suppose the consumer takes prices \( P_i \) and \( P \) as given and find his optimal choice of production \( Y_i \).

**Solution:** The budget constraint for the consumer can be written as \( PC_i = P_i N_i \), thus the Lagrangian function is

\[
\mathcal{L} = C_i - \frac{N_i^{1+\phi}}{1 + \phi} + \lambda [P_i N_i - PC_i]
\]

FOCs:

\[
1 - \lambda P = 0
\]

\[
-N_i^\phi + \lambda P_i = 0
\]

So we have

\[
Y_i = N_i = \left(\frac{P_i}{P}\right)^{\frac{1}{\phi}}
\]

(b) Let \( y_i = \log Y_i \) and \( y = \log Y \) where \( Y \) is aggregate output, and suppose that the demand for good \( i \) is given by:

\[
y_i = y + z_i - \eta (p_i - p),
\]

where \( z_i \) is a demand shock for firm \( i \). Find the equilibrium level of production \( y_i \) and price \( p_i \) in terms of the aggregates \( p \) and \( y \).

**Solution:** The supply for good \( i \) is

\[
y_i^s = \log Y_i = \frac{1}{\phi} (p_i - p)
\]

since the demand for good \( i \) is given by:

\[
y_i^d = y + z_i - \eta (p_i - p)
\]
we have

\[ p_i = \frac{\phi (y + z_i)}{1 + \phi \eta} + p \]

\[ y_i = \frac{y + z_i}{1 + \phi \eta} \]

(c) Now suppose that the consumer cannot distinguish between movements in the aggregates \((y, p)\) and a shock to his own demand \(z_i\). He must forecast aggregate prices, and uses the same decision rule for production as in part (a), but substitutes the expected price \(E(p)\) for the observed price \(p\). Write the consumer’s production choice for \(y_i\) in terms of \(p_i\) and \(E(p)\). Now define the aggregate (log) price \(p\) as the average of individual prices \(p_i\) and aggregate (log) output \(y\) as the average of \(y_i\). Find an expression for \(y\) in terms of \(p\) and \(E(p)\).

**Solution:** Now we have

\[ y_i = \frac{1}{\phi} [p_i - E(p)] \]

As the question defines for simplicity, the aggregate (log) price \(p\) is the average of individual prices \(p_i\) and aggregate (log) output \(y\) is the average of \(y_i\), we have

\[ y = \frac{1}{\phi} [p - E(p)] \]

(d) Suppose that velocity is constant at 1 so we can write \(Y = MP\) or \(y = m - p\) where \(m = \log M\). Using the results of the previous part, find the equilibrium \(p\) and \(y\) in terms of \(m\) and \(E(p)\).

**Solution:** Since

\[ y = \frac{1}{\phi} [p - E(p)] \]

\[ y = m - p \]

we have

\[ p = \frac{\phi m + E(p)}{1 + \phi} \]

\[ y = \frac{m - E(p)}{1 + \phi} \]

(e) Suppose there is an unanticipated increase in the money supply \(m\). How will output respond?

**Solution:** In the short run, from the expression of output \(y = \frac{m - E(p)}{1 + \phi}\), when there is an unanticipated increase in the money supply \(m\), \(y\) increases.

In the long run, rational agents adjust their expectations \(E(p)\), such that the output will go back to the initial level.