Consumption and Savings

Infinite Horizon Model

\[
\max_{\{c_t, a_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \\
\text{s.t. } c_t + a_{t+1} = x_t + (1 + r_t)a_t, \forall t
\]

Flow budget constraint: \(c_t\) consumption at date \(t\), \(a_t\) assets on hand at start of \(t\), 
\(a_{t+1}\) assets chosen at \(t\), carried over to \(t+1\), \(r_t\) interest rate between \(t-1\) and \(t\), \(x_t\) income.

At the beginning of period \(t\) the stock of financial assets (and firm capital, which is not a variable chosen by households) is given. Thus households must choose \(\{c_t, a_{t+1}\}\) in period \(t\), \(\{c_{t+1}, a_{t+2}\}\) in period \(t+1\), and so on. This is equivalent to choosing the complete path of consumption, i.e., current and all future consumption, \(\{c_t, c_{t+1}, \ldots\}\)

- Method 1: choose current and all future consumption, \(\{c_t, c_{t+1}, \ldots\}\)

Under the NPG restriction, we can write the present value budget constraint (PVBC):

Lagrangian:

First order condition for consumption at any dates \(t, t+1\):
The consumption Euler equation:

- Method 2: choose \( \{c_t, a_{t+1}\}, \forall t \).

Lagrangian:

First order condition for \( \{c_t, a_{t+1}\}, \forall t \):

The consumption Euler equation:
Exercise 1 (Review)

The representative household is assumed to choose \( \{c_t, c_{t+1}, \ldots\} \) to solve

\[
\max_{\{c_t, c_{t+1}, \ldots\}} \quad \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

\[
s.t. \quad c_t + a_{t+1} = x_t + (1 + r_t)a_t, \forall t
\]

where \( 0 < \beta = \frac{1}{1+\theta} < 1 \), \( c_t \) is consumption, \( x_t \) is exogenous income, \( a_t \) is the (net) stock of financial assets at the beginning of period \( t \) and \( r_t = r \) is the (constant) real interest rate.

(a) Assuming that \( r = \theta \) and using the first-order Taylor series expansion

\[
U'(c_{t+1})/U'(c_t) \simeq 1 + \frac{U''}{U'} \Delta c_{t+1} = 1 - \sigma \frac{\Delta c_{t+1}}{c_t}
\]

where \( \sigma = -c U''/U' \) is the coefficient of relative risk aversion (CRRA). Show that optimal consumption is constant.

(b) Does this mean that changes in income will have no effect on consumption? Explain.
Exercise 2

(a) Derive the dynamic path of optimal household consumption when the utility function reflects exogenous habit persistence $h_t$ and the utility function is $u(c_t) = \frac{(c_t-h_t)^{1-\sigma}}{1-\sigma}$, and household budget constraint is $c_t + a_{t+1} = x_t + (1+r)a_t$.

(b) Hence, obtain the consumption function making the assumption that $\beta(1+r) = 1$.

Comment on the case where expected future levels of habit persistence are the same as those in the current period, i.e. $h_{t+s} = h_t$ for $s \geq 0$. 