ANSWER THREE OF THE FOLLOWING FOUR QUESTIONS

1. Consider the simplified real business cycle model studied directly as a planner’s problem. Households supply labor inelastically and have preferences over consumption:

$$\sum_{t=0}^{\infty} \beta^t \log C_t.$$

Output is produced via a Cobb-Douglas production function:

$$Y_t = z_t K_t,$$

where $z_t$ is the level of total factor productivity. Suppose that capital depreciates fully each period ($\delta = 1$), so that the aggregate feasibility (or goods market clearing) condition is:

$$Y_t = C_t + K_{t+1}.$$

The planner maximizes the household utility subject to the feasibility condition.

(a) Find the Euler equation characterizing the optimal consumption allocation.

**Solution:** The budget constraint for the households can be written as $C_t + K_{t+1} = z_t K_t$, thus the Lagrangian function is

$$\mathcal{L} = \beta^t \log C_t + \lambda_t (z_t K_t - C_t - K_{t+1})$$

The FOCs w.r.t. $C_t, C_{t+1}, K_{t+1}$ are:

$$\beta^t \frac{1}{C_t} - \lambda_t = 0$$

$$\frac{\beta^{t+1}}{C_{t+1}} - \lambda_{t+1} = 0$$

$$-\lambda_t + \lambda_{t+1} z_{t+1} = 0$$

So the Euler Equation is:

$$\frac{1}{C_t} = \beta \frac{z_{t+1}}{C_{t+1}}$$

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(b) Show that the optimal decision rule is to save a constant fraction of output consume a constant fraction of output $K_{t+1} = sY_t = sz_tK_t$ and consume a constant fraction $C_t = (1 - s)z_tK_t$ and find an expression for the constant $s$.

**Solution:** Substitute $C_t = (1 - s)z_tK_t$ and $C_{t+1} = (1 - s)z_{t+1}K_{t+1}$ into the Euler Equation, we have

$$
\frac{1}{(1 - s)z_tK_t} = \beta \frac{z_{t+1}}{(1 - s)z_{t+1}K_{t+1}} = \beta \frac{1}{(1 - s)K_{t+1}}
$$

Since $K_{t+1} = sz_tK_t$, we have

$$\frac{1}{1 - s} = \beta \frac{1}{s(1 - s)}$$

Thus

$$s = \beta$$

(c) How do increases in productivity at date $t$ affect output at date $t + 1$?

**Solution:** If $z_t$ increases, then $K_{t+1} = sz_tK_t$ increases, and $Y_{t+1} = z_{t+1}K_{t+1}$ increases.

2. Suppose that the economy is initially in equilibrium and then a short-lived war breaks out which requires a temporary increase in government spending. However rather than increase taxes to fund the expenditure, the government decides to simply print more money in the current period. Answer the following using the Lucas model where prices are flexible but agents may not be able to distinguish changes in the price level from changes in productivity.

(a) If the public understands this policy and knows that the money supply is increased, how do these changes affect the equilibrium levels of output, interest rates, employment, real wages, and prices?

(Answer) If public understand this policy, money is neutral so that money increase has no effect on real variables. With an increase in $G$, employment, output, and interest increase while real wage decreases. Since both output and interest increase, the effect on money demand is not clear. If money demand curve shifts to the left, the price level would increase. On the other hand, if money demand shifts to the right, the price level could either rise or fall, depending on how much money supply has been increased.

$$G \uparrow: w \downarrow, N \uparrow, Y \uparrow, r \uparrow, P? \quad (1)$$

Note that since the temporary increase in $M$ is used to finance spending, this is a one-time seignorage increase. So this operates like a tax. But since it’s a one-time change it doesn’t have the distortionary effects (on consumption vs. leisure) that increases in trend inflation do, which results from a change in the growth rate of money supply.
(b) Now suppose that the public cannot directly observe the money supply or the general price level and so tries to make inference about these via observing the nominal wage. Suppose that households were expecting the increase in spending to be met by changes in (lump sum) taxes, so that the money supply increase was unanticipated. Now what happens to the equilibrium levels of output, interest rates, employment, real wages, and prices?

(Answer) We know that an unanticipated increase in money supply leads to increase in output, employment, and price level and decrease in wage and interest rate. That is

$$M \uparrow: w \downarrow, N \uparrow, Y \uparrow, r \downarrow, P \uparrow$$  \hspace{1cm} (2)

Therefore the final effects on the economy would be a combination of this and the results from the previous part. Clearly, output and employment would increase and real wage would decrease. In principle, the effect on real interest rates and the price level would be ambiguous.

3. Suppose that a household has separable utility over consumption and next period’s real balances \(m' = M'/P'\) which are given by:

$$U(C, C', m') = \frac{C^{1-\gamma}}{1-\gamma} + \beta \left( \frac{(C')^{1-\gamma}}{1-\gamma} + \frac{(m')^{1-\gamma}}{1-\gamma} \right)$$

The household gets constant income \(Y\) in each period and faces the real intertemporal budget constraint:

$$C + \frac{1}{1+r}C' + \frac{R}{1+r}m' = Y + \frac{Y}{1+r}$$

Suppose that this is a representative agent endowment economy, so the goods market equilibrium conditions are \(C = Y\) and \(C' = Y\).

(a) From the household optimality conditions and the goods market equilibrium, find an aggregate money demand relationship of the form \(M'/P' = L(Y, R)\), recalling the relationship \(R = (1 + r)(1 + \pi) - 1\).

(Answer) The Lagrangian is given by

$$L = U(C, C', m') - \lambda \left[ C + \frac{1}{1+r}C' + \frac{R}{1+r}m' - \left( 1 + \frac{1}{1+r} \right) Y \right]$$

FOCs:

$$C : \hspace{1cm} C^{-\gamma} = \lambda$$  \hspace{1cm} (3)

$$C' : \hspace{1cm} \beta C'^{-\gamma} = \frac{1}{1+r} \lambda$$  \hspace{1cm} (4)

$$m' : \hspace{1cm} \beta m'^{-\gamma} = \frac{R}{1+r} \lambda.$$  \hspace{1cm} (5)
Note that goods market clearing conditions must hold in each period. Therefore it must be true that $C = Y$ and $C' = Y$. This implies that $C = C'$ and $\beta(1 + r) = 1$ from the FOCs. The money demand function is obtained as

$$\frac{M'}{P'} = YR^{-\frac{1}{\gamma}}.$$  

(b) Implicitly here the government is raising seignorage revenue and rebating it to the consumers lump sum. Find an expression for the level of inflation which maximizes seignorage revenue.

(Answer) The seignorage revenue $S$ is given by

$$S = \pi \frac{M'}{P'} = \pi YR^{-\frac{1}{\gamma}} = Y\pi \{(1 + r)(1 + \pi) - 1\}^{-\frac{1}{\gamma}}.$$  

The FOC w.r.t inflation is

$$\frac{\partial S}{\partial \pi} = Y \{(1 + r)(1 + \pi) - 1\}^{-\frac{1}{\gamma}} - \frac{1}{\gamma} Y(1 + r) = 0 \iff \left(\frac{1 + r}{\gamma}\right) \pi = r + (1 + r)\pi$$  

Therefore the level of inflation that maximizes seignorage is:

$$\pi = (1 - \beta) \left(\frac{\gamma}{1 - \gamma}\right).$$

4. This problem uses a Keynesian model with sticky prices and efficiency wages to compare the effects of a recession in Europe and the US.

(a) In both Europe and the US housing prices have fallen, reducing the wealth of consumers. Describe the effects of this fall on consumption, output, interest rates, and unemployment in the short run.

Solution: In the short run, from the negative wealth effect, consumption demand decreases, and thus the IS curve shifts to the left, which leads to a decrease in consumption, output, and real interest rate. Labor supply shifts to the left (from the substitution effect: real interest rate decreases), labor demand doesn’t change, thus unemployment decreases.

(b) Both economies are in a recession. In the US, the equilibrium real interest rate is likely negative, and inflation expectations are near zero (a liquidity trap). In Europe, equilibrium real interest rates are low but positive, and inflation expectations are low. How effective will monetary policy be in these economies to restoring full employment by changing interest rates (or increasing the money supply)? Illustrate and explain your results.
**Solution:** Since nominal interest rates are bounded at zero, the LM curve is very flat near zero nominal interest rates. In the US, with likely negative real interest rate and nearly zero inflation expectation, increases in money supply have little effect at low rates. Therefore, the monetary policy is less effective.

In Europe, since the real interest rates and the inflation expectations are positive, the monetary policy is more effective.