Problem Set 1
Due in class on February 12.

1. Suppose that instead of simply being a waste of output, government purchases are used for infrastructure which increases current productivity. In particular, suppose that the government increases its current purchases ($G$ increases) and that this in turn increases total factor productivity ($z$ increases). The government funds the higher expenditure by lump sum taxes. Using the static general equilibrium model from class (and diagrams when possible), discuss the effects this policy change will have on the equilibrium levels of output, employment, and consumption.

2. Suppose that there is a progressive tax on labor income. We model this by supposing that for a given wage if $N \leq N^*$ then the household faces a tax rate $t$, but if $N > N^*$ the tax rate increases to $t' > t$ for hours worked above $N^*$. That is, with $N > N^*$ after-tax labor income is $(1 - t)wN^* + (1 - t')w(N - N^*)$.

   (a) Suppose a worker has unearned income $\pi$ and faces this tax schedule. Draw the worker’s budget constraint. Suppose that preferences ($MRS$) differ across workers, and illustrate the optimal choice of consumption and leisure for different individuals. Show that there is likely to be a mass of workers who choose $N = N^*$.

   (b) Now suppose that there is an increase in the top labor tax rate $t'$, but $t$ is unchanged. What happens to the labor supply choices of different workers?

3. Consider the optimal growth model from class, but add government spending. That is, there is now a specified amount of government spending $G$ which must be funded every period via lump sum taxes.

   (a) Compared to the case of no government spending, how does having $G > 0$ affect the optimal steady state levels of consumption and capital $c^*$ and $k^*$?

   (b) How are the dynamics affected? That is, suppose the economy is initially in the steady state $(k_0^*, c_0^*)$ associated with $G = 0$. Then there is announcement that there will be government spending $G > 0$ for all future dates. How do consumption and capital respond, both immediately upon the announcement and then in the succeeding periods?
4. Suppose that instead of labor being supplied inelastically, households value leisure. That is, in the optimal growth model we now have preferences:

\[ \sum_{t=0}^{\infty} \beta^t [U(c_t) + v(1 - N_t)], \]

where households have 1 unit of time each period, and \( N_t \) is labor, so \( 1 - N_t \) is leisure. Both \( U \) and \( v \) are strictly increasing and strictly concave. Firms produce using a constant returns to scale production function \( F(k, N) \), so the aggregate feasibility condition is now:

\[ F(k_t, N_t) = c_t + k_{t+1} - (1 - \delta)k_t. \]

Consider the social planner’s problem in this environment.

(a) Write down the Lagrangian and find the optimality conditions for the choices of \( c_t, k_{t+1} \) and \( N_t \) at any date \( t \).

(b) Taking ratios of your optimality conditions, find the household Euler equation relating the marginal utilities of consumption at dates \( t \) and \( t + 1 \) to the marginal product of capital. Also find the equation relating the marginal rate of substitution between consumption and leisure to the marginal product of labor.

(c) Find the equations determining the steady state, and characterize the steady state as sharply as you can. How does having elastic labor supply \((v \neq 0)\) affect the steady state levels of consumption and capital?

(d) Now consider the special case (like in class) with \( \delta = 1 \) and \( F(k, N) = A k^\alpha N^{1-\alpha} \) and \( U(c) + v(1 - N) = \log c + \gamma \log(1 - N) \). Show that the optimal solution is to save a constant fraction \( s \) of output, \( c_t = (1 - s)F(k_t, N_t) \) and work a constant number of hours \( N_t = \bar{N} \). Find expressions for \( s \) and \( \bar{N} \).

(e) Continuing with the example from the previous part, suppose that there is an increase in productivity \( A \). How will that affect the optimal levels of consumption, labor, and capital, both on impact of the change and over time?