Economics 312
Macroeconomics
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Problem Set 2
Due on February 28, either in discussion or in my mailbox by 9:30 AM.

1. This problem considers a variation on the Solow model. Suppose that instead of the population growing at a constant rate, that people have fewer children as they become wealthier. In particular, as always suppose that output is produced via a Cobb-Douglas production function $Y = K^\alpha N^{1-\alpha}$, consumers save a constant fraction $s$ of their income, and the capital stock depreciates at rate $d$. But now instead of being a constant, the population growth rate is proportional to the marginal product of capital ($MPK$):

$$\begin{align*}
\frac{\dot{N}}{N} &= n \cdot MPK
\end{align*}$$

where $n > 0$. Since the $MPK$ falls as capital increases, this captures the declining population growth rates of wealthier nations.

(a) Determine the steady state per-capita quantities of capital, output, consumption, and population growth.

(b) What are the short and long run effects of an increase in $n$ on the per-capita quantities of capital, output, consumption, and population growth?

2. This problem considers the qualitative effects of a growth slowdown. Consider an economy with population growth at rate $n$ and technological change at rate $g$, so that we write household preferences in terms of consumption per unit of effective labor $c$ as:

$$\begin{align*}
\sum_{t=0}^{\infty} \left[ \beta (1 + n) (1 + g)^{1-\gamma} \right]^t \frac{c_t^{1-\gamma}}{1-\gamma}.
\end{align*}$$

As in class, we can write the capital evolution equation in terms of capital per effective labor $k_t$ as:

$$\begin{align*}
(1 + n) (1 + g) k_{t+1} &= (1 - \delta) k_t + f (k_t) - c_t.
\end{align*}$$

(1)

Suppose the economy is on the balanced growth path, and then there is a fall in the rate of technological change $g$.

(a) How, if at all, does this affect the $\Delta k = 0$ curve?

(b) How, if at all, does this affect the $\Delta c = 0$ curve?

(c) What happens to consumption per worker $c_t$ at the time of the change?

(d) What are the short-run and long-run effects of the fall in productivity on output per capita, total output, and output growth?
3. In 1861 many southern US states seceded and formed the Confederacy. Treat the North and South as two separate countries, and analyze the effects of the secession and the Civil War using the Solow model. Suppose that the rate of population growth is the same in the North and South, but the North had a higher rate of productivity growth and a higher savings rate.

(a) What does the model predict about the long-run comparative economic performance (both levels of output and growth rates) of the North and South?

(b) During the Civil War, much of the capital stock in the South was destroyed, and after the war the country was reunited. Suppose that after war the whole US had the same (high) TFP growth and savings rates that were predominant in the North. What does the model predict about the comparative economic performance (both levels of output and growth rates) of the North and South after re-unification? Consider both the long run and transitional dynamics.

4. Consider an infinite horizon model of consumption and savings where consumers have “habits”, meaning that they care about consumption relative to their own past consumption. Thus preferences are given by:

\[ \sum_{t=0}^{\infty} \beta^t u(c_t - c_{t-1}) \]

Suppose the consumers face a constant interest rate and so face the flow constraint:

\[ c_t + a_{t+1} = x_t + (1 + r)a_t \]

where \( a_t \) are assets and \( x_t \) is income as of date \( t \)

(a) Write down the Lagrangian for the consumer’s optimization problem, and find the first order conditions for the choice of consumption at arbitrary dates \( t \) and \( t + 1 \).

(b) How does the Euler equation with habits compare to the case from class without habits?

(c) As in class, suppose that income and hence consumption are now random, so introduce expectations appropriately. Further, suppose \( \beta(1 + r) = 1 \) and

\[ u(c_t - c_{t-1}) = c_t - c_{t-1} - \frac{a}{2}(c_t - c_{t-1})^2. \]

What does the Euler equation look like now? Now what information at date \( t \) helps predict consumption at \( t + 1 \)?