Problem Set 6

4. This problem considers monetary policy in a simplified version of the New Keynesian model. Let $\Pi_t$ be inflation, $E_{\Pi_t+1}$ expected inflation, $x_t$ the output gap, and $R_t$ the nominal interest rate. We simplify the Euler equation (IS curve) so that the output gap depends only on the real interest rate:

$$x_t = -\phi(R_t - E_{\Pi_t+1})$$

The second equation is the New Keynesian Phillips curve relating inflation and real activity:

$$\Pi_t = \lambda x_t + \beta E_{\Pi_t+1} + u_t,$$

where

$$u_t = \rho u_{t-1} + \epsilon_t,$$

with $\epsilon_t$ i.i.d. In particular, this means $E_{\Pi_{t+1}} = \rho u_t$. Suppose that monetary policy is set according to an interest rate rule of the following form:

$$R_t = f(\Pi_t)$$

where $f$ is a constant.

(a) Solve for the (rational expectations) equilibrium values of inflation and the output gap for an arbitrary interest rate rule $f$. To do so, first use the equations above to find an equation relating $\Pi_t$, $E_{\Pi_t+1}$, and $u_t$. Then guess that $\Pi_t = ku_t$ for some $k$, which in turn means $E_{\Pi_t+1} = k\rho u_t$. Substitute this into your equation and solve for $k$, then find $\Pi_t$ and $x_t$.

Solution: First, we substitute the equation for $x_t$ and the policy rule for $R_t$ into the Phillips curve, then simplify:

$$\Pi_t = \lambda x_t + \beta E_{\Pi_t+1} + u_t$$

$$= -\lambda \phi (f(\Pi_t) - E_{\Pi_t+1}) + \beta E_{\Pi_t+1} + u_t$$

$$(1 + \lambda \phi f)\Pi_t = (\lambda \phi + \beta)E_{\Pi_t+1} + u_t$$

Then we use the guess and the form of expectations:

$$k(1 + \lambda \phi f)u_t = (\lambda \phi + \beta)k\rho u_t + u_t$$

Clearly the $u_t$ terms cancel, which confirms that our guess is of the right form. We can then solve for $k$:

$$k(1 + \lambda \phi f - \rho(\lambda \phi + \beta)) = 1$$

$$k = \frac{1}{1 + \lambda \phi(f - \rho) - \beta \rho}$$

1
Then we substitute back to solve for $x_t$:

$$x_t = -\phi(f_{t-1} - E_{t+1}u_t)$$
$$= -\phi(fk - \rho k)u_t$$
$$= -\phi k(f - \rho)u_t$$

(b) Suppose that $f = \rho$. What will be the equilibrium levels of inflation and the output gap? Suppose that $\beta$ and $\rho$ are numbers close to 1. What does this policy rule imply for the variability of inflation?

**Solution:** If $f = \rho$ then we see from above that $x_t = 0$. In addition we have:

$$\Pi_t = \frac{1}{1 - \rho \beta} u_t.$$

If $\rho$ and $\beta$ are very close to 1 then the denominator of $k$ is very close to zero, and so $k$ is very large. Therefore inflation will respond very strongly to shocks, and thus be highly volatile. The policy works to close the output gap, but at the cost of having very volatile inflation.

(c) Now suppose instead that $f$ is set to a very large positive constant (i.e. let $f \to \infty$). What does this imply for the equilibrium levels and variability of inflation and the output gap?

**Solution:** Now if $f$ is very large, the denominator of $k$ goes to infinity, and thus $k$ goes to zero. Therefore inflation will be almost completely stabilized at zero. In other words, inflation will respond very little to shocks. For the output gap, we must look at $\phi k(f - \beta)$ The first term will converge to a constant ($k \to 0$ but $f \to \infty$), which we can work out to be $-\lambda$. The second term will converge to zero with $k$. Thus the output gap will still be variable, although not overly so, while inflation will be stabilized.

$$\lim_{f \to \infty} -\phi k f = \lim_{f \to \infty} \frac{-\phi f}{1 + \lambda \phi (f - \beta) - \beta \rho}$$

$$= \lim_{f \to \infty} \frac{-\phi}{\lambda \phi} = -\frac{1}{\lambda}$$

L'Hôpital's rule.