Lecture 20
Hyperinflations
Time Consistency

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Economics 312
- Conduct of monetary policy may have dramatic implications for economic outcomes.
- Main example in US: The “Great Inflation” of the 1970s. Sustained double digit annual inflation rates, accompanied by slow economic growth and recession.
Argentina: Inflation and Real Money

Inflation

Real balances

Log value of monthly gross inflation rate

Log value of real (base-money) balances


−1 0 1 2


4 5 6 7
Brazil: Inflation and Real Money

![Graph showing the log value of the gross inflation rate and the log value of real (base-money) balances from 1980 to 2005.](image)

- **Inflation**
- **Real balances**
## Highest Monthly Inflation Rates in History

<table>
<thead>
<tr>
<th>Country</th>
<th>Month</th>
<th>Monthly rate</th>
<th>Daily</th>
<th>Doubling Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hungary</td>
<td>July 1946</td>
<td>$4.19 \times 10^{16}%$</td>
<td>207%</td>
<td>15 hours</td>
</tr>
<tr>
<td>Zimbabwe</td>
<td>Nov. 2008</td>
<td>$7.96 \times 10^{10}%$</td>
<td>98%</td>
<td>24.7 hours</td>
</tr>
<tr>
<td>Yugoslavia</td>
<td>Jan. 1994</td>
<td>$3.13 \times 10^{8}%$</td>
<td>64.6%</td>
<td>1.4 days</td>
</tr>
<tr>
<td>Germany</td>
<td>Oct. 1923</td>
<td>29,500 %</td>
<td>20.9%</td>
<td>3.7 days</td>
</tr>
<tr>
<td>Greece</td>
<td>Oct. 1944</td>
<td>13,800%</td>
<td>17.9%</td>
<td>4.3 days</td>
</tr>
<tr>
<td>China</td>
<td>May 1949</td>
<td>2,178 %</td>
<td>11%</td>
<td>6.7 days</td>
</tr>
</tbody>
</table>
German Hyperinflation
Zimbabwe Hyperinflation
Zimbabwe Hyperinflation

- Most recent hyperinflation episode was Zimbabwe.
- Problem began with land reforms (land confiscation), leading to collapse in output.
- From January to December 2008, the money supply growth rose from 81,143% to 658 billion percent.
August 2006: New dollar = 1000 old dollars
New ZW $1000 = $1.50 US
Jan 1. 2008: ZW$10 million = $1.66 US
Jul 1. 2008: ZW$100 billion = $0.13 US
Zimbabwe Currency

Aug. 1, 2008: New currency ZW$10 billion = ZWR$1
New ZWR $100 trillion = ZW$10^{14} = ZW$10^{27} pre-2006
New ZWR $100 trillion = $300 US on 1/1/09, $30 US on 1/16
Now selling for US$3.50 on e-Bay as collector’s item.
Zimbabwe Exchange Rates

- In terms of new currency (ZWR):

<table>
<thead>
<tr>
<th>Date</th>
<th>New ZWR per USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sept 2008</td>
<td>1,000</td>
</tr>
<tr>
<td>Oct 2008</td>
<td>90,000</td>
</tr>
<tr>
<td>Nov 2008</td>
<td>1,200,000</td>
</tr>
<tr>
<td>Mid Dec 2008</td>
<td>60,000,000</td>
</tr>
<tr>
<td>End Dec 2008</td>
<td>2,000,000,000</td>
</tr>
<tr>
<td>Mid Jan 2009</td>
<td>1,000,000,000,000</td>
</tr>
<tr>
<td>Feb 2 2009</td>
<td>300,000,000,000,000</td>
</tr>
</tbody>
</table>


- Zimbabwe currency suspended in April 2009.
Zimbabwe Exchange Rates

In terms of original currency

ZWD vs USD Exchange Rate History

Official
Parallel
OMR/UN

Date
Hyperinflations largely an issue of government finance. **Seignorage**: government revenue from creation of money.

Consider money demand where inflation equal to expected, real interest rate and output constant:

$$\frac{M}{P} = L(\bar{r} + \pi, \bar{Y})$$

where $\pi = \frac{\dot{M}}{M}$ growth of money = inflation rate.

Seignorage $S$ given by real increase in money:

$$S = \frac{\dot{M}}{P} = \frac{\dot{M}M}{MP} = \pi \frac{M}{P}$$

Seignorage is the **inflation tax**: revenue = tax rate ($\pi$) times base $M/P$. 
Substituting back money demand:

\[ S = \pi L(\bar{r} + \pi, \bar{Y}) \]

How does seignorage revenue depend on inflation rate?

\[ S_\pi = L(\bar{r} + \pi, \bar{Y}) + \pi L_R(\bar{r} + \pi, \bar{Y}) \]

\( L > 0 \) but \( L_R < 0 \). For small \( \pi \) first term dominates, and \( S_\pi > 0 \). For large \( \pi \), \( S_\pi < 0 \). Hence get a Laffer curve.

Cagan (1956) studied hyperinflations, used a particular functional form:

\[ \log \frac{M}{P} = a - bR + \log Y \]

hence \( S_\pi = (1 - b\pi) \exp(-b\pi) \). Maximal level \( S^* \).
Figure 15.07a  The determination of real seignorage revenue

(a) Determination of real seignorage revenue for $\pi = 8\%$
Figure 15.07b  The determination of real seignorage revenue

(b) Determination of real seignorage revenue for $\pi = 1\%$ and $\pi = 15\%$
Figure 15.08  The relation of real seignorage revenue to the rate of inflation

Real seignorage revenue, $R$ (in billions of dollars)

Inflation, $\pi$ (percent per year)
Laffer Curve

Even moderate seignorage needs can lead to high inflation.
Seigniorage: ratio of Diff money to nominal GDP

- 0.05
- 0
- 0.05
- 0.1
- 0.15
- 0.2
- 0.25
- 0.3


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Seigniorage Revenue: Brazil

Seigniorage: ratio of Diff money to nominal gdp

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Seignorage and Hyperinflation

- If seignorage revenue maximized at hundreds of percents (or lower), why have we seen even higher inflation rates?

- Cagan’s answer: gradual adjustment of expected inflation and money holdings. Assume $r$ and $Y$ are constant so desired real money holdings are:

$$\hat{m} = \exp(a - b(r + \pi))Y = B \exp(-b\pi)$$

- Suppose actual real money holdings $m$ adjusts toward desired:

$$\frac{\dot{m}}{m} = \beta \left[ \log \hat{m}(t) - \log m(t) \right]$$

$$= \beta \left[ \log B - b\pi(t) - \log m(t) \right]$$

- This is a partial adjustment model, perhaps due to adaptive expectations.
Hyperinflations

- No longer have inflation equal to expected inflation.

\[
\dot{m} = \frac{\dot{M}}{P} - \frac{M}{P^2} \dot{P} = \frac{\dot{M}}{P} \frac{M}{M} - \frac{M}{P} \frac{\dot{P}}{P}
\]

\[
\Rightarrow \frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \pi
\]

\[
S = \frac{\dot{M}}{P} = \frac{\dot{M}}{P} \frac{M}{M} = m \frac{\dot{M}}{M}
\]

- So \( \pi = S/m - \frac{\dot{m}}{m} \) substitute in and solve:

\[
\frac{\dot{m}}{m} = \beta \left[ \log B - b\left( \frac{S}{m} - \frac{\dot{m}}{m} \right) - \log m(t) \right]
\]

\[
= \frac{\beta}{1 - b \beta} \left[ \log B - b \frac{S(t)}{m(t)} - \log m(t) \right]
\]
The evolution of real money is:

\[
\frac{\dot{m}}{m} = \frac{\beta}{1 - b\beta} \left[ \log B - b \frac{S(t)}{m(t)} - \log m(t) \right]
\]

As long as \( S < S^* \) there are two steady states \( \bar{m} \):

\[
\frac{\dot{m}}{m} = 0 \Rightarrow \bar{m} = B \exp(-b \frac{S}{\bar{m}}), \quad \bar{\pi} = \frac{S}{\bar{m}}
\]

If in a lower \( \bar{m} \) steady state where \( S < S^* \) but then a shock to money demand hits, will converge to the higher \( \bar{m} \) steady state. That is, the high \( \bar{m} \), low \( \bar{\pi} \) steady state is stable.
Increase in required seignorage may lead to hyperinflation.
Suppose economy initially in a steady state where required seignorage is less than $S^\star$. But then required $S$ increases to more than $S^\star$. If there were immediate adjustment, this equilibrium would not be sustainable. But with partial adjustment, this implies ever accelerating inflation and money growth and declining real balances.

In Germany after WWI, inflation reached 322% per month, but Cagan estimated the inflation rate that maximizes seignorage at only 20% per month.

Start with an expectations augmented Phillips curve,

\[ \pi_t = \pi_t^e + a(Y_t - Y_t^T), \]

motivated by the Lucas misperceptions model, where

- \( \pi_t \) is inflation, \( \pi_t^e \) is expected inflation,
- \( Y_t \) is output, and \( Y_t^T \) is trend output.

The central bank cares about price stability and output.

\[ \min_{Y_t, \pi_t} \pi_t^2 + \omega(Y_t - Y_t^*)^2 \]

- The first term represents the price stability goal.
- The second term represents the output gap.
Figure 17.13  The Fed’s Preferences Over Inflation Rates and Output
Represent the central bank’s problem as a Lagrangian:

\[ L(Y_t, \pi_t) = \pi_t^2 + \omega(Y_t - Y_t^*)^2 + \lambda(\pi_t^e + a(Y_t - Y_t^T) - \pi_t) \]

The first order conditions are

\[ 2\pi_t - \lambda = 0 \text{ and } 2\omega(Y_t - Y_t^*) + \lambda a = 0 \]

or \[ \omega(Y_t - Y_t^*) + a\pi_t = 0 \]

Eliminate \( Y_t \) using the Phillips curve constraint:

\[ \pi_t = \pi_t^e + a \left( Y_t^* - \frac{a}{\omega} \pi_t - Y_t^T \right) \Rightarrow \pi_t = \frac{\beta(\pi_t^e + a(Y_t^* - Y_t^T))}{\omega + a^2} \]
Assume \( Y_t^* > Y_{t,T} \), so \( \pi_t > \frac{\omega \pi_t^e}{\omega + a^2} \).

Suppose in 1960, people expected price stability, \( \pi_{1960}^e = 0 \).
- The central bank decides to ‘exploit’ the Phillips curve.
- \( \pi_{1960} > 0 \) and \( Y_{1960,T} < Y_{1960} < Y_{1960}^* \).
- In the short-run, this makes people better off.

But in a few years, people figure out what is happening.
- Adaptive expectations: \( \pi_{1965}^e = \pi_{1960} > 0 \).
- Exploiting the Phillips curve requires still higher inflation.
- \( \pi_{1965} > \pi_{1965}^e, Y_{1965,T} < Y_{1965} < Y_{1965}^* \).

Eventually expectations again catch up with reality.
Figure 17.14 The Fed Exploits the Phillips Curve
This process stops only when $\pi_t = \pi_t^e$.

$$\pi_t^e = \frac{\omega(\pi_t^e + a(Y_t^* - Y_t^T))}{\omega + a^2} \Rightarrow \pi_t = \pi_t^e = \frac{\beta(Y_t^* - Y_t^T)}{a}$$

Moreover, since $\pi_t = \pi_t^e$, $Y_t = Y_t^T$ as well.

In the long-run, this yields higher inflation with no output gain.

Known as the time-consistency problem.

Under rational expectations, this process occurs immediately.

- If $\pi_t^e < \frac{\omega(Y_t^* - Y_t^T)}{a}$, households expect $\pi_t > \pi_t^e$.
- This is inconsistent with rational expectations.
- But this requires households to understand Fed’s incentives.
Figure 17.15 The Fed Attempts to Increase Y Permanently
How can the Fed reduce inflationary expectations?

- **Adaptive expectations**: stop trying to exploit Phillips curve.
- **Rational expectations**: commit not to exploit the Phillips curve.
  - Isolate the Fed from pressure to make $Y_t^* > Y_t^T$.
  - Appoint “conservative” central bankers.
  - Force the Fed to follow a narrow set of rules.
  - Reputation may substitute for explicit commitment.
- **Reality probably lies somewhere in between**.
  - It is possible but costly to simply lower money growth.
  - Disinflation is less costly if it is credibly pre-announced.
Figure 17.16 The Commitment Problem
Fed funds rate is very short term. In itself has little effect, as long-term rates are key determinant of spending decisions.

But long rate depends on future path of short rate. So not just current policy matters, but expected future policy matters.

Modern theory emphasizes the role of nominal rigidities. Some prices and wages are pre-set in nominal terms.

So in short run Fed has some influence over aggregate demand. By lowering rates, it makes borrowing cheaper for households which boosts aggregate demand. Raising rates opposite.

But prices and wages set understanding Fed behavior, hence importance of expectations.
The shaded region shows the Humphrey-Hawkins CPI inflation range. Beginning in January 2000, the Humphrey-Hawkins inflation range was reported using the PCE price index and therefore is not shown on this graph.

See the notes section for an explanation of the chart.

Treasury Security Yield Spreads
Yield to maturity

Real Interest Rates
Percent, Real rate = Nominal rate less year-over-year CPI inflation