Lecture 6
Economic Growth

Noah Williams
University of Wisconsin - Madison

Economics 312
Now will turn to analysis of the process of economic growth. A main determinant of living standards.

Differences across countries:
- Out of 6.4 billion people, 0.8 do not have access to enough food, 1 to safe drinking water, and 2.4 to sanitation.
- Life expectancy in rich countries is 77 years, 67 years in middle income countries, and 53 years in poor countries.

Differences across time:
- Japanese boy born in 1880 had a life expectancy of 35 years, today 81 years.
- An American worked 61 hours per week in 1870, today 34.
I do not see how one can look at figures like these without seeing them as representing possibilities. Is there some action a government could take that would lead the Indian economy to grow like Indonesia’s or Egypt’s? If so, what exactly? If not, what is it about the “nature of India” that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else (Lucas 1988, p. 5).
Will develop theories to address growth over time and across countries. First decompose into growth in inputs and growth in productivity.

As will see later, growth in productivity may be sustainable. Growth in inputs may be important but can’t be sustained.

Decompose output growth into $A$, $K$, and $N$.

Starting point: Production Function

$Y(t) = A(t)F(K(t), N(t))$. Now view as continuous time: $t \in (0, \infty)$. 
Assume $F(K, N) = K^\alpha N^{1-\alpha}$. Take logs:

$$\log Y(t) = \log A(t) + \alpha \log K(t) + (1 - \alpha) \log N(t)$$

Let $\dot{x}(t) = \frac{dx}{dt}$. Differentiate respect to $t$:

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{A}(t)}{A(t)} + \alpha \frac{\dot{K}(t)}{K(t)} + (1 - \alpha) \frac{\dot{N}(t)}{N(t)}$$

We can measure $\dot{Y}/Y$, $\dot{K}/K$, $\dot{N}/N$, and $\alpha$. $\dot{A}/A$ is the Solow residual.
Using Factor Markets to Get $\alpha$

- Assume the labor and credit markets are competitive:
- Credit Market:

\[ r(t) = A(t)F_K(K(t), N(t)) = A(t)\alpha K(t)^{\alpha-1}N(t)^{1-\alpha} = \frac{\alpha Y(t)}{K(t)} \]

\[ \Rightarrow \alpha = \frac{r(t)K(t)}{Y(t)}, \text{ Capital Share.} \]

- Labor Market:

\[ w(t) = A(t)F_N(K(t), N(t)) = A(t)(1-\alpha)K(t)^{\alpha}N(t)^{-\alpha} \]

\[ \Rightarrow 1 - \alpha = \frac{w(t)N(t)}{Y(t)}, \text{ Labor Share.} \]

- Standard Estimates for U.S.: $\alpha = 0.33 - 0.36$. 
Figure 6.22
Natural Log of the Solow Residual, 1948–2007
### Table 6.1
Average Annual Growth Rates in the Solow Residual

<table>
<thead>
<tr>
<th>Years</th>
<th>Average Annual Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950–1960</td>
<td>1.42</td>
</tr>
<tr>
<td>1960–1970</td>
<td>1.61</td>
</tr>
<tr>
<td>1970–1980</td>
<td>0.50</td>
</tr>
<tr>
<td>1980–1990</td>
<td>1.05</td>
</tr>
<tr>
<td>1990–2000</td>
<td>1.36</td>
</tr>
<tr>
<td>2000–2007</td>
<td>0.76</td>
</tr>
</tbody>
</table>
Notice that there was big reduction in productivity growth in the 1970s. Following are some reasons:

- Sharp increases in the price of oil in 70’s
- Structural changes: more services and less and less manufacturing goods produced
- Slowdown in resources spent on R&D in the late 60’s.
- TFP was abnormally high in the 50’s and 60’s
- Information technology (IT) revolution in the 70’s
One key question: was fast growth in East Asian growth miracles mostly due to technological progress or mostly due to capital accumulation?

East Asia: in late 1980s-early 1990s growth in East Asia declared a “miracle” by many observers.

Average growth in real GDP 1966-1990: Hong Kong 7%, Singapore 8.5%, South Korea and Taiwan over 8%. But was growth due to growth in inputs or productivity?

Alwyn Young (Chicago GSB), summarized by Paul Krugman “The Myth of Asia’s Miracle”

Krugman draws parallel to Soviet economy. Similar also post-WWII Japan.
<table>
<thead>
<tr>
<th>Country</th>
<th>Per.</th>
<th>$g_Y$</th>
<th>$\alpha$</th>
<th>$\alpha g_K$</th>
<th>$(1 - \alpha)g_N$</th>
<th>$g_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>60-90</td>
<td>3.2</td>
<td>0.4</td>
<td>59%</td>
<td>$-8%$</td>
<td>49%</td>
</tr>
<tr>
<td>Italy</td>
<td>60-90</td>
<td>4.1</td>
<td>0.38</td>
<td>49%</td>
<td>3%</td>
<td>48%</td>
</tr>
<tr>
<td>UK</td>
<td>60-90</td>
<td>2.5</td>
<td>0.39</td>
<td>52%</td>
<td>$-4%$</td>
<td>52%</td>
</tr>
<tr>
<td>Argentina</td>
<td>40-80</td>
<td>3.6</td>
<td>0.54</td>
<td>43%</td>
<td>26%</td>
<td>31%</td>
</tr>
<tr>
<td>Brazil</td>
<td>40-80</td>
<td>6.4</td>
<td>0.45</td>
<td>51%</td>
<td>20%</td>
<td>29%</td>
</tr>
<tr>
<td>Chile</td>
<td>40-80</td>
<td>3.8</td>
<td>0.52</td>
<td>34%</td>
<td>26%</td>
<td>40%</td>
</tr>
<tr>
<td>Mexico</td>
<td>40-80</td>
<td>6.3</td>
<td>0.63</td>
<td>41%</td>
<td>23%</td>
<td>36%</td>
</tr>
<tr>
<td>Japan</td>
<td>60-90</td>
<td>6.8</td>
<td>0.42</td>
<td>57%</td>
<td>14%</td>
<td>29%</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>66-90</td>
<td>7.3</td>
<td>0.37</td>
<td>42%</td>
<td>28%</td>
<td>30%</td>
</tr>
<tr>
<td>Singapore</td>
<td>66-90</td>
<td>8.5</td>
<td>0.53</td>
<td>73%</td>
<td>31%</td>
<td>$-4%$</td>
</tr>
<tr>
<td>South Korea</td>
<td>66-90</td>
<td>10.3</td>
<td>0.32</td>
<td>46%</td>
<td>42%</td>
<td>12%</td>
</tr>
<tr>
<td>Taiwan</td>
<td>66-90</td>
<td>9.1</td>
<td>0.29</td>
<td>40%</td>
<td>40%</td>
<td>20%</td>
</tr>
</tbody>
</table>
Growth “Miracles”? 

- Table illustrates finding that most of the growth in East Asian countries was due to growth in inputs.
- Singapore is a prime example. The employed share of the population surged from 27 to 51 percent.
- Increased education: in 1966 more than 1/2 workers had no formal education at all, by 1990 2/3 had completed secondary education.
- Huge investment in physical capital: investment as a share of output rose from 11% to more than 40%. After accounting for inputs, found negative TFP growth.
- Similar arguments for Japan (post WWII) and China (current), not as dire. Was an improvement in Japanese TFP, although slowed. China started from low position, has large population, so may grow for a long time before diminishing returns set in.
So far have analyzed the sources of growth in an economy, argued that only TFP growth provides sustainable growth.

Now develop a model showing this and addressing:

1. What is the relationship between the long-run standard of living and the saving rate, population growth rate, and rate of technical progress?

2. How does economic growth change over time? (Speed up, slow down, stabilize?)

3. Are there forces that allow poorer countries to catch up to richer ones?
Basic Assumptions of the Solow Growth Model

- Continuous time. (Book does discrete time.)
- Single good in the economy produced, constant technology.
- No government or international trade.
- All factors of production are fully employed.
- Labor force (population) grows at constant rate

\[ n = \frac{\dot{N}}{N} \]

- Initial values for capital, \( K_0 \) and labor, \( N_0 \) given.
Cobb-Douglas aggregate production function:

\[ Y(t) = A(t)F(K(t), N(t)) = A(t)K(t)^\alpha N(t)^{1-\alpha} \]

For now suppose no TFP growth: \( A(t) = 1 \).

Define **per worker** variables: \( y = \frac{Y}{N}, k = \frac{K}{N} \). Then:

\[ y = \frac{Y}{N} = \frac{K^\alpha N^{1-\alpha}}{N} = \left( \frac{K}{N} \right)^\alpha \left( \frac{N}{N} \right)^{1-\alpha} = k^\alpha = f(k) \]

Per worker production function has decreasing returns to scale.

Again constant returns implies \( Y = rK + wN \).
Suppose household’s don’t value leisure, inelastically supply 1 unit of labor. Aggregate labor supply then $N$.

Assume result of household maximization problem is to save a constant fraction $s$ of income, consume $1 - s$.

\[ C = (1 - s)[rK + wN] = (1 - s)Y \]

Capital evolution:

\[ K_{t+1} = (1 - \delta)K_t + sY_t \]

Rearrange and take limits as $\Delta t \to 0$:

\[ K_{t+1} - K_t = -\delta K_t + sY_t \]
\[ \Rightarrow \dot{K} = sY - \delta K \]
Divide by $N$ in the capital accumulation equation:

$$\frac{\dot{K}}{N} = sy - \delta k = sk^\alpha - d\delta$$

Now remember that $k(t) = K(t)/N(t)$, so:

$$\dot{k} = \frac{\dot{K}}{N} - \frac{\dot{N}K}{NN}$$

$$= \frac{\dot{K}}{N} - nk$$

$$= sk^\alpha - (\delta + n)k$$

The last line is the **fundamental equation** of the Solow Model

$$\dot{k} = sk^\alpha - (\delta + n)k$$
Graphical Analysis

- Change in $k$, $\dot{k}$ is given by difference of $sk^\alpha$ and $(\delta + n)k$
- If $sk^\alpha > (\delta + n)k$, then $k$ increases.
- If $sk^\alpha < (\delta + n)k$, then $k$ decreases.
- Unique positive steady state. (Trivial steady state at $k = 0$.)
- Positive steady state stable: if move away, will come back to it.
Steady state in the Solow model
Steady State Analysis

- Steady State: \( \dot{k} = 0 \)
- Solve for steady state

\[
0 = s \, (k^*)^\alpha - (n + d)k^* \Rightarrow k^* = \left( \frac{s}{n + \delta} \right)^{\frac{1}{1-\alpha}}
\]

- Steady state output per worker \( y^* = \left( \frac{s}{n + \delta} \right)^{\frac{\alpha}{1-\alpha}} \)
- Steady state consumption per worker:

\[
c^* = (1 - s)(k^*)^\alpha
\]
Comparative Statics

- Suppose that of all a sudden saving rate $s$ increases to $s' > s$. Suppose that economy was initially at its old steady state with saving rate $s$.
- $(n + d)k$ curve does not change.
- $sf(k)$ shifts up to $s'f(k)$
- New steady state: higher capital and output per worker.
- Capital stock increases monotonically from old to new steady state.
Increase in savings rate in the Solow model.
Why are some countries rich (have high per worker GDP) and others are poor (have low per worker GDP)?

Solow model: if all countries are in their steady states, then:

1. Rich countries have higher saving (investment) rates than poor countries.
2. Rich countries have lower population growth rates than poor countries.

Data seem to support this prediction of the model.

But no growth in the steady state: only transitional dynamics.

Illuminates why capital accumulation has an inherit limitation as a source of economic growth. (Recall Singapore example.)
Figure 6.2  Output per Worker vs. Investment Rate
Figure 6.3  Output per Worker vs. the Population Growth Rate
• Re-introduce TFP. No limits to innovation.
• Slightly simpler to introduce as labor-augmenting technical change. Aggregate production function:

\[ Y = K^\alpha (AN)^{1-\alpha} \]

• Equivalent to \( zK^\alpha N^{1-\alpha} \) with \( z = A^{1-\alpha} \).
• Key assumption: constant rate of technological progress:

\[ \frac{\dot{A}}{A} = g > 0 \]

• TFP growth is \textit{exogenous} here.
Balanced Growth Path

- Situation in which output per worker, capital per worker, and consumption per worker grow at constant rates.
- For Solow model, \( y, k, c \) all grow same at constant rate \( g \).
- Why? In balanced growth path \( g_k \) constant, but:

\[
\dot{k} = sy - (n + d)k \Rightarrow g_k = sy/k - (n + d)
\]

So \( y/k \) constant \( \Rightarrow g_y = g_k \). But then:

\[
y = \frac{Y}{N} = \frac{K^\alpha (AN)^{1-\alpha}}{N} = k^\alpha A^{1-\alpha}
\]

- Take logs and differentiate:

\[
g_y = \alpha g_k + (1 - \alpha)g_A
\]

But we showed \( g_k = g_y \) and assumed \( g_A = g \), so

\[
g_k = g_y = g_A = g.
\]
In BGP all per-capita variables grow at rate $g$. Want to work with variables that are constant in long run. Define:

$$\tilde{y} = \frac{y}{A} = \frac{Y}{AN}$$

$$\tilde{k} = \frac{k}{A} = \frac{K}{AN}$$

Repeat the analysis with new variables:

$$\tilde{y} = \tilde{k}^\alpha$$

$$\dot{\tilde{k}} = s\tilde{y} - (n + g + \delta)\tilde{k}$$

$$\dot{\tilde{k}} = s\tilde{k}^\alpha - (n + g + \delta)\tilde{k}$$
Solve for $\tilde{k}^*$ analytically

$$0 = s\tilde{k}^* - (n + g + \delta)\tilde{k}^*$$

$$\tilde{k}^* = \left(\frac{s}{n + g + \delta}\right)^{\frac{1}{1-\alpha}}$$

Therefore

$$\tilde{y}^* = \left(\frac{s}{n + g + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$
Balanced Growth Path Analysis II

\[ k(t) = A(t) \left( \frac{s}{n + g + \delta} \right)^{1 \frac{1}{1-\alpha}} \]

\[ y(t) = A(t) \left( \frac{s}{n + g + \delta} \right)^{1 \frac{\alpha}{1-\alpha}} \]

\[ K(t) = N(t) A(t) \left( \frac{s}{n + g + \delta} \right)^{1 \frac{1}{1-\alpha}} \]

\[ Y(t) = N(t) A(t) \left( \frac{s}{n + g + \delta} \right)^{1 \frac{\alpha}{1-\alpha}} \]