A Change in the Growth Rate of Money

- An increase in the money supply raises prices.
- An increase in money supply growth raises inflation as prices continue to rise.
- Suppose $M' = (1 + m)M$.
- $M = PL(Y', R)$ and $M' = P'L(Y', R')$.
- Suppose the real economy is the same in every period ($Y = Y'$, $R = R'$):
  \[ \frac{M'}{M} = \frac{P'}{P} \Rightarrow 1 + m = 1 + \pi \Rightarrow m = \pi \]
- Inflation due growth in money supply.
- But increase in money growth may have real effects. (We’ll return to this later.)
Quantity Theory of Money

- **Velocity** of money is a measure of how quickly money changes hands. Basic *quantity theory* equation:

  \[ MV = PY \]

- Serves as definition of \( V = PY/M = Y/L(Y, R) \).

- Quantity theory or *monetarism* says that velocity is stable, so changes in \( M^s \) lead to changes in nominal income \( PY \).


- Problems: Increased financial innovation led to shifts in money demand, hence changes in velocity. Made implementation of policy difficult and led to volatility.
M1 Velocity: 1959-1981

Velocity of Circulation: GDP divided by M1

Economic Chart Dispenser

http://www.Econographic.com
M1 Velocity: 1959-2013

Source: Federal Reserve Bank of St. Louis

Velocity of M1 Money Stock

Shaded areas indicate US recessions - 2014 research.stlouisfed.org
Figure 10.17 Central Bank Does Not Observe the Price Level Response to a Shift in Demand for Money
Money in the Utility Function (MIU)

- Rather than assuming cash in advance, another way to introduce money is to assume real money balances \( m_t = M_t/P_t \) enter directly in the utility function.
- Given suitable restrictions on the utility function, such an approach can guarantee that, in equilibrium, agents choose to hold positive amounts of money so that money will be positively valued.
- Representative household takes prices as given and maximizes

\[
E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, m_{t+i}, l_{t+i})
\]

subject to a budget constraint.
Period Budget Constraints

- Budget constraint, in nominal terms:

\[
W_t N_t + (r_t + 1 - \delta) P_t K_{t-1} + (1 + R_{t-1}) B_{t-1} + M_{t-1} - P_t T_t = P_t C_t + P_t K_t + B_t + M_t
\]

where \( T_t \) are real, lump-sum taxes.

- In real terms:

\[
\omega_t N_t + (r_t + 1 - \delta) K_{t-1} + (1 + R_{t-1}) \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} - T_t = C_t + K_t + \frac{B_t}{P_t} + \frac{M_t}{P_t}
\]

or

\[
\omega_t N_t + (r_t + 1 - \delta) K_{t-1} + (1 + R_{t-1}) \frac{P_{t-1}}{P_t} b_{t-1} + \frac{P_{t-1}}{P_t} m_{t-1} - T_t = C_t + K_t + b_t + m_t
\]
Optimality Conditions

- First order conditions:

\[ U_C - \lambda_t = 0 \]

\[ U_l - w_t \lambda_t = 0 \Rightarrow \frac{U_l}{U_C} = w_t \]

\[-\lambda_t + \beta E_t (r_{t+1} + 1 - \delta) \lambda_{t+1} = 0 \]

\[-\lambda_t + \beta (1 + R_t) E_t \left( \frac{P_t}{P_{t+1}} \right) \lambda_{t+1} = 0 \]

\[ U_m - \lambda_t + \beta E_t \left( \frac{P_t}{P_{t+1}} \right) \lambda_{t+1} = 0 \]

- First three look just as before (except \( U_C \) and \( U_l \) may depend on \( m \)).
The FOC for money can be written

$$\frac{\lambda_t}{P_t} = \frac{U_m}{P_t} + \beta E_t \left( \frac{\lambda_{t+1}}{P_{t+1}} \right)$$

Solve forward to yield

$$\frac{1}{P_t} = \left( \frac{1}{\lambda_t} \right) E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{U_m (C_{t+i}, m_{t+i}, l_{t+i})}{P_{t+i}} \right]$$

Value of money equals expected present value of discounted utility return.
Optimality Conditions

- Combining the first and third conditions yields the Euler condition:

\[ U_C(C_t, m_t, l_t) = \beta E_t (r_{t+1} + 1 - \delta) U_C(C_{t+1}, m_{t+1}, l_{t+1}) \]

- Similarly,

\[ U_C(C_t, m_t, l_t) = \beta (1 + R_t) E_t \left( \frac{P_t}{P_{t+1}} \right) U_C(C_{t+1}, m_{t+1}, l_{t+1}) \]

- From fourth and fifth,

\[ \frac{U_m}{U_C} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) \frac{U_C(C_{t+1}, m_{t+1}, l_{t+1})}{U_C(C_t, m_t, l_t)} \]

- But

\[ 1 = \beta (1 + R_t) E_t \left( \frac{P_t}{P_{t+1}} \right) \frac{U_C(C_{t+1}, m_{t+1}, l_{t+1})}{U_C(C_t, m_t, l_t)} \]

so

\[ \frac{U_m}{U_C} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) \frac{U_C(C_{t+1}, m_{t+1}, l_{t+1})}{U_C(C_t, m_t, l_t)} = \frac{R_t}{1 + R_t}. \]
Equilibrium Conditions

\[ \frac{U_l(C_t, m_t, 1 - N_t)}{U_C(C_t, m_t, 1 - N_t)} = w_t \]
\[ \frac{U_m(C_t, m_t, 1 - N_t)}{U_C(C_t, m_t, 1 - N_t)} = \frac{R_t}{1 + R_t}. \]
\[ U_C(C_t, m_t, 1 - N_t) = \beta E_t (r_{t+1} + 1 - \delta) U_C(C_{t+1}, m_{t+1}, 1 - N_{t+1}) \]
\[ U_C(C_t, m_t, 1 - N_t) = \beta (1 + R_t) E_t \left( \frac{P_t}{P_{t+1}} \right) U_C(C_{t+1}, m_{t+1}, 1 - N_{t+1}) \]

\[ Y_t = F(N_t, K_{t-1}) \]
\[ F_N(N_t, K_{t-1}) = w_t \]
\[ F_K(N_t, K_{t-1}) = r_t \]
\[ Y_t = C_t + K_t - (1 - \delta)K_{t-1} \]
\[ P_t m_t = M_t \]
The Steady State

- Standard relationships plus

\[ 1 = \beta(r + 1 - \delta) \]

\[ 1 = \beta \left( \frac{1 + R}{1 + \pi} \right) \]

\[ \frac{U_m(C, m, 1 - N)}{U_C(C, m, 1 - N)} = \frac{R}{1 + R} = 1 - \frac{\beta}{1 + \pi} \]

\[ P_t = \frac{M_0(1 + \pi)^t}{m} \]
Superneutrality

- If $U_L$ and $U_C$ independent of $m$:
  
  $U(C, m, l) = U(c, l) + v(m)$, real side identical to basic RBC model. (Steady state and dynamics)

  - Neutrality: One time changes in the nominal quantity of money affect only the price level.
  - Superneutrality: Changes in growth rate of money do not affect real variables.

- Changes in the rate of growth of money affect only the inflation rate, the nominal interest rate, and real money balances.
  
  - If $F$ displays constant returns to scale and $U_l$ and $U_C$ depend on $m$, still get steady-state capital-labor ratio independent of money growth.
Because money holdings yield direct utility and higher inflation reduces real money balances, inflation generates a welfare loss.

Questions:

- How large is the welfare cost of inflation?
- Is there an optimal rate of inflation that maximizes the steady-state welfare of the representative household?
Optimal Inflation Rate

- The optimal rate of inflation addressed by Bailey (1956) and M. Friedman (1969).

- Basic intuition: the private opportunity cost of holding money depends on the nominal rate of interest. The social marginal cost of producing money is essentially zero.

- The wedge that arises between the private marginal cost and the social marginal cost when the nominal rate of interest is positive generates an inefficiency.

- This inefficiency would be eliminated if the nominal rate of interest were zero.

- So the optimal rate of inflation is a rate of deflation approximately equal to the real interest rate:

\[ R = 0 \Rightarrow \pi = -r \]
In CIA models, inflation acts as a tax on goods or activities whose purchase requires cash.

This tax then introduces a distortion by creating a wedge between the marginal rates of transformation implied by the economy’s technology and the marginal rates of substitution faced by consumers.

Since the CIA model, like the MIU model, offers no reason for such a distortion to be introduced (there is no inefficiency that calls for Pigovian taxes or subsidies on particular activities, and the government’s revenue needs can be met through lump-sum taxation), optimality calls for setting the inflation tax equal to zero.

The inflation tax is directly related to the nominal rate of interest; a zero inflation tax is achieved when the nominal rate of interest is equal to zero.
Money in the Long Run

- There is a high (almost unity) correlation between the rate of growth of monetary supply and the rate of inflation.
- There is no correlation between the growth rates of money and real output.
- There in no correlation between inflation and real output.
- These are consistent with the quantity theory.
Money Growth and Inflation: A High, Positive Correlation

Average Annual Rates of Growth in M2 and in Consumer Prices During 1960–90 in 110 Countries
Money and Real Output Growth: No Correlation in the Full Sample . . .

Average Annual Rates of Growth in M2 and in Nominal Gross Domestic Product, Deflated by Consumer Prices During 1960–90 in 110 Countries
Inflation and Real Output Growth: No Correlation

Average Annual Rates of Growth in Consumer Prices and in Nominal Gross Domestic Product, Deflated by Consumer Prices During 1960–90 in 110 Countries
Money and Business Cycles

- Relation between output and money. Money is pro-cyclical, more so prior to 1985.

- Other pieces of evidence:
  - Volcker’s recessions: Tightening of monetary policy in early 1980s seemed to lead to recessions.
  - Friedman and Schwartz (1963), *A Monetary History of the United States*: independent fluctuations in money supply were followed by changes in real output.

- Phillips Curve: tend to observe negative relationship between inflation rate and unemployment rate. Relationship not stable.
Growth in M1 and GDP

M1 Money Stock (M1SL)
Real Gross Domestic Product, 1 Decimal (GDPC1)

Shaded areas indicate US recessions.
2013 research.stlouisfed.org
Phillips Curve, 1956-2004
Phillips Curve, 1976-1979
Phillips Curve, 1993-2000

Unemployment vs. Inflation


Williams
Economics 312
In model so far money is neutral. Change in money has no effect on economic activity.

Seems to hold in long-run, less so in short run. How to reconcile?

Response #1: RBC denial. Evidence of effect of money is not causal. Observe $M$ leading $Y$ but also observe carrying of umbrellas leading rainfall. Increase in $M$ may reflect expectations of higher future $Y$.

Response #2: Lucas (1972) imperfect-information model. Short-run effects of money supply caused by confusing changes in relative and aggregate prices. More on this next.

Figure 11.5 Procyclical Money Supply in the Real Business Cycle Model with Endogenous Money