Second Midterm Examination Solutions

Instructions: This is a 75 minute examination worth 100 total points. Each question is worth 25 points. Choose FOUR out of the following SIX questions. DO NOT ANSWER MORE THAN FOUR QUESTIONS. If you do, your grade will be based on the LOWEST four questions.

In order to get full credit, you must give a clear, concise, and correct answer, including all necessary calculations. Notes and books will not be permitted. Explain your answers clearly and use graphs when helpful.

1. Suppose that there is a temporary change in consumer sentiment, so that households cut back on consumption spending, but this has no other direct effects on the economy. Consider the Keynesian model with sticky prices, and discuss the short and long run effects on output and interest rates of the following policy options.

Solution: The shock causes a leftward shift in the IS curve. Recall that the SRAS is horizontal and LRAS is vertical. Also, in the long run $Y$ always returns to its original level, and in the short run, $P$ does not change (sticky prices), so I won’t mention these explicitly below. The question doesn’t mention efficiency wages, so we’ll assume that the FE curve is our usual output supply curve, and so is not vertical. However answers with a vertical FE curve are also acceptable.

(a) The money supply is adjusted to return the economy to full employment.

Solution: If the money supply is adjusted, the LM curve shifts to the right, $Y$ falls slightly (due to the slope of the FE curve), $r$ falls, and $P$ is of course unchanged. This holds in the LR too, as the policy adjusted things immediately. (If the FE curve were vertical, then there would be no change in output.)

(b) Government spending is adjusted to return the economy to full employment.

Solution: If $G$ is expanded, this shifts the IS curve back to its original level. No change in $Y, r, P$ in either the short or the long run compared to the equilibrium before the shock.

2. Consider the Solow model with the addition of government spending. Suppose productivity is constant, so the only source of growth is the population (and hence labor force) which grows at rate $n = \dot{N}/N$. The government spends a constant amount per capita each period, so $G = gN$ where $g$ is a positive constant. Spending is financed via lump sum taxes, and the government balances its budget each period, so $G = T$. Consumers save a constant fraction $s$ of after-tax income, so $C = (1 - s)(Y - T)$. 


(a) Show that there can be two steady state levels of the per-capita capital $k = K/N$.

**Solution:** First we work out the analytics of the model. By assumption, we know that $T = G$, and so we can express total investment in equilibrium as:

$$I = s(Y - G) = s(Y - gN) = sN(y - g).$$

Therefore the usual equation of the Solow model now becomes:

$$\dot{k} = sf(k) - sg - (n + d)k.$$

So in steady state we have:

$$sf(k) = sg + (n + d)k$$

This equilibrium condition is depicted in Figure 3 below, which also shows that there can be two steady states.

(b) Ignore the low capital steady state, and analyze the effects of an increase in $g$.

What are the effects on per-capita capital and output in the steady state?

**Solution:** The effects of an increase in $g$ are depicted in the figure below. Capital and output per capita both decline in the steady state.
3. Suppose that problems in the banking sector lead to an unanticipated reduction in the money supply. In each of the following scenarios consider the effects of this shock on output and real interest rates. In addition, suppose that one option to respond to this shock would be to increase government spending temporarily. If the policy goal is to stabilize output, is this fiscal policy response a good idea?

(a) Use the Lucas-type misperceptions model.

**Solution:** In this model, an unanticipated fall in the money supply is interpreted by households as at least partly a reduction in total factor productivity. Labor supply and hence output supply fall, leading to a reduction in output and an increase in the real interest rate. The effects are just the opposite direction of the following figure.

![Figure 11.1 The Effects of an Unanticipated Increase in the Money Supply in the Money Surprise Model](image)

The response of the economy to an increase in government spending is just the same as in the real intertemporal model above, with output and interest rates increasing. If the goal is to stabilize output, this policy response would be a good idea, as it could offset the negative shock to output supply. Interest rates would increase even further in comparison to the original equilibrium. (While fiscal policy may help stabilize output, it may not increase welfare, as the government spending leads to lower private consumption and less leisure.)

(b) Use the Keynesian model

**Solution:** In the Keynesian model, the reduction in money supply would lead to a shift to the left of the $LM$ curve. Therefore in the short run there would be a fall output and an increase in the real interest rate. The effects are just the opposite direction of the following figure. Note: the figure uses a vertical FE curve, answers with an upward sloping FE curve are also acceptable. This only matters for the effects of the policy.
In this model, an increase in government spending shifts the IS curve out. In the short run, output $Y$ and the real interest rate $r$ increase. Thus this policy could help stabilize output, as again it could offset the negative shock and return output to its full employment level, with a higher interest rates. The net effects are as in this figure (but the change in $LM$ is not due to a price increase). If the FE curve were not vertical, then output would be slightly higher at the higher interest rate.

4. Suppose that a household does not face a cash in advance constraint, but instead money is the only asset for transferring income over time. That is, suppose that household preferences are given by:

$$\log C + \beta \log C'.$$

The household has real unearned income $Y$ in period 1, and uses money $M$ to transfer
assets to the future, thus facing the budget constraints:

\[ PC + M = PY \]
\[ P'C' = M \]

(a) Find the household’s money demand function and show that it is decreasing in the inflation rate.

**Solution:** We first eliminate \( M \) and solve for the present value budget constraint, which can be written:

\[ PC + P'C' = PY \]

or:
\[ C + (1 + i)C' = Y \]

To solve the household problem we can then form a Lagrangian with multiplier \( \lambda > 0 \).
\[
\mathcal{L} = \log(C) + \beta \log(C') + \lambda (Y - C - C'(1 + i))
\]

FOC:
\[
\frac{1}{C} = \lambda \\
\frac{\beta}{C'} = \lambda(1 + i)
\]

Eliminating \( \lambda \) between these, then using the budget constraint gives:

\[
C = \frac{Y}{1+\beta}
\]

\[
C' = \frac{\beta}{(1+i)(1+\beta)}Y
\]

\[
M = P'C' = P' \frac{\beta}{(1+i)(1+\beta)}Y
\]

Since \( i \) is in the denominator of this last expression, we clearly see that money demand is decreasing in \( i \).

(b) Relate your results to the quantity theory of money \( MV = PY \). What is the velocity of money in this problem?

**Solution:** Based on our previous expressions, we get:

\[
V = \frac{PY}{M} = \frac{PY}{P' \frac{\beta}{(1+i)(1+\beta)}Y} = \frac{P(1 + i)(1 + \beta)}{P' \beta} = \frac{1 + \beta}{\beta}
\]

Thus velocity is constant here, so the quantity theory of money always holds.

5. Suppose that everyone knows that government spending is to be increased for one period, but because of delays in Congress it may take effect either in the current year or the next. Using our real dynamic general equilibrium model, find the effects of this increase on the current values of output, interest rates, employment, and real wages in the following two scenarios.
(a) The increase is announced in the current period and takes effect immediately. Next year spending returns to its original level.

**Solution:** This is as we discussed in class and is summarized in Figure 9.17 in textbook, where it discusses the equilibrium effects of an increase in $G$. Output, employment, and interest rates increase. The real wage decreases.

(b) The increase is announced in the current period but takes effect next year. This year spending remains at its original level.

**Solution:** The increase in $G'$ would reduce lifetime income, and consequently both the labor and output supply curve shift to the right. The current consumption also decreases. On the other hand, current investment would rise because of an increase in future labor supply due to the reduced lifetime wealth. Therefore, the effect on output demand curve is ambiguous. It could either shift in, shift out, or remain at the original position. It’s reasonable to think that the shift of output demand is not large relative to the shift of output supply.

Therefore real interest rate decreases and output increases. Since output increases, employment also must increase. On the other hand, the real wage decreases as labor supply has increased.

6. Consider a real business cycle type model without labor. The sole input in production is capital $k_t$, which is owned by households and rented to firms at a rental rate $r_t$. That is output is produced via $y_t = A_t k_t$ where $A_t$ is the technology factor. Capital depreciates at rate $\delta$, and investment is given by $I_t$ so the law of motion for capital is:

$$k_{t+1} = (1 - \delta)k_t + I_t.$$
Households have logarithmic utility over consumption and discount with factor $\beta$:

$$U(c_0, c_1, \ldots) = \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

The household budget constraint each period is thus:

$$c_t + I_t = r_t k_t$$

(a) Write down the firm’s profit maximization problem, and derive an expression for the real interest rate at any date $t$.

**Solution:** The firm’s problem is to maximize profits, with a linear production function and capital as the only input:

$$\max_{k_t} \ A_t k_t - r_t k_t$$

Taking first order conditions gives the interest rate $r_t = A_t$. (Note: the firm’s problem is linear in $k$, so technically first order conditions don’t characterize the optimal choice. The firm’s demand has a “bang-bang” property: it is either zero if $r_t > A_t$, infinite if $r_t < A_t$ and anything if $r_t = A_t$. In order to have an equilibrium with positive, finite capital we must have $r_t = A_t$.)

(b) Write down the household utility maximization problem, and considering the choice of consumption at two arbitrary dates $t$ and $t+1$, find the Euler equation.

**Solution:** The household’s problem is to maximize utility subject to the budget constraint and the law of motion for capital. You could either write a Lagrangian, or substitute out for consumption. Let’s do the latter.

$$c_t = r_t k_t - I_t = r_t k_t - (k_{t+1} - (1 - \delta)k_t) = (1 + r_t - \delta)k_t - k_{t+1}.$$ 

Rather than rewrite this all, let’s just write $c_t = c(k_t, k_{t+1})$. Then the household problem is:

$$\max_{\{k_t\}} \sum_{t=0}^{\infty} \beta^t \log c(k_t, k_{t+1})$$

So the first order condition for a choice of $k_{t+1}$ at some date will be will be:

$$-\beta^t \frac{1}{c_t} + \beta^{t+1} \frac{1}{c_{t+1}} (1 + r_{t+1} - \delta) = 0$$

or rearranging, we get the Euler equation:

$$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}} (1 + r_{t+1} - \delta)$$
(c) Suppose that $A_t = A$ is constant. Under what conditions is equilibrium consumption increasing?

**Solution:** In equilibrium then $r_t = A$ for all $t$, so the Euler equation can be rearranged to give:

$$c_{t+1} = \beta(1 + A - \delta)c_t.$$

Therefore consumption is increasing if $\beta(1 + A - \delta) > 1$. (This means that the market rate of interest, net of depreciation, is greater than the subjective rate of discount.)