Lecture 1: Additional Material
More on Preferences

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Preferences

- Preferences: binary relation $\succeq$ defined over pairs $(c, l)$:
  \[(c_i, l_i) \succeq (c_j, l_j)\]

- Basic assumptions on preferences:
  1. Complete: for $\forall (c_i, l_i), (c_j, l_j) \in \mathbb{R}_+ \times [0, h]$ either $(c_i, l_i) \succeq (c_j, l_j)$ or $(c_j, l_j) \succeq (c_i, l_i)$.
  2. Reflexive: for $\forall (c_i, l_i) \in \mathbb{R}_+ \times [0, h]$ $(c_i, l_i) \succeq (c_i, l_i)$.
  3. Transitive: for $\forall (c_i, l_i), (c_j, l_j), (c_k, l_k) \in \mathbb{R}_+ \times [0, h]$, if $(c_i, l_i) \succeq (c_j, l_j)$ and $(c_j, l_j) \succeq (c_k, l_k)$ $\Rightarrow (c_i, l_i) \succeq (c_k, l_k)$. 
The Transitivity Axiom

- Violations of transitivity lead to “Dutch books,” sequence of trades where agent loses money for sure.
- Suppose $c_i \succeq c_j$ and $c_j \succeq c_k$ but $c_k \succeq c_i$. If agent starts with $c_k$, willing to pay something to get $c_j$ (since $c_j \succeq c_k$).
- Then with $c_j$ is willing to pay something more to get $c_i$ (since $c_i \succeq c_j$).
- But then with $c_i$, is willing to pay something to get $c_k$ (since $c_k \succeq c_i$).
- After these trades, is back at starting point of $c_k$, but lost costs of trades.
- Would continue to trade until money is all gone. Agents without transitive preferences will be driven out of market.
Indifference Curves

- Loci of pairs such that:

\[(c_i, l_i) \succeq (c_j, l_j), (c_j, l_j) \succeq (c_i, l_i) \Rightarrow (c_j, l_j) \sim (c_i, l_i)\]

- Additional assumptions on preferences:
  1. Monotonicity: If \(c_i \geq c_j, l_i \geq l_j\) then \((c_i, l_i) \succeq (c_j, l_j)\).
  2. Convexity: If \((c_i, l_i) \succeq (c_k, l_k)\) and \((c_j, l_j) \succeq (c_k, l_k)\) then 
     \[\forall \lambda \in [0, 1] \quad \lambda (c_i, l_i) + (1 - \lambda) (c_j, l_j) \succeq (c_k, l_k)\]

- Under these assumptions, the indifference curves are:
  1. Negatively sloped.
  2. Convex.
Figure 4.1  Indifference Curves
Preferences

- Preferences: binary relation \( \succeq \) defined over pairs \((c, l)\):

\[
(c_i, l_i) \succeq (c_j, l_j)
\]

- Working directly with binary relations difficult.

**Definition:** a real-valued function \( u : \mathbb{R}^2 \to \mathbb{R} \) is called a utility function representing the binary relation \( \succeq \) defined over pairs \((c, l)\) if for \( \forall (c_i, l_i), (c_j, l_j) \in \mathbb{R}_+ \times [0, h], \)

\[
(c_i, l_i) \succeq (c_j, l_j) \iff u(c_i, l_i) \geq u(c_j, l_j).
\]

**Theorem:** if the binary relation \( \succeq \) is complete, reflexive, transitive, strictly monotone and continuous, there exist a continuous real-valued function \( u \) that represents \( \succeq \).
Properties of Utility Functions

- $u$ simply represents the indifference curves. Indifference curves are level sets $\{(c, l) : u(c, l) = \bar{u}\}$.
- We’ll always assume $u$ is continuous and differentiable.
- $u(c, l)$ is a function of two variables. To consider properties of $u$ and so optimal choices, we’ll need to consider how it varies separately with $c$ and $l$.
- To do so, we’ll use partial derivatives of the utility function. We’ll write these in one of two ways:

$$u_c(c, l) = \frac{\partial u}{\partial c}(c, l), \quad u_l(c, l) = \frac{\partial u}{\partial l}(c, l)$$
Examples:

\[ u(c, l) = \log c + \log l \]
\[ u(c, l) = c^a l^b, \quad a, b > 0 \]
\[ u(c, l) = (c^\rho + l^\rho)^{\frac{1}{\rho}} \]

Find \( u_c \) and \( u_l \).

The properties of preferences imply properties of \( u \):

1. Monotonicity \( \Rightarrow \) \( u_c \geq 0, u_l \geq 0 \).
2. Convexity \( \Rightarrow \) \( u_{cc} \leq 0, u_{ll} \leq 0 \)
Utility Function and Indifference Curves

Utility Function

Indifference Curves
Marginal Rate of Substitution

- Indifference curve \( c(l) : u(c(l), l) = \bar{u} \).
- The slope of an indifference curve \( c(l) \) is given by minus the ratio of marginal utilities:

\[
\begin{align*}
 u(c(l), l) &= \bar{u} \\
 u_c(c, l)c'(l) + u_l(c, l) &= 0 \\
 c'(l) &= -\frac{u_l}{u_c}
\end{align*}
\]

- Minus this slope is called the marginal rate of substitution.

\[ MRS = \frac{u_l}{u_c}. \]

- Monotonicity \( \Rightarrow MRS \geq 0 \).
- Convexity \( \Rightarrow MRS \) decreasing in \( l \).