Market Competitiveness
and Interdependent Preferences*

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Abstract

Does encouraging trader participation enhance market competitiveness? When trader preferences are interdependent, markets do not necessarily become more competitive as the number of traders grows and traders need not become price takers in large markets. In the linear-normal model, this paper provides the necessary and sufficient conditions for a large class of preference interdependencies under which market competitiveness is monotone in market size. Conditions are given when the rational expectations equilibrium, which is typically not fully revealing within the considered class of preference interdependencies, obtains in large markets.

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1 Introduction

Two central lessons from industrial organization and auction theory are that markets with a larger number of traders are more competitive: (1) greater participation reduces the impact of each individual trader on the market as a whole; and (2) competitive equilibrium obtains

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in large markets. An implication of these results for market design is the recommendation to encourage market participation, which is viewed as an enhancement to competition, liquidity and trade efficiency. The two predictions hold robustly for markets with complete information, or independent private values (e.g., Rustichini, Satterthwaite and Williams (1994)). This paper investigates how interdependence of trader values affects market power in small and large markets. Relative to the existing literature on strategic trading with common values, we examine a richer class of interdependencies in preferences that are present in relevant economic settings in which the two predictions need not hold in general.

For markets with interdependent values, (non)competitiveness has been studied in a number of influential papers. In strategic settings, Wilson (1977), Milgrom (1981), Pesendorfer and Swinkels (1997, 2000), Reny and Perry (2006), and Vives (2009) established the convergence of Nash equilibrium to the competitive rational expectations equilibrium. The question of monotonicity of price impact in small markets with interdependent preferences has received less attention, with the exception of Vives (2009) in the context of markets with one-sided market power. Research on small and large markets with interdependent values has primarily focused on markets in which there is an underlying fundamental value that defines, for all agents, the values derived from the exchanged good—a common assumption in asset-pricing and macroeconomic models. In particular, the preferences of all market participants comove only through aggregate shocks to the fundamental value, such as monetary shocks, shocks to an aggregate endowment, productivity or other asset fundamentals. In these markets, the effect of an additional trader on market competitiveness is unambiguous: the interdependence in trader values through aggregate shocks does not alter conclusions (1) and (2) about price impact.

In many markets, the preferences of some agents comove more closely than others. For instance, consumer endowments and producer technology tend to comove differently within and across the two groups; liquidity needs of institutional investors covary based on the types of portfolios held; a growing body of empirical research shows that trading strategies vary with geographical proximity or social affiliation—cultural or linguistic. Heterogeneity in preference comovement is ubiquitous in markets. Indeed, “local” comovements may provide the sole source of interdependence among values derived by market participants from the traded good; for instance, when diversity in preference comovement originates from spatial dependence. Crucially, when trader values are correlated heterogeneously, unlike markets characterized by aggregate comovements in values alone, a new trader may affect the informativeness of market prices and strengthen the market power of all traders. Moreover,

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under extreme conditions, this mechanism may give rise to price-making, even in (infinitely) large markets.

MODEL. To investigate the relationship between non-competitiveness and interdependencies in values, we model the market of a perfectly divisible good\textsuperscript{2} as a uniform-price double (Walrasian) auction with an arbitrary, possibly small, number of traders. To achieve tractability, the model is cast in a linear-normal setting; we analyze the unique symmetric linear Bayesian Nash equilibrium. Thus, in contrast to the large-market rational-expectations literature on information aggregation, we deal with a more modest class of quadratic utilities, but are able to analyze a relatively rich class of interdependencies in trader values for markets of any size. Considering interdependencies among values beyond those resulting from fundamental shocks allows accommodating a variety of environments, including: markets with value dependence within, but not across groups of agents; markets in which correlation among values varies with distance between agents; or networks with size externalities on interdependence among values. Moreover, unlike models with aggregate shocks, negative dependence of values is permitted here. A notable feature of the model is that all traders—buyers and sellers—are Bayesian and strategic in that they (endogenously) have price impact and take it into account in their trading decisions. In particular, there are no noise traders, uninformed or (by assumption) price-taking traders.\textsuperscript{3} That all market participants are fully strategic is a feature of the model that is shared by the double-auction models of Dubey, Geanakoplos and Shubik (1987) and Reny and Perry (2006).\textsuperscript{4}

\textsuperscript{2}Examples of perfectly divisible goods include: assets, electricity, gold, emission permits, etc.

\textsuperscript{3}This (among other features) distinguishes the present paper’s model from the classical small-market model by Kyle (1989) and a recent important model by Vives (2009). As Kyle (1989) demonstrated, with pure common values, price is too informative for equilibrium to exist; to re-establish existence, Kyle lowered the informativeness of price by introducing noise traders, who are neither Bayesian nor do they account for price impact. Vives (2009) considered a market with one-sided market power, with strategic Bayesian sellers and price-taking buyers.

\textsuperscript{4}Dubey, Geanakoplos and Shubik (1987) model a market as a dynamic strategic market game, in which every period traders choose nominal spending that is not contingent on price. In a static strategic market game, information revealed in price cannot be incorporated in decisions; with multiple trading rounds, traders are able to use information contained in prices from prior trading rounds. A Walrasian (double) auction accounts for feedback between inference and market depth, even though the game is static. By allowing choices that are contingent on prices, downward-sloping demands enable traders to take advantage of the information contained in prices even though they choose strategies before (without) knowing the equilibrium price. This feature of the Walrasian auction contrasts also with the Cournot competition in quantities (e.g., Vives (1988)), where traders can learn from prices, but cannot incorporate the information conveyed by prices into their bids.

Reny and Perry (2006) provide a fully strategic foundation for a fully revealing rational expectations equilibrium in a model of a large ($I \to \infty$) double auction with unit demands, more general utility functions than quadratic and dependence in values that stems from a fundamental value of the good. Instead, the model presented in this paper adopts a linear-normal setting and multi-unit demands (divisible goods), while permitting an arbitrary number of bidders and more general preference interdependencies.
Results. Analysis reveals that when trader preferences are interdependent, enlarging the market need not advance competitiveness. Consequently, policy implications and empirical predictions in markets with heterogeneously correlated values may differ markedly from those for markets with a fundamental value of the good. The first of the paper’s two main results establishes the necessary and sufficient condition under which non-competitiveness decreases with the introduction of a new trader. Specifically, the price impact of each trader decreases provided that a new market participant does not increase too much the commonality in values of all market participants, measured by the average correlation. We examine the monotonicity of market power for various canonical types of interdependencies in trader values.

The second result demonstrates that, under mild assumptions, as the number of bidders increases, the linear Bayesian Nash equilibrium converges to the unique competitive rational expectations equilibrium, in which bidders become price takers. Thus, for the linear-normal model, the present paper provides a strategic foundation for the competitive rational expectations equilibrium in markets with perfectly divisible goods and heterogeneous interdependencies in preferences. Since price typically does not fully aggregate information in the considered class of auctions, even though inefficiency due to market power disappears in large (limit) markets, private information inefficiency does not. Our result contributes to the literature: (1) by providing a (fully strategic) foundation for not fully privately revealing equilibria; and (2) by separating full revelation from price-taking behavior—for the class of interdependencies considered, price taking is predicted robustly in large markets for all but perfectly correlated values (pure common values for almost all traders), whereas price is fully revealing only in the absence of heterogeneity in the comovement of values of all market participants, for both small and large markets. We provide an example of a market that violates our convergence condition and features price-making in the limit.

Structure of the Paper. Section 2 lays out the model of a double auction. Section 3 studies the monotonicity of price impact in market size and convergence to the competitive rational expectations equilibrium. Section 4 discusses additional implications of our results. Proofs of all results are contained in the Appendices.

5 “Privately” means that the private information of a trader (his signal) and price provide a sufficient statistic for the joint information in the market (profile of all signals). In our model, in markets with (only) aggregate and idiosyncratic preference shocks, equilibrium is fully privately revealing.
2 A Model of a Double Equicommonal Auction

We model the market as a double auction in the familiar linear-normal setting. $I \geq 2$ agents trade a divisible good. Trader $i$ has a quasilinear and quadratic utility function

$$U_i(q_i, m_i) = \theta_i q_i - \frac{\lambda}{2} q_i^2 + m_i,$$

where $q_i$ is the obtained quantity of the good auctioned, $m_i$ is money, and $\lambda > 0$. Each bidder is uncertain about the utility he derives from the good and observes only a noisy signal about his own value $\theta_i$, $s_i = \theta_i + \varepsilon_i$. Asymmetric information is captured by random intercepts of marginal utility functions $\{\theta_i\}_{i \in I}$, referred to as values, and interpreted to arise from shocks to preferences, endowment or any other shock that shifts the marginal utility of a trader. The affine information structure allows maintaining the linearity of the model: Random vector $\{\theta_i, \varepsilon_i\}_{i \in I}$ is jointly normally distributed, noise $\varepsilon_i$ is mean-zero i.i.d. with variance $\sigma_\varepsilon^2$, and the expectation $E(\theta_i)$ and the variance $\sigma^2_\theta$ of $\theta_i$ are the same for all $i$. The variance ratio $\sigma^2 = \sigma^2_\varepsilon/\sigma^2_\theta$ measures the relative importance of noise in the signal. We define an index of market size as a monotone function of the number of traders,

$$\gamma = \frac{I - 2}{I - 1}. \quad (2)$$

$\gamma$ ranges between zero for $I = 2$ and one as $I \to \infty$; $\gamma \in \Gamma$, where

$$\Gamma \equiv \{\gamma \in [0, 1]| \gamma = (I - 2) / (I - 1) \text{ for } I = 2, 3, \ldots \}.$$

(3)

Throughout, we refer to auctions with $\gamma < 1$ as small and reserve the term large for limits as $\gamma \to 1$.

Preference Interdependencies. As long as the true values $\theta_i$ are not independent across bidders, the signals of $i$’s trading partners $\{s_j\}_{j \neq i}$ contain useful information about $\theta_i$. Much of the literature on trading with private information, in strategic and rational-expectations models alike, has examined the following specification of preference interdependence

$$\theta_i = \theta + \tilde{\theta}_i, \quad (4)$$

where $\theta$ is a fundamental shock and $\tilde{\theta}_i$ is an idiosyncratic shock that is independent across traders, and $\theta$ and $\tilde{\theta}_i$ are independent as well. Important variants of specification (4) include the pure common value model of a double auction, $\tilde{\theta}_i = 0$ for all $i$, and the independent (private) value (IPV) model, $\theta = 0$. More generally, (4) captures markets in which an underlying fundamental value of the good ($\theta$) affects, but does not necessary determine, the values
derived from the good by all market participants. The specification of preferences based on a fundamental value is commonly adopted in asset-pricing or macroeconomic literature to study the impact of aggregate shocks to fundamentals. In terms of the correlation matrix of the joint distribution of values \( \{\theta_i\}_{i \in I} \) (i.e., the variance-covariance matrix of \( \{\theta_i\}_{i \in I} \) normalized by variance \( \sigma^2_\theta \)),

\[
C \equiv \begin{pmatrix}
1 & \rho_{1,2} & \cdots & \rho_{1,I} \\
\rho_{2,1} & 1 & \cdots & \rho_{2,I} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{I,1} & \rho_{I,2} & \cdots & 1
\end{pmatrix},
\]  

specification (4) implies that pairwise correlation \( \rho_{i,j} = \tilde{\rho} \in [0,1] \) for all \( i \neq j \), where \( \tilde{\rho} = \frac{\text{Var}(\theta)}{\text{Var}(\theta) + \text{Var}(\tilde{\theta}_i)} \) (e.g., Kyle (1989); Vives (2009))\(^6\). Thus, while trader preferences may only be imperfectly aligned, comovements in values are identical for all pairs of traders in the market. We call the class of markets that satisfy this condition *Uniform Correlations Models*.

To analyze markets in which values comove more strongly for some traders than others, this paper allows \( \rho_{i,j} \) to differ across pairs of traders, possibly for all pairs. For the equilibrium price to be equally informative across agents, which is necessary for the symmetry of equilibrium and, hence, tractability of the model, we require that each trader \( i \)'s value \( \theta_i \) be on average correlated with other traders’ values \( \theta_j, i \neq j \), in the same way,

\[
\frac{1}{I-1} \sum_{j \neq i} \rho_{i,j} = \tilde{\rho},
\]  

for some \( \tilde{\rho} \in [-1,1] \); that is, in each row (and column) in \( C \), the average of the off-diagonal elements is the same. \( \tilde{\rho} \) measures how a trader’s value comoves on average with the values of all other traders in the market. Given the assumption that the average correlation be the same across traders, statistic \( \tilde{\rho} \) can be interpreted as a measure of *commonality* in values of the traded good to market participants. We call the family of auctions that satisfy condition (6) *equicommonal*. While restrictive, by accommodating interdependencies in values through local shocks, the class of equicommonal auctions encompasses a variety of economic environments beyond those with identical preference comovements for all agents, such as (4). In particular, relative to specification (4), an equicommonal auction permits correlations among \( {\tilde{\theta}_j}_{j \neq i} \) as long as the average correlation of \( \tilde{\theta}_i \) with \( {\tilde{\theta}_j}_{j \neq i} \) is the same for all agents \( i \), which introduces various forms of local—apart from aggregate and idiosyncratic—comovements in

\(^6\)Like ours, these small-market models are also set in a linear-normal environment. Kyle (1989) considered \( \tilde{\rho} = 1 \), assuming the presence of noise traders as well as strategic traders. Vives (2009) allowed \( \tilde{\rho} < 1 \).
values. Throughout the analysis, the results are illustrated in a family of equicommonal auctions with groups of traders where the comovement of values among group members is stronger than across groups. For instance, our example captures markets in which (utility) parameters of consumers and (cost) parameters of producers comove more strongly within than across the two groups; similarly, income, endowment or liquidity needs are typically more correlated among traders from the same city (or country, or social network), rather than across cities.

Example 1 (Twin Cities Model) There are two groups of traders of equal size, A and B, referred to as cities, and interpreted as neighborhoods, clubs, social affiliations, etc.; the total number of traders adds up to an even number $I$. The values that members of a given city derive from the good are perfectly correlated ($\rho_{i,j} = 1$); cross-group correlation may depend on market size $\gamma$ and can be positive or negative, or values can be independent; $\rho_{i,j} = \alpha_1 + \alpha_2(\gamma^{\alpha_3} - 1)$, where $\alpha_1, \alpha_2$ and $\alpha_3$ are such that $\rho_{i,j} \in [-1, 1]$ for all $\gamma$. $\alpha_1$ measures the size-independent cross-city correlation, $\alpha_2$ and $\alpha_3$ measure the strength and convexity of the correlation’s dependence on market size. The behavior of cross-city correlation $\rho_{i,j}$ for different parameters is depicted in Figure 1.B.

The dependence of cross-city correlation on market size ($\alpha_2, \alpha_3 \neq 0$) allows capturing the size externality that results from the increased or decreased number and strength of linkages between the cities as populations grow. The Twin Cities model allows negative (average) interdependence of values. With negative correlations (e.g., without externalities, $\alpha_1 < -\frac{1}{3}, \alpha_2 = 0$), the model can accommodate interactions in which groups of traders compete for a pool of resources (for instance, government transfers) outside the market, and the division of the pool is uncertain during the trade stage. In the extreme case of $\alpha_1 = -1$ and $\alpha_2 = 0$, the pool is fixed. Section 4 applies our results to other economically relevant examples of equicommonal markets. In a companion paper, Rostek and Weretka (2010, hereafter RW) study price inference and information aggregation in the small-markets model presented in this paper.

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7The model with utility maximizers extends in a straightforward way to double auctions with profit-maximizing producers and utility-maximizing consumers.

8Consider the following example. Let trades be characterized by a quasilinear utility function

$$U_i(q_i, m_i) = (q_i + t_i) - \frac{1}{2} \lambda (q_i + t_i)^2 + m_i,$$

where $q_i$ is the quantity of a good obtained from trade in the market and $t_i$ is the uncertain-at-the-time-of-trade transfer of a commodity determined by the government. This model gives rise to preferences as in (1), up to a constant, where $\theta_i = 1 - \lambda t_i$. A model with a balanced government budget (a fixed pool of resources), $t_A = -t_B$, corresponds to $\alpha_1 = -1$ and $\alpha_2 = 0$. An imperfect negative correlation of transfers gives rise to $\alpha \in (-1, 0)$. In a model where $t_A$ and $t_B$ are determined independently, $\alpha = 0$. 

A Sequence of Auctions. Since market-size effects are of primary interest, instead of taking an auction with a fixed number of traders as the object of analysis, we analyze sequences of auctions indexed by the number of market participants \( \{A^I\}_{I=1}^\infty \). In all auctions in the sequence, the utility function remains unchanged. Along with the number of traders \( I \), the correlation matrix \( C \) may change in the sequence in an arbitrary way provided that, for a given market size, the average correlation is the same across traders; the commonality \( \bar{\rho} \) itself may vary with market size, as may other details of the correlation matrix. A sequence of auctions can be conveniently summarized by the commonality function \( \tilde{\rho} : \Gamma \rightarrow [-1, 1] \), which specifies commonality for any market size and may be a non-trivial function of \( \gamma \).

In the model with values described by (4) or, more generally, in the Uniform Correlations model, a new trader is neutral for the commonality of values in that the correlation of an entrant with each incumbent is equal to \( \tilde{\rho} \); the commonality function is given by \( \tilde{\rho}^{UC}(\gamma) = \tilde{\rho} \).

In the Twin Cities model, additional traders increase populations in both neighborhoods, and the commonality function is given by

\[
\tilde{\rho}^{TC}(\gamma) = \frac{1}{2} [\gamma + (2 - \gamma) (\alpha_1 - \alpha_2 + \alpha_2 \gamma^{\alpha_3})],
\]

and depicted for various parameters in Figure 2.

Double Auction. We study double auctions based on the classical uniform-price (Walrasian) mechanism. Traders submit strictly downward-sloping (net) demand schedules, \( \{q_i(p)\}_{i \in I} \); the part of a bid with negative quantities is interpreted as a supply schedule. The market-clearing price is the price \( p^* \) at which the aggregate demand \( Q(p) = \sum_{i=1}^I q_i(p) \) equals zero, \( Q(p^*) = 0 \). Trader \( i \) obtains the quantity determined by his submitted bid evaluated at the equilibrium price, \( q_i^* = q_i(p^*) \), for which he pays \( q_i^* \cdot p^* \), and his payoff is given by the utility function (1) evaluated at \( (q_i^*, p^*) \). As a solution concept, we use the unique symmetric linear\(^9\) Bayesian Nash equilibrium.

One can show that, in equicommonal double auctions, statistic \( \tilde{\rho} \) is sufficient for \( C \) in the linear Bayesian Nash equilibrium, i.e., \( ceteris paribus \), any two equicommonal auctions with the same commonality and otherwise arbitrary correlation matrices have the same Bayesian Nash equilibrium (Lemma 1, RW). It follows that equilibrium bids \( \{q_i(p)\}_{i \in I} \) are the same in the class of all auctions characterized by a given profile \( (\gamma, \tilde{\rho}) \in \Gamma \times [-1, 1] \).\(^10\) As is

\(^9\)The assumption that bids be strictly downward-sloping rules out trivial (no-trade) equilibria. “Symmetric linear” in our setting means that bids have the functional form of

\[
q_i(p) = \alpha_0 + \alpha_s s_i + \alpha_p p,
\]

and that coefficients \( \alpha_0 \), \( \alpha_s \), and \( \alpha_p \) are the same across traders.

\(^10\)For the questions studied in this paper, let us note that auctions in the class characterized by a given
well known, when trader preferences comove too strongly, price may be too informative for equilibrium to exist. In the model presented in this paper, for any level of market size $\gamma$, one can find an upper bound $\tilde{p}^+ (\gamma, \sigma^2)$ and a lower bound $\tilde{p}^- (\gamma)$ on commonality with the property that equilibrium exists if, and only if, commonality of an auction is strictly between the two bounds. These two bounds are given in Lemma 2 in the Appendix and are depicted in Figure 1.A.

3 Market Power with Interdependent Values

We begin by looking into the determinants of market power in equicommonal double auctions (Section 3.1). We then give conditions under which non-competitiveness, measured by price impact, is monotone in the number of traders (Section 3.2). Next, we study competitiveness in large markets (Section 3.3). Finally, we apply our results to equicommonal models other than the Twin Cities model (Section 3.4).

3.1 Origins of Market Power

In this paper, the market power of trader $i$ is measured by his price impact—a price change resulting from a unilateral deviation of trader $i$ from his equilibrium bid at the margin. In a symmetric linear Bayesian Nash equilibrium, the price impact of trader $i$ corresponds to the slope of his residual supply and is deterministic,

$$\mu = (1 - \gamma) \frac{\partial q_i(p)}{\partial p}, \quad (10)$$

where $\partial q_i(p)/\partial p$ is the slope of bidder $i$’s submitted demand schedule.$^{11,12}$

Our model allows delineation of three sources of price impact in markets in which all traders, buyers and sellers, are strategic and Bayesian. In equilibrium, trader $i$ accounts for his price impact $\mu$ by equalizing his expected marginal utility with his marginal expenditure, profile $(\gamma, \hat{p})$ exhibit the same equilibrium price impact. Nevertheless, $\hat{p}$ is not a sufficient statistic for other properties of equilibrium, for instance, informational efficiency (see RW).

$^{11}$The residual supply of trader $i$ obtains as a horizontal sum of the other traders’ bids schedules $\{q_j(p)\}_{j \neq i}$, the slopes of which coincide with $\partial q_i(p)/\partial p$ for all $j \neq i$ in a symmetric equilibrium. The slope of the horizontal sum of $I - 1 = \frac{1}{1 - \gamma}$ bids is given by (10).

$^{12}$Alternatively, one could measure market power as the equilibrium elasticity of the residual supply faced by each trader. The advantage that slope has over elasticity in our model is that, in a linear equilibrium, the slope is deterministic and the same across bidders, whereas elasticity is bidder-specific and, for each trader $i$, depends on the realization of the valuations of his trading partners $\{\theta_j\}_{j \neq i}$. 


for every price. Thus, his demand schedule \( q_i(p) \) derives from the first-order condition

\[
E(\theta_i|s_i, p) - \lambda q_i = p + \mu q_i.
\] (11)

Central to the non-competitiveness of the market is that traders are Bayesian. Indeed, it is through inference from prices that the interdependence of values affects market non-competitiveness. Knowing the map from equilibrium prices to states of the world, traders take advantage of the information contained in price about their own values by conditioning their bids on prices—and, hence, states of the world—by updating their expectations \( E(\theta_i|s_i, p) \).

Given the affine information structure, the conditional expectation of \( \theta_i \) is linear in the bidder’s own signal \( s_i \) and price \( p \),

\[
E(\theta_i|s_i, p) = c_0 E(\theta_i) + c_p p + c_s s_i.
\] (12)

Equilibrium inference coefficient \( c_p \) can be derived in closed form in terms of exogenous parameters (see (23) in Appendix),

\[
c_p = \frac{2 - \gamma}{1 - \gamma + \bar{\rho}} \frac{\rho \sigma^2}{1 - \bar{\rho} + \sigma^2}.
\] (13)

The stochastic process that governs joint distribution of valuations \( \{\theta_i\}_{i \in I} \) affects price impact whenever commonality satisfies \( \bar{\rho} \neq 0 \), in which case bidders learn from prices. The equilibrium price \( p^* \) is positively correlated with each trader’s value \( \theta_i \), even in markets with negative commonality \( \bar{\rho} < 0 \). Note, however, that the coefficient of price inference \( c_p \) in (12) can be negative. This occurs if (and only if) the comovement in trader values \( \bar{\rho} \) is negative.

Why? Heuristically, a positive correlation of price \( p^* \) with value \( \theta_i \) indicates a high realization of \( \theta_i \) and induces traders to revise their conditional expectations of \( \theta_i \) upward for high price realizations. On the other hand, given the realization of signal \( s_i = \theta_i + \varepsilon_i \) observed by trader \( i \), a positive correlation of price \( p^* \) with noise \( \varepsilon_i \) triggers a downward revision of expectation about \( \theta_i \). It can be shown that which countervailing effect is dominant determines the sign of \( c_p \), and depends precisely on the sign of \( \bar{\rho} \).

In our model, Bayesian updating and optimization by all market participants jointly determine the equilibrium price impact of each trader.

**Lemma 1 (Determinants of Price Impact)** In the linear Bayesian Nash equilibrium, trader price impact can be decomposed as follows,

\[
\mu = \frac{\gamma}{\gamma - c_p} \times \frac{1}{1 - \gamma} \times \frac{1 - \gamma}{\lambda}.
\] (14)
In a finite market \((\gamma < 1)\) with decreasing marginal utility \((\lambda > 0)\), each trader has a positive equilibrium price impact. In response to market power, buyers (sellers) reduce their demands (supplies) below (above) marginal utility.

Decomposition of equilibrium price impact into three effects (14) disentangles the determinants of non-competitiveness that derive exclusively from interdependencies among bidder values from those that are present even if values are independent private. Regardless of whether bidder values are correlated, the decreasing marginal utility of \(i\)’s trading partners induces price impact. To see this, let us shut down interdependencies in values and consider a model with \(\rho_{i,j} = 0\) for all \(i \neq j\). Suppose that a trader \(i\) marginally deviates from equilibrium by submitting a more aggressive bid. The price impact of trader \(i\) is given by the resulting change of price normalized by the quantity obtained from a deviation. Given that their marginal utilities are decreasing in quantity, \(i\)’s trading partners are willing to sell additional quantity to trader \(i\) only at price concessions. The resultant price change corresponds to the average marginal utility of traders \(j \neq i\) and equals \((1 - \gamma) \lambda\). Such an adjustment of price completely describes the trade-induced price change in markets with a monopsony buying from \(I-1\) price-taking sellers \((monopsony\ effect)\).\(^{13}\) With bilateral (buyer and seller) market power, the change is magnified via mutual reinforcement: by reducing demands in response to their market power, traders increase price impact for one another, which enhances order reduction and amplifies price impact, etc. Relative to markets with one-sided market power, the strategic reinforcement effect in a double auction boosts the monopsony effect by \(1/\gamma\).

When trader values are interdependent \((\bar{\rho} \neq 0)\), the two effects, which jointly quantify trader price impact in an \(IPV\) model, are strengthened or weakened by a change of the posterior beliefs that they elicit. When bidder \(i\) unilaterally deviates from equilibrium, other traders interpret (incorrectly) the changed price as corresponding to a different realization of the average signal \(\bar{s}\). Revision of conditional expectations of \(\{\theta_j\}_{j \neq i}\) and market clearing imply an additional change to price adjustment, which, by the same mechanism, revises expectations, etc. The inference effect in (14) measures the overall impact of the revision.

All in all, a trader’s market power in a double auction originates from the decreasing marginal utility (of his trading partners) and is further shaped by the bilaterally strategic nature of market interactions—accounting for price impact by all traders in a Nash equilibrium—and Bayesian updating. The analysis yields several insights: It is not the interdependence of values \(\{\theta_i\}_{i \in I}\) \(per\ se\), but the non-zero average correlation \(\bar{\rho}\) that affects market non-competitiveness in (14), nor is it correlation between a price and trader values. For instance, in the Twin Cities model with \(\alpha_1 \simeq -1\) and \(\alpha_2 = 0\), price is uncorrelated with each bidder’s

\(^{13}\)In models of financial market microstructure, this effect is sometimes called an “inventory effect.”
value $\theta_i$ and yet bidders learn from prices, which increases competitiveness compared to the IPV setting. It is noteworthy that whether learning about values from prices makes the market more or less competitive depends on whether bidders make inferences about their values (net) from the correlation of price $p^*$ with noise $\varepsilon_i$ or value $\theta_i$. Specifically, inference through prices enhances market competitiveness if, and only if, the correlation of price $p^*$ with noise $\varepsilon_i$ dominates its correlation with value $\theta_i$; then, $c_p < 0$. Thus, with interdependent values, in small markets, market power can be below the IPV level. By assessing market power relative to the price impact that traders would have if the values were independent private, an outside market observer could infer the source of learning and the sign of commonality in values $\bar{\rho}$.

### 3.2 Price Impact in Small Markets

Market power reduces trade volume, amplifies aggregate price volatility and adversely affects welfare. Consequently, market competitiveness is one of the key concerns in market design. Absent private information, encouraging participation by a market designer monotonically increases competitiveness with every new trader, as does it in markets where traders have private information and their values are independent ($\rho_{i,j} = 0$ for all $i \neq j$). In light of decomposition (14), the mechanisms underlying monotonicity are aggregation of a greater number of bids (diminished monopsony effect in (14)) along with a reduction in the reinforcement of bilateral market power (lessened reinforcement effect in (14)) that is brought by a new bidder. Robust in the absence of interdependencies in values, the monotonicity result need not carry over to markets in which trader values comove. By altering informativeness of the equilibrium price, new market participants may increase the market power of all traders.

Proposition 1 provides the necessary and sufficient condition for enlarging the market to enhance competitiveness in an arbitrary equicommonal auction. For any sequence of equicommonal auctions, define $\Delta \rho(\gamma)$ as the change in commonality that results from increasing the number of traders by one. We refer to $\Delta \rho(\gamma)$ as a commonality impact.

**Proposition 1 (Monotonicity of Price Impact)** For any market size $\gamma$ and commonality $\bar{\rho}$, there exists a threshold $\pi_{\gamma,\bar{\rho}} > 0$, such that price impact decreases with an additional trader if, and only if, $\Delta \rho(\gamma) < \pi_{\gamma,\bar{\rho}}$.

The threshold $\pi_{\gamma,\bar{\rho}}$ is characterized in terms of exogenous parameters of an auction in the Appendix. Proposition 1 asserts that whether market becomes more competitive with a new bidder depends precisely on the induced change in the (average) comovement in preferences that his participation brings about in the market. By decomposition (14), if a new trader sufficiently increases commonality, the impact of price inference on market power
outweighs the joint countervailing impact of the aggregation of larger number of bids and the bilaterally strategic nature of the exchange (monopsony and reinforcement effects), and the larger market is less competitive.

Proposition 1 has a transparent geometric interpretation. For any level of price impact \( \mu \in (0, \infty) \), let a \( \mu \)-curve comprise all profiles \( (\gamma, \bar{\rho}) \in \Gamma \times [-1, 1] \) such that auctions of size \( \gamma \) and commonality \( \bar{\rho} \) are characterized by equilibrium price impact \( \mu \) (Figure 1.A). The positive slope of \( \mu \)-curves reflects that an inclusion of a trader while maintaining the same level of commonality in the market lowers price impact, whereas increasing commonality for a given number of traders increases price impact. The map of price-impact curves in Figure 1.A spans price impact ranging from \( 0 \) (\( \mu \)-curves approach the lower bound \( \bar{\rho}^- (\gamma) \) as \( \mu \to 0 \)) to \( 1 \) (\( \mu \)-curves approach the upper bound \( \bar{\rho}^+ (\gamma, \sigma^2) \) as \( \mu \to \infty \)).

The condition from Proposition 1 can now be seen in terms of “crossing from above.” Consider any sequence of equicommonal auctions represented by a commonality function \( \bar{\rho} (\gamma) \). For any auction of size \( \gamma \) in the sequence, the market becomes more competitive with a new bidder if the schedule \( \bar{\rho} (\gamma) \) crosses the corresponding \( \mu \)-curve at point \( (\gamma, \bar{\rho}(\gamma)) \) from above. The (model-independent) bound \( \pi_{\gamma, \bar{\rho}} \) in Proposition 1 corresponds to the change in commonality induced by a new bidder that just suffices to leave price impact intact (see Figure 1.A).\(^{14}\)

Being invariant to changes in \( \lambda \), the map of \( \mu \)-curves allows comparing price impact across models with different convexity parameters \( \lambda \). More precisely, a model with any \( \lambda > 0 \) gives rise to the same \( \mu \)-curve map as with \( \lambda = 1 \), with the value of price impact for each curve normalized by \( \lambda \). It follows that Proposition 1 holds quantitatively as well as qualitatively in models with various convexities \( \lambda \); if price impact is monotone for some \( \lambda \), it is also monotone in a model with any (fixed) \( \lambda' > 0 \).

A general lesson from Proposition 1 is that equilibrium price impact in equicommonal markets may feature essentially arbitrary, possibly non-monotone, behavior as markets grow. While arbitrary, the behavior of market power in a sequence of equicommonal auctions \( \{A_t\}_{t=2,3,...} \) is fully characterized by an evolution of the sufficient statistic given by commonality. For more specific implications, suppose one wished to determine the market size that maximizes market competitiveness. In all equicommonal markets with non-increasing \( \bar{\rho}(\gamma) \), such as the Uniform Correlations model, price impact strictly decreases in market size as in models with independent private values. Therefore, in markets where comovement in trader preferences arises from aggregate (fundamental) and/or idiosyncratic shocks alone, price im-

\(^{14}\)Note that since the domain of the commonality function \( \bar{\rho} (\cdot), \Gamma \subset [0, 1] \), is countable, the threshold \( \pi_{\gamma, \bar{\rho}} \) in the proposition corresponds to the change in (rather than the slope of) commonality along the appropriate \( \mu \)-curve that follows from the change in \( \gamma \) due to an inclusion of one trader, \( \Delta \gamma = 1 / (I - 1) \).
pact is monotone. As Proposition 1 indicates, market competitiveness may be monotone in the number of traders even when preferences become more aligned in a larger market. This is, for example, the case in the Twin Cities model with constant cross-city correlation ($\alpha_2 = 0$). Here, the commonality function increases in market size even without the size externality (Figure 2.A.1), yet commonality impact is not sufficiently large for price impact to increase for any market size (Figure 2.A.2). The monotonicity of price impact holds also when the cross-city correlation decreases in market size; this could capture, for example, negative size externalities such as congestion effects. However, when the size externality on the cross-city correlation is positive and sufficiently strong, for instance, as a result of positive taste synergies in a market, then the Twin Cities model gives rise to a non-monotone price impact for a range of market sizes. Figure 2.B depicts the commonality and price impact for $\alpha_2, \alpha_3 > 0$: the market is least competitive at an intermediate market size and, thus, for larger markets, a market designer could foster competitiveness by discouraging entry.

### 3.3 Price Impact in Large Markets

A model of economic interactions that is central to both finance and economics is based on a competitive equilibrium, in which each trader assumes that he has no price impact. A voluminous body of research has developed to address the question as to whether strategic traders become price takers in large markets and, hence, whether models based on competitive equilibrium accurately describe large-market interactions. Accordingly, we turn to an analysis of competitiveness in large equicommonal markets. In the presence of asymmetric information, the appropriate solution concept in a large market is a (competitive) rational expectations equilibrium, in which traders have correct conditional expectations about values $\theta_i$ given observed prices. A rational expectations equilibrium is a profile of demands $\{q_i^{REE}(p)\}_i$ such that for each $i$,

$$q_i^{REE}(p) \equiv \arg \max_q E\left(U_i(q, p \times q) \mid p, s_i\right).$$

(15)

Analogous to the Bayesian Nash equilibrium, the competitive rational expectations equilibrium entails the knowledge of the map between the equilibrium price and states of the world $p^{REE}(s)$ by each trader, and conditional expectations reflect this knowledge. Contrary to the Nash equilibrium, in the competitive equilibrium, traders do not recognize their price impact and, for all prices, their demands coincide with marginal utility.

In models where only one (buying or selling) side of the market is strategic (e.g., Vives (2009)), order reduction by only one side of the market changes the equilibrium price—and the map from states to prices—relative to the competitive model; in turn, this distorts
bidders’ (models of) expectations. Conveniently, in the double auction studied in this paper, symmetric traders reduce their demands and supplies for every price by the same factor (cf. (24) in the Appendix) and, as a result, the market clears at the competitive price \( p^* (s) = p^{REE} (s) \), regardless of the size of the market, even though the equilibrium trade is non-competitive. Note that price equality does not stem from lack of, but rather the balancing of market power (and of order reduction, for every price) between the buyer and seller sides of the market. In particular, in a Nash equilibrium, price impacts are strictly positive, demand schedules are below the expected marginal utility and the equilibrium allocation is inefficient for all \( \gamma < 1 \).

Proposition 2 gives the conditions under which market power vanishes in large auctions and the demand schedules of strategic Bayesian players converge (pointwise) to bids (15) in equicommonal markets.

**Proposition 2 (Convergence to Competitive REE)** In the symmetric Bayesian Nash equilibrium, each bidder \( i \)'s bid converges pointwise to bid \( a^{REE}_i (p) \), if

\[
\lim_{\gamma \to 1} \sup \rho (\gamma) < 1. \tag{16}
\]

The competitive rational expectations equilibrium obtains under relatively weak conditions: As long as values do not become perfectly aligned in large markets (i.e., pure common values for almost all traders), traders treat prices parametrically and markets become competitive. Conversely, only when commonality converges to one might price impact in large markets prevail. Note that condition (16) does not require that the commonality function converges in a given model.

Our result contributes to the literature that seeks the foundations for the competitive REE in two ways. First, in our model, all bidders are strategic, Bayesian players. There are no noise traders, uninformed or, by assumption, price-taking traders. Perhaps more significantly for applications, the existing results have been obtained for markets with particular types of interdependencies in trader values; namely, those in which there is a fundamental value of the good that determines the values derived from the good for all bidders or, more generally, in which bidder values derive from aggregate and idiosyncratic shocks. Such a shock structure generates a Uniform Correlations matrix \( C \) and, in large markets, rules out comovements in values in groups of traders except for those present in the market as a whole. Proposition 2 provides a foundation for the competitive REE in models with equicommonal correlation structures \( C \). Additionally, in equicommonal markets, price fully privately reveals all the available information only with uniform correlations (Proposition 3 in RW), which are non-generic within the class of equicommonal auctions. Proposition 2 holds in equicom-
monal markets with identical and heterogeneous interdependencies; and therefore, provides a foundation also for a not fully privately revealing rational expectations equilibrium in equicommonal markets.\footnote{Partial revelation of information in the competitive limit of our model does not result from the presence of noise traders (e.g., Hellwig (1980)), or uninformed traders (e.g., Ausubel (1990)), or uncertainty of dimension greater than that of price (e.g., Jordan (1983); Messner and Vives (2001)). In particular, in any equicommonal auction, for any agent, a one-dimensional statistic exists that is sufficient for the payoff-relevant information contained in the signals of other agents. Nevertheless, for all but uniform correlations, the statistic differs from the average signal that the equilibrium price reveals.}

Proposition 2 can also be interpreted geometrically in the map of $\mu$-curves from Figure 1.A. For any level of price impact $\mu > 0$, the $\mu$-curves approach $\bar{p} = 1$ as markets grow large. Therefore, in any market where condition (16) holds, the commonality function eventually crosses from above each $\mu$-curve and price impact becomes negligible in the limit.

The significance of condition (16) can be seen in the Twin Cities model with $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = 2.5$; the corresponding commonality function and price impact are depicted in Figure 2.C. As $\gamma \to 1$, price impact $\mu$ of trader $i$ is bounded away from zero. In fact, properly choosing $\alpha_1$ and the positive externality coefficients $\alpha_2$ and $\alpha_3$ may yield an arbitrary equilibrium price impact in the large, limit market. Figure 2 depicts examples of markets in the Twin Cities family with distinct monotonicity and convergence properties of price impact: markets with monotonically decreasing in the number of traders or non-monotone price impact that is positive in the limit large market (Figure 2.C and 2.D); a market with non-monotone price impact and price-taking behavior with a large number of traders (Figure 2.B); and a market with $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = 2$, where price impact is maximal in the limit large market (Figure 2.D).

In a market with infinitely many traders, how can an individual bidder affect prices? As the market becomes effectively one of pure common values ($\hat{\rho}(\gamma) \to 1$), price becomes the principal source of inference in expectations $E(\theta_i|s_i, p)$; since traders interpret price changes as resulting from changes in the fundamental value, with pure common values, this translates (almost) one-to-one into an adjustment of conditional expectation $E(\theta_i|s_i, p)$. Thus, while the monopsony and strategic effects in (14) are negligible, so is the difference $\gamma - c_p$; the inference effect grows without bound and the joint impact of the three effects is bounded away from zero.

### 3.4 Other Equicommonal Markets

So far, we examined the behavior of market non-competitiveness in the models of Uniform Correlations and Twin Cities, in which the values of two distinct groups of bidders comove more closely within than across groups. The model of equicommonal auctions extends to
markets with asymmetric sizes of the two groups of agents in which the correlation of trader values in the smaller city exceeds the small-city correlation to preserve equicommonality; for instance, a small number of producers with strongly correlated cost parameters trades with a large number of consumers whose preferences comove only weakly. Then, the residual market is not \textit{ex ante} identical for each agent. As in the case of symmetric Twin Cities, although commonality is increasing, the commonality impact is not sufficient to upset the monotonicity property of price impact. In addition, the competitive rational expectations equilibrium is predicted in large markets. We now describe two other economic environments that are common in economic applications and can give rise to equicommonal models.

\textbf{Uniform Correlations Model with Size Dependence.} Consider a model as in (4), where the variances of the aggregate and idiosyncratic components of \( \{\theta_i\}_{i \in I} \) are affected by market size. This specification subsumes an important variant in which the true value \( \theta_i \) is a (weighted) sum of the signals of all traders. The variance of \( \theta_i \) and commonality are then not constant, but rather vary in market size. Depending on the shape of the commonality function in the particular model, price impact may be non-monotone and price-making behavior might be predicted for the large limit market.

\textbf{Circle City Model.} In many markets, correlation in endowments, preferences or productivity varies with geographical or cultural proximity in a systematic manner. This motivates a spatial model in which the correlation of values decays according to distance among the agents. Such interdependence of values can be represented by the spatial Circle City model in which \( I \) traders are located on a circle. The distance between any two immediate neighbors is normalized to one. Let \( d_{i,j} \) be the shorter of the two distances between traders \( i \) and \( j \) (measured along the circle). To capture that the values of closer neighbors comove more, correlation between any two traders \( \rho_{i,j} \) is assumed to decay with distance, \( \rho_{i,j} = \beta^{d_{i,j}} \), where \( \beta \in (0, 1) \) is the decay rate. The model takes the decay rate \( \beta \) as a primitive and assumes that a new trader enlarges the market by increasing the circumference of the circle by one. This allows for analysis of market interactions where trader preferences become less and less alike as the market expands. The commonality function is given by \( \tilde{\rho}^{CC}(\gamma) = 2(1 - \gamma)\beta \left(1 - \beta^{\frac{1}{1-\gamma}}\right) / (1 - \beta) \), assuming that \( I \) is an odd number, and is monotonically decreasing to zero in market size. In this model, price impact is monotone and competitive equilibrium obtains in the large market.\footnote{In this experiment, we assume that a new trader increases the circumference of the circle by one so that a city with \( I \) traders has a circumference equal to \( I \). W.l.o.g. a trader can be added at an arbitrary position on the circle. Alternatively, one could assume that the circumference is fixed and that additional traders increase the density of the population. The latter formulation would imply that preferences comove more closely in pairs of traders in larger markets and, therefore, would not capture the decaying in distance.} Since equilibrium price in the large market is deterministic, in the competitive limit market, price reveals no information...
about the values of \( \{\theta_i\}_{i \in I} \) to traders (even though some of the information is available in the market).

## 4 Discussion

Our double auction model allows us to investigate how the monotonicity of market power is shaped by information aggregation through price inference in a class of models that have heterogeneous independencies in values. Analysis of information aggregation remains implicit in the present paper, as it is studied in detail the companion paper, RW. In light of the results about information aggregation from RW, the results established in the present paper reveal additional insights, discussed now.

(1) A number of studies from the information aggregation literature examined the relative contribution of market power versus private information to the inefficiency of equilibrium allocation. This paper finds that the extent to which information aggregation increases, market power is governed by commonality in trader preferences. In turn, RW show that how much of the total information that is available in the market fails to be transmitted through prices to bidders (and, therefore, the potential to learn via non-market modes of learning) depends instead on the heterogeneity in preference comovements: information is aggregated into prices if, and only if, \( \bar{\rho}_{i,j} = \bar{\rho} \) for all \( i \neq j \). This sheds light on the sources of inefficiency: commonality in values leads to—and, indeed, determines—inefficiency due to market power, whereas heterogeneity in preference interdependencies induces inefficiency due to private information.

(2) Our results qualify the robustness of predictions regarding price-taking behavior in large markets and the ability of prices to aggregate information (in small as well as large markets). In markets with Uniform Correlations, bidders become price takers and markets are informationally efficient (i.e., price is fully privately revealing) in equilibrium of the limit large market. In equicommonal markets, price taking is predicted robustly in large markets and independent of details of information structure other than a uniform bound on the large-market commonality. On the other hand, even in large markets, aggregation of information is not generic and obtains only in markets with Uniform Correlations.

(3) In Uniform Correlations models, such as those with a fundamental value of the exchanged good, additional market participants lower price impact and increase price informativeness. In this sense, there is a comonotone relationship between market competitiveness and learning from prices. Our analysis suggests that this relationship is not inherent in commonality, which we intend to analyze. In this case, price impact would still be monotonic and, because \( \lim_{\gamma \to 1} \bar{\rho}(\gamma) < 1 \), the competitive equilibrium would obtain in the large market.
double auctions and, in particular, does not extend to equicommonal markets. Rather, the relation is shaped by how the comovement in trader values evolves with new market participants, in the following way. For moderate changes in commonality of values, one should expect price informativeness to increase and price impact to decrease with additional bidders. Large enough increases in commonality increase both price impact as well as price informativeness, whereas sufficiently large decreases diminish both price impact and the informational content of prices.

(4) Learning from prices may or may not improve welfare for a given market size: improved estimations of agent values improves individual decisions, but may also increase market power. There is a trade-off between price informativeness and welfare if, and only if, \( \rho > 0 \); in markets where agents learn through noise, more informative prices are unambiguously better.

Tractability motivated certain modeling assumptions in our paper, namely the quadratic functional form for the payoffs as well as the affine information structure. While admittedly specialized, the linear-normal model with the linear Bayesian Nash equilibrium is the workhorse model of market microstructure (e.g., mean-variance preferences) and industrial organization. We believe that our model and results provide a useful benchmark for analysis of non-competitiveness in more general environments with a unique equilibrium. The main limitation of equicommonality is that price is equally informative for all trades. Allowing some agents to learn more from prices than others would introduce asymmetries to equilibrium market power. Exploring the effects of asymmetries on monotonicity of competitiveness remains an important, but also challenging, objective for future research.

References


Appendix

Lemma 2 (Existence of Equilibrium, Rostek and Weretka (2010)) In an equicommonal auction of size $\gamma$ and commonality $\bar{p}$, a symmetric linear Bayesian Nash equilibrium exists, if and only if, $\bar{p}^-(\gamma) < \bar{p} < \bar{p}^+(\gamma, \sigma^2)$, where

$$\bar{p}^+(\gamma, \sigma^2) = \frac{\gamma^2 - 2(1 - \gamma)\sigma^2 + (1 - \gamma)\sqrt{4\sigma^4 + (\gamma\frac{2-\gamma}{1-\gamma})^2}}{2\gamma},$$

(17)
\[ \hat{\rho}^- (\gamma) = - (1 - \gamma). \] (18)

**Proof 1 Lemma 1 (Determinants of Price Impact)** In the symmetric linear equilibrium, bids take the functional form of \( q_i (p) = \alpha_0 + \alpha_s s_i + \alpha_p p \), where constants \( \alpha_0 \), \( \alpha_s \), and \( \alpha_p \) are the same across all traders. Given linear strategies, trader \( i \) faces a residual supply with a deterministic slope \( \mu \) and a stochastic intercept that is a function of other traders’ signals. The best response of trader \( i \) to the residual supply is given by the first-order (necessary and sufficient) condition: for any \( p \),

\[ E (\theta_n | s_i, p) - \lambda q_i = p + \mu q_i. \] (19)

By market clearing and condition (19), the equilibrium price is equal to \( p^* = \frac{1}{\lambda} \sum_{i \in I} E (\theta_i | s_i, p^*) \).

Given an affine information structure, conditional expectations are linear, \( E (\theta_i | s_i, p) = c_\theta E (\theta_i) + c_s s_i + c_p p \), where coefficients \( c_\theta, c_s, c_p \) are identical across traders and, since \( E (\theta_i) = E (s_i) = E (p^*) \), also satisfy \( c_\theta = 1 - c_s - c_p \). It follows that the equilibrium price is given by

\[ p^* = \frac{c_\theta E (\theta_i) + c_s s_i}{1 - c_p} + \frac{c_p}{1 - c_p} \bar{s}, \] (20)

where \( \bar{s} = \frac{1}{I} \sum_{i \in I} s_i \). Using (20), the random vector \((\theta_i, s_i, p^*)\) is jointly normally distributed,

\[
\begin{pmatrix}
\theta_i \\
 s_i \\
 p^*
\end{pmatrix} = N
\begin{pmatrix}
 E (\theta_i) \\
 E (\theta_i) \\
 E (\theta_i)
\end{pmatrix},
\begin{pmatrix}
 \sigma^2_\theta & \sigma^2_{\theta, \theta^*} & \text{cov} (\theta_i, p^*) \\
 \sigma^2_{\theta, \theta^*} & \sigma^2_\theta + \sigma^2_\varepsilon & \text{cov} (s_i, p^*) \\
 \text{cov} (p^*, \theta_i) & \text{cov} (p^*, s_i) & \text{Var} (p^*)
\end{pmatrix}.
\] (21)

(20) allows determination of variances and covariances in (21). Applying the projection theorem, the method of undetermined coefficients yields the inference coefficients \( c_s \) and \( c_p \),

\[
c_s = \frac{1 - \bar{\rho}}{1 - \bar{\rho} + \sigma^2}, \] (22)

\[
c_p = \frac{(2 - \gamma) \bar{\rho}}{1 - \gamma + \bar{\rho} (1 - \bar{\rho} + \sigma^2)}. \] (23)

Using (19), the equilibrium bid is

\[ q_i (p) = \frac{1}{(\lambda + \mu)} [c_\theta E (\theta_i) + c_s s_i + (1 - c_p) p]. \] (24)

The equilibrium slope of the residual supply of trader \( i \), defined as a horizontal sum of the
bids of traders other than \( i \), satisfies

\[
\mu = 1/(I - 1)(\partial q_i(p)/\partial p)^{-1} = (1 - \gamma)(\partial q_i(p)/\partial p)^{-1}.
\] (25)

Using (24) and solving for \( \mu \) gives (14).

**Proof 2**  **Proposition 1** (Monotonicity of Price Impact) Equations (14) and (23) implicitly define \( \bar{p} \), for any value of \( \lambda/\mu \) and \( \gamma \), through a quadratic equation, the roots of which are

\[
\bar{p} = \frac{1}{2} \left[ \gamma + \sigma^2 - \frac{(2 - \gamma) \sigma^2}{\gamma - (1 - \gamma) \lambda/\mu} \right] \pm \frac{1}{2} \left[ \left( \gamma + \sigma^2 - \frac{(2 - \gamma) \sigma^2}{\gamma - (1 - \gamma) \lambda/\mu} \right)^2 + 4 \left[ 1 + \sigma^2 \right][1 - \gamma] \right]^{\frac{1}{2}}.
\] (26)

Each root is discontinuous in \( \gamma \) at the market size \( \gamma^* = (\lambda/\mu)/(1 + \lambda/\mu) \), for which \( c_p = 0 \) and inference from prices does not occur. Next, it is demonstrated that for any given price impact \( \mu \), a unique \( \mu \)-curve exists within the bounds (18) and (17) that is increasing in \( \gamma \) and defined by the negative root for \( \gamma < \gamma^* \) and by the positive root for \( \gamma > \gamma^* \). Fix \( \mu \). This also determines the threshold \( \gamma^* = \lambda/\mu/(1 + \lambda/\mu) \). We first argue that for \( \gamma < \gamma^* \), the positive root exceeds one, and therefore, cannot be part of a \( \mu \)-curve: when \( \gamma < \gamma^* \), the denominator in (26) is negative and, hence, the root is strictly increasing in \( \sigma^2 \). The value of the positive root is thus bounded from below by the value of (26) at \( \sigma^2 = 0 \), which is equal to \( \frac{1}{2} \gamma + \frac{1}{2} \left[ \gamma^2 + 4 \left[ 1 - \gamma \right] \right]^{\frac{1}{2}} \). This bound itself is, in turn, monotonically increasing in \( \gamma \) and, hence, bounded from below by one (at \( \gamma = 0 \)). We now argue that, for any \( \mu \in (0, \infty) \) and \( \gamma < \gamma^* \), the negative root gives a value of \( \bar{p} \) within the bounds (18) and (17). The negative root is non-positive for all \( \gamma < \gamma^* \) and one can write

\[
\frac{1}{2} \bar{p} = x - \sqrt{x^2 + c},
\] (27)

where \( c = 4 \left[ 1 + \sigma^2 \right][1 - \gamma] \) and

\[
x = \gamma + \sigma^2 - \frac{(2 - \gamma) \sigma^2}{\gamma - (1 - \gamma) \lambda/\mu}.
\] (28)

From (27), the negative root is increasing in \( x \). Since \( x \) is decreasing in \( \lambda/\mu \), for any \( \gamma \) the value of \( \bar{p} \) is bounded from below by the negative root evaluated at the limit \( \lambda/\mu \to \infty \). For any fixed \( \gamma \), the limit equals \( -(1 - \gamma) \) and, therefore, coincides with \( \bar{p}^- \). It follows that for any \( \gamma < \gamma^* \), exactly one value of \( \bar{p} \in (\bar{p}^- (\gamma), \bar{p}^+ (\gamma, \sigma^2)) \) exists such that the price impact is equal to \( \mu \) and given by the negative root (26). By (26), a \( \mu \)-curve is continuous and strictly increasing in \( \gamma \). Mimicking the argument for \( \gamma > \gamma^* \), the \( \mu \)-curve and its properties
are uniquely determined by the positive root (26). Finally, that $c_p = 0$ for $\gamma = \gamma^*$ implies $\bar{\rho} = 0$. Note that since for $\gamma < \gamma^*$ and $\gamma > \gamma^*$, the $\mu$-curves converge to 0 as $\gamma \rightarrow \gamma^*$ and, hence, the uniquely defined $\mu$-curve is continuous on the whole interval $[0,1]$. Notice that the $\mu$-curve converges to the upper bound (17) as $\mu \rightarrow \infty$ and to the lower bound (18) as $\mu \rightarrow 0$. Tedious algebra (see the Supplementary Material) reveals that for all profiles of $(\gamma, \bar{\rho})$ for which equilibrium exists except $(\gamma^*, \bar{\rho})$, the following derivatives of price impact can be established: $\partial \mu / \partial \bar{\rho} > 0$ and $\partial \mu / \partial \gamma < 0$. By the implicit function theorem, it follows that the $\mu$-curve is monotonically increasing in $\gamma$. Threshold $\pi_{\gamma, \bar{\rho}}$ equals the increase of commonality that preserves the same price impact after an increase of $\gamma$ that results from increasing the number of traders by one, $\Delta \gamma = 1 / (1 - 1)$. For $\gamma < \gamma^* - \Delta \gamma$, the threshold is given by

$$
\pi_{\gamma, \bar{\rho}} = \frac{1}{2} \left[ \gamma + \Delta \gamma - \frac{(2 - \gamma) \sigma^2}{\gamma - (1 - \gamma) \lambda / \mu} + \frac{(2 - \gamma - \Delta \gamma) \sigma^2}{\gamma + \Delta \gamma - (1 - \gamma - \Delta \gamma) \lambda / \mu} \right] ^{\frac{1}{2}}
- \frac{1}{2} \left[ \left( \gamma + \Delta \gamma + \sigma^2 - \frac{(2 - \gamma - \Delta \gamma) \sigma^2}{\gamma + \Delta \gamma - (1 - \gamma - \Delta \gamma) \lambda / \mu} \right) ^{2} + 4 \left[ 1 + \sigma^2 \right] \left[ 1 - \gamma - \Delta \gamma \right] \right] ^{\frac{1}{2}}
+ \frac{1}{2} \left[ \left( \gamma + \sigma^2 - \frac{2 - \gamma) \sigma^2}{\gamma - (1 - \gamma) \lambda / \mu} \right) ^{2} + 4 \left[ 1 + \sigma^2 \right] \left[ 1 - \gamma \right] \right] ^{\frac{1}{2}}.
$$

For $\gamma > \gamma^*$, the sign of the last two expressions is reverted, while for $\gamma \in (\gamma^* - \Delta \gamma, \gamma^*)$ the two last expressions have positive signs. By the monotonicity of $\mu$-curves, $\pi_{\gamma, \bar{\rho}} > 0$.

**Proof 3** Proposition 2 (Convergence to Competitive REE) If $\lim_{\gamma \rightarrow 1} \sup \bar{\rho} (\gamma) < 1$, then

$$
\lim_{\gamma \rightarrow 1} \sup \frac{c_p}{1 - \lim_{\gamma \rightarrow 1} \sup \bar{\rho} (\gamma) + \sigma^2} < 1,
$$

which when combined with

$$
\lim_{\gamma \rightarrow 1} \sup \mu = \frac{1}{1 - \lim_{\gamma \rightarrow 1} \sup c_p (1 - \gamma) \lambda}
$$

gives $\lim_{\gamma \rightarrow 1} \sup \mu = 0$, and the first of the two elements in (31) is bounded. It follows that the optimal bids (24) pointwise converge to the rational expectation bids (15), given by (24) with $\mu = 0$. 

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**Figure 1.A: Price Impact Curves**

- **Price Too Informative**
- **Price-Making in Large Market**
- "Crossing from Above"

\[ \rho(\gamma) \]
\[ \rho^*(\gamma, \sigma^2) \]

\[ \mu = \infty \]
\[ \mu = 6 \]
\[ \mu = 2 \]
\[ \mu = 1 \]
\[ \mu = 0.5 \]
\[ \mu = 0 \]

**Auctions do not exist**

**Figure 1.B: Cross-City Correlation**

\[ \rho_{i,j} \]

- \( \alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 2 \)
- \( \alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 2.5 \)
- \( \alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0 \)
- \( \alpha_1 = -0.5, \alpha_2 = 0.5, \alpha_3 = 0.5 \)
Figure 2: Market Power in Twin Cities Model

A.1) Commonality: $\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0$

A.2) Price Impact: $\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0$

B.1) Commonality: $\alpha_1 = -0.5, \alpha_2 = 0.5, \alpha_3 = 0.5$

B.2) Price Impact: $\alpha_1 = -0.5, \alpha_2 = 0.5, \alpha_3 = 0.5$

C.1) Commonality: $\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 2.5$

C.2) Price Impact: $\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 2.5$

D.1) Commonality: $\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 2$

D.2) Price Impact: $\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 2$