**Econ 301**
**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+15+20+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

### Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, $x_1$, and clothing, $x_2$.

a) The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$ . Show geometrically Lila's budget set if her income is $m = 30$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^2 (x_2)^1 .$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = 2 \ln x_1 + \ln x_2.$$

Explain the idea behind "monotonic transformation" (one sentence) and derive function $V$ from $U$.

c) Assuming utility function $V(x_1, x_2) = 2 \ln x_1 + \ln x_2$

-Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(8, 8)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(8, 8)$?

-Write down two secrets of happiness that determine optimal choice given parameters $p_1, p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each).

-Using secrets of happiness derive optimal consumption $x_1$,$x_2$, given values of $p_1 = 2, p_2 = 4$ and $m = 30$. Is the solution corner or interior (chose one)

d) (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary; 2) normal; and 3) neither gross complements nor gross substitutes (one sentence for each property).

### Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education, $x_1$ and health care, $x_2$. Your preferences over the two are represented by function

$$U(x_1, x_2) = 3x_1 + x_2.$$

a) Find marginal rate of substitution (give number).

b) Find optimal consumption of $x_1$ and $x_2$, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 50$ (two numbers).

c) Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

### Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 30$) but her future (period two) is not so bright ($m_2 = 20$)

a) Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

b) Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

c) Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 2$. Using magic formulas, find optimal consumption plan $(C_1, C_2)$ (two numbers) and the corresponding level of savings/borrowing, $S$.

d) Is Zoe tilting her consumption over time? (yes-no answer).

**Problem 4 (25p) (Short questions)**

a) Given utility function $U(C, R) = \min(2C, R)$, daily endowment of time $24h$, price and wage $p_c = w = 2$, find optimal choice of $C$, relaxation time $R$ and labor supply $L$. (three numbers, use secrets of happiness for perfect complements).

b) Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 100 \ln x_2$, prices $p_1 = \$8, p_2 = \$1$ and income $m = \$60$. Is your solution corner or interior?

c) Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond) that pays annually $\$1000$ forever, starting next year or cash $\$12,000$ now (compare PV).

d) Your annual income when working (age 21-60) is $\$60,000$ and then you are are going to live for the next 30 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 2\%$.

**Bonus question (Just for fun)**

a) Derive magic formulas for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$ that give optimal choices $x_1, x_2$ as a function of $a, b, p_1, p_2, m$.

b) Provide economic intuition for the magic formula for perfect complements (economic interpretation for the numerator and denominator).

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## Midterm 1 (B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+15+20+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

### Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, $x_1$, and clothing, $x_2$.

a) The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$ . Show geometrically Lila's budget set if her income is $m = 60$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^2 (x_2)^1 .$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = 2 \ln x_1 + \ln x_2.$$

Explain the idea behind "monotonic transformation" (one sentence) and derive function $V$ from $U$.

c) Assuming utility function $V(x_1, x_2) = 2 \ln x_1 + \ln x_2$

-Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

-Write down two secrets of happiness that determine optimal choice given parameters $p_1, p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each).

-Using secrets of happiness derive optimal consumption $x_1$,$x_2$, given values of $p_1 = 2, p_2 = 4$ and $m = 60$. Is the solution corner or interior (chose one)

d) (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary; 2) normal; and 3) neither gross complements nor gross substitutes (one sentence for each property).

### Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education, $x_1$ and health care, $x_2$. Your preferences over the two are represented by function

$$U(x_1, x_2) = x_1 + x_2.$$

a) Find marginal rate of substitution (give number).

b) Find optimal consumption of $x_1$ and $x_2$, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 50$ (two numbers).

c) Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

### Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 40$) but her future (period two) is not so bright ($m_2 = 20$)

a) Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

b) Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

c) Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 3$. Using magic formulas, find optimal consumption plan $(C_1, C_2)$ (two numbers) and the corresponding level of savings/borrowing, $S$.

d) Is Zoe tilting her consumption over time? (yes-no answer).

**Problem 4 (25p) (Short questions)**

a) Given utility function $U(C, R) = \min(2C, R)$, daily endowment of time $24h$, price and wage $p_c = w = 1$, find optimal choice of $C$, relaxation time $R$ and labor supply $L$. (three numbers, use secrets of happiness for perfect complements).

b) Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 100 \ln x_2$, prices $p_1 = \$8, p_2 = \$1$ and income $m = \$80$. Is your solution corner or interior?

c) Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond) that pays annually $\$1000$, starting next year or cash $\$8,000$ now (compare PV).

d) Your annual income when working (age 21-60) is $\$70,000$ and then you are are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

**Bonus question (Just for fun)**

a) Derive magic formulas for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$ that give optimal choices $x_1, x_2$ as a function of $a, b, p_1, p_2, m$.

b) Provide economic intuition for the magic formula for perfect complements (economic interpretation for the numerator and denominator).

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## Midterm 1 (C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+15+20+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

### Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, $x_1$, and clothing, $x_2$.

a) The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$ . Show geometrically Lila's budget set if her income is $m = 60$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^1 (x_2)^2 . \tag{1}$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = \ln x_1 + 2 \ln x_2. \tag{2}$$

Explain the idea behind "monotonic transformation" (one sentence) and derive function $V$ from $U$.

c) Assuming utility function $V(x_1, x_2) = \ln x_1 + 2 \ln x_2$

-Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

-Write down two secrets of happiness that determine optimal choice given parameters $p_1, p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each).

-Using secrets of happiness derive optimal consumption $x_1$,$x_2$, given values of $p_1 = 2, p_2 = 4$ and $m = 60$. Is the solution corner or interior (chose one)

d) (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary; 2) normal; and 3) neither gross complements nor gross substitutes (one sentence for each property).

### Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education, $x_1$ and health care, $x_2$. Your preferences over the two are represented by function

$$U(x_1, x_2) = 2x_1 + 2x_2.$$

a) Find marginal rate of substitution (give number).

b) Find optimal consumption of $x_1$ and $x_2$, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 100$ (two numbers).

c) Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

### Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 80$) but her future (period two) is not so bright ($m_2 = 40$)

a) Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

b) Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

c) Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 3$. Using magic formulas, find optimal consumption plan $(C_1, C_2)$ (two numbers) and the corresponding level of savings/borrowing, $S$.

d) Is Zoe tilting her consumption over time? (yes-no answer).

**Problem 4 (25p) (Short questions)**

a) Given utility function $U(C, R) = \min(C, 2R)$, daily endowment of time $24h$, price and wage $p_c = w = 1$, find optimal choice of $C$, relaxation time $R$ and labor supply $L$. (three numbers, use secrets of happiness for perfect complements).

b) Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 50 \ln x_2$, prices $p_1 = \$8, p_2 = \$1$ and income $m = \$40$. Is your solution corner or interior?

c) Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond) that pays annually $\$500$ starting next year or cash $\$4,000$ now (compare PV).

d) Your annual income when working (age 21-60) is $\$70,000$ and then you are are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

Bonus question (Just for fun)

a) Derive magic formulas for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$ that give optimal choices $x_1, x_2$ as a function of $a, b, p_1, p_2, m$.

b) Provide economic intuition for the magic formula for perfect complements (economic interpretation for the numerator and denominator).

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## Midterm 1 (D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+15+20+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

### Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, $x_1$, and clothing, $x_2$.

a) The price food is $p_1 = 4$ and one piece of clothing costs $p_2 = 8$ . Show geometrically Lila's budget set if her income is $m = 120$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^1 (x_2)^2 . \tag{3}$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = \ln x_1 + 2 \ln x_2. \tag{4}$$

Explain the idea behind "monotonic transformation" (one sentence) and derive function $V$ from $U$.

c) Assuming utility function $V(x_1, x_2) = \ln x_1 + 2 \ln x_2$

-Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

-Write down two secrets of happiness that determine optimal choice given parameters $p_1, p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each).

-Using secrets of happiness derive optimal consumption $x_1$,$x_2$, given values of $p_1 = 4, p_2 = 8$ and $m = 120$. Is the solution corner or interior (chose one)

d) (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary; 2) normal; and 3) neither gross complements nor gross substitutes (one sentence for each property).

### Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education, $x_1$ and health care, $x_2$. Your preferences over the two are represented by function

$$U(x_1, x_2) = 3x_1 + 3x_2.$$

a) Find marginal rate of substitution (give number).

b) Find optimal consumption of $x_1$ and $x_2$, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 100$ (two numbers).

c) Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

### Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young" is high ($m_1 = 80$) but her future (period two) is not so bright ($m_2 = 40$)

a) Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

b) Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

c) Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 4$. Using magic formulas, find optimal consumption plan $(C_1, C_2)$ (two numbers) and the corresponding level of savings/borrowing, $S$.

d) Is Zoe tilting her consumption over time? (yes-no answer).

**Problem 4 (25p) (Short questions)**

a) Given utility function $U(C, R) = \min(C, 2R)$, daily endowment of time $24h$, price and wage $p_c = w = 2$, find optimal choice of $C$, relaxation time $R$ and labor supply $L$. (three numbers, use secrets of happiness for perfect complements).

b) Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 50 \ln x_2$, prices $p_1 = \$8, p_2 = \$1$ and income $m = \$60$. Is your solution corner or interior?

c) Assume Interest rate $r = 10\%$. Choose one of the two: Choose one of the two: a consol (a type of British government bond) that pays annually $\$100$, starting next year or $\$1,200$ now (compare PV).

d) Your annual income when working (age 21-60) is $\$70,000$ and then you are are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

Bonus question (Just for fun)

a) Derive magic formulas for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$ that give optimal choices $x_1, x_2$ as a function of $a, b, p_1, p_2, m$.

b) Provide economic intuition for the magic formula for perfect complements (economic interpretation for the numerator and denominator).

# Midterm 1 Solution for Version A

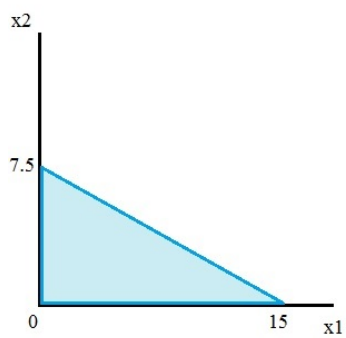
[Econ 301] Spring 2014

## Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x1, and clothing, x2.

**a) 6pts** The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$. Show geometrically Lila's budget set if her income is $m = 30$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

**Ans)** The relative price food in terms of clothing is $\frac{p_1}{p_2} = \frac{2}{4} = \frac{1}{2}$. Absolute value of slope of budget line is the relative price.




**b) 7pts** Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^2(x_2)^1$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = 2\ln x_1 + \ln x_2$$

Explain the idea behind "monotonic transformation"(one sentence) and derive function V from U.

**Ans)** Taking logarithm of $U$ becomes $V$. Logarithm is a positive monotone transformation, and monotonic transformation of utility function does not distort original preference order. That means, whenever $U(a, b) > U(c, d)$, it is true that $V(a, b) > V(c, d)$ and vice versa.

$$V(x_1, x_2) = 2\ln x_1 + \ln x_2 = \ln x_1^2 + \ln x_2 = \ln (x_1)^2(x_2)^1 = \ln U(x_1, x_2)$$

**c) 15pts** Assuming utility function $V(x_1, x_2) = 2\ln x_1 + \ln x_2$

- Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(8, 8)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption (8, 8)?

**Ans)** $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{2x_2}{x_1}$. At $(8, 8)$, $MRS = -\frac{16}{8} = -2$. $MRS_{12}$ means the amount of $x_2$ which consumer is willing to give up in order to get additional unit of $x_1$ while holding same utility level. $x_1$ is more (twice) valuable than $x_2$.

- Write down two secrets of happiness that determine optimal choice given parameters $p_1$,$p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each).

**Ans)**

$$\begin{aligned} x_1p_1 + x_2p_2 &= m \\ \frac{MU_1}{P_1} = \frac{MU_2}{P_2} & \quad or \quad MRS_{12} = -\frac{P_1}{P_2} \end{aligned}$$

The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, $x_1$ and $x_2$. The second condition is about efficient combination of two goods. Mathematically, the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. Economically, marginal utility per dollar of each good should become equal at the optimal consumption bundle, or consumer's relative evaluation about two goods in terms of marginal utility should coincide market's relative evaluation in terms of price.

- Using secrets of happiness derive optimal consumption $x_1$,$x_2$, given values of $p_1 = 2$, $p_2 = 4$ and $m = 30$. Is the solution corner or interior (chose one)

**Ans)** Since utility function is Cobb-Douglas, optimal consumption is following.

$$\begin{aligned} x_1 &= \frac{a}{a+b}\frac{m}{p_1} = \frac{2}{3}\frac{30}{2} = 10 \\ x_2 &= \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{3}\frac{30}{4} = 2.5 \end{aligned}$$

Both consumptions are positive, so it is interior solution.

**d) 12pts** (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary, 2) normal, and 3) neither gross complements nor gross substitutes (one sentence for each property).

**Ans)** When $p_1$ increases $x_1$ decreases, and when $p_2$ increases $x_2$ decreases: $\frac{\partial x_1}{\partial p_1} = -\frac{a}{a+b}\frac{m}{p_1^2} < 0$, $\frac{\partial x_2}{\partial p_2} = -\frac{b}{a+b}\frac{m}{p_2^2} < 0$, so both are ordinary. When $m$ increases, both $x_1$ and $x_2$ increases: $\frac{\partial x_1}{\partial m} = \frac{a}{(a+b)p_1} > 0$, $\frac{\partial x_2}{\partial m} = \frac{b}{(a+b)p_2} > 0$. Therefore, both are normal. When $p_1$ increases $x_2$ doesn't change, and when $p_2$ increases $x_1$ doesn't change: $\frac{\partial x_1}{\partial p_2} = 0$, $\frac{\partial x_2}{\partial p_2} = 0$. Therefore, $x_1$ and $x_2$ are gross neutral (neither gross complements nor gross substitutes).

## Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education,$x_1$ and health care, $x_2$. Your preferences over the two are represented by function

$$U(x_1, x_2) = 3x_1 + x_2$$

**a) 3pts** Find marginal rate of substitution (give number).

**Ans)** $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{3}{1} = -3$.

**b) 4pts** Find optimal consumption of $x_1$ and $x_2$, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 50$ (two numbers).

**Ans)** Compare marginal utility per dollars. Since $\frac{MU_1}{P_1} = \frac{3}{2} = 1.5$ is larger than $\frac{MU_2}{P_2} = \frac{1}{1} = 1$, optimal choice is consuming only $x_1$. $x_1 = \frac{m}{p_1} = \frac{50}{2} = 25$ and $x_2 = 0$.

**c) 8pts** Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

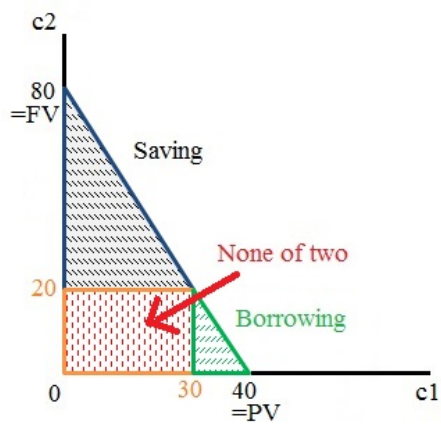
**Ans)** No, it is a corner solution since I consume zero $x_2$. No, $\frac{MU_1}{P_1} = \frac{3}{2} = 1.5 \neq \frac{MU_2}{P_2} = \frac{1}{1} = 1$.

## Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young"is high ($m_1 = 30$) but her future (period two) is not so bright ($m_2 = 20$).

**a) 4pts** Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

**Ans)** For $c_1 \leq 30, c_2 \leq 20$, neither borrowing nor saving is required. $c_1 < 30, c_2 > 20$ means saving and $c_1 > 30, c_2 < 20$ means borrowing.




**b) 6pts** Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

**Ans)** With $r = 1$, $PV = m_1 + \frac{m_2}{1+r} = 30 + 20/2 = 40$, $FV = (1+r)m_1 + m_2 = 2 * 30 + 20 = 80$. PV is x-intercept of budget line and FV is y-intercept of budget line. PV means Zoe's life time income evaluated at time 1 and it represents the maximum amount of $c_1$ Zoe can afford by borrowing $\frac{m_2}{1+r} = 10$.

**c) 6pts** Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 2$. Using magic formulas, find optimal consumption plan $(C_1, C_2)$ (two numbers) and the corresponding level of savings/borrowing, S.

**Ans)** Given utility function represents same preference of $V(C_1, C_2) = C_1^{(1+\delta)} C_2 = C_1^3 C_2$. Define $P_1 = 1$, $P_2 = 1/(1+r) = 1/2$, and $m = PV = 40$. Magic formulas of Cobb-Douglas tell us to consume $C_1 = 30$ and $C_2 = 20$. There is no needs for borrowing or saving, so $S = 0$.

**Alternative approach** $PV(m) = PV(c)$ and $MU_1 = (1+r)MU_2$ are

two conditions for optimal choice.

$$m_1 + \frac{m_2}{1+r} = C_1 + \frac{C_2}{1+r}$$
$$\frac{1}{C_1} = \frac{1+r}{(1+\delta)C_2} \quad \Leftrightarrow \quad C_1 = \frac{(1+\delta)C_2}{1+r}$$

Plug in numbers, then we get $C_1 = \frac{3}{2}C_2$ and $40 = C_1 + C_2/2$. Therefore $C_1 = 30$ and $C_2 = 20$. $S = m_1 - C_1 = 0$

**d) 4pts** Is Zoe tilting her consumption over time? (yes-no answer).

**Ans)** Yes, since time preference parameter $\delta = 2$ whereas interest rate $r = 1$, i.e. since $\delta > r$, it is optimal to consume more today ($C_1 = 30$) and to consume less tomorrow ($C_2 = 20$). Therefore, $C_1 \neq C_2$, and it means she tilts her consumption over time.

## Problem 4 (25p) (Short questions)

**a) 6pts** Given utility function $U(C, R) = \min(2C, R)$, daily endowment of time $24h$, price and wage $p_c = w = 2$, find optimal choice of $C$, relaxation time $R$ and labor supply $L$. (three numbers, use secrets of happiness for perfect complements).

**Ans)** Two formulas are $ax_1 = bx_2$ and $p_1x_1 + p_2x_2 = m$. Here, $2C = R$ and $2C + 2R = 48$ are optimal conditions. $C = 8$, $R = 16$ and $L = 24 - R = 8$.

**b) 7pts** Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 100 \ln x_2$, prices $p_1 = \$8$, $p_2 = \$1$ and income $m = \$60$. Is your solution corner or interior?

**Ans)** $\frac{MU_1}{P_1} = \frac{MU_2}{P_2}$ gives me threshold of non-linear good ($x_2$), and budget constraint $p_1x_1 + p_2x_2 = m$ need to be satisfied.

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} \Leftrightarrow \frac{1}{8} = \frac{100}{x_2} \Leftrightarrow x_2 = 800$$

I want to buy 800 units of $x_2$ first, and then want to buy $x_1$ as much as I can afford. However, I have only \$60, so I cannot afford both goods (800 units of $x_2$ costs \$800). Optimal choice is buying only $x_2$ good. $x_1 = 0$, $x_2 = \frac{m}{p_2} = \frac{60}{1} = 60$. My solution is corner because I don't buy any $x_1$ good.

**c) 6pts** Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond)that pays annually $\$1,000$, starting next year or cash $\$12,000$ now (compare PV).

**Ans)** Consol bond pays coupon $\$1,000$ forever. $r = 0.1$. The cash flow of consol bond is same with perpetuity. Recall that the present value of perpetuity is

$$\sum_{n=1}^{\infty} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}}{1-\frac{1}{1+r}} = \frac{x}{r}.$$

$$\begin{aligned} PV(\texttt{consol}) &= \frac{x}{r} = \frac{1,000}{0.1} = 10,000 \\ PV(\texttt{cash}) &= 12,000 \end{aligned}$$

Current cash doesn't require discount. Taking cash is better: $12,000 > 10,000$.

**d) 6pts** Your annual income when working (age 21-60) is $\$60,000$ and then you are going to live for the next 30 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 2\%$.

**Ans)** For first 40 years, I will earn $\$60,000$ annually, and I will consume constant level $c$ annually for 70 years. Recall that the present value of annuity cash flow is

$$\sum_{n=1}^{T} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}\left[1-(\frac{1}{1+r})^T\right]}{1-\frac{1}{1+r}} = \frac{x}{r}\left(1-\frac{1}{(1+r)^T}\right),$$

and $r = 0.02$.

$$\begin{aligned} PV(\texttt{income}) &= \frac{60,000}{0.02}\left(1-\left[\frac{1}{1.02}\right]^{40}\right) \\ PV(\texttt{consumption}) &= \frac{c}{0.02}\left(1-\left[\frac{1}{1.02}\right]^{70}\right) \\ \therefore \quad \frac{60,000}{0.02}\left(1-\left[\frac{1}{1.02}\right]^{40}\right) &= \frac{c}{0.02}\left(1-\left[\frac{1}{1.02}\right]^{70}\right) \end{aligned}$$

# Midterm 1 Solution for Version B

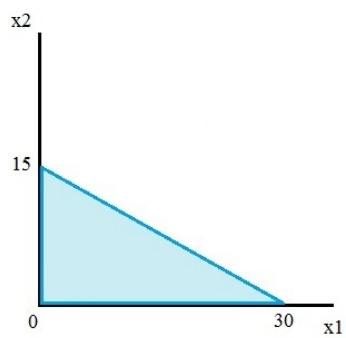
[Econ 301] Spring 2014

## Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x1, and clothing, x2.

**a) 6pts** The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$. Show geometrically Lila's budget set if her income is $m = 60$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

**Ans)** The relative price food in terms of clothing is $\frac{p_1}{p_2} = \frac{2}{4} = \frac{1}{2}$. Absolute value of slope of budget line is the relative price.




**b) 7pts** Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^2 (x_2)^1$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = 2 \ln x_1 + \ln x_2$$

Explain the idea behind "monotonic transformation"(one sentence) and derive function V from U.

**Ans)** Taking logarithm of $U$ becomes $V$. Logarithm is a positive monotone transformation, and monotonic transformation of utility function does not distort original preference order. That means, whenever $U(a, b) > U(c, d)$, it is true that $V(a, b) > V(c, d)$ and vice versa.

$$V(x_1, x_2) = 2 \ln x_1 + \ln x_2 = \ln x_1^2 + \ln x_2 = \ln (x_1)^2 (x_2)^1 = \ln U(x_1, x_2)$$

**c) 15pts** Assuming utility function $V(x_1, x_2) = 2\ln x_1 + \ln x_2$

- Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

**Ans)** $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{2x_2}{x_1}$. At $(2, 2)$, $MRS = -\frac{4}{2} = -2$. $MRS_{12}$ means the amount of $x_2$ which consumer is willing to give up in order to get additional unit of $x_1$ while holding same utility level. $x_1$ is more (twice) valuable than $x_2$.

- Write down two secrets of happiness that determine optimal choice given parameters $p_1$,$p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each).

**Ans)**

$$\begin{aligned} x_1p_1 + x_2p_2 &= m \\ \frac{MU_1}{P_1} = \frac{MU_2}{P_2} & \quad or \quad MRS_{12} = -\frac{P_1}{P_2} \end{aligned}$$

The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, $x_1$ and $x_2$. The second condition is about efficient combination of two goods. Mathematically, the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. Economically, marginal utility per dollar of each good should become equal at the optimal consumption bundle, or consumer's relative evaluation about two goods in terms of marginal utility should coincide market's relative evaluation in terms of price.

- Using secrets of happiness derive optimal consumption $x_1$,$x_2$, given values of $p_1 = 2$, $p_2 = 4$ and $m = 60$. Is the solution corner or interior (chose one)

**Ans)** Since utility function is Cobb-Douglas, optimal consumption is following.

$$\begin{aligned} x_1 &= \frac{a}{a+b}\frac{m}{p_1} = \frac{2}{3}\frac{60}{2} = 20 \\ x_2 &= \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{3}\frac{60}{4} = 5 \end{aligned}$$

Both consumptions are positive, so it is interior solution.

**d) 12pts** (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary, 2) normal, and 3) neither gross complements nor gross substitutes (one sentence for each property).

**Ans)** When $p_1$ increases $x_1$ decreases, and when $p_2$ increases $x_2$ decreases: $\frac{\partial x_1}{\partial p_1} = -\frac{a}{a+b}\frac{m}{p_1^2} < 0$, $\frac{\partial x_2}{\partial p_2} = -\frac{b}{a+b}\frac{m}{p_2^2} < 0$, so both are ordinary. When $m$ increases, both $x_1$ and $x_2$ increases: $\frac{\partial x_1}{\partial m} = \frac{a}{(a+b)p_1} > 0$, $\frac{\partial x_2}{\partial m} = \frac{b}{(a+b)p_2} > 0$. Therefore, both are normal. When $p_1$ increases $x_2$ doesn't change, and when $p_2$ increases $x_1$ doesn't change: $\frac{\partial x_1}{\partial p_2} = 0$, $\frac{\partial x_2}{\partial p_2} = 0$. Therefore, $x_1$ and $x_2$ are gross neutral (neither gross complements nor gross substitutes).

## Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education,$x_1$ and health care, $x_2$. Your preferences over the two are represented by function

$$U(x_1, x_2) = x_1 + x_2$$

**a) 3pts** Find marginal rate of substitution (give number).

**Ans)** $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{1}{1} = -1$.

**b) 4pts** Find optimal consumption of $x_1$ and $x_2$, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 50$ (two numbers).

**Ans)** Compare marginal utility per dollars. Since $\frac{MU_1}{P_1} = \frac{1}{2} = 0.5$ is smaller than $\frac{MU_2}{P_2} = \frac{1}{1} = 1$, optimal choice is consuming only $x_2$. $x_1 = 0$ and $x_2 = \frac{m}{p_2} = \frac{50}{1} = 50$.

**c) 8pts** Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

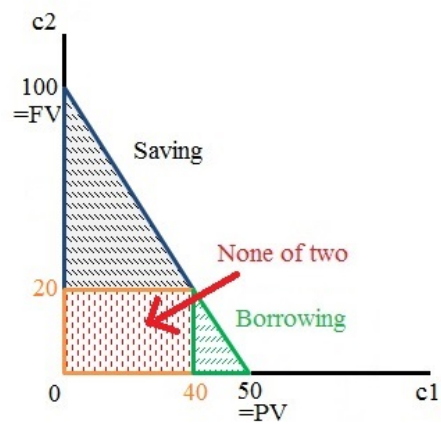
**Ans)** No, it is corner solution since I consume zero $x_1$. No, $\frac{MU_1}{P_1} = \frac{1}{2} = 0.5 \neq \frac{MU_2}{P_2} = \frac{1}{1} = 1$.

## Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young"is high ($m_1 = 40$) but her future (period two) is not so bright ($m_2 = 20$).

**a) 4pts** Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

**Ans)** For $c_1 \leq 40, c_2 \leq 20$, neither borrowing nor saving is required. $c_1 < 40, c_2 > 20$ means saving and $c_1 > 40, c_2 < 20$ means borrowing.




**b) 6pts** Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

**Ans)** With $r = 1$, $PV = m_1 + \frac{m_2}{1+r} = 40 + 20/2 = 50$, $FV = (1+r)m_1 + m_2 = 2 * 40 + 20 = 100$. PV is x-intercept of budget line and FV is y-intercept of budget line. PV means Zoe's life time income evaluated at time 1 and it represents the maximum amount of $c_1$ Zoe can afford by borrowing $\frac{m_2}{1+r} = 10$.

**c) 6pts** Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 3$. Using magic formulas, find optimal consumption plan $(C_1, C_2)$ (two numbers) and the corresponding level of savings/borrowing, S.

**Ans)** Given utility function represents same preference of $V(C_1, C_2) = C_1^{(1+\delta)} C_2 = C_1^4 C_2$. Define $P_1 = 1$, $P_2 = 1/(1+r) = 1/2$, and $m = PV = 50$. Magic formulas of Cobb-Douglas tell us to consume $C_1 = 40$ and $C_2 = 20$. There is no needs for borrowing or saving, so $S = 0$.

**Alternative approach** $PV(m) = PV(c)$ and $MU_1 = (1+r)MU_2$ are

two conditions for optimal choice.

$$\begin{aligned} m_1 + \frac{m_2}{1+r} &= C_1 + \frac{C_2}{1+r} \\ \frac{1}{C_1} = \frac{1+r}{(1+\delta)C_2} &\Leftrightarrow \quad C_1 = \frac{(1+\delta)C_2}{1+r} \end{aligned}$$

Plug in numbers, then we get $C_1 = \frac{4}{2}C_2$ and $50 = C_1 + C_2/2$. Therefore $C_1 = 40$ and $C_2 = 20$. $S = m_1 - C_1 = 0$.

**d) 4pts** Is Zoe tilting her consumption over time? (yes-no answer).

**Ans)** Yes, since time preference parameter $\delta = 3$ whereas interest rate $r = 1$, i.e. since $\delta > r$, it is optimal to consume more today ($C_1 = 40$) and to consume less tomorrow ($C_2 = 20$). Therefore, $C_1 \neq C_2$, and it means she tilts her consumption over time.

## Problem 4 (25p) (Short questions)

**a) 6pts** Given utility function $U(C, R) = \min(2C, R)$, daily endowment of time $24h$, price and wage $p_c = w = 1$, find optimal choice of $C$, relaxation time $R$ and labor supply $L$. (three numbers, use secrets of happiness for perfect complements).

**Ans)** Two formulas are $ax_1 = bx_2$ and $p_1x_1 + p_2x_2 = m$. Here, $2C = R$ and $C + R = 24$ are optimal conditions. $C = 8$, $R = 16$ and $L = 24 - R = 8$.

**b) 7pts** Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 100 \ln x_2$, prices $p_1 = \$8$, $p_2 = \$1$ and income $m = \$80$. Is your solution corner or interior?

**Ans)** $\frac{MU_1}{P_1} = \frac{MU_2}{P_2}$ gives me threshold of non-linear good ($x_2$), and budget constraint $p_1x_1 + p_2x_2 = m$ need to be satisfied.

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} \Leftrightarrow \frac{1}{8} = \frac{100}{x_2} \Leftrightarrow x_2 = 800$$

I want to buy 800 units of $x_2$ first, and then want to buy $x_1$ as much as I can afford. However, I have only \$80, so I cannot afford both goods (800 units of $x_2$ costs \$800). Optimal choice is buying only $x_2$ good. $x_1 = 0$, $x_2 = \frac{m}{p_2} = \frac{80}{1} = 80$. My solution is corner because I don't buy any $x_1$ good.

**c) 6pts** Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond)that pays annually $\$1,000$, starting next year or cash $\$8,000$ now (compare PV).

**Ans)** A consol bond pays coupon $\$1,000$ forever. $r = 0.1$. The cash flow of a consol bond is same with perpetuity. Recall that the present value of perpetuity is

$$\sum_{n=1}^{\infty} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}}{1 - \frac{1}{1+r}} = \frac{x}{r}.$$

$$\begin{aligned} PV(\texttt{consol}) &= \frac{x}{r} = \frac{1,000}{0.1} = 10,000 \\ PV(\texttt{cash}) &= 8,000 \end{aligned}$$

Current cash doesn't require discount. Taking a consol is better: $10,000 > 8,000$.

**d) 6pts** Your annual income when working (age 21-60) is $\$70,000$ and then you are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

**Ans)** For first 40 years, I will earn $\$70,000$ annually, and I will consume constant level $c$ annually for 75 years. Recall that the present value of annuity cash flow is

$$\sum_{n=1}^{T} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}\left[1 - (\frac{1}{1+r})^T\right]}{1 - \frac{1}{1+r}} = \frac{x}{r}\left(1 - \frac{1}{(1+r)^T}\right),$$

and $r = 0.1$.

$$\begin{aligned} PV(\texttt{income}) &= \frac{70,000}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{40}\right) \\ PV(\texttt{consumption}) &= \frac{c}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{75}\right) \\ \therefore \quad \frac{70,000}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{40}\right) &= \frac{c}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{75}\right) \end{aligned}$$

# Midterm 1 Solution for Version C

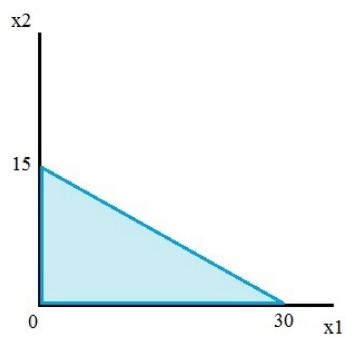
[Econ 301] Spring 2014

## Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x1, and clothing, x2.

**a) 6pts** The price food is $p_1 = 2$ and one piece of clothing costs $p_2 = 4$. Show geometrically Lila's budget set if her income is $m = 60$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

**Ans)** The relative price food in terms of clothing is $\frac{p_1}{p_2} = \frac{2}{4} = \frac{1}{2}$. Absolute value of slope of budget line is the relative price.




**b) 7pts** Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^1 (x_2)^2$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = \ln x_1 + 2 \ln x_2$$

Explain the idea behind "monotonic transformation"(one sentence) and derive function V from U.

**Ans)** Taking logarithm of $U$ becomes $V$. Logarithm is a positive monotone transformation, and monotonic transformation of utility function does not distort original preference order. That means, whenever $U(a, b) > U(c, d)$, it is true that $V(a, b) > V(c, d)$ and vice versa.

$$V(x_1, x_2) = \ln x_1 + 2 \ln x_2 = \ln x_1 + \ln x_2^2 = \ln (x_1)^1 (x_2)^2 = \ln U(x_1, x_2)$$

**c) 15pts** Assuming utility function $V(x_1, x_2) = \ln x_1 + 2 \ln x_2$

- Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

**Ans)** $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{x_2}{2x_1}$. At $(2, 2)$, $MRS = -\frac{2}{4} = -0.5$. $MRS_{12}$ means the amount of $x_2$ which consumer is willing to give up in order to get additional unit of $x_1$ while holding same utility level. $x_2$ is more (twice) valuable than $x_1$.

- Write down two secrets of happiness that determine optimal choice given parameters $p_1$,$p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each).

**Ans)**

$$\begin{aligned} x_1 p_1 + x_2 p_2 &= m \\ \frac{MU_1}{P_1} = \frac{MU_2}{P_2} & \quad or \quad MRS_{12} = -\frac{P_1}{P_2} \end{aligned}$$

The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, $x_1$ and $x_2$. The second condition is about efficient combination of two goods. Mathematically, the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. Economically, marginal utility per dollar of each good should become equal at the optimal consumption bundle, or consumer's relative evaluation about two goods in terms of marginal utility should coincide market's relative evaluation in terms of price.

- Using secrets of happiness derive optimal consumption $x_1$,$x_2$, given values of $p_1 = 2$, $p_2 = 4$ and $m = 60$. Is the solution corner or interior (chose one)

**Ans)** Since utility function is Cobb-Douglas, optimal consumption is following.

$$\begin{aligned} x_1 &= \frac{a}{a+b}\frac{m}{p_1} = \frac{1}{3}\frac{60}{2} = 10 \\ x_2 &= \frac{b}{a+b}\frac{m}{p_2} = \frac{2}{3}\frac{60}{4} = 10 \end{aligned}$$

Both consumptions are positive, so it is interior solution.

**d) 12pts** (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary, 2) normal, and 3) neither gross complements nor gross substitutes (one sentence for each property).

**Ans)** When $p_1$ increases $x_1$ decreases, and when $p_2$ increases $x_2$ decreases: $\frac{\partial x_1}{\partial p_1} = -\frac{a}{a+b}\frac{m}{p_1^2} < 0$, $\frac{\partial x_2}{\partial p_2} = -\frac{b}{a+b}\frac{m}{p_2^2} < 0$, so both are ordinary. When $m$ increases, both $x_1$ and $x_2$ increases: $\frac{\partial x_1}{\partial m} = \frac{a}{(a+b)p_1} > 0$, $\frac{\partial x_2}{\partial m} = \frac{b}{(a+b)p_2} > 0$. Therefore, both are normal. When $p_1$ increases $x_2$ doesn't change, and when $p_2$ increases $x_1$ doesn't change: $\frac{\partial x_1}{\partial p_2} = 0$, $\frac{\partial x_2}{\partial p_2} = 0$. Therefore, $x_1$ and $x_2$ are gross neutral (neither gross complements nor gross substitutes).

## Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education,$x_1$ and health care, $x_2$. Your preferences over the two are represented by function

$$U(x_1, x_2) = 2x_1 + 2x_2$$

**a) 3pts** Find marginal rate of substitution (give number).

**Ans)** $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{2}{2} = -1$.

**b) 4pts** Find optimal consumption of $x_1$ and $x_2$, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 100$ (two numbers).

**Ans)** Compare marginal utility per dollars. Since $\frac{MU_1}{P_1} = \frac{2}{2} = 1$ is smaller than $\frac{MU_2}{P_2} = \frac{2}{1} = 2$, optimal choice is consuming only $x_2$. $x_1 = 0$ and $x_2 = \frac{m}{p_2} = \frac{100}{1} = 100$.

**c) 8pts** Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

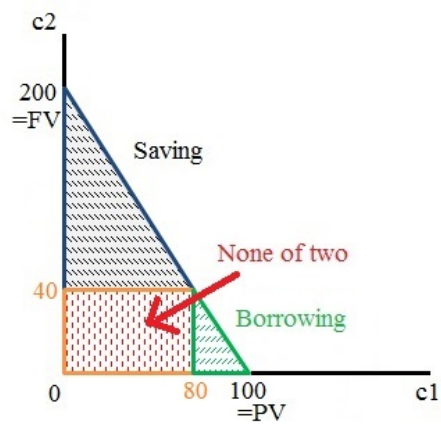
**Ans)** No, it is corner solution since I consume zero $x_1$. No, $\frac{MU_1}{P_1} = \frac{2}{2} = 1 \neq \frac{MU_2}{P_2} = \frac{2}{1} = 2$.

## Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young"is high ($m_1 = 80$) but her future (period two) is not so bright ($m_2 = 40$).

**a) 4pts** Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

**Ans)** For $c_1 \leq 80, c_2 \leq 40$, neither borrowing nor saving is required. $c_1 < 80, c_2 > 40$ means saving and $c_1 > 80, c_2 < 40$ means borrowing.




**b) 6pts** Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

**Ans)** With $r = 1$, $PV = m_1 + \frac{m_2}{1+r} = 80 + 40/2 = 100$, $FV = (1+r)m_1 + m_2 = 2 * 80 + 40 = 200$. PV is x-intercept of budget line and FV is y-intercept of budget line. PV means Zoe's life time income evaluated at time 1 and it represents the maximum amount of $c_1$ Zoe can afford by borrowing $\frac{m_2}{1+r} = 20$.

**c) 6pts** Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 3$. Using magic formulas, find optimal consumption plan $(C_1, C_2)$ (two numbers) and the corresponding level of savings/borrowing, S.

**Ans)** Given utility function represents same preference of $V(C_1, C_2) = C_1^{(1+\delta)} C_2 = C_1^4 C_2$. Define $P_1 = 1$, $P_2 = 1/(1+r) = 1/2$, and $m = PV = 100$. Magic formulas of Cobb-Douglas tell us to consume $C_1 = 80$ and $C_2 = 40$. There is no needs for borrowing or saving, so $S = 0$.

**Alternative approach** $PV(m) = PV(c)$ and $MU_1 = (1+r)MU_2$ are

two conditions for optimal choice.

$$m_1 + \frac{m_2}{1+r} = C_1 + \frac{C_2}{1+r}$$
$$\frac{1}{C_1} = \frac{1+r}{(1+\delta)C_2} \quad \Leftrightarrow \quad C_1 = \frac{(1+\delta)C_2}{1+r}$$

Plug in numbers, then we get $C_1 = \frac{4}{2}C_2$ and $100 = C_1 + C_2/2$. Therefore $C_1 = 80$ and $C_2 = 40$. $S = m_1 - C_1 = 0$.

**d) 4pts** Is Zoe tilting her consumption over time? (yes-no answer).

**Ans)** Yes, since time preference parameter $\delta = 3$ whereas interest rate $r = 1$, i.e. since $\delta > r$, it is optimal to consume more today ($C_1 = 80$) and to consume less tomorrow ($C_2 = 40$). Therefore, $C_1 \neq C_2$, and it means she tilts her consumption over time.

## Problem 4 (25p) (Short questions)

**a) 6pts** Given utility function $U(C, R) = \min(C, 2R)$, daily endowment of time $24h$, price and wage $p_c = w = 1$, find optimal choice of $C$, relaxation time $R$ and labor supply $L$. (three numbers, use secrets of happiness for perfect complements).

**Ans)** Two formulas are $ax_1 = bx_2$ and $p_1x_1 + p_2x_2 = m$. Here, $C = 2R$ and $C + R = 24$ are optimal conditions. $C = 16$, $R = 8$ and $L = 24 - R = 16$.

**b) 7pts** Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1 + 50 \ln x_2$, prices $p_1 = \$8$, $p_2 = \$1$ and income $m = \$40$. Is your solution corner or interior?

**Ans)** $\frac{MU_1}{P_1} = \frac{MU_2}{P_2}$ gives me threshold of non-linear good ($x_2$), and budget constraint $p_1x_1 + p_2x_2 = m$ need to be satisfied.

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} \Leftrightarrow \frac{1}{8} = \frac{50}{x_2} \Leftrightarrow x_2 = 400$$

I want to buy 400 units of $x_2$ first, and then want to buy $x_1$ as much as I can afford. However, I have only \$40, so I cannot afford both goods (400 units of $x_2$ costs \$400). Optimal choice is buying only $x_2$ good. $x_1 = 0$, $x_2 = \frac{m}{p_2} = \frac{40}{1} = 40$. My solution is corner because I don't buy any $x_1$ good.

**c) 6pts** Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond)that pays annually \$500, starting next year or cash \$4,000 now (compare PV).

**Ans)** A consol bond pays coupon \$500 forever and $r = 0.1$. The cash flow of a consol bond is same with perpetuity. Recall that the present value of perpetuity is

$$\sum_{n=1}^{\infty} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}}{1 - \frac{1}{1+r}} = \frac{x}{r}.$$

$$\begin{aligned} PV(\texttt{consol}) &= \frac{x}{r} = \frac{500}{0.1} = 5,000 \\ PV(\texttt{cash}) &= 4,000 \end{aligned}$$

Current cash doesn't require discount. Taking a consol is better: $5,000 > 4,000$.

**d) 6pts** Your annual income when working (age 21-60) is \$70,000 and then you are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

**Ans)** For first 40 years, I will earn \$70,000 annually, and I will consume constant level $c$ annually for 75 years. Recall that the present value of annuity cash flow is

$$\sum_{n=1}^{T} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}\left[1 - (\frac{1}{1+r})^T\right]}{1 - \frac{1}{1+r}} = \frac{x}{r}\left(1 - \frac{1}{(1+r)^T}\right),$$

and $r = 0.1$.

$$\begin{aligned} PV(\texttt{income}) &= \frac{70,000}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{40}\right) \\ PV(\texttt{consumption}) &= \frac{c}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{75}\right) \\ \therefore \quad \frac{70,000}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{40}\right) &= \frac{c}{0.1}\left(1 - \left[\frac{1}{1.1}\right]^{75}\right) \end{aligned}$$

# Midterm 1 Solution for Version D

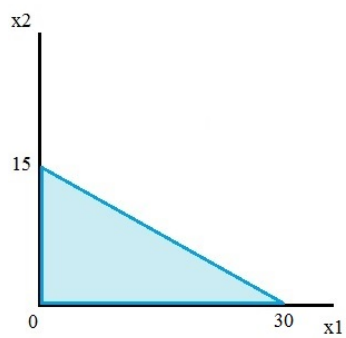
[Econ 301] Spring 2014

## Problem 1 (40p) (Well-behaved preferences)

Lila spends her income on two goods: food, x1, and clothing, x2.

**a) 6pts** The price food is $p_1 = 4$ and one piece of clothing costs $p_2 = 8$. Show geometrically Lila's budget set if her income is $m = 120$. Find the relative price food in terms of clothing (one number)? Where can the relative price be seen in the graph of a budget set? (one sentence)

**Ans)** The relative price food in terms of clothing is $\frac{p_1}{p_2} = \frac{4}{8} = \frac{1}{2}$. Absolute value of slope of budget line is the relative price.




**b) 7pts** Lila's preferences are represented by utility function

$$U(x_1, x_2) = (x_1)^1 (x_2)^2$$

We know that her preferences can be alternatively represented by function

$$V(x_1, x_2) = \ln x_1 + 2 \ln x_2$$

Explain the idea behind "monotonic transformation"(one sentence) and derive function V from U.

**Ans)** Taking logarithm of $U$ becomes $V$. Logarithm is a positive monotone transformation, and monotonic transformation of utility function does not distort original preference order. That means, whenever $U(a, b) > U(c, d)$, it is true that $V(a, b) > V(c, d)$ and vice versa.

$$V(x_1, x_2) = \ln x_1 + 2 \ln x_2 = \ln x_1 + \ln x_2^2 = \ln (x_1)^1 (x_2)^2 = \ln U(x_1, x_2)$$

**c) 15pts** Assuming utility function $V(x_1, x_2) = \ln x_1 + 2 \ln x_2$

- Find marginal rate of substitution (MRS) for all bundles (derive formula). For bundle $(2, 2)$ find the value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable given consumption $(2, 2)$?

**Ans)** $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{x_2}{2x_1}$. At $(2, 2)$, $MRS = -\frac{2}{4} = -0.5$. $MRS_{12}$ means the amount of $x_2$ which consumer is willing to give up in order to get additional unit of $x_1$ while holding same utility level. $x_2$ is more (twice) valuable than $x_1$.

- Write down two secrets of happiness that determine optimal choice given parameters $p_1$,$p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each).

**Ans)**

$$\begin{aligned} x_1 p_1 + x_2 p_2 &= m \\ \frac{MU_1}{P_1} = \frac{MU_2}{P_2} & \quad or \quad MRS_{12} = -\frac{P_1}{P_2} \end{aligned}$$

The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, $x_1$ and $x_2$. The second condition is about efficient combination of two goods. Mathematically, the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. Economically, marginal utility per dollar of each good should become equal at the optimal consumption bundle, or consumer's relative evaluation about two goods in terms of marginal utility should coincide market's relative evaluation in terms of price.

- Using secrets of happiness derive optimal consumption $x_1$,$x_2$, given values of $p_1 = 4$, $p_2 = 8$ and $m = 120$. Is the solution corner or interior (chose one)

**Ans)** Since utility function is Cobb-Douglas, optimal consumption is following.

$$\begin{aligned} x_1 &= \frac{a}{a+b}\frac{m}{p_1} = \frac{1}{3}\frac{120}{4} = 10 \\ x_2 &= \frac{b}{a+b}\frac{m}{p_2} = \frac{2}{3}\frac{120}{8} = 10 \end{aligned}$$

Both consumptions are positive, so it is interior solution.

**d) 12pts** (Harder) Using magic formulas for Cobb-Douglass preferences argue that the two commodities are 1) ordinary, 2) normal, and 3) neither gross complements nor gross substitutes (one sentence for each property).

**Ans)** When $p_1$ increases $x_1$ decreases, and when $p_2$ increases $x_2$ decreases: $\frac{\partial x_1}{\partial p_1} = -\frac{a}{a+b}\frac{m}{p_1^2} < 0$, $\frac{\partial x_2}{\partial p_2} = -\frac{b}{a+b}\frac{m}{p_2^2} < 0$, so both are ordinary. When $m$ increases, both $x_1$ and $x_2$ increases: $\frac{\partial x_1}{\partial m} = \frac{a}{(a+b)p_1} > 0$, $\frac{\partial x_2}{\partial m} = \frac{b}{(a+b)p_2} > 0$. Therefore, both are normal. When $p_1$ increases $x_2$ doesn't change, and when $p_2$ increases $x_1$ doesn't change: $\frac{\partial x_1}{\partial p_2} = 0$, $\frac{\partial x_2}{\partial p_2} = 0$. Therefore, $x_1$ and $x_2$ are gross neutral (neither gross complements nor gross substitutes).

## Problem 2 (15p) (Perfect substitutes)

You are planing a budget for the state of Wisconsin. The two major budget positions include education,$x_1$ and health care, $x_2$. Your preferences over the two are represented by function

$$U(x_1, x_2) = 3x_1 + 3x_2$$

**a) 3pts** Find marginal rate of substitution (give number).

**Ans)** $MRS_{12} = -\frac{MU_1}{MU_2} = -\frac{3}{3} = -1$.

**b) 4pts** Find optimal consumption of $x_1$ and $x_2$, given prices $p_1 = 2$ and $p_2 = 1$ and available funds $m = 100$ (two numbers).

**Ans)** Compare marginal utility per dollars. Since $\frac{MU_1}{P_1} = \frac{3}{2} = 1.5$ is smaller than $\frac{MU_2}{P_2} = \frac{3}{1} = 3$, optimal choice is consuming only $x_2$. $x_1 = 0$ and $x_2 = \frac{m}{p_2} = \frac{100}{1} = 100$.

**c) 8pts** Is solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

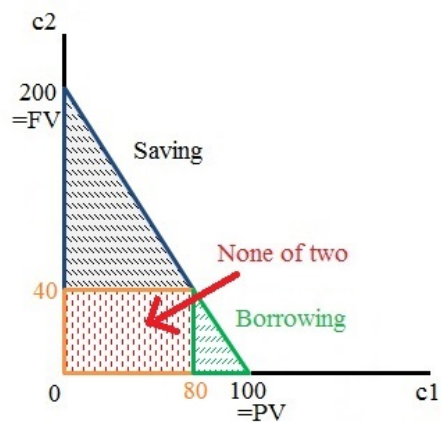
**Ans)** No, it is corner solution since I consume zero $x_1$. No, $\frac{MU_1}{P_1} = \frac{3}{2} = 1.5 \neq \frac{MU_2}{P_2} = \frac{3}{1} = 3$.

## Problem 3 (20p) (Intertemporal choice)

Zoe is a professional Olympic skier. Her income when "young"is high ($m_1 = 80$) but her future (period two) is not so bright ($m_2 = 40$).

**a) 4pts** Depict Zoe's budget set, assuming that she can borrow and save at the interest rate $r = 100\%$. Partition her budget set into three regions: the area that involves saving, borrowing and none of the two.

**Ans)** For $c_1 \leq 80, c_2 \leq 40$, neither borrowing nor saving is required. $c_1 < 80, c_2 > 40$ means saving and $c_1 > 80, c_2 < 40$ means borrowing.




**b) 6pts** Find PV and FV of Zoe's lifetime income (two numbers) and show the two values in the graph. Interpret economically PV.

**Ans)** With $r = 1$, $PV = m_1 + \frac{m_2}{1+r} = 80 + 40/2 = 100$, $FV = (1+r)m_1 + m_2 = 2 * 80 + 40 = 200$. PV is x-intercept of budget line and FV is y-intercept of budget line. PV means Zoe's life time income evaluated at time 1 and it represents the maximum amount of $c_1$ Zoe can afford by borrowing $\frac{m_2}{1+r} = 20$.

**c) 6pts** Zoe's utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$ where discount rate is $\delta = 4$. Using magic formulas, find optimal consumption plan $(C_1, C_2)$ (two numbers) and the corresponding level of savings/borrowing, S.

**Ans)** Given utility function represents same preference of $V(C_1, C_2) = C_1^{(1+\delta)} C_2 = C_1^5 C_2$. Define $P_1 = 1$, $P_2 = 1/(1+r) = 1/2$, and $m = PV = 100$. Magic formulas of Cobb-Douglas tell us to consume $C_1 = \frac{500}{6} = \frac{250}{3} \approx 83.33$ and $C_2 = \frac{100}{3} \approx 33.33$. She needs to borrow $B = 10/3$ (because $c_1 > m_1$), or her saving is $S = -\frac{10}{3}$.

**Alternative approach** $PV(m) = PV(c)$ and $MU_1 = (1+r)MU_2$ are

two conditions for optimal choice.

$$\begin{aligned} m_1 + \frac{m_2}{1+r} &= C_1 + \frac{C_2}{1+r} \\ \frac{1}{C_1} = \frac{1+r}{(1+\delta)C_2} &\Leftrightarrow \quad C_1 = \frac{(1+\delta)C_2}{1+r} \end{aligned}$$

Plug in numbers, then we get $C_1 = \frac{5}{2}C_2$ and $100 = C_1 + C_2/2$. Therefore $C_1 = \frac{250}{3}$ and $C_2 = \frac{100}{3}$. Saving is $S = m_1 - C_1 = -\frac{10}{3}$ (or borrowing $B = C_1 - m_1 = \frac{10}{3}$).

**d) 4pts** Is Zoe tilting her consumption over time? (yes-no answer).

**Ans)** Yes, since time preference parameter $\delta = 4$ whereas interest rate $r = 1$, i.e. since $\delta > r$, it is optimal to consume more today ($C_1 \approx 83.33$) and to consume less tomorrow ($C_2 \approx 33.33$). Therefore, $C_1 \neq C_2$, and it means she tilts her consumption over time.

## Problem 4 (25p) (Short questions)

**a) 6pts** Given utility function $U(C, R) = \min(C, 2R)$, daily endowment of time $24h$, price and wage $p_c = w = 2$, find optimal choice of $C$, relaxation time $R$ and labor supply $L$. (three numbers, use secrets of happiness for perfect complements).

**Ans)** Two formulas are $ax_1 = bx_2$ and $p_1x_1 + p_2x_2 = m$. Here, $C = 2R$ and $2C+2R = 48$ are optimal conditions. $C = 16$, $R = 8$ and $L = 24-R = 16$.

**b) 7pts** Find optimal choice given quasilinear preferences $U(x_1, x_2) = x_1+50\ln x_2$, prices $p_1 = \$8$, $p_2 = \$1$ and income $m = \$60$. Is your solution corner or interior?

**Ans)** $\frac{MU_1}{P_1} = \frac{MU_2}{P_2}$ gives me threshold of non-linear good ($x_2$), and budget constraint $p_1x_1 + p_2x_2 = m$ need to be satisfied.

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2} \Leftrightarrow \frac{1}{8} = \frac{50}{x_2} \Leftrightarrow x_2 = 400$$

I want to buy 400 units of $x_2$ first, and then want to buy $x_1$ as much as I can afford. However, I have only \$60, so I cannot afford both goods (400 units of $x_2$ costs \$400). Optimal choice is buying only $x_2$ good. $x_1 = 0$, $x_2 = \frac{m}{p_2} = \frac{60}{1} = 60$. My solution is corner because I don't buy any $x_1$ good.

**c) 6pts** Assume Interest rate $r = 10\%$. Choose one of the two: a consol (a type of British government bond)that pays annually \$100, starting next year or cash $\$1,200$ now (compare PV).

**Ans)** A consol bond pays coupon \$100 forever and $r = 0.1$. The cash flow of a consol bond is same with perpetuity. Recall that the present value of perpetuity is

$$\sum_{n=1}^{\infty} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}}{1-\frac{1}{1+r}} = \frac{x}{r}.$$

$$\begin{aligned} PV(\texttt{consol}) &= \frac{x}{r} = \frac{100}{0.1} = 1,000 \\ PV(\texttt{cash}) &= 1,200 \end{aligned}$$

Current cash doesn't require discount. Taking cash is better since $1,200 > 1,000$.

**d) 6pts** Your annual income when working (age 21-60) is $\$70,000$ and then you are going to live for the next 35 years. Write down equation that determines constant (maximal) level of consumption during your lifetime. Assume annual interest rate $r = 10\%$.

**Ans)** For first 40 years, I will earn $\$70,000$ annually, and I will consume constant level $c$ annually for 75 years. Recall that the present value of annuity cash flow is

$$\sum_{n=1}^{T} \frac{x}{(1+r)^n} = \frac{\frac{x}{1+r}\left[1-(\frac{1}{1+r})^T\right]}{1-\frac{1}{1+r}} = \frac{x}{r}\left(1-\frac{1}{(1+r)^T}\right),$$

and $r = 0.1$.

$$\begin{aligned} PV(\texttt{income}) &= \frac{70,000}{0.1}\left(1-\left[\frac{1}{1.1}\right]^{40}\right) \\ PV(\texttt{consumption}) &= \frac{c}{0.1}\left(1-\left[\frac{1}{1.1}\right]^{75}\right) \\ \therefore \quad \frac{70,000}{0.1}\left(1-\left[\frac{1}{1.1}\right]^{40}\right) &= \frac{c}{0.1}\left(1-\left[\frac{1}{1.1}\right]^{75}\right) \end{aligned}$$

**Econ 301**
**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

### Problem 1 (45p) (Well-behaved preferences)

Freddie Frolic consumes only two types of commodities: cheese curds, $x_1$, and diet coke, $x_2$.

a) The price of one portion of cheese curds is $p_1 = 5$ an one diet coke is $p_2 = 2$. Freddie's income spent entirely on the two commodities is $m = 60$. Show geometrically Freddie's budget set. Find relative price of cheese curds in terms of coke (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Suppose that due to shortages in cheese supply, cheese curds are rationed, i.e., each consumer can buy at most five portions. Show the new budget set on the graph.

c) Freddie' s utility is

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

-Find Marginal Rate of Substitution (MRS) as a function of parameters $a, b$ and $x_1, x_2$ (derive formula). For parameters $a = 4, b = 2$ and bundle $(7, 7)$ find value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable at the bundle $(7, 7)$? What is the geometric interpretation of MRS? (one sentence)

-Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions (two sentences for each).

-Derive optimal choice of $x_1$ and $x_2$ as a function of $a, b, p_1, p_2$ and $m$ (show the derivation of magic formulas). Is your solution corner or interior (chose one and provide mathematical argument)

d) Assume $a = 4, b = 2$ and $p_2 = 2, m = 60$. Find geometrically and determine analytically price offer curve and demand curve (give two functions). Is $x_1$ and ordinary or Giffen good and why (one sentence)?

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (derive one function from the other).

### Problem 2 (15p) (Quasilinear Preferences)

You are asked to plan a budget of University of Wisconsin, Madison for the next year. The two major expenses involve computers, $x_1$ and classroom furniture, $x_2$. The university' utility function is given by

$$U(x_1, x_2) = 2 \ln x_1 + x_2.$$

a) Find marginal rate of substitution as a function of $(x_1, x_2)$ (give formula).

b) Using two secrets of happiness find optimal "consumption" of computers and furniture if corresponding prices are $p_1 = 2$ and $p_2 = 4$ and the available funds are $m = 40$ (give two numbers).

c) Suppose the price of a computer goes down to $p_1 = 1$. Find optimal choice after the price change (two numbers). Decompose the change in $x_1$ into a substitution and income effect (two numbers).

d) Find optimal consumption for $p_1 = 2$ , $p_2 = 4$ and $m = 4$ (give two numbers). Is your solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

### Problem 3 (15p) (Perfect complements, Intertemporal choice)

Casper is a manager in a small startup firm. His income today is relatively small ($m_1 = 50$) but in the future (period two) he expects to become very rich ($m_2 = 200$)

a) Depict Casper's budget set assuming that he can borrow and save at the interest rate $r = 100\%$. Mark consumption plans on the budget line that involve savings and the plans that require borrowing. Find Present and Future Value of Casper's income (two numbers) and show the two in the graph.

b) Casper's utility function is $U(C_1, C_2) = \min(C_1, C_2)$. In the commodity space plot Casper's' indifference curves.

c) Find optimal consumption plan $(C_1, C_2)$ (give two numbers: use two secrets of happiness for perfect complements and the fact that $p_1 = 1$ and $p_2 = 1/(1+r)$). Find the level of savings/borrowing in equilibrium (one number). Is Casper smoothing his consumption over time? (yes-no answer)

**Problem 4 (25p) (Short questions)**

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption $C$, relaxation time $R$ and labor supply $L$ as a function of real wage rate $w/p_c$.(three nubmers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2) = 3x_1 + x_2$, prices $p_1 = \$8, p_2 = \$2$ and income $m = 100$. Is your solution corner or interior?

c) You are going to save $\$10,000$ when working (age 21-70) and then you are going to live for the next 30 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate $r = 3\%$.

d) Derive Present Value formula for perpetuity.

**Bonus question (Just for fun)**

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

**Econ 301**
**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

### Problem 1 (45p) (Well-behaved preferences)

Freddie Frolic consumes only two types of commodities: cheese curds, $x_1$, and diet coke, $x_2$.

a) The price of one portion of cheese curds is $p_1 = 10$ an one diet coke is $p_2 = 2$. Freddie's income spent entirely on the two commodities is $m = 120$. Show geometrically Freddie's budget set. Find relative price of cheese curds in terms of coke (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Suppose that due to shortages in cheese supply, cheese curds are rationed, i.e., each consumer can buy at most five portions. Show the new budget set on the graph.

c) Freddie's utility is

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

-Find Marginal Rate of Substitution (MRS) as a function of parameters $a, b$ and $x_1, x_2$ (derive formula). For parameters $a = 4, b = 2$ and bundle $(7, 14)$ find value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable at the bundle $(7, 7)$? What is the geometric interpretation of MRS? (one sentence)

-Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions (two sentences for each).

-Derive optimal choice of $x_1$ and $x_2$ as a function of $a, b, p_1, p_2$ and $m$ (show the derivation of magic formulas). Is your solution corner or interior (chose one and provide mathematical argument)

d) Assume $a = 4, b = 2$ and $p_2 = 2, m = 120$. Find geometrically and determine analytically price offer curve and demand curve (give two functions). Is $x_1$ and ordinary or Giffen good and why (one sentence)?

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$$U(x_1, x_2) = 2 \ln x_1 + x_2.$$

a) Find marginal rate of substitution as a function of $(x_1, x_2)$ (give formula).

b) Using two secrets of happiness find optimal "consumption" of computers and furniture if corresponding prices are $p_1 = 4$ and $p_2 = 8$ and the available funds are $m = 80$ (give two numbers).

c) Suppose the price of a computer goes down to $p_1 = 2$. Find optimal choice after the price change (two numbers). Decompose the change in $x_1$ into a substitution and income effect (two numbers).

d) Find optimal consumption for $p_1 = 4$, $p_2 = 8$ and $m = 8$ (give two numbers). Is your solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

### Problem 3 (15p) (Perfect complements, Intertemporal choice)

Casper is a manager in a small startup firm. His income today is relatively small ($m_1 = 100$) but in the future (period two) he expects to become very rich ($m_2 = 300$)

a) Depict Casper's budget set assuming that he can borrow and save at the interest rate $r = 200\%$. Mark consumption plans on the budget line that involve savings and the plans that require borrowing. Find Present and Future Value of Casper's income (two numbers) and show the two in the graph.

b) Casper's utility function is $U(C_1, C_2) = \min(C_1, C_2)$. In the commodity space plot Casper's' indifference curves.

c) Find optimal consumption plan $(C_1, C_2)$ (give two numbers: use two secrets of happiness for perfect complements and the fact that $p_1 = 1$ and $p_2 = 1/(1+r)$). Find the level of savings/borrowing in equilibrium (one number). Is Casper smoothing his consumption over time? (yes-no answer)

**Problem 4 (25p) (Short questions)**

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption $C$, relaxation time $R$ and labor supply $L$ as a function of real wage rate $w/p_c$.(three nubmers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2) = 5x_1 + x_2$, prices $p_1 = \$10, p_2 = \$1$ and income $m = 100$. Is your solution corner or interior?

c) You are going to save $\$30,000$ when working (age 21-50) and then you are going to live for the next 40 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate $r = 2\%$.

d) Derive Present Value formula for perpetuity.

**Bonus question (Just for fun)**

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

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## Midterm 1 (C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

### Problem 1 (45p) (Well-behaved preferences)

Freddie Frolic consumes only two types of commodities: cheese curds, $x_1$, and diet coke, $x_2$.

a) The price of one portion of cheese curds is $p_1 = 3$ an one diet coke is $p_2 = 1$. Freddie's income spent entirely on the two commodities is $m = 60$. Show geometrically Freddie's budget set. Find relative price of cheese curds in terms of coke (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Suppose that due to shortages in cheese supply, cheese curds are rationed, i.e., each consumer can buy at most five portions. Show the new budget set on the graph.

c) Freddie's utility is

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

-Find Marginal Rate of Substitution (MRS) as a function of parameters $a, b$ and $x_1, x_2$ (derive formula). For parameters $a = 4, b = 2$ and bundle $(3, 9)$ find value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable at the bundle $(7, 7)$? What is the geometric interpretation of MRS? (one sentence)

-Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions (two sentences for each).

-Derive optimal choice of $x_1$ and $x_2$ as a function of $a, b, p_1, p_2$ and $m$ (show the derivation of magic formulas). Is your solution corner or interior (chose one and provide mathematical argument)

d) Assume $a = 4, b = 2$ and $p_2 = 1, m = 60$. Find geometrically and determine analytically price offer curve and demand curve (give two functions). Is $x_1$ and ordinary or Giffen good and why (one sentence)?

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (derive one function from the other).

### Problem 2 (15p) (Quasilinear Preferences)

You are asked to plan a budget of University of Wisconsin, Madison for the next year. The two major expenses involve computers, $x_1$ and classroom furniture, $x_2$. The university' utility function is given by

$$U(x_1, x_2) = 4 \ln x_1 + x_2.$$

a) Find marginal rate of substitution as a function of $(x_1, x_2)$ (give formula).

b) Using two secrets of happiness find optimal "consumption" of computers and furniture if corresponding prices are $p_1 = 4$ and $p_2 = 2$ and the available funds are $m = 20$ (give two numbers).

c) Suppose the price of a computer goes down to $p_1 = 2$. Find optimal choice after the price change (two numbers). Decompose the change in $x_1$ into a substitution and income effect (two numbers).

d) Find optimal consumption for $p_1 = 4$, $p_2 = 2$ and $m = 4$ (give two numbers). Is your solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

### Problem 3 (15p) (Perfect complements, Intertemporal choice)

Casper is a manager in a small startup firm. His income today is relatively small ($m_1 = 50$) but in the future (period two) he expects to become very rich ($m_2 = 150$)

a) Depict Casper's budget set assuming that he can borrow and save at the interest rate $r = 100\%$. Mark consumption plans on the budget line that involve savings and the plans that require borrowing. Find Present and Future Value of Casper's income (two numbers) and show the two in the graph.

b) Casper's utility function is $U(C_1, C_2) = \min(C_1, C_2)$. In the commodity space plot Casper's' indifference curves.

c) Find optimal consumption plan $(C_1, C_2)$ (give two numbers: use two secrets of happiness for perfect complements and the fact that $p_1 = 1$ and $p_2 = 1/(1+r)$). Find the level of savings/borrowing in equilibrium (one number). Is Casper smoothing his consumption over time? (yes-no answer)

**Problem 4 (25p) (Short questions)**

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption $C$, relaxation time $R$ and labor supply $L$ as a function of real wage rate $w/p_c$.(three nubmers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2) = 5x_1 + x_2$, prices $p_1 = \$8, p_2 = \$2$ and income $m = 100$. Is your solution corner or interior?

c) You are going to save $\$50,000$ when working (age 21-80) and then you are going to live for the next 20 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate $r = 2\%$.

d) Derive Present Value formula for perpetuity.

**Bonus question (Just for fun)**

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

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## Midterm 1 (D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

### Problem 1 (45p) (Well-behaved preferences)

Freddie Frolic consumes only two types of commodities: cheese curds, $x_1$, and diet coke, $x_2$.

a) The price of one portion of cheese curds is $p_1 = 3$ an one diet coke is $p_2 = 3$. Freddie's income spent entirely on the two commodities is $m = 90$. Show geometrically Freddie's budget set. Find relative price of cheese curds in terms of coke (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Suppose that due to shortages in cheese supply, cheese curds are rationed, i.e., each consumer can buy at most five portions. Show the new budget set on the graph.

c) Freddie's utility is

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

-Find Marginal Rate of Substitution (MRS) as a function of parameters $a, b$ and $x_1, x_2$ (derive formula). For parameters $a = 4, b = 2$ and bundle $(3, 9)$ find value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable at the bundle $(7, 7)$? What is the geometric interpretation of MRS? (one sentence)

-Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions (two sentences for each).

-Derive optimal choice of $x_1$ and $x_2$ as a function of $a, b, p_1, p_2$ and $m$ (show the derivation of magic formulas). Is your solution corner or interior (chose one and provide mathematical argument)

d) Assume $a = 4, b = 2$ and $p_2 = 3, m = 90$. Find geometrically and determine analytically price offer curve and demand curve (give two functions). Is $x_1$ and ordinary or Giffen good and why (one sentence)?

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (derive one function from the other).

### Problem 2 (15p) (Quasilinear Preferences)

You are asked to plan a budget of University of Wisconsin, Madison for the next year. The two major expenses involve computers, $x_1$ and classroom furniture, $x_2$. The university' utility function is given by

$$U(x_1, x_2) = 4 \ln x_1 + x_2.$$

a) Find marginal rate of substitution as a function of $(x_1, x_2)$ (give formula).

b) Using two secrets of happiness find optimal "consumption" of computers and furniture if corresponding prices are $p_1 = 4$ and $p_2 = 4$ and the available funds are $m = 80$ (give two numbers).

c) Suppose the price of a computer goes down to $p_1 = 2$. Find optimal choice after the price change (two numbers). Decompose the change in $x_1$ into a substitution and income effect (two numbers).

d) Find optimal consumption for $p_1 = 4$, $p_2 = 4$ and $m = 4$ (give two numbers). Is your solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

### Problem 3 (15p) (Perfect complements, Intertemporal choice)

Casper is a manager in a small startup firm. His income today is relatively small ($m_1 = 100$) but in the future (period two) he expects to become very rich ($m_2 = 400$)

a) Depict Casper's budget set assuming that he can borrow and save at the interest rate $r = 100\%$. Mark consumption plans on the budget line that involve savings and the plans that require borrowing. Find Present and Future Value of Casper's income (two numbers) and show the two in the graph.

b) Casper's utility function is $U(C_1, C_2) = \min(C_1, C_2)$. In the commodity space plot Casper's' indifference curves.

c) Find optimal consumption plan $(C_1, C_2)$ (give two numbers: use two secrets of happiness for perfect complements and the fact that $p_1 = 1$ and $p_2 = 1/(1+r)$). Find the level of savings/borrowing in equilibrium (one number). Is Casper smoothing his consumption over time? (yes-no answer)

**Problem 4 (25p) (Short questions)**

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption $C$, relaxation time $R$ and labor supply $L$ as a function of real wage rate $w/p_c$.(three nubmers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2) = 5x_1 + x_2$, prices $p_1 = \$8, p_2 = \$2$ and income $m = 40$. Is your solution corner or interior?

c) You are going to save $\$6,000$ when working (age 21-80) and then you are going to live for the next 20 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate $r = 4\%$.

d) Derive Present Value formula for perpetuity.

**Bonus question (Just for fun)**

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

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## Makeup exam

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

### Problem 1 (45p) (Well-behaved preferences)

Freddie Frolic consumes only two types of commodities: cheese curds, $x_1$, and diet coke, $x_2$.

a) The price of one portion of cheese curds is $p_1 = 3$ an one diet coke is $p_2 = 3$. Freddie's income spent entirely on the two commodities is $m = 90$. Show geometrically Freddie's budget set. Find relative price of cheese curds in terms of coke (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of a budget set? (one sentence)

b) Suppose that due to shortages in cheese supply, cheese curds are rationed, i.e., each consumer can buy at most five portions. Show the new budget set on the graph.

c) Freddie's utility is

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

-Find Marginal Rate of Substitution (MRS) as a function of parameters $a, b$ and $x_1, x_2$ (derive formula). For parameters $a = 4, b = 2$ and bundle $(3, 9)$ find value of MRS (one number). Give economic interpretation of MRS (one sentence). Which of the goods is more valuable at the bundle $(7, 7)$? What is the geometric interpretation of MRS? (one sentence)

-Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions (two sentences for each).

-Derive optimal choice of $x_1$ and $x_2$ as a function of $a, b, p_1, p_2$ and $m$ (show the derivation of magic formulas). Are two goods gross substitutes, gross complements or none?

d) Assume $a = 4, b = 2$ and $p_1 = 3, p_2 = 6$. Find geometrically and determine analytically income offer curve and Engel curve (give two functions). Is $x_1$ and normal or inferior, and why (one sentence)?

e) Show that utility functions $V(x_1, x_2) = x_1^{3a} x_2^{3b}$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (derive one function from the other).

### Problem 2 (15p) (Quasilinear Preferences)

You are asked to plan a budget of University of Wisconsin, Madison for the next year. The two major expenses involve computers, $x_1$ and classroom furniture, $x_2$. The university' utility function is given by

$$U(x_1, x_2) = \ln x_1 + \frac{1}{2} x_2.$$

a) Find marginal rate of substitution as a function of $(x_1, x_2)$ (give formula).

b) Using two secrets of happiness find optimal "consumption" of computers and furniture if corresponding prices are $p_1 = 4$ and $p_2 = 4$ and the available funds are $m = 80$ (give two numbers).

c) Suppose the price of a computer goes down to $p_1 = 2$. Find optimal choice after the price change (two numbers). Decompose the change in $x_1$ into a substitution and income effect (two numbers).

d) Find optimal consumption for $p_1 = 4$, $p_2 = 4$ and $m = 4$ (give two numbers). Is your solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)

### Problem 3 (15p) (Perfect complements, Intertemporal choice)

Casper is a manager in a small startup firm. His income today is relatively small ($m_1 = 100$) but in the future (period two) he expects to become very rich ($m_2 = 600$)

a) Depict Casper's budget set assuming that he can borrow and save at the interest rate $r = 100\%$. Mark consumption plans on the budget line that involve savings and the plans that require borrowing. Find Present and Future Value of Casper's income (two numbers) and show the two in the graph.

b) Casper's utility function is $U(C_1, C_2) = 2\min(C_1, C_2)$. In the commodity space plot Casper's' indifference curves.

c) Find optimal consumption plan $(C_1, C_2)$ (give two numbers: use two secrets of happiness for perfect complements and the fact that $p_1 = 1$ and $p_2 = 1/(1+r)$). Find the level of savings/borrowing in equilibrium (one number). Is Casper smoothing his consumption over time? (yes-no answer)

**Problem 4 (25p) (Short questions)**

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption $C$, relaxation time $R$ and labor supply $L$ as a function of real wage rate $w/p_c$.(three nubmers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2) = 10x_1 + 5x_2$, prices $p_1 = \$8, p_2 = \$2$ and income $m = 40$. Is your solution corner or interior?

c) You are going to save $\$6,000$ when working (age 21-80) and then you are going to live for the next 20 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate $r = 4\%$.

d) Derive Present Value formula for perpetuity.

**Bonus question (Just for fun)**

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

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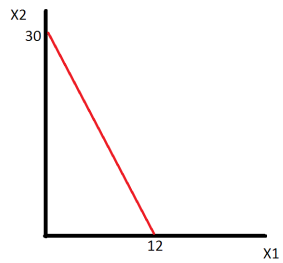
## Midterm 1 (A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

### Problem 1 (45p) (Well-behaved preferences)

a) Show geometrically Freddie's budget set. (5 points)



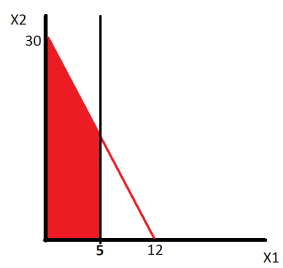

Find relative price of cheese curds in terms of coke. (2.5 points) **1 unit of cheese curds = 2.5 units of diet coke**

Give economic interpretation of the relative price. (2.5 points)

**This is the REAL PRICE of the two goods, meaning that in an exchange economy this would be the rate of exchange between the two goods**

Where can the relative price be seen in the graph? (2.5 points)
**The relative price can be seen (geometrically) as the slope of the budget constraint.**

b) Cheese curds are rationed, show the new budget set on the graph.(5 points)
**The area shaded red is the new budget set.**




c) Find (MRS) as a function of parameters (2.5 points)

$$MRS = -\frac{ax_2}{bx_1}$$

For parameters $a = 4, b = 2$ and bundle $(7, 7)$ find value of MRS. (2.5 points)
**MRS(7,7) = -2**
Give economic interpretation of MRS. (1 point)
**The marginal rate of substitution measure the rate at which the consumer is willing to substitute less consumption of good $x_1$ for more consumption of good $x_2$ while remaining at the same level of utility.**
Which of the goods is locally more valuable? ($\frac{1}{2}$ point)
**$x_1$ is locally more valuable at the bundle (7,7)**
What is the geometric interpretation of MRS? (1 point)
**The MRS is the slope of the indifference curve**

Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions. (5 points)

$$M = x_1p_1 + x_2p_2$$
$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{P_{x_1}}{P_{x_2}}$$

**The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, $x_1$ and $x_2$. The second condition is a utility maximization condition that requires the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. The second condition will effectively require the marginal utility per dollar of each good to equate at the optimal consumption bundle.**

Derive optimal choice of $x_1$ and $x_2$ as a function of $a, b, p_1, p_2$ and $m$ (show the derivation of magic formulas). Is your solution corner or interior (5 points)

$$M = x_1p_1 + x_2p_2$$
$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{P_{x_1}}{P_{x_2}}$$

**Solve for $P_1x_1$ in the second equation and substitute that into the budget constraint and solve for $X_1$. To get $x_2$ simply solve for $P_2x_2$ in the second equation and substitute that into the budget constraint.**

$$\frac{ax_2}{bx_1} = \frac{P_{x_1}}{P_{x_2}} \tag{1}$$
$$P_{x_1}x_1 = \frac{a}{b}x_2P_2 \tag{2}$$
$$M = \frac{a}{b}x_2P_2 + x_2P_2 \tag{3}$$
$$M = (\frac{a+b}{b}P_2)x_2 \tag{4}$$
$$x_2 = \frac{b}{a+b} \times \frac{m}{P_2} \tag{5}$$

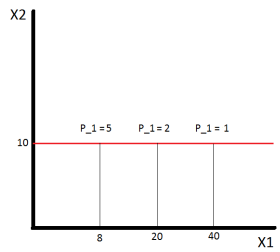
and

$$\frac{ax_2}{bx_1} = \frac{P_{x_1}}{P_{x_2}} \tag{6}$$
$$\frac{b}{a}P_{x_1}x_1 = x_2P_2 \tag{7}$$
$$M = \frac{b}{a}x_1P_1 + x_1P_1 \tag{8}$$
$$M = (\frac{a+b}{a}P_1)x_1 \tag{9}$$
$$x_1 = \frac{a}{a+b} \times \frac{m}{P_1}$$

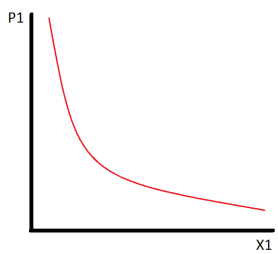
$$x_1 = \frac{a}{a+b} \times \frac{m}{P_1} \implies x_1 = \frac{2m}{3P_1}$$
$$x_2 = \frac{b}{a+b} \times \frac{m}{P_2} \implies x_2 = \frac{m}{3P_2}$$

d) Assume $a = 4, b = 2$ and $p_2 = 2, m = 60$. Find geometrically and determine analytically price offer curve (2.5 points) and demand curve (2.5 points). **The price offer curve can be plotted by taking the partial derivative of the demands for $x_1$ and $x_2$ with respect to $P_1$. This will give us the slope of the price offer curve, with the partial derivative of the demand for $x_2$ being the "rise" and the partial derivative of the demand for $x_1$ being the "run". Since the demand for $x_2$ does not depend on $P_1$ there is no vertical change in the**

**price offer curve, making it a horizontal line plotted at 10 units of $x_2$**




**The demand curve for $x_1$ can be plotted by taking the partial derivative of the demand for $x_1$ with respect to $P_1$. The slope of this curve is $\frac{\partial x_1(M,P_1}{\partial P_1)} = -\frac{2M}{3P_1^2}$**




Is $x_1$ and ordinary or Giffen good and why? (2 points)

**This is an ordinary good because there is a negative relationship between price and the quantity of the good demanded. This can be seen by taking the partial derivative of the demand function with respect to its own price:**

$$\frac{\partial x_1(M, P_1)}{\partial P_1} = -\frac{M}{2P_1^2}$$

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (3 points).

**The function U() appears to be the result of applying the natural logarithm to the function V(). That means that we can define the function F(x) = ln(x) so that $U(x_1, x_2) = F(V(x_1, x_2))$. The natural logarithm is a concave function (strictly increasing), meaning that applying the natural logarithm to a function will preserve the ordinality of that function. Thus, the function U() is merely a monotonic transformation of the function V(), meaning they represent the same preference order.**

**Problem 2 (15p) (Quasilinear Preferences)**

a) Find marginal rate of substitution as a function of $(x_1, x_2)$ (1 point).

$$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{2}{x_1}}{1} = \frac{2}{x_1}$$

b) Using two secrets of happiness find optimal consumption choices (4 points)

1)$40 = 2x_1 + 4x_2$

2)$\frac{2}{x_1} = \frac{2}{4} \implies x_1 = 4$

**Plug $x_1$ = 4 into the first secret (budget constraint) to reveal:** $x_1 = 4, x_2 = 8$

c) Suppose the price of a computer goes down to $p_1$= 1. Find optimal choice after the price change (1 point).

To find the new optimal choice repeat the above steps with $P_1 = 1$:

1)$40 = x_1 + 4x_2$

2)$\frac{2}{x_1} = \frac{1}{4} \implies x_1 = 8$

**Plug $x_1$ = 8 into the first secret (budget constraint) to reveal:** $x_1 = 8, x_2 = 8$

Decompose the change in $x_1$ into a substitution effect (2 points) and income effect (2 points).

To calculate the substitution and income effects we need to find an 'intermediate point'. This is the optimal consumption choice given the purchasing power necessary to buy the original optimal consumption bundle (4,8) given the new prices $P_1 = 1$, $P_2 = 4$:

M′ = (1)4 + (4)8 $\implies$ M′ = 36

To find the 'intermediate' optimal choice repeat the above steps with $P_1 = 1$ and M = 36:
1) $36 = x_1 + 4x_2$

2) $\frac{2}{x_1} = \frac{1}{4} \implies x_1 = 8$

**Plug $x_1$ = 8 into the first secret (budget constraint) to reveal:** $x_1 = 8, x_2 = 7$

**The substitution effect is the change in consumption of $x_1$ due only to the price change (holding purchasing power constant). That means the substitution effect is the difference between the amount of $x_1$ originally consumed (4 units) and how much is consumed after the price change (8 units). The substitution effect is then 4 units of $x_1$, while the income effect is zero (when we increase purchasing power from M = 36 to M = 40 we do not change our level of $x_1$ consumption, only $x_2$.**

d) Find optimal consumption for $p_1$= 2 , $p_2$= 4 and $m$ = 4 (1 point). 1) $4 = 2x_1 + 4x_2$

2) $\frac{2}{x_1} = \frac{2}{4} \implies x_1 = 4$

**Plug $x_1$ = 4 into the first secret (budget constraint) reveals that:** $x_1 = 4, x_2 = -1$**. We cannot consumer negative amounts of a consumption good, so this indicates we would prefer to give up 1 unit of $x_2$ in order to consume more $x_1$. This is not an option, however, as the lower bound on $x_2$ is zero.**

Is your solution interior? (1 point).

**Our solution is $x_1 = 2$, $x_2 = 0$, which is not interior (corner solution). Interior solutions are those at which consumption of each good is strictly greater than zero, which is not the case here.**

Is marginal utility of a dollar equalized? (3 points)

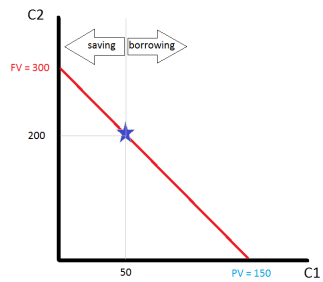
**No, we will very rarely see marginal utility per dollar of each consumption good equalized at a corner solution. In this case we have:**

$$\frac{MU_{x_1}}{P_1} = \frac{\frac{2}{x_1}}{2} = \frac{1}{2} \neq \frac{1}{4} = \frac{MU_{x_2}}{P_2}$$

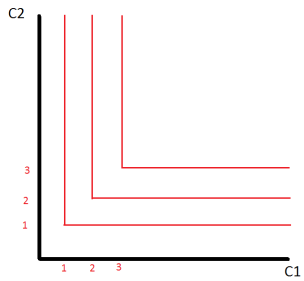
**Problem 3 (15p) (Perfect complements, Intertemporal choice)**

Casper is a manager in a small startup firm. His income today is relatively small ($m_1$= 50) but in the future (period two) he expects to become very rich ($m_2$= 200)

a) Depicts Casper's budget set assuming that he can borrow and save at the interest rate $r$ = 100%(1 point). Mark consumptions plans on the budget line that involve savings and the plans that require borrowing.(1 point) Find Present and Future Value of Casper's income and show the two in the graph.(2 points)




b) In the commodity space plot Casper's' indifference curves.(2 points)




c) Find optimal consumption plan $(C_1, C_2)$ (5 points). Find the level of savings/borrowing in equilibrium (2 points). Is Casper smoothing his consumption over time? (2 points)

1) $150 = C_1 + \frac{1}{1+r}C_2$

2) $C_1 = C_2$

**Plug $C_2$ into the budget constraint for $C_1$ gives Casper the equation:** $150 = \frac{1+r}{1+r}C_2 + \frac{1}{1+r}C_2$**. Plug in $r = 1$ and we see that this determines $C_2 = 100$ and looking back at the second secret of happiness, it also determines $C_1 = 100$. We find that $C_1 = C_2$, which means that Casper is consumption smoothing. In order to do this, he borrows \$50 from his second period income in the first period (reducing his second period consumption by \$100 and increasing his first period consumption by \$50 in the process.**

**Problem 4 (25p) (Short questions)**

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption $C$, relaxation time $R$ and labor supply $L$ as a function of real wage rate $w/p_c$.(three nubmers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2 = 3x_1 + x_2)$, prices $p_1$=\$8 and $p_2$=\$2 as well as income m=100. Is your solution corner or interior?

$$\frac{MU_{x_1}}{P_1} = \frac{3}{8} < \frac{1}{2} = \frac{MU_{x_2}}{P_2}$$

**The marginal utility per dollar is constant for both goods, but higher for good two, so in this case all income is spent on $x_2$. This makes the solution (0,50).**

c) You are going to save \$10,000 when working (age 21-70) and then you are going to live for the next 30 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate r=3%.

$$\frac{\$10,000}{r}(1 - (\frac{1}{1+r})^{50}) = \frac{C}{r}(1 - (\frac{1}{1+r})^{30}(\frac{1}{1+r})^{50}$$

d) Derive Present Value formula for perpetuity.

**The formula of a stream of payments that never ends is:**

$$\begin{aligned} PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + .... \\ &= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ....] \\ &= \frac{1}{(1+r)}[x + PV] \end{aligned}$$

**so we can solve for PV to get a more concise solution:**

$$\begin{aligned} (1 - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{1+r}{1+r} - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{r}{1+r})PV &= \frac{1}{1+r}x \\ PV &= \frac{x}{r} \end{aligned}$$

**Bonus question (Just for fun)**

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

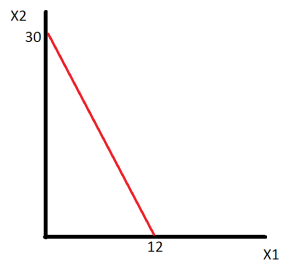
b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

**Midterm 1 (B)**

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

**Problem 1 (45p) (Well-behaved preferences)**

a) Show geometrically Freddie's budget set. (5 points)



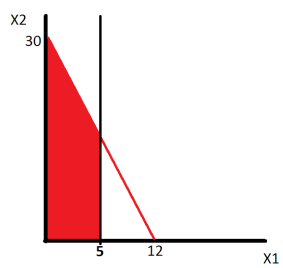

Find relative price of cheese curds in terms of coke. (2.5 points) **1 unit of cheese curds = 2.5 units of diet coke**

Give economic interpretation of the relative price. (2.5 points)

**This is the REAL PRICE of the two goods, meaning that in an exchange economy this would be the rate of exchange between the two goods**

Where can the relative price be seen in the graph? (2.5 points)
**The relative price can be seen (geometrically) as the slope of the budget constraint.**

b) Cheese curds are rationed, show the new budget set on the graph.(5 points)
**The area shaded red is the new budget set.**




c) Find (MRS) as a function of parameters (2.5 points)

$$MRS = -\frac{ax_2}{bx_1}$$

For parameters $a = 4, b = 2$ and bundle $(7, 7)$ find value of MRS. (2.5 points)
**MRS(7,7) = -2**
Give economic interpretation of MRS. (1 point)
**The marginal rate of substitution measure the rate at which the consumer is willing to substitute less consumption of good $x_1$ for more consumption of good $x_2$ while remaining at the same level of utility.**
Which of the goods is locally more valuable? ($\frac{1}{2}$ point)
**$x_1$ is locally more valuable at the bundle (7,7)**
What is the geometric interpretation of MRS? (1 point)
**The MRS is the slope of the indifference curve**
Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions. (5 points)

$$M = x_1p_1 + x_2p_2$$
$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{P_{x_1}}{P_{x_2}}$$

**The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, $x_1$ and $x_2$. The second condition is a utility maximization condition that requires the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. The second condition will effectively require the marginal utility per dollar of each good to equate at the optimal consumption bundle.**

Derive optimal choice of $x_1$ and $x_2$ as a function of $a, b, p_1, p_2$ and $m$ (show the derivation of magic formulas). Is your solution corner or interior (5 points)

$$M = x_1 p_1 + x_2 p_2$$
$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{P_{x_1}}{P_{x_2}}$$

**Solve for $P_1 x_1$ in the second equation and substitute that into the budget constraint and solve for $X_1$. To get $x_2$ simply solve for $P_2 x_2$ in the second equation and substitute that into the budget constraint.**

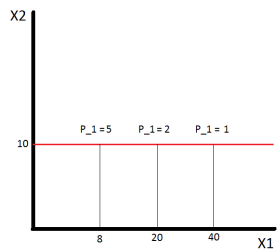
$$\frac{ax_2}{bx_1} = \frac{P_{x_1}}{P_{x_2}} \tag{10}$$
$$P_{x_1} x_1 = \frac{a}{b} x_2 P_2 \tag{11}$$
$$M = \frac{a}{b} x_2 P_2 + x_2 P_2 \tag{12}$$
$$M = (\frac{a+b}{b} P_2) x_2 \tag{13}$$
$$x_2 = \frac{b}{a+b} \times \frac{m}{P_2} \tag{14}$$

and

$$\frac{ax_2}{bx_1} = \frac{P_{x_1}}{P_{x_2}} \tag{15}$$
$$\frac{b}{a} P_{x_1} x_1 = x_2 P_2 \tag{16}$$
$$M = \frac{b}{a} x_1 P_1 + x_1 P_1 \tag{17}$$
$$M = (\frac{a+b}{a} P_1) x_1 \tag{18}$$
$$x_1 = \frac{a}{a+b} \times \frac{m}{P_1}$$

$$x_1 = \frac{a}{a+b} \times \frac{m}{P_1} \implies x_1 = \frac{2m}{3P_1}$$
$$x_2 = \frac{b}{a+b} \times \frac{m}{P_2} \implies x_2 = \frac{m}{3P_2}$$

d) Assume $a = 4, b = 2$ and $p_2 = 2, m = 60$. Find geometrically and determine analytically price offer curve (2.5 points) and demand curve (2.5 points). **The price offer curve can be plotted by taking the partial derivative of the demands for $x_1$ and $x_2$ with respect to $P_1$. This will give us the slope of the price offer curve, with the partial derivative of the demand for $x_2$ being the "rise" and the partial derivative of the demand for $x_1$ being the "run". Since the demand for $x_2$ does not depend on $P_1$ there is no vertical change in the price offer curve, making it a horizontal line plotted at 10 units of $x_2$**




**The demand curve for $x_1$ can be plotted by taking the partial derivative of the demand for $x_1$ with respect to $P_1$. The slope of this curve is $\frac{\partial x_1(M,P_1}{\partial P_1)} = -\frac{2M}{3P_1^2}$**


Is $x_1$ and ordinary or Giffen good and why? (2 points)

**This is an ordinary good because there is a negative relationship between price and the quantity of the good demanded. This can be seen by taking the partial derivative of the demand function with respect to its own price:**

$$\frac{\partial x_1(M, P_1)}{\partial P_1} = -\frac{M}{2P_1^2}$$

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (3 points).

**The function U() appears to be the result of applying the natural logarithm to the function V(). That means that we can define the function F(x) = ln(x) so that $U(x_1, x_2) = F(V(x_1, x_2))$. The natural logarithm is a concave function (strictly increasing), meaning that applying the natural logarithm to a function will preserve the ordinality of that function. Thus, the function U() is merely a monotonic transformation of the function V(), meaning they represent the same preference order.**

**Problem 2 (15p) (Quasilinear Preferences)**
a) Find marginal rate of substitution as a function of $(x_1, x_2)$ (1 point).

$$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{2}{x_1}}{1} = \frac{2}{x_1}$$

b) Using two secrets of happiness find optimal consumption choices (4 points)

1)$40 = 2x_1 + 4x_2$

2)$\frac{2}{x_1} = \frac{2}{4} \implies x_1 = 4$

**Plug $x_1$ = 4 into the first secret (budget constraint) to reveal:** $x_1 = 4, x_2 = 8$

c) Suppose the price of a computer goes down to $p_1$= 1. Find optimal choice after the price change (1 point).

To find the new optimal choice repeat the above steps with $P_1 = 1$:
1)$40 = x_1 + 4x_2$

2) $\frac{2}{x_1} = \frac{1}{4} \implies x_1 = 8$

**Plug $x_1$ = 8 into the first secret (budget constraint) to reveal:** $x_1 = 8, x_2 = 8$

Decompose the change in $x_1$ into a substitution effect (2 points) and income effect (2 points).

To calculate the substitution and income effects we need to find an 'intermediate point'. This is the optimal consumption choice given the purchasing power necessary to buy the original optimal consumption bundle (4,8) given the new prices $P_1 = 1$, $P_2 = 4$:

M′ = (1)4 + (4)8 $\implies$ M′ = 36

To find the 'intermediate' optimal choice repeat the above steps with $P_1 = 1$ and M = 36:
1) $36 = x_1 + 4x_2$

2) $\frac{2}{x_1} = \frac{1}{4} \implies x_1 = 8$

**Plug $x_1$ = 8 into the first secret (budget constraint) to reveal:** $x_1 = 8, x_2 = 7$

**The substitution effect is the change in consumption of $x_1$ due only to the price change (holding purchasing power constant). That means the substitution effect is the difference between the amount of $x_1$ originally consumed (4 units) and how much is consumed after the price change (8 units). The substitution effect is then 4 units of $x_1$, while the income effect is zero (when we increase purchasing power from M = 36 to M = 40 we do not change our level of $x_1$ consumption, only $x_2$.**

d) Find optimal consumption for $p_1$= 2 , $p_2$= 4 and $m$ = 4 (1 point). 1) $4 = 2x_1 + 4x_2$

2) $\frac{2}{x_1} = \frac{2}{4} \implies x_1 = 4$

**Plug $x_1$ = 4 into the first secret (budget constraint) reveals that:** $x_1 = 4, x_2 = -1$**. We cannot consumer negative amounts of a consumption good, so this indicates we would prefer to give up 1 unit of $x_2$ in order to consume more $x_1$. This is not an option, however, as the lower bound on $x_2$ is zero.**

Is your solution interior? (1 point).

**Our solution is $x_1 = 2$, $x_2 = 0$, which is not interior (corner solution). Interior solutions are those at which consumption of each good is strictly greater than zero, which is not the case here.**

Is marginal utility of a dollar equalized? (3 points)

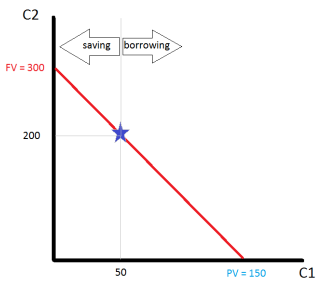
**No, we will very rarely see marginal utility per dollar of each consumption good equalized at a corner solution. In this case we have:**

$$\frac{MU_{x_1}}{P_1} = \frac{\frac{2}{x_1}}{2} = \frac{1}{2} \neq \frac{1}{4} = \frac{MU_{x_2}}{P_2}$$

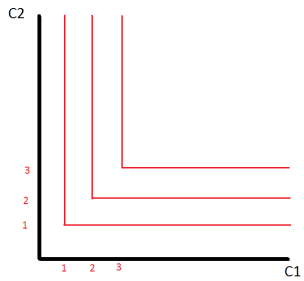
**Problem 3 (15p) (Perfect complements, Intertemporal choice)**

Casper is a manager in a small startup firm. His income today is relatively small ($m_1$= 50) but in the future (period two) he expects to become very rich ($m_2$= 200)

a) Depicts Casper's budget set assuming that he can borrow and save at the interest rate $r$ = 100%(1 point). Mark consumptions plans on the budget line that involve savings and the plans that require borrowing.(1 point) Find Present and Future Value of Casper's income and show the two in the graph.(2 points)




b) In the commodity space plot Casper's' indifference curves.(2 points)




c) Find optimal consumption plan $(C_1, C_2)$ (5 points). Find the level of savings/borrowing in equilibrium (2 points). Is Casper smoothing his consumption over time? (2 points)

1) $150 = C_1 + \frac{1}{1+r}C_2$

2) $C_1 = C_2$

**Plug $C_2$ into the budget constraint for $C_1$ gives Casper the equation:** $150 = \frac{1+r}{1+r}C_2 + \frac{1}{1+r}C_2$**. Plug in $r = 1$ and we see that this determines $C_2 = 100$ and looking back at the second secret of happiness, it also determines $C_1 = 100$. We find that $C_1 = C_2$, which means that Casper is consumption smoothing. In order to do this, he borrows \$50 from his second period income in the first period (reducing his second period consumption by \$100 and increasing his first period consumption by \$50 in the process.**

**Problem 4 (25p) (Short questions)**

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption $C$, relaxation time $R$ and labor supply $L$ as a function of real wage rate $w/p_c$.(three nubmers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

**The two secrets to happiness are:**

$$1) \quad (24 - R)W = P_c C$$
$$2) \quad \frac{MU_{x_1}}{MU_{x_2}} = \frac{P_1}{P_2}$$

$$\implies P_c C = WR$$
$$\implies 24W = P_c C + P_c C$$
$$\implies C = \frac{12W}{P_c} \; R = 12 \; L = 12$$

**We find that labor supply is perfectly inelastic because it does not vary with the real wage rate.**

b) Find optimal choice given utility function $U(x_1, x_2 = 5x_1 + x_2)$, prices $p_1$=\$10 and $p_2$=\$1 as well as income m=100. Is your solution corner or interior?

$$\frac{MU_{x_1}}{P_1} = \frac{5}{10} < \frac{1}{1} = \frac{MU_{x_2}}{P_2}$$

**The marginal utility per dollar is constant for both goods, but higher for good two, so in this case all income is spent on $x_2$. This makes the solution (0,100).**

c) You are going to save \$10,000 when working (age 21-70) and then you are going to live for the next 30 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate r=3%.

$$\frac{\$10,000}{r}(1-(\frac{1}{1+r})^{50}) = \frac{C}{r}(1-(\frac{1}{1+r})^{30}(\frac{1}{1+r})^{50}$$

d) Derive Present Value formula for perpetuity.

**The formula of a stream of payments that never ends is:**

$$\begin{aligned} PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + .... \\ &= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ....] \\ &= \frac{1}{(1+r)}[x + PV] \end{aligned}$$

**so we can solve for PV to get a more concise solution:**

$$\begin{aligned} (1-\frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{1+r}{1+r}-\frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{r}{1+r})PV &= \frac{1}{1+r}x \\ PV &= \frac{x}{r} \end{aligned}$$

**Bonus question (Just for fun)**

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

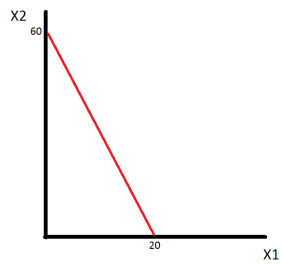
b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

**Midterm 1 (C)**

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

**Problem 1 (45p) (Well-behaved preferences)**

a) Show geometrically Freddie's budget set. (5 points)



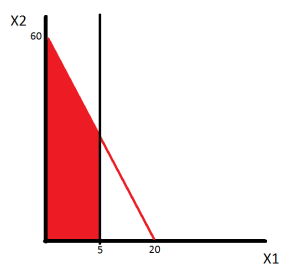

Find relative price of cheese curds in terms of coke. (2.5 points) **1 unit of cheese curds = 3 units of diet coke**

Give economic interpretation of the relative price. (2.5 points)

**This is the REAL PRICE of the two goods, meaning that in an exchange economy this would be the rate of exchange between the two goods**

Where can the relative price be seen in the graph? (2.5 points)
**The relative price can be seen (geometrically) as the slope of the budget constraint.**

b) Cheese curds are rationed, show the new budget set on the graph.(5 points)
**The area shaded red is the new budget set.**




c) Find (MRS) as a function of parameters (2.5 points)

$$MRS = -\frac{ax_2}{bx_1}$$

For parameters $a = 4, b = 2$ and bundle $(3, 9)$ find value of MRS. (2.5 points)
**MRS(3,9) = -6**
Give economic interpretation of MRS. (1 point)
**The marginal rate of substitution measure the rate at which the consumer is willing to substitute less consumption of good $x_1$ for more consumption of good $x_2$ while remaining at the same level of utility.**
Which of the goods is locally more valuable? ($\frac{1}{2}$ point)
$x_1$ **is locally more valuable at the bundle (7,7)**
What is the geometric interpretation of MRS? (1 point)
**The MRS is the slope of the indifference curve**
Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions. (5 points)

$$M = x_1p_1 + x_2p_2$$
$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{P_{x_1}}{P_{x_2}}$$

**The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, $x_1$ and $x_2$. The second condition is a utility maximization condition that requires the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. The second condition will effectively require the marginal utility per dollar of each good to equate at the optimal consumption bundle.**

Derive optimal choice of $x_1$ and $x_2$ as a function of $a, b, p_1, p_2$ and $m$ (show the derivation of magic formulas). Is your solution corner or interior (5 points)

$$M = x_1 p_1 + x_2 p_2$$
$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{P_{x_1}}{P_{x_2}}$$

**Solve for $P_1 x_1$ in the second equation and substitute that into the budget constraint and solve for $X_1$. To get $x_2$ simply solve for $P_2 x_2$ in the second equation and substitute that into the budget constraint.**

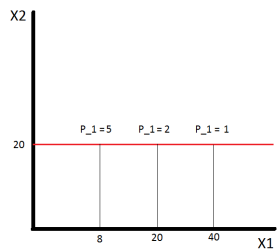
$$\frac{ax_2}{bx_1} = \frac{P_{x_1}}{P_{x_2}} \tag{19}$$
$$P_{x_1} x_1 = \frac{a}{b} x_2 P_2 \tag{20}$$
$$M = \frac{a}{b} x_2 P_2 + x_2 P_2 \tag{21}$$
$$M = (\frac{a+b}{b} P_2) x_2 \tag{22}$$
$$x_2 = \frac{b}{a+b} \times \frac{m}{P_2} \tag{23}$$

and

$$\frac{ax_2}{bx_1} = \frac{P_{x_1}}{P_{x_2}} \tag{24}$$
$$\frac{b}{a} P_{x_1} x_1 = x_2 P_2 \tag{25}$$
$$M = \frac{b}{a} x_1 P_1 + x_1 P_1 \tag{26}$$
$$M = (\frac{a+b}{a} P_1) x_1 \tag{27}$$
$$x_1 = \frac{a}{a+b} \times \frac{m}{P_1}$$

$$x_1 = \frac{a}{a+b} \times \frac{m}{P_1} \implies x_1 = \frac{2m}{3P_1}$$
$$x_2 = \frac{b}{a+b} \times \frac{m}{P_2} \implies x_2 = \frac{m}{3P_2}$$

d) Assume $a = 4, b = 2$ and $p_2 = 1, m = 60$. Find geometrically and determine analytically price offer curve (2.5 points) and demand curve (2.5 points). **The price offer curve can be plotted by taking the partial derivative of the demands for $x_1$ and $x_2$ with respect to $P_1$. This will give us the slope of the price offer curve, with the partial derivative of the demand for $x_2$ being the "rise" and the partial derivative of the demand for $x_1$ being the "run". Since the demand for $x_2$ does not depend on $P_1$ there is no vertical change in the price offer curve, making it a horizontal line plotted at 20 units of $x_2$**




**The demand curve for $x_1$ can be plotted by taking the partial derivative of the demand for $x_1$ with respect to $P_1$. The slope of this curve is $\frac{\partial x_1(M,P_1}{\partial P_1)} = -\frac{2M}{3P_1^2} \implies x_1 = \frac{-40}{P_1}$**


Is $x_1$ and ordinary or Giffen good and why? (2 points)

**This is an ordinary good because there is a negative relationship between price and the quantity of the good demanded. This can be seen by taking the partial derivative of the demand function with respect to its own price:**

$$\frac{\partial x_1(M, P_1)}{\partial P_1} = -\frac{M}{2P_1^2}$$

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (3 points).

**The function U() appears to be the result of applying the natural logarithm to the function V(). That means that we can define the function F(x) = ln(x) so that $U(x_1, x_2) = F(V(x_1, x_2))$. The natural logarithm is a concave function (strictly increasing), meaning that applying the natural logarithm to a function will preserve the ordinality of that function. Thus, the function U() is merely a monotonic transformation of the function V(), meaning they represent the same preference order.**

**Problem 2 (15p) (Quasilinear Preferences)**
a) Find marginal rate of substitution as a function of $(x_1, x_2)$ (1 point).

$$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{4}{x_1}}{1} = \frac{4}{x_1}$$

b) Using two secrets of happiness find optimal consumption choices (4 points)

1)$20 = 4x_1 + 2x_2$

2)$\frac{4}{x_1} = \frac{4}{2} \implies x_1 = 2$

**Plug $x_1$ = 2 into the first secret (budget constraint) to reveal:** $x_1 = 2, x_2 = 6$

c) Suppose the price of a computer goes down to $p_1$= 2. Find optimal choice after the price change (1 point).

To find the new optimal choice repeat the above steps with $P_1 = 1$:
1)$20 = 2x_1 + 2x_2$

2) $\frac{4}{x_1} = \frac{2}{2} \implies x_1 = 4$

**Plug $x_1$ = 4 into the first secret (budget constraint) to reveal:** $x_1 = 4, x_2 = 6$

Decompose the change in $x_1$ into a substitution effect (2 points) and income effect (2 points).

To calculate the substitution and income effects we need to find an 'intermediate point'. This is the optimal consumption choice given the purchasing power necessary to buy the original optimal consumption bundle (2,6) given the new prices $P_1 = 2$, $P_2 = 2$:

M′ = (2)2 + (2)6 $\implies$ M′ = 116

To find the 'intermediate' optimal choice repeat the above steps with $P_1 = 2$ and M = 16:
1) $16 = 2x_1 + 2x_2$

2) $\frac{4}{x_1} = \frac{2}{2} \implies x_1 = 4$

**Plug $x_1$ = 4 into the first secret (budget constraint) to reveal:** $x_1 = 4, x_2 = 4$

**The substitution effect is the change in consumption of $x_1$ due only to the price change (holding purchasing power constant). That means the substitution effect is the difference between the amount of $x_1$ originally consumed (2 units) and how much is consumed after the price change (4 units). The substitution effect is then 2 units of $x_1$, while the income effect is zero (when we increase purchasing power from M = 16 to M = 20 we do not change our level of $x_1$ consumption, only $x_2$.**

d) Find optimal consumption for $p_1$= 4 , $p_2$= 2 and $m$ = 4 (1 point). 1) $4 = 4x_1 + 2x_2$

2) $\frac{4}{x_1} = \frac{4}{2} \implies x_1 = 2$

**Plug $x_1$ = 2 into the first secret (budget constraint) reveals that:** $x_1 = 2, x_2 = -2$**. We cannot consumer negative amounts of a consumption good, so this indicates we would prefer to give up 2 unit of $x_2$ in order to consume more $x_1$. This is not an option, however, as the lower bound on $x_2$ is zero.**

Is your solution interior? (1 point).

**Our solution is $x_1 = 1$, $x_2 = 0$, which is not interior (corner solution). Interior solutions are those at which consumption of each good is strictly greater than zero, which is not the case here.**

Is marginal utility of a dollar equalized? (3 points)

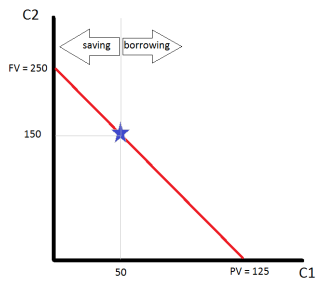
**No, we will very rarely see marginal utility per dollar of each consumption good equalized at a corner solution. In this case we have:**

$$\frac{MU_{x_1}}{P_1} = \frac{\frac{4}{x_1}}{4} = \frac{4}{4} \neq \frac{1}{2} = \frac{MU_{x_2}}{P_2}$$

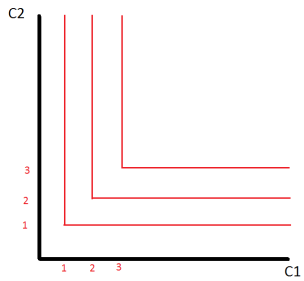
**Problem 3 (15p) (Perfect complements, Intertemporal choice)**

Casper is a manager in a small startup firm. His income today is relatively small ($m_1$= 50) but in the future (period two) he expects to become very rich ($m_2$= 150)

a) Depicts Casper's budget set assuming that he can borrow and save at the interest rate $r$ = 100%(1 point). Mark consumptions plans on the budget line that involve savings and the plans that require borrowing.(1 point) Find Present and Future Value of Casper's income and show the two in the graph.(2 points)




b) In the commodity space plot Casper's' indifference curves.(2 points)




c) Find optimal consumption plan $(C_1, C_2)$ (5 points). Find the level of savings/borrowing in equilibrium (2 points). Is Casper smoothing his consumption over time? (2 points)

1) $125 = C_1 + \frac{1}{1+r}C_2$

2) $C_1 = C_2$

**Plug $C_2$ into the budget constraint for $C_1$ gives Casper the equation: $125 = \frac{1+r}{1+r}C_2 + \frac{1}{1+r}C_2$. Plug in $r = 1$ and we see that this determines $C_2 = \frac{250}{3}$ and looking back at the second secret of happiness, it also determines $C_1 = \frac{250}{3}$. We find that $C_1 = C_2$, which means that Casper is consumption smoothing. In order to do this, he borrows $\frac{100}{3}$ from his second period income in the first period (reducing his second period consumption by $\frac{00}{3}$ and increasing his first period consumption by 33.33 dollars in the process.**

**Problem 4 (25p) (Short questions)**

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption $C$, relaxation time $R$ and labor supply $L$ as a function of real wage rate $w/p_c$.(three nubmers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2 = 5x_1 + x_2)$, prices $p_1$=\$8 and $p_2$=\$2 as well as income m=100. Is your solution corner or interior?

$$\frac{MU_{x_1}}{P_1} = \frac{5}{8} > \frac{1}{2} = \frac{MU_{x_2}}{P_2}$$

**The marginal utility per dollar is constant for both goods, but higher for good one, so in this case all income is spent on $x_1$. This makes the solution (12.5,0).**

c) You are going to save \$50,000when working (age 21-80) and then you are going to live for the next 20 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate r=2%.

$$\frac{\$50,000}{r}(1 - (\frac{1}{1+r})^{60}) = \frac{C}{r}(1 - (\frac{1}{1+r})^{20}(\frac{1}{1+r})^{60}$$

d) Derive Present Value formula for perpetuity.

**The formula of a stream of payments that never ends is:**

$$\begin{aligned} PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + .... \\ &= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ....] \\ &= \frac{1}{(1+r)}[x + PV] \end{aligned}$$

**so we can solve for PV to get a more concise solution:**

$$\begin{aligned} (1 - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{1+r}{1+r} - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{r}{1+r})PV &= \frac{1}{1+r}x \\ PV &= \frac{x}{r} \end{aligned}$$

**Bonus question (Just for fun)**

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

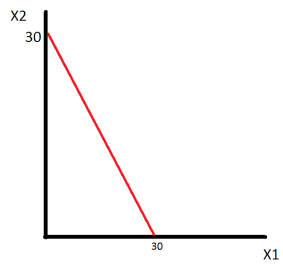
b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

**Midterm 1 (D)**

You have 70 minutes to complete the exam. The midterm consists of 4 questions (45+15+15+25=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

**Problem 1 (45p) (Well-behaved preferences)**

a) Show geometrically Freddie's budget set. (5 points)



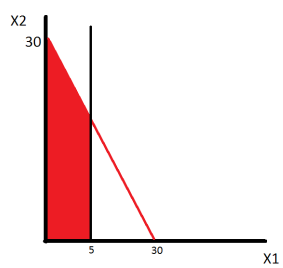

Find relative price of cheese curds in terms of coke. (2.5 points) **1 unit of cheese curds = 1 unit of diet coke**

Give economic interpretation of the relative price. (2.5 points)

**This is the REAL PRICE of the two goods, meaning that in an exchange economy this would be the rate of exchange between the two goods**

Where can the relative price be seen in the graph? (2.5 points)
**The relative price can be seen (geometrically) as the slope of the budget constraint.**

b) Cheese curds are rationed, show the new budget set on the graph.(5 points)
**The area shaded red is the new budget set.**




c) Find (MRS) as a function of parameters (2.5 points)

$$MRS = -\frac{ax_2}{bx_1}$$

For parameters $a = 4, b = 2$ and bundle $(3, 9)$ find value of MRS. (2.5 points)
**MRS(3,9) = -6**
Give economic interpretation of MRS. (1 point)
**The marginal rate of substitution measure the rate at which the consumer is willing to substitute less consumption of good $x_1$ for more consumption of good $x_2$ while remaining at the same level of utility.**
Which of the goods is locally more valuable? ($\frac{1}{2}$ point)
$x_1$ **is locally more valuable at the bundle (7,7)**
What is the geometric interpretation of MRS? (1 point)
**The MRS is the slope of the indifference curve**
Write down two secrets of happiness that determine optimal choice. Explain economic intuition behind the two conditions. (5 points)

$$M = x_1p_1 + x_2p_2$$
$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{P_{x_1}}{P_{x_2}}$$

**The first equation is a budget constraint which requires consumers to spend all of their income on the consumption of two goods, $x_1$ and $x_2$. The second condition is a utility maximization condition that requires the slope of the indifference curve passing through the optimal consumption bundle to be tangent to the price ratio. The second condition will effectively require the marginal utility per dollar of each good to equate at the optimal consumption bundle.**

Derive optimal choice of $x_1$ and $x_2$ as a function of $a, b, p_1, p_2$ and $m$ (show the derivation of magic formulas). Is your solution corner or interior (5 points)

$$M = x_1 p_1 + x_2 p_2$$
$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{P_{x_1}}{P_{x_2}}$$

**Solve for $P_1 x_1$ in the second equation and substitute that into the budget constraint and solve for $X_1$. To get $x_2$ simply solve for $P_2 x_2$ in the second equation and substitute that into the budget constraint.**

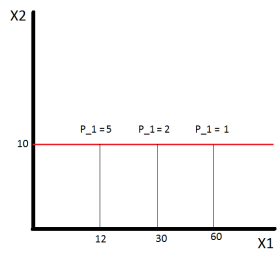
$$\frac{ax_2}{bx_1} = \frac{P_{x_1}}{P_{x_2}} \tag{28}$$
$$P_{x_1} x_1 = \frac{a}{b} x_2 P_2 \tag{29}$$
$$M = \frac{a}{b} x_2 P_2 + x_2 P_2 \tag{30}$$
$$M = (\frac{a+b}{b} P_2) x_2 \tag{31}$$
$$x_2 = \frac{b}{a+b} \times \frac{m}{P_2} \tag{32}$$

and

$$\frac{ax_2}{bx_1} = \frac{P_{x_1}}{P_{x_2}} \tag{33}$$
$$\frac{b}{a} P_{x_1} x_1 = x_2 P_2 \tag{34}$$
$$M = \frac{b}{a} x_1 P_1 + x_1 P_1 \tag{35}$$
$$M = (\frac{a+b}{a} P_1) x_1 \tag{36}$$
$$x_1 = \frac{a}{a+b} \times \frac{m}{P_1}$$

$$x_1 = \frac{a}{a+b} \times \frac{m}{P_1} \implies x_1 = \frac{2m}{3P_1}$$
$$x_2 = \frac{b}{a+b} \times \frac{m}{P_2} \implies x_2 = \frac{m}{3P_2}$$

d) Assume $a = 4, b = 2$ and $p_2 = 3, m = 90$. Find geometrically and determine analytically price offer curve (2.5 points) and demand curve (2.5 points). **The price offer curve can be plotted by taking the partial derivative of the demands for $x_1$ and $x_2$ with respect to $P_1$. This will give us the slope of the price offer curve, with the partial derivative of the demand for $x_2$ being the "rise" and the partial derivative of the demand for $x_1$ being the "run". Since the demand for $x_2$ does not depend on $P_1$ there is no vertical change in the price offer curve, making it a horizontal line plotted at 10 units of $x_2$**




**The demand curve for $x_1$ can be plotted by taking the partial derivative of the demand for $x_1$ with respect to $P_1$. The slope of this curve is $\frac{\partial x_1(M,P_1}{\partial P_1)} = -\frac{2M}{3P_1^2} \implies x_1 = \frac{-60}{P_1}$**


Is $x_1$ and ordinary or Giffen good and why? (2 points)

**This is an ordinary good because there is a negative relationship between price and the quantity of the good demanded. This can be seen by taking the partial derivative of the demand function with respect to its own price:**

$$\frac{\partial x_1(M, P_1)}{\partial P_1} = -\frac{M}{2P_1^2}$$

e) Show that utility functions $V(x_1, x_2) = x_1^a x_2^b$ and $U(x_1, x_2) = a \ln x_1 + b \ln x_2$ represent the same preferences (3 points).

**The function U() appears to be the result of applying the natural logarithm to the function V(). That means that we can define the function F(x) = ln(x) so that $U(x_1, x_2) = F(V(x_1, x_2))$. The natural logarithm is a concave function (strictly increasing), meaning that applying the natural logarithm to a function will preserve the ordinality of that function. Thus, the function U() is merely a monotonic transformation of the function V(), meaning they represent the same preference order.**

**Problem 2 (15p) (Quasilinear Preferences)**
a) Find marginal rate of substitution as a function of $(x_1, x_2)$ (1 point).

$$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{4}{x_1}}{1} = \frac{4}{x_1}$$

b) Using two secrets of happiness find optimal consumption choices (4 points)

1)$80 = 4x_1 + 4x_2$

2)$\frac{4}{x_1} = \frac{4}{4} \implies x_1 = 4$

**Plug $x_1$ = 4 into the first secret (budget constraint) to reveal:** $x_1 = 4, x_2 = 16$

c) Suppose the price of a computer goes down to $p_1$= 2. Find optimal choice after the price change (1 point).

To find the new optimal choice repeat the above steps with $P_1 = 2$:
1)$80 = 2x_1 + 4x_2$

2) $\frac{4}{x_1} = \frac{2}{4} \implies x_1 = 8$

**Plug $x_1$ = 8 into the first secret (budget constraint) to reveal:** $x_1 = 8, x_2 = 16$

Decompose the change in $x_1$ into a substitution effect (2 points) and income effect (2 points).

To calculate the substitution and income effects we need to find an 'intermediate point'. This is the optimal consumption choice given the purchasing power necessary to buy the original optimal consumption bundle (2,6) given the new prices $P_1 = 2$, $P_2 = 2$:

M′ = (2)4 + (4)16 $\implies$ M′ = 72

To find the 'intermediate' optimal choice repeat the above steps with $P_1 = 2$ and M = 72:
1) $72 = 2x_1 + 4x_2$

2) $\frac{4}{x_1} = \frac{2}{4} \implies x_1 = 8$

**Plug $x_1$ = 8 into the first secret (budget constraint) to reveal:** $x_1 = 8, x_2 = 14$

**The substitution effect is the change in consumption of $x_1$ due only to the price change (holding purchasing power constant). That means the substitution effect is the difference between the amount of $x_1$ originally consumed (4 units) and how much is consumed after the price change (8 units). The substitution effect is then 4 units of $x_1$, while the income effect is zero (when we increase purchasing power from M = 72 to M = 80 we do not change our level of $x_1$ consumption, only $x_2$.**

d) Find optimal consumption for $p_1$= 4 , $p_2$= 4 and $m$ = 4 (1 point). 1) $4 = 4x_1 + 4x_2$

2) $\frac{4}{x_1} = \frac{4}{4} \implies x_1 = 4$

**Plug $x_1$ = 2 into the first secret (budget constraint) reveals that:** $x_1 = 2, x_2 = -3$**. We cannot consumer negative amounts of a consumption good, so this indicates we would prefer to give up 3 unit of $x_2$ in order to consume more $x_1$. This is not an option, however, as the lower bound on $x_2$ is zero.**

Is your solution interior? (1 point).

**Our solution is $x_1 = 1$, $x_2 = 0$, which is not interior (corner solution). Interior solutions are those at which consumption of each good is strictly greater than zero, which is not the case here.**

Is marginal utility of a dollar equalized? (3 points)

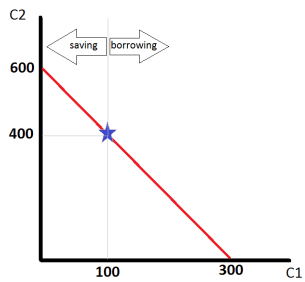
**No, we will very rarely see marginal utility per dollar of each consumption good equalized at a corner solution. In this case we have:**

$$\frac{MU_{x_1}}{P_1} = \frac{\frac{4}{x_1}}{4} = \frac{4}{4} \neq \frac{1}{4} = \frac{MU_{x_2}}{P_2}$$

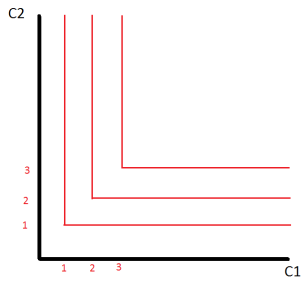
**Problem 3 (15p) (Perfect complements, Intertemporal choice)**

Casper is a manager in a small startup firm. His income today is relatively small ($m_1$= 100) but in the future (period two) he expects to become very rich ($m_2$= 400)

a) Depicts Casper's budget set assuming that he can borrow and save at the interest rate $r$ = 100%(1 point). Mark consumptions plans on the budget line that involve savings and the plans that require borrowing.(1 point) Find Present and Future Value of Casper's income and show the two in the graph.(2 points)




b) In the commodity space plot Casper's' indifference curves.(2 points)




c) Find optimal consumption plan $(C_1, C_2)$ (5 points). Find the level of savings/borrowing in equilibrium (2 points). Is Casper smoothing his consumption over time? (2 points)

1) $300 = C_1 + \frac{1}{1+r}C_2$

2) $C_1 = C_2$

**Plug $C_2$ into the budget constraint for $C_1$ gives Casper the equation:** $300 = \frac{1+r}{1+r}C_2 + \frac{1}{1+r}C_2$**. Plug in** $r = 1$ **and we see that this determines** $C_2 = 200$ **and looking back at the second secret of happiness, it also determines** $C_1 = 200$**. We find that** $C_1 = C_2$**, which means that Casper is consumption smoothing. In order to do this, he borrows from his second period income in the first period (reducing his second period consumption by \$200 and increasing his first period consumption by \$100 in the process.**

**Problem 4 (25p) (Short questions)**

a) Assume utility function $U(C, R) = C \times R$ and the daily endowment of time equal to $24h$. Find optimal choice of consumption $C$, relaxation time $R$ and labor supply $L$ as a function of real wage rate $w/p_c$.(three nubmers) use magic formula). Is labor supply elastic or inelastic (one sentence)?

b) Find optimal choice given utility function $U(x_1, x_2 = 5x_1 + x_2)$, prices $p_1$=\$8 and $p_2$=\$2 as well as income m=40. Is your solution corner or interior?

$$\frac{MU_{x_1}}{P_1} = \frac{5}{8} > \frac{1}{2} = \frac{MU_{x_2}}{P_2}$$

**The marginal utility per dollar is constant for both goods, but higher for good one, so in this case all income is spent on $x_1$. This makes the solution (5,0).**

c) You are going to save \$6,000when working (age 21-80) and then you are going to live for the next 20 years. Write down equation that determines constant (maximal) level of consumption during retirement age given your savings. Assume annual interest rate r=4%.

$$\frac{\$6,000}{r}(1 - (\frac{1}{1+r})^{60}) = \frac{C}{r}(1 - (\frac{1}{1+r})^{20}(\frac{1}{1+r})^{60}$$

d) Derive Present Value formula for perpetuity.

**The formula of a stream of payments that never ends is:**

$$\begin{aligned} PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + .... \\ &= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ....] \\ &= \frac{1}{(1+r)}[x + PV] \end{aligned}$$

**so we can solve for PV to get a more concise solution:**

$$\begin{aligned} (1 - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{1+r}{1+r} - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{r}{1+r})PV &= \frac{1}{1+r}x \\ PV &= \frac{x}{r} \end{aligned}$$

**Bonus question (Just for fun)**

a) Prove that for perfect complements $U(x_1, x_2) = \min(ax_1, bx_2)$, MRS is equal to zero for all bundles below the optimal proportion line and equal to $-\infty$ for bundles above it.

b) Explain in words why the solution to a linear optimization problem such as with perfect substitutes is called a bang bang solution.

**Econ 301**
**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

**Problem 1 (50p)** (Standard choice)

Jim has two pleasures in his life, drinking Port wine $(x_1)$ and reading books $(x_2)$.

a) Port wine $x_1$ costs $p_1 = \$20$ a bottle and each book $x_2$, $p_2 = \$20$, and his daily budget is $m = \$600$. Show geometrically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set would be affected by introduction of ad valorem tax on imported wine (but not books) assuming tax rate $\tau = 100\%$.

b) Jim seems to be a fairly sophisticated fellow with quite a complicated utility function

$$U(x_1, x_2) = 123 \times \log\left[\exp\left[(6\ln x_1 + 3\ln x_2)^2\right] + 10\right]^{322}$$

Argue that in fact Jim is not really that sophisticated as his preferences can represented by a significantly simpler utility function. (one sentence + simpler utility function). If you are unable to answer point b) in the remaining part of Problem 1, you can assume $U(x_1, x_2) = x_1^2 x_2$.

c) Find Jim's marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give a formula for MRS). What is the value of MRS at consumption bundle $(1, 2)$ (give a number)? Which of the two commodities is more precious to Jim?

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).
- Describe how the two "secrets of happiness" can be seen in the graph.(graph + 2 sentences)
- Derive the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation).

e) Using your formula from d), find the optimal consumption levels of both types of commodities $(x_1, x_2)$ for:

- $p_1 = \$40, p_2 = \$20$ and $m = \$600$ (give two numbers).

and after the price of Port wine decreased:

- for $p_1 = \$20, p_2 = \$20$ and $m = \$600$ (give two numbers).

What is the total change in consumption of Port wine? (give a number). Illustrate the change on the graph. Is Port wine an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of $x_1$ from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

**Problem 2 (20p)** (Intertemporal choice)

Marky Mark is a rap singer who earns $m_1 = \$100$ when young and $m_2 = \$300$ when old.

a) What is the present value (in terms of \$ from period one) of Marky Mark's income (give one number).

b) Write down Marky Mark's budget constraint (one inequality) and plot budget set given interest rate $r = 200\%$ in the graph. Mark all consumption plans on the budget line that require borrowing and the ones that require saving.

c) Marky Mark's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$ and discount rate is $\delta = 2$. Provide an economic interpretation of parameter $\delta$ (one sentence). Using magic formulas, find the optimal consumption plan and the optimal saving strategy. (give three numbers $C_1, C_2, S$). Does Sam smooth his consumption? (yes no + one sentence) Is Sam tilting his consumption? (yes no + one sentence)

d) (Perpetuity) Your sister has just promised to send you pocket money of \$500 each month starting next month and she will keep doing it forever. What is the present value of "having such a sister" if monthly interest rate is equal to 1% (one number).

e) (Annuity) You are going to work for 45 years (from 20 to 65) earning income of \$1000,000 a year. Then you are going to retire and going to live for another 35 years (till you are 100). Assume that the annual interest is given by $r = 5\%$. Write down equation that would allow you to determine the maximal constant level of consumption $C$ throughout your whole life (80 years). (write down equation that determines PV of income and consumption, but you need not calculate $C$).

**Problem 3 (15p)** (Perfect complements)

The old recipe for Pierogies (Polish dumplings) requires that sauerkraut $x_1$ is mixed with portabella mushrooms $x_2$ in a fixed (and sacred) proportion of 4:1.

a) Propose a utility function over sauerkraut and mushrooms (function $U(x_1, x_2)$) assuming that pierogies is a good.

b) In the commodity space, carefully depict indifference curves (marking the optimal proportion line).

c) Assume that $p_1 = 2$ and $p_2 = 2$ and income is $m = 40$. Write down two secrets of happiness (give two equations) that determine the optimal choice (two numbers). Explain the economic intuition behind the conditions (one sentence for each secret). Is your solution interior (yes or no)?

d) Without any calculations, in two separate graphs plot the price offer curve and income offer curve (just plot two curves).

**Problem 4 (15p)** (Perfect substitutes)

Sam does not have any money but is endowed with $\omega_1 = 10$ apples and $\omega_1 = 30$ oranges and prices of the two are $p_1 = p_2 = 1$. His utility function is given by $U(x_1, x_2) = 2x_1 + x_2$

a) Find Sam's budget line, marking the endowment point.

b) Find the optimal choice of the two commodities if his utility is given by $U(x_1, x_2) = 2x_1 + x_2$ (give two numbers). Is your solution interior?

c) Find net demands in optimum (give two numbers). Is Sam a net buyer or a net seller of apples. How about oranges? (chose one for each commodity)

d) In mathematics, the solution to a problem with perfect substitutes is called "bang, bang" solution. Provide intuition why (one sentence).

**Bonus question** (Just for fun)

Derive PV formula for annuity and perpetuity.

**Econ 301**
**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

**Problem 1 (50p)** (Standard choice)

Jim has two pleasures in his life, drinking Port wine $(x_1)$ and reading books $(x_2)$.

a) Port wine $x_1$ costs $p_1 = \$10$ a bottle and each book $x_2$, $p_2 = \$10$, and his daily budget is $m = \$120$. Show geometrically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set would be affected by introduction of ad valorem tax on imported wine (but not books) assuming tax rate $\tau = 100\%$.

b) Jim seems to be a fairly sophisticated fellow with quite a complicated utility function

$$U(x_1, x_2) = 12 \times \log\left[\exp\left[(4\ln x_1 + 2\ln x_2)^2\right] + 10\right]^{322}$$

Argue that in fact Jim is not really that sophisticated as his preferences can represented by a significantly simpler utility function. (one sentence + simpler utility function). If you are unable to answer point b) in the remaining part of Problem 1, you can assume $U(x_1, x_2) = x_1^2 x_2^1$.

c) Find Jim's marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give a formula for MRS). What is the value of MRS at consumption bundle $(1, 2)$ (give a number)? Which of the two commodities is more precious to Jim?

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).
- Describe how the two "secrets of happiness" can be seen in the graph.(graph + 2 sentences)
- Derive the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation).

e) Using your formula from d), find the optimal consumption levels of both types of commodities $(x_1, x_2)$ for:

- $p_1 = \$20, p_2 = \$10$ and $m = \$120$ (give two numbers).

and after the price of Port wine decreased:

- for $p_1 = \$10, p_2 = \$10$ and $m = \$120$ (give two numbers).

What is the total change in consumption of Port wine? (give a number). Illustrate the change on the graph. Is Port wine an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of $x_1$ from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

**Problem 2 (20p)** (Intertemporal choice)

Marky Mark is a rap singer who earns $m_1 = \$20$ when young and $m_2 = \$60$ when old.

a) What is the present value (in terms of \$ from period one) of Marky Mark's income (give one number).

b) Write down Marky Mark's budget constraint (one inequality) and plot budget set given interest rate $r = 200\%$ in the graph. Mark all consumption plans on the budget line that require borrowing and the ones that require saving.

c) Marky Mark's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$ and discount rate is $\delta = 2$. Provide an economic interpretation of parameter $\delta$ (one sentence). Using magic formulas, find the optimal consumption plan and the optimal saving strategy. (give three numbers $C_1, C_2, S$). Does Sam smooth his consumption? (yes no + one sentence) Is Sam tilting his consumption? (yes no + one sentence)

d) (Perpetuity) Your sister has just promised to send you pocket money of \$500 each month starting next month and she will keep doing it forever. What is the present value of "having such a sister" if monthly interest rate is equal to 10% (one number).

e) (Annuity) You are going to work for 45 years (from 20 to 65) earning income of \$500,000 a year. Then you are going to retire and going to live for another 35 years (till you are 100). Assume that the annual interest is given by $r = 1\%$. Write down equation that would allow you to determine the maximal constant level of consumption $C$ throughout your whole life (80 years). (write down equation that determines PV of income and consumption, but you need not calculate $C$).

**Problem 3 (15p)** (Perfect complements)

The old recipe for Pierogies (Polish dumplings) requires that sauerkraut $x_1$ is mixed with portabella mushrooms $x_2$ in a fixed (and sacred) proportion of 1:4.

a) Propose a utility function over sauerkraut and mushrooms (function $U(x_1, x_2)$) assuming that pierogies is a good.

b) In the commodity space, carefully depict indifference curves (marking the optimal proportion line).

c) Assume that $p_1 = 2$ and $p_2 = 2$ and income is $m = 40$. Write down two secrets of happiness (give two equations) that determine the optimal choice (two numbers). Explain the economic intuition behind the conditions (one sentence for each secret). Is your solution interior (yes or no)?

d) Without any calculations, in two separate graphs plot the price offer curve and income offer curve (just plot two curves).

**Problem 4 (15p)** (Perfect substitutes)

Sam does not have any money but is endowed with $\omega_1 = 100$ apples and $\omega_1 = 200$ oranges and prices of the two are $p_1 = p_2 = 1$. His utility function is given by $U(x_1, x_2) = x_1 + 3x_2$

a) Find Sam's budget line, marking the endowment point.

b) Find the optimal choice of the two commodities if his utility is given by $U(x_1, x_2) = x_1 + 3x_2$ (give two numbers). Is your solution interior?

c) Find net demands in optimum (give two numbers). Is Sam a net buyer or a net seller of apples. How about oranges? (chose one for each commodity)

d) In mathematics, the solution to a problem with perfect substitutes is called "bang, bang" solution. Provide intuition why (one sentence).

**Bonus question** (Just for fun)

Derive PV formula for annuity and perpetuity.

**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

**Problem 1 (50p)** (Standard choice)

Jim has two pleasures in his life, drinking Port wine $(x_1)$ and reading books $(x_2)$.

a) Port wine $x_1$ costs $p_1 = \$5$ a bottle and each book $x_2, p_2 = \$5$, and his daily budget is $m = \$60$. Show geometrically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set would be affected by introduction of ad valorem tax on imported wine (but not books) assuming tax rate $\tau = 100\%$.

b) Jim seems to be a fairly sophisticated fellow with quite a complicated utility function

$$U(x_1, x_2) = 12 \times \log\left[\exp\left[(10\ln x_1 + 5\ln x_2)^2\right] + 10\right]^{322}$$

Argue that in fact Jim is not really that sophisticated as his preferences can represented by a significantly simpler utility function. (one sentence + simpler utility function). If you are unable to answer point b) in the remaining part of Problem 1, you can assume $U(x_1, x_2) = x_1^2 x_2^1$.

c) Find Jim's marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give a formula for MRS). What is the value of MRS at consumption bundle $(1, 2)$ (give a number)? Which of the two commodities is more precious to Jim?

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).
- Describe how the two "secrets of happiness" can be seen in the graph.(graph + 2 sentences)
- Derive the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation).

e) Using your formula from d), find the optimal consumption levels of both types of commodities $(x_1, x_2)$ for:

- $p_1 = \$10, p_2 = \$5$ and $m = \$60$ (give two numbers).

and after the price of Port wine decreased:

- for $p_1 = \$5, p_2 = \$5$ and $m = \$60$ (give two numbers).

What is the total change in consumption of Port wine? (give a number). Illustrate the change on the graph. Is Port wine an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of $x_1$ from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

**Problem 2 (20p)** (Intertemporal choice)

Marky Mark is a rap singer who earns $m_1 = \$40$ when young and $m_2 = \$120$ when old.

a) What is the present value (in terms of \$ from period one) of Marky Mark's income (give one number).

b) Write down Marky Mark's budget constraint (one inequality) and plot budget set given interest rate $r = 200\%$ in the graph. Mark all consumption plans on the budget line that require borrowing and the ones that require saving.

c) Marky Mark's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$ and discount rate is $\delta = 2$. Provide an economic interpretation of parameter $\delta$ (one sentence). Using magic formulas, find the optimal consumption plan and the optimal saving strategy. (give three numbers $C_1, C_2, S$). Does Sam smooth his consumption? (yes no + one sentence) Is Sam tilting his consumption? (yes no + one sentence)

d) (Perpetuity) Your sister has just promised to send you pocket money of \$1000 each month starting next month and she will keep doing it forever. What is the present value of "having such a sister" if monthly interest rate is equal to 2% (one number).

e) (Annuity) You are going to work for 40 years (from 20 to 60) earning income of \$100,000 a year. Then you are going to retire and going to live for another 30 years (till you are 90). Assume that the annual

interest is given by $r = 1\%$. Write down equation that would allow you to determine the maximal constant level of consumption $C$ throughout your whole adult life (70 years). (write down equation that determines PV of income and consumption, but you need not calculate $C$).

**Problem 3 (15p)** (Perfect complements)

The old recipe for Pierogies (Polish dumplings) requires that sauerkraut $x_1$ is mixed with portabella mushrooms $x_2$ in a fixed (and sacred) proportion of 1:3.

a) Propose a utility function over sauerkraut and mushrooms (function $U(x_1, x_2)$) assuming that pierogies is a good.

b) In the commodity space, carefully depict indifference curves (marking the optimal proportion line).

c) Assume that $p_1 = 2$ and $p_2 = 2$ and income is $m = 80$. Write down two secrets of happiness (give two equations) that determine the optimal choice (two numbers). Explain the economic intuition behind the conditions (one sentence for each secret). Is your solution interior (yes or no)?

d) Without any calculations, in two separate graphs plot the price offer curve and income offer curve (just plot two curves).

**Problem 4 (15p)** (Perfect substitutes)

Sam does not have any money but is endowed with $\omega_1 = 100$ apples and $\omega_1 = 200$ oranges and prices of the two are $p_1 = p_2 = 1$. His utility function is given by $U(x_1, x_2) = x_1 + 4x_2$

a) Find Sam's budget line, marking the endowment point.

b) Find the optimal choice of the two commodities if his utility is given by $U(x_1, x_2) = x_1 + 4x_2$ (give two numbers). Is your solution interior?

c) Find net demands in optimum (give two numbers). Is Sam a net buyer or a net seller of apples. How about oranges? (chose one for each commodity)

d) In mathematics, the solution to a problem with perfect substitutes is called "bang, bang" solution. Provide intuition why (one sentence).

**Bonus question** (Just for fun)

Derive PV formula for annuity and perpetuity.

**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

**Problem 1 (50p)** (Standard choice)

Jim has two pleasures in his life, drinking Port wine $(x_1)$ and reading books $(x_2)$.

a) Port wine $x_1$ costs $p_1 = \$20$ a bottle and each book $x_2$, $p_2 = \$20$, and his daily budget is $m = \$240$. Show geometrically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set would be affected by introduction of ad valorem tax on imported wine (but not books) assuming tax rate $\tau = 100\%$.

b) Jim seems to be a fairly sophisticated fellow with quite a complicated utility function

$$U(x_1, x_2) = 12 \times \sqrt{\left[\ln\left[(20\ln x_1 + 10\ln x_2)^2\right] + 10\right]^{322}}$$

Argue that in fact Jim is not really that sophisticated as his preferences can represented by a significantly simpler utility function. (one sentence + simpler utility function). If you are unable to answer point b) in the remaining part of Problem 1, you can assume $U(x_1, x_2) = x_1^2 x_2^1$.

c) Find Jim's marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give a formula for MRS). What is the value of MRS at consumption bundle $(1, 2)$ (give a number)? Which of the two commodities is more precious to Jim?

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).
- Describe how the two "secrets of happiness" can be seen in the graph.(graph + 2 sentences)
- Derive the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation).

e) Using your formula from d), find the optimal consumption levels of both types of commodities $(x_1, x_2)$ for:

- $p_1 = \$40, p_2 = \$20$ and $m = \$240$ (give two numbers).

and after the price of Port wine decreased:

- for $p_1 = \$20, p_2 = \$20$ and $m = \$240$ (give two numbers).

What is the total change in consumption of Port wine? (give a number). Illustrate the change on the graph. Is Port wine an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of $x_1$ from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

**Problem 2 (20p)** (Intertemporal choice)

Marky Mark is a rap singer who earns $m_1 = \$80$ when young and $m_2 = \$240$ when old.

a) What is the present value (in terms of \$ from period one) of Marky Mark's income (give one number).

b) Write down Marky Mark's budget constraint (one inequality) and plot budget set given interest rate $r = 200\%$ in the graph. Mark all consumption plans on the budget line that require borrowing and the ones that require saving.

c) Marky Mark's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$ and discount rate is $\delta = 2$. Provide an economic interpretation of parameter $\delta$ (one sentence). Using magic formulas, find the optimal consumption plan and the optimal saving strategy. (give three numbers $C_1, C_2, S$). Does Sam smooth his consumption? (yes no + one sentence) Is Sam tilting his consumption? (yes no + one sentence)

d) (Perpetuity) Your sister has just promised to send you pocket money of \$200 each month starting next month and she will keep doing it forever. What is the present value of "having such a sister" if monthly interest rate is equal to 10% (one number).

e) (Annuity) You are going to work for 40 years (from 20 to 60) earning income of \$200,000 a year. Then you are going to retire and going to live for another 30 years (till you are 90). Assume that the annual interest is given by $r = 1\%$. Write down equation that would allow you to determine the maximal constant level of consumption $C$ throughout your whole adult life (70 years). (write down equation that determines PV of income and consumption, but you need not calculate $C$).

**Problem 3 (15p)** (Perfect complements)

The old recipe for Pierogies (Polish dumplings) requires that sauerkraut $x_1$ is mixed with portabella mushrooms $x_2$ in a fixed (and sacred) proportion of 3:1.

a) Propose a utility function over sauerkraut and mushrooms (function $U(x_1, x_2)$) assuming that pierogies is a good.

b) In the commodity space, carefully depict indifference curves (marking the optimal proportion line).

c) Assume that $p_1 = 2$ and $p_2 = 2$ and income is $m = 160$. Write down two secrets of happiness (give two equations) that determine the optimal choice (two numbers). Explain the economic intuition behind the conditions (one sentence for each secret). Is your solution interior (yes or no)?

d) Without any calculations, in two separate graphs plot the price offer curve and income offer curve (just plot two curves).

**Problem 4 (15p)** (Perfect substitutes)

Sam does not have any money but is endowed with $\omega_1 = 100$ apples and $\omega_1 = 200$ oranges and prices of the two are $p_1 = p_2 = 1$. His utility function is given by $U(x_1, x_2) = 2x_1 + x_2$

a) Find Sam's budget line, marking the endowment point.

b) Find the optimal choice of the two commodities if his utility is given by $U(x_1, x_2) = 2x_1 + x_2$ (give two numbers). Is your solution interior?

c) Find net demands in optimum (give two numbers). Is Sam a net buyer or a net seller of apples. How about oranges? (chose one for each commodity)

d) In mathematics, the solution to a problem with perfect substitutes is called "bang, bang" solution. Provide intuition why (one sentence).

**Bonus question** (Just for fun)

Derive PV formula for annuity and perpetuity.

# Problem 1 (50pts)

(a) (5pts) The budget constraint is given by $20x_1 + 20x_2 = 600$. With the tax $\tau = 100\%$, it becomes $20(1+\tau)x_1 + 20x_2 = 40x_1 + 20x_2 = 600$. Since it plays a role in increasing $p_1$ by 100%, the budget line must shift in as displayed in Figure 1.

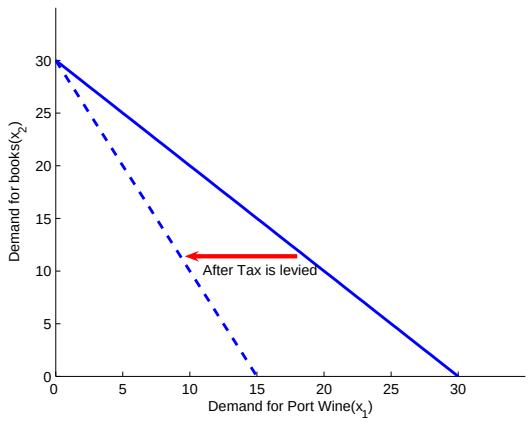



Figure 1: The Budget Set

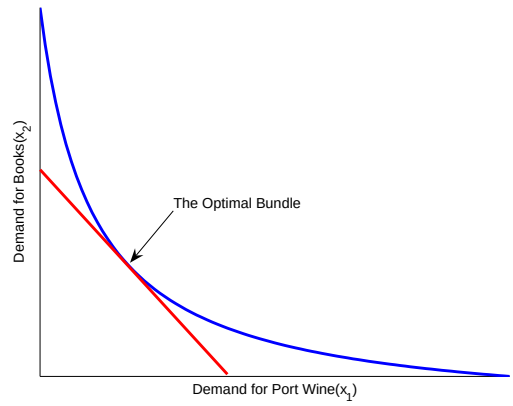



Figure 2: Two Secrets of Happiness

(b) (5pts) His sophisticated utility function is simply a monotone transformation of $2\ln x_1 + \ln x_2$.

(c) (5pts) From the previous part, we shall write his utility function $U(x_1, x_2) = 2\ln x_1 + \ln x_2$ without loss of generality. Differentiating with respect to $x_1$ and $x_2$ gives us

$$\mathrm{MU}_1 = \frac{2}{x_1} \quad \text{and} \quad \mathrm{MU}_2 = \frac{1}{x_2}$$

Hence the marginal rate of substitution, defined as the (negative) ratio of two marginal utilities, is given by $\mathrm{MRS} = -\frac{2/x_1}{1/x_2} = -\frac{2x_2}{x_1}$. At the bundle $(1,2)$, it takes $-\frac{2\cdot 2}{1} = -4$. Since its absolute value is greater than 1, the port wine is more valuable from his perspective.

(d) (10pts) Two secrets of happiness state

$$(i) \;\; p_1x_1 + p_2x_2 = m$$
$$(ii) \;\; MRS\left(= -\frac{2x_2}{x_1}\right) = -\frac{p_1}{p_2}$$

The first condition indicates Jim's budget line; he has to utilize all his monetary resource to maximize utility. And the second condition requires the MRS coincide with the relative price. It implies that the consumer's subjective exchange ratio(MRS) must be the same as the market's objective exchange ratio at the optimal bundle. Figure 2 illustrates it in the graph. For a bundle to be optimal, it must be on the budget line and the indifference curve through it must be tangent to the budget line.

Note that solving (ii) for $x_1$ gives $x_1 = 2\frac{p_2}{p_1}x_2$. Plugging it in (i) for $x_1$ gives

$$p_1\left(2\frac{p_2}{p_1}x_2\right) + p_2x_2 = m \rightarrow x_2 = \frac{m}{3p_2}$$

Consequently, $x_1 = 2\frac{p_2}{p_1}\frac{m}{3p_2} = \frac{2m}{3p_1}$.

(e) (15pts) When $p_1 = 40$, $p_2 = 20$, $m = 600$,

$$x_1 = \frac{2 \cdot 600}{3 \cdot 40} = 10 \quad \text{and} \quad x_2 = \frac{600}{3 \cdot 20} = 10.$$

When $p_1$ falls down to $p_1 = 20$,

$$x_1 = \frac{2 \cdot 600}{3 \cdot 20} = 20 \quad \text{and} \quad x_2 = \frac{600}{3 \cdot 20} = 10.$$

The total change in $x_1$ is $20 - 10 = 10$. Note that the demand for Port wine is increasing as $p_1$ decreases. It is an ordinary good. Figure 3 illustrate the total change in $x_1$.

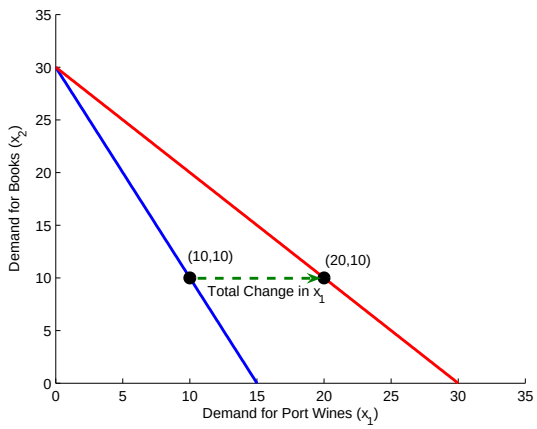



Figure 3: Total Change in $x_1$

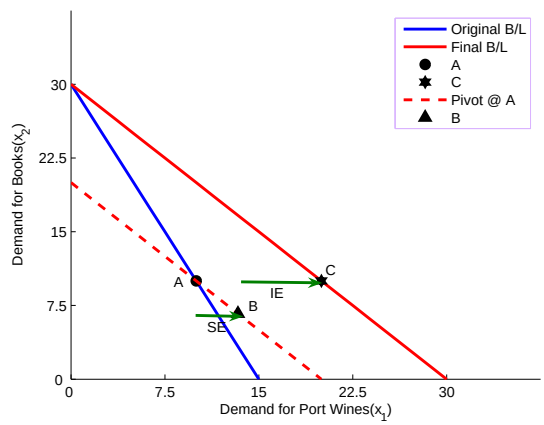



Figure 4: Substitution and Income Effect

(f) (10pts) In order to decompose it, we have to adjust money income $m'$, which represents the amount of money income that is necessary to make the original bundle(i.e. before $p_1$ changes it was (10,10)) affordable at the new price.

$$m' = 20 \cdot 10 + 20 \cdot 10 = 400.$$

After $p_1$ changes, Jim needs $m' = 400$ to purchase the original commodity bundle. Then we need figure out what is the optimal choice for Port wine at the new price and this $m'$. Plugging $p_1 = 20$ and $m = 400$ into the formula $x_1 = \frac{2m}{3p_1}$ gives us $x_1 = \frac{40}{3}$. The substitution effect is given by the difference between the new demand $\frac{40}{3}$ and the original one 10

$$\text{SE} = \frac{40}{3} - 10 = \frac{10}{3}$$

The associated income effect immediately follows from Slutsky's equation, which tells us that the total change can be decomposed into the substitution and income effect.

$$\text{IE} = \text{Total Change} - \text{SE} = 10 - \frac{10}{3} = \frac{20}{3}$$

In figure 4, I denote by $A$,$B$ and $C$ the original optimal choice, the final optimal choice, and the intermediate optimal choice we obtained from $m'$, respectively. The substitution effect is the gap between $A$ and $C$ on the horizontal axis while the income effect is between $C$ and $B$.

## Problem 2 (20pts)

(a) (2pts) $\text{PV} = 100 + \dfrac{300}{1+r} = 100 + \dfrac{300}{3} = 200$

(b) (3pts) The budget constraint is written by

$$C_1 + \frac{1}{3}C_2 = 200$$

To wit briefly, the left-hand side is the present value of consumption stream $(C_1, C_2)$. Observe that I discounted $C_2$ by $\frac{1}{1+r} = \frac{1}{3}$ to express it in terms of present value. The right-hand side simply represents the present value of his income we got in part (a). The budget line tells us that they should be balanced. See figure 5.

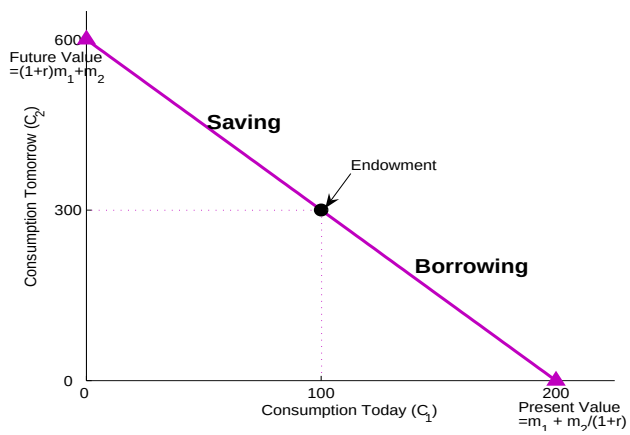



Figure 5: Intertemporal Choice Model

(c) (7pts) For $\delta = 2$, observe that the utility function comes down to

$$U(C_1, C_2) = \ln C_1 + \frac{1}{3}\ln C_2$$

which is a Cobb-Douglas function. Using its magic formula, we obtain

$$C_1 = \frac{a}{a+b} \cdot \frac{m}{p_1} = \frac{1}{1+\frac{1}{3}} \cdot \frac{200}{1} = 150$$

$$C_2 = \frac{b}{a+b} \cdot \frac{m}{p_2} = \frac{\frac{1}{3}}{1+\frac{1}{3}} \cdot \frac{200}{\frac{1}{3}} = 150.$$

To consume 150 today, he must save $100 - 150 = -50$, that is, he must borrow 50. He does smooth his consumption plan because $C_1 = C_2 = 150$.

(d) (3pts) $\text{PV} = \dfrac{x}{r} = \dfrac{500}{0.01} = 50,000.$

(e) (5pts) The characterization equation is given by

$$\frac{1,000,000}{0.05}\left[1 - \frac{1}{(1+0.05)^{45}}\right] = \frac{C}{0.05}\left[1 - \frac{1}{(1+0.05)^{80}}\right]$$

The left-hand side and the right-hand side represents the present value of earnings for next 45 years and the present value of consumptions for next 80 years, respectively. And the constant level of consumption $C$ comes from the equation in which these two values are balanced.

## Problem 3 (15pts)

(a) (2pts) $U(x_1, x_2) = \min\{x_1, 4x_2\}$

(b) (3pts) See figure 6.

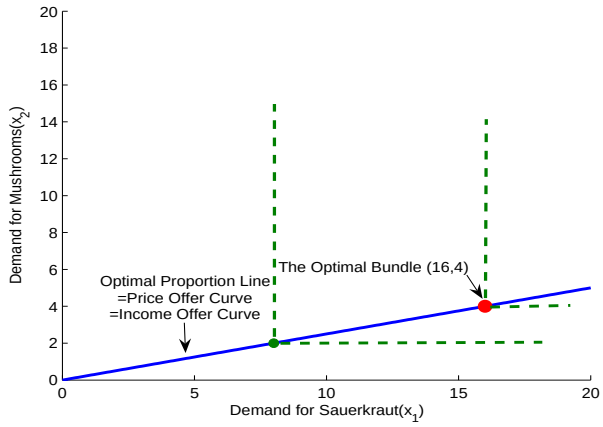



Figure 6: Perfect Complements

(c) (6pts) The associated "two secrets of happiness" with perfect complements is

$$(i)\ \ 2x_1 + 2x_2 = 40$$
$$(ii)\ \ x_1 = 4x_2.$$

The first condition indicates the consumer's balanced budget, and its economic intuition is the same as before: All income must be exhausted. The second condition represents the optimal proportion $(4:1)$ between quantities demanded for sauerkraut and mushrooms.

To figure out the optimal choice, we plug in $4x_2$ for $x_1$ in the budget line and obtain

$$2(4x_2) + 2x_2 = 10x_2 = 40 \Rightarrow x_2 = 4.$$

Substituting into $x_1 = 4x_2$ we get $x_1 = 16$. The solution is interior as $x_1$ and $x_2$ are strictly positive.

(d) (4pts) Since the two goods are perfect complements, both the price offer curve and the income offer curve would coincide with the optimal proportion line. Refer Figure 6.

## Problem 4 (15pts)

(a) (4pts) $x_1 + x_2 = 40$. The value of endowment, the total resources available for consumption, is $p_1\omega_1 + p_2\omega_2 = 10 + 30 = 40$. See Figure 7.

(b) (4pts) Since $p_1 = p_2 = 1$ and the two goods are perfect substitutes, Sam is willing to consume the good with more marginal utilities. In this example, it follows from the utility function that apples(good 1) yield more marginal utilities. Therefore, $x_1 = 40$ and $x_2 = 0$. Since $x_2 = 0$, the solution is not interior but it is at the corner.

(c) (4pts) The net demand for apples is $x_1 - \omega_1 = 40 - 10 = 30$, which is positive. It means that Sam is a net buyer of apples. The net demand for oranges, on the other hand, is $x_2 - \omega_2 = 0 - 30 = -30 < 0$. Hence he is a net seller for oranges.

(d) (3pts) It is because apples and oranges, in this example, are perfectly substitutable. Hence the consumer would choose either one good only, depending on the marginal utility from a dollar.

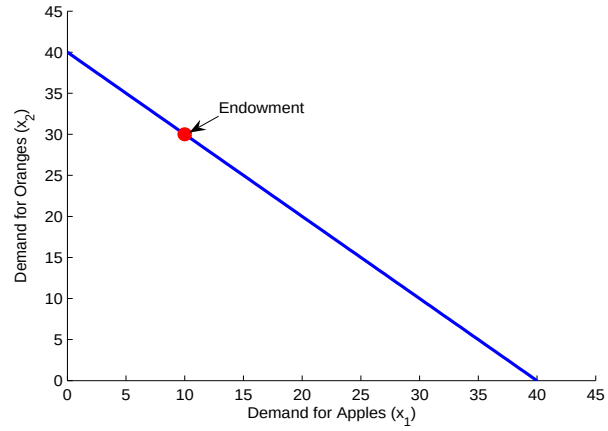



Figure 7: Budget Line and Endowment Point

# Bonus Problem

## Perpetuity

A perpetuity in an annuity that has no definite end, or a stream of cash payments, say $x$, that continues indefinitely. With the (risk-free) interest rate $r$, we can write its present value as

$$\begin{aligned}
\text{PV} &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots \\
&= \frac{x}{(1+r)} + \left[\frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots\right] \\
&= \frac{x}{(1+r)} + \frac{1}{(1+r)} \underbrace{\left[\frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \cdots\right]}_{=\text{PV}}
\end{aligned}$$

In the second step, I just partitioned the sum into the first term and the remainder using the parenthesis. The key step is the next one, factoring $\frac{1}{1+r}$ out of the remainder. Then we come up with the same sequence as the original PV. It leads to the following simple equation

$$\text{PV} = \frac{x}{(1+r)} + \frac{1}{(1+r)} \cdot \text{PV}$$

Solving for PV gives us the desired formula, $\text{PV} = \dfrac{x}{r}$.

## Annuity

The annuity is an asset that promises a terminating stream of fixed payments over a prespecified period of time. Its value is closely linked to *"No Arbitrage Principle"*, the most fundamental principle in finance. It states that "two assets with identical cash flows must trade at the same price." With this principle in hand, consider one annuity which guarantees to annually give us payment $x$ over $T$ years. Now it is the key idea that with the two perpetuities we can replicate the identical cash flows as the annuity.

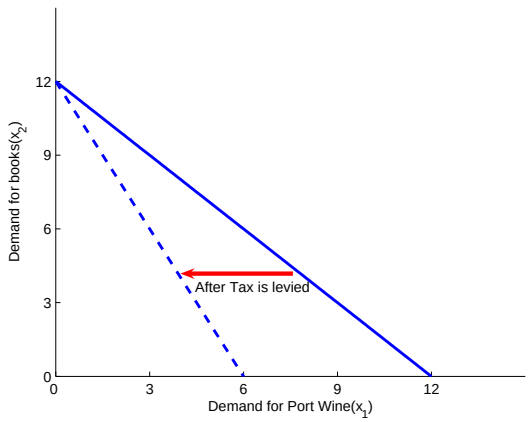
- Perpetuity 1 - It promises $x$ payment forever from now on.
- Perpetuity 2 - It promises $x$ payment forever but $T+1$ years from now.

Observe that the cash flow of the annuity is the same as the cash flow of "Perpetuity 1 − Perpetuity 2". By no arbitrage condition, then, it must be the case that

$$\begin{aligned}
\text{PV(Annuity)} &= \text{PV(Perpetuity} \quad 1) - \text{PV(Perpetuity} \quad 2) \\
&= \frac{x}{r} - \frac{x}{r} \cdot \frac{1}{(1+r)^T} \\
&= \frac{x}{r}\left[1 - \frac{1}{(1+r)^T}\right].
\end{aligned}$$

# Problem 1 (50pts)

(a) (5pts) The budget constraint is given by $10x_1 + 10x_2 = 120$. With the tax $\tau = 100\%$, it becomes $10(1+\tau)x_1 + 10x_2 = 20x_1 + 10x_2 = 120$. Since it plays a role in increasing $p_1$ by 100%, the budget line must shift in as displayed in Figure 1.

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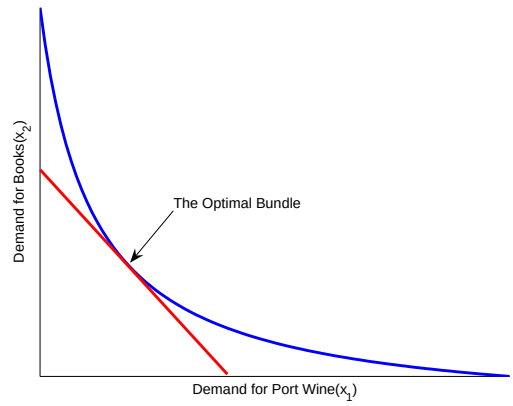



Figure 2: Two Secrets of Happiness

(b) (5pts) His sophisticated utility function is simply a monotone transformation of $2\ln x_1 + \ln x_2$.

(c) (5pts) From the previous part, we shall write his utility function $U(x_1, x_2) = 2\ln x_1 + \ln x_2$ without loss of generality. Differentiating with respect to $x_1$ and $x_2$ gives us

$$\text{MU}_1 = \frac{2}{x_1} \quad \text{and} \quad \text{MU}_2 = \frac{1}{x_2}$$

Hence the marginal rate of substitution, defined as the (negative) ratio of two marginal utilities, is given by $\text{MRS} = -\frac{2/x_1}{1/x_2} = -\frac{2x_2}{x_1}$. At the bundle $(1,2)$, it takes $-\frac{2\cdot 2}{1} = -4$. Since its absolute value is greater than 1, the port wine is more valuable from his perspective.

(d) (10pts) Two secrets of happiness state

$$(i) \;\; p_1x_1 + p_2x_2 = m$$
$$(ii) \;\; MRS\left(= -\frac{2x_2}{x_1}\right) = -\frac{p_1}{p_2}$$

The first condition indicates Jim's budget line; he has to utilize all his monetary resource to maximize utility. And the second condition requires the MRS coincide with the relative price. It implies that the consumer's subjective exchange ratio(MRS) must be the same as the market's objective exchange ratio at the optimal bundle. Figure 2 illustrates it in the graph. For a bundle to be optimal, it must be on the budget line and the indifference curve through it must be tangent to the budget line.

Note that solving (ii) for $x_1$ gives $x_1 = 2\frac{p_2}{p_1}x_2$. Plugging it in (i) for $x_1$ gives

$$p_1\left(2\frac{p_2}{p_1}x_2\right) + p_2x_2 = m \rightarrow x_2 = \frac{m}{3p_2}$$

Consequently, $x_1 = 2\frac{p_2}{p_1}\frac{m}{3p_2} = \frac{2m}{3p_1}$.

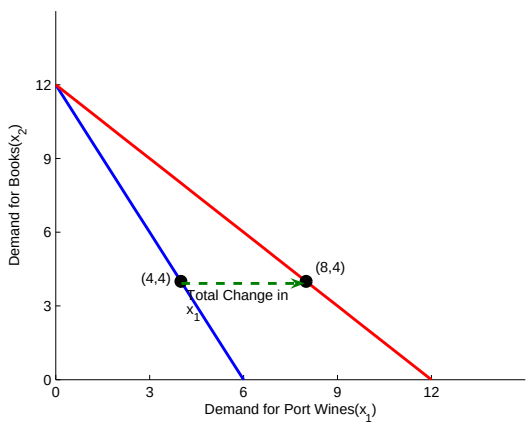
(e) (15pts) When $p_1 = 20$, $p_2 = 10$, $m = 120$,

$$x_1 = \frac{2 \cdot 120}{3 \cdot 20} = 4 \quad \text{and} \quad x_2 = \frac{120}{3 \cdot 10} = 4.$$

When $p_1$ falls down to $p_1 = 10$,

$$x_1 = \frac{2 \cdot 120}{3 \cdot 10} = 8 \quad \text{and} \quad x_2 = \frac{120}{3 \cdot 10} = 4.$$

The total change in $x_1$ is $8 - 4 = 4$. Note that the demand for Port wine is increasing as $p_1$ decreases. It is an ordinary good. Figure 3 illustrate the total change in $x_1$.

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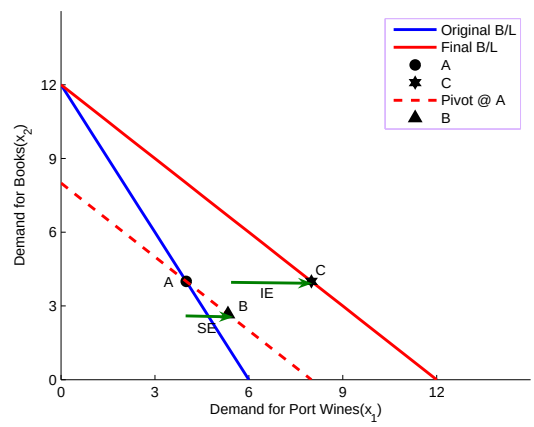


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(f) (10pts) In order to decompose it, we have to adjust money income $m'$, which represents the amount of money income that is necessary to make the original bundle(i.e. before $p_1$ changes it was (4,4)) affordable at the new price.

$$m' = 4 \cdot 10 + 4 \cdot 10 = 80.$$

After $p_1$ changes, Jim needs $m' = 80$ to purchase the original commodity bundle. Then we need figure out what is the optimal choice for Port wine at the new price and this $m'$. Plugging $p_1 = 10$ and $m = 80$ into the formula $x_1 = \frac{2m}{3p_1}$ gives us $x_1 = \frac{16}{3}$. The substitution effect is given by the difference between the new demand $\frac{16}{3}$ and the original one 4

$$\text{SE} = \frac{16}{3} - 4 = \frac{4}{3}$$

The associated income effect immediately follows from Slutsky's equation, which tells us that the total change can be decomposed into the substitution and income effect.

$$\text{IE} = \text{Total Change} - \text{SE} = 4 - \frac{4}{3} = \frac{8}{3}$$

In figure 4, I denote by $A$,$B$ and $C$ the original optimal choice, the final optimal choice, and the intermediate optimal choice we obtained from $m'$, respectively. The substitution effect is the gap between $A$ and $C$ on the horizontal axis while the income effect is between $C$ and $B$.

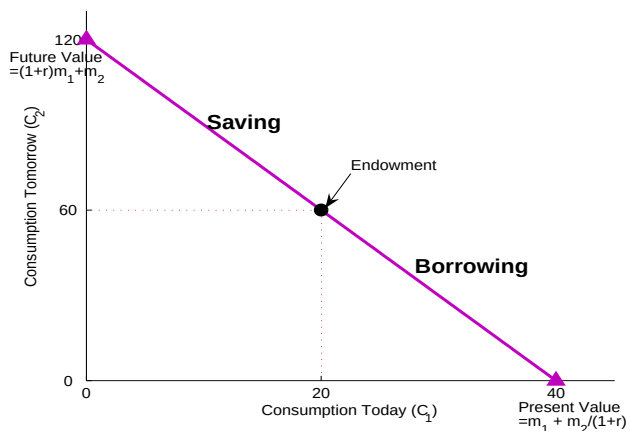
## Problem 2 (20pts)

(a) (2pts) $\text{PV} = 20 + \dfrac{60}{1+r} = 20 + \dfrac{60}{3} = 40$

(b) (3pts) The budget constraint is written by

$$C_1 + \frac{1}{3}C_2 = 40$$

To wit briefly, the left-hand side is the present value of consumption stream $(C_1, C_2)$. Observe that I discounted $C_2$ by $\frac{1}{1+r} = \frac{1}{3}$ to express it in terms of present value. The right-hand side simply represents the present value of his income we got in part (a). The budget line tells us that they should be balanced. See figure 5.

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$$U(C_1, C_2) = \ln C_1 + \frac{1}{3}\ln C_2$$

which is a Cobb-Douglas function. Using its magic formula, we obtain

$$C_1 = \frac{a}{a+b} \cdot \frac{m}{p_1} = \frac{1}{1+\frac{1}{3}} \cdot \frac{40}{1} = 30$$

$$C_2 = \frac{b}{a+b} \cdot \frac{m}{p_2} = \frac{\frac{1}{3}}{1+\frac{1}{3}} \cdot \frac{40}{\frac{1}{3}} = 30.$$

To consume 30 today, he must save $20 - 30 = -10$, that is, he must borrow 10. He does smooth his consumption plan because $C_1 = C_2 = 30$.

(d) (3pts) $\text{PV} = \dfrac{x}{r} = \dfrac{500}{0.1} = 5,000.$

(e) (5pts) The characterization equation is given by

$$\frac{500,000}{0.01}\left[1 - \frac{1}{(1+0.01)^{45}}\right] = \frac{C}{0.01}\left[1 - \frac{1}{(1+0.01)^{80}}\right]$$

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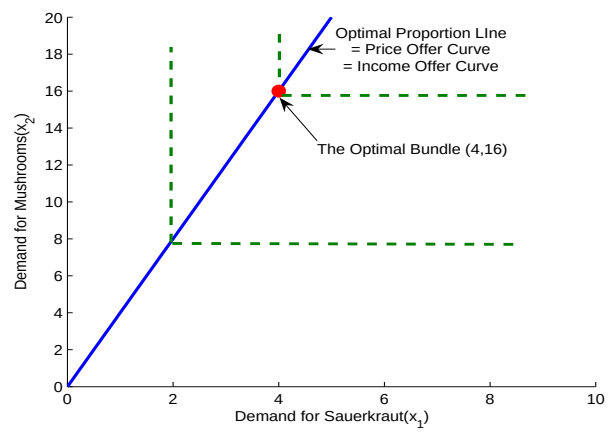


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(c) (4pts) The net demand for apples is $x_1 - \omega_1 = 0 - 100 = -100$, which is negative. It means that Sam is a net seller of apples. The net demand for oranges, on the other hand, is $x_2 - \omega_2 = 300 - 200 = 100 > 0$. Hence he is a net buyer for oranges.

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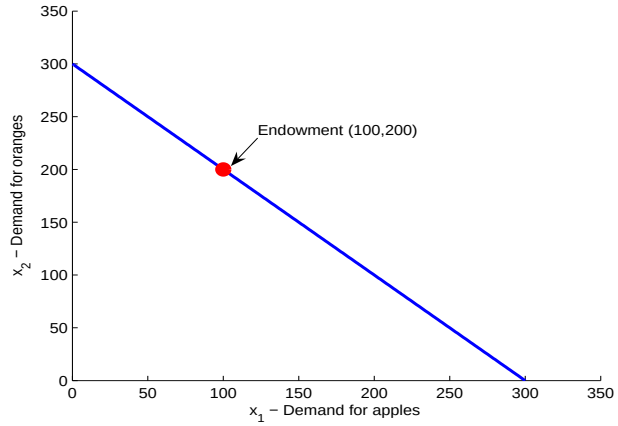


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# Bonus Problem

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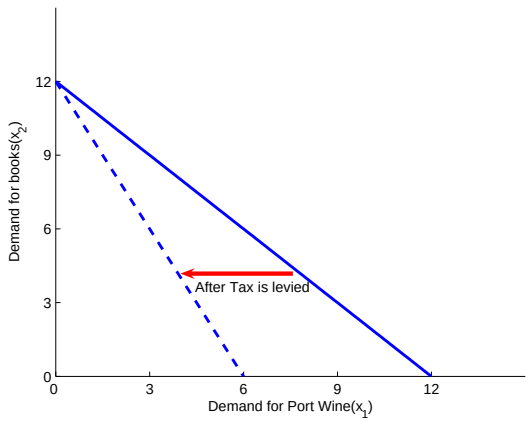


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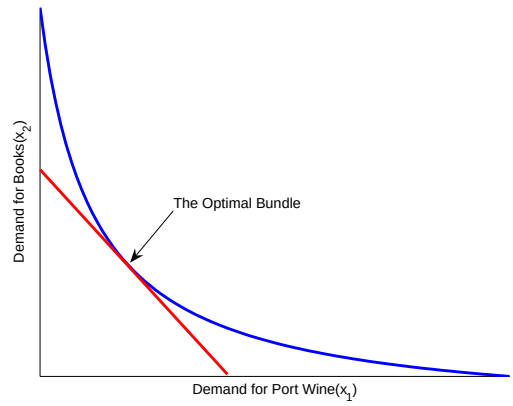


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Consequently, $x_1 = 2\frac{p_2}{p_1}\frac{m}{3p_2} = \frac{2m}{3p_1}$.

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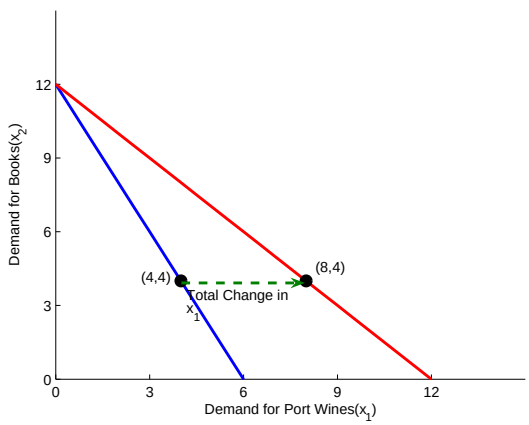
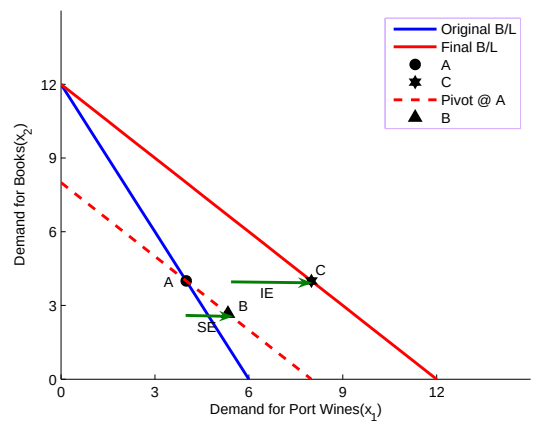



Figure 3: Total Change in $x_1$

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$$m' = 4 \cdot 5 + 4 \cdot 5 = 40.$$

After $p_1$ changes, Jim needs $m' = 40$ to purchase the original commodity bundle. Then we need figure out what is the optimal choice for Port wine at the new price and this $m'$. Plugging $p_1 = 5$ and $m = 40$ into the formula $x_1 = \frac{2m}{3p_1}$ gives us $x_1 = \frac{16}{3}$. The substitution effect is given by the difference between the new demand $\frac{16}{3}$ and the original one 4

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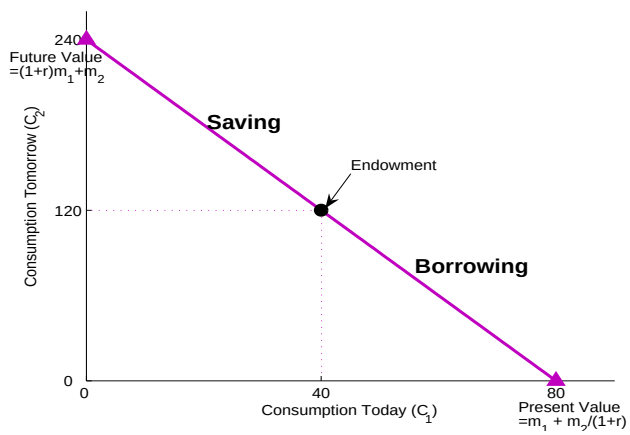
To wit briefly, the left-hand side is the present value of consumption stream $(C_1, C_2)$. Observe that I discounted $C_2$ by $\frac{1}{1+r} = \frac{1}{3}$ to express it in terms of present value. The right-hand side simply represents the present value of his income we got in part (a). The budget line tells us that they should be balanced. See figure 5.

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$$U(C_1, C_2) = \ln C_1 + \frac{1}{3}\ln C_2$$

which is a Cobb-Douglas function. Using its magic formula, we obtain

$$C_1 = \frac{a}{a+b} \cdot \frac{m}{p_1} = \frac{1}{1+\frac{1}{3}} \cdot \frac{80}{1} = 60$$

$$C_2 = \frac{b}{a+b} \cdot \frac{m}{p_2} = \frac{\frac{1}{3}}{1+\frac{1}{3}} \cdot \frac{80}{\frac{1}{3}} = 60.$$

To consume 60 today, he must save $60 - 40 = -20$, that is, he must borrow 20. He does smooth his consumption plan because $C_1 = C_2 = 60$.

(d) (3pts) $\text{PV} = \dfrac{x}{r} = \dfrac{1000}{0.02} = 50,000.$

(e) (5pts) The characterization equation is given by

$$\frac{100,000}{0.01}\left[1 - \frac{1}{(1+0.01)^{40}}\right] = \frac{C}{0.01}\left[1 - \frac{1}{(1+0.01)^{70}}\right]$$

The left-hand side and the right-hand side represents the present value of earnings for next 40 years and the present value of consumptions for next 70 years, respectively. And the constant level of consumption $C$ comes from the equation in which these two values are balanced.

## Problem 3 (15pts)

(a) (2pts) $U(x_1, x_2) = \min\{3x_1, x_2\}$

(b) (3pts) See figure 6.

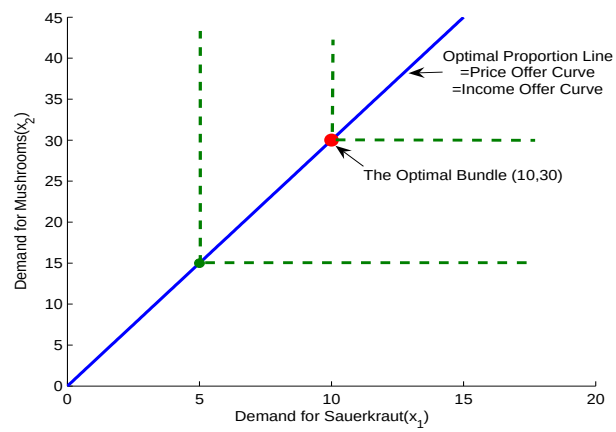


Figure 6: Perfect Complements

(c) (6pts) The associated "two secrets of happiness" with perfect complements is

$(i)$ $2x_1 + 2x_2 = 80$

$(ii)$ $3x_1 = x_2$.

The first condition indicates the consumer's balanced budget, and its economic intuition is the same as before: All income must be exhausted. The second condition represents the optimal proportion $(1:3)$ between quantities demanded for sauerkraut and mushrooms.

To figure out the optimal choice, we plug in $3x_1$ for $x_2$ in the budget line and obtain

$$2x_1 + 2(3x_1) = 8x_1 = 80 \Rightarrow x_1 = 8.$$

Substituting into $x_2 = 3x_1$ we get $x_2 = 24$. The solution is interior as $x_1$ and $x_2$ are strictly positive.

(d) (4pts) Since the two goods are perfect complements, both the price offer curve and the income offer curve would coincide with the optimal proportion line. Refer Figure 6.

## Problem 4 (15pts)

(a) (4pts) $x_1 + x_2 = 300$. The value of endowment, the total resources available for consumption, is $p_1\omega_1 + p_2\omega_2 = 100 + 200 = 300$. See Figure 7.

(b) (4pts) Since $p_1 = p_2 = 1$ and the two goods are perfect substitutes, Sam is willing to consume the good with more marginal utilities. In this example, it follows from the utility function that oranges(good 2) yield more marginal utilities. Therefore, $x_2 = 300$ and $x_1 = 0$. Since $x_1 = 0$, the solution is not interior but it is at the corner.

(c) (4pts) The net demand for apples is $x_1 - \omega_1 = 0 - 100 = -100$, which is negative. It means that Sam is a net seller of apples. The net demand for oranges, on the other hand, is $x_2 - \omega_2 = 300 - 200 = 100 > 0$. Hence he is a net buyer for oranges.

(d) (3pts) It is because apples and oranges, in this example, are perfectly substitutable. Hence the consumer would choose either one good only, depending on the marginal utility from a dollar.

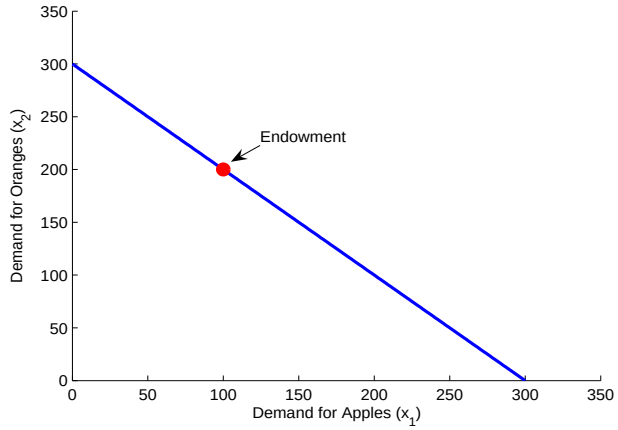


Figure 7: Budget Line and Endowment Point

# Bonus Problem

## Perpetuity

A perpetuity in an annuity that has no definite end, or a stream of cash payments, say $x$, that continues indefinitely. With the (risk-free) interest rate $r$, we can write its present value as

$$\begin{aligned}
\text{PV} &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots \\
&= \frac{x}{(1+r)} + \left[\frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots\right] \\
&= \frac{x}{(1+r)} + \frac{1}{(1+r)} \underbrace{\left[\frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \cdots\right]}_{=\text{PV}}
\end{aligned}$$

In the second step, I just partitioned the sum into the first term and the remainder using the parenthesis. The key step is the next one, factoring $\frac{1}{1+r}$ out of the remainder. Then we come up with the same sequence as the original PV. It leads to the following simple equation

$$\text{PV} = \frac{x}{(1+r)} + \frac{1}{(1+r)} \cdot \text{PV}$$

Solving for PV gives us the desired formula, $\text{PV} = \dfrac{x}{r}$.

## Annuity

The annuity is an asset that promises a terminating stream of fixed payments over a prespecified period of time. Its value is closely linked to *"No Arbitrage Principle"*, the most fundamental principle in finance. It states that "two assets with identical cash flows must trade at the same price." With this principle in hand, consider one annuity which guarantees to annually give us payment $x$ over $T$ years. Now it is the key idea that with the two perpetuities we can replicate the identical cash flows as the annuity.

- Perpetuity 1 - It promises $x$ payment forever from now on.
- Perpetuity 2 - It promises $x$ payment forever but $T+1$ years from now.

Observe that the cash flow of the annuity is the same as the cash flow of "Perpetuity 1 − Perpetuity 2". By no arbitrage condition, then, it must be the case that

$$\begin{aligned}
\text{PV(Annuity)} &= \text{PV(Perpetuity} \quad 1) - \text{PV(Perpetuity} \quad 2) \\
&= \frac{x}{r} - \frac{x}{r} \cdot \frac{1}{(1+r)^T} \\
&= \frac{x}{r}\left[1 - \frac{1}{(1+r)^T}\right].
\end{aligned}$$

# Problem 1 (50pts)

(a) (5pts) The budget constraint is given by $20x_1 + 20x_2 = 240$. With the tax $\tau = 100\%$, it becomes $20(1+\tau)x_1 + 20x_2 = 40x_1 + 20x_2 = 240$. Since it plays a role in increasing $p_1$ by 100%, the budget line must shift in as displayed in Figure 1.

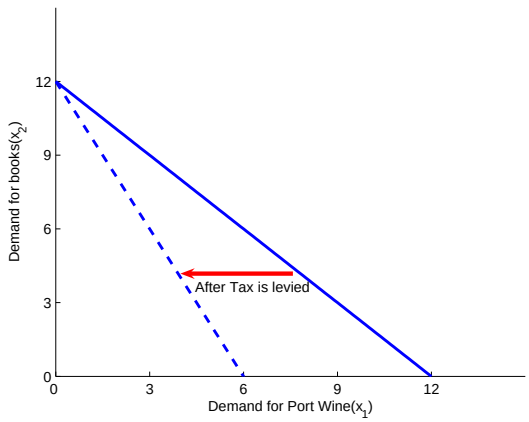


Figure 1: The Budget Set

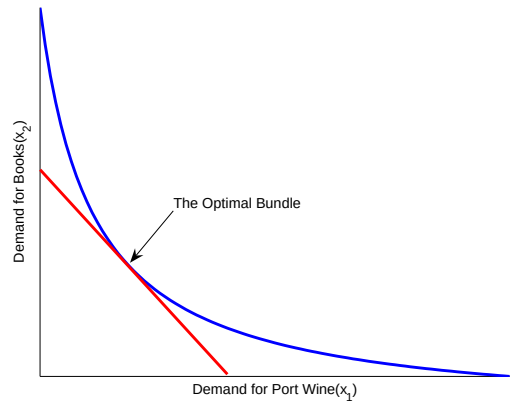


Figure 2: Two Secrets of Happiness

(b) (5pts) His sophisticated utility function is simply a monotone transformation of $2\ln x_1 + \ln x_2$.

(c) (5pts) From the previous part, we shall write his utility function $U(x_1, x_2) = 2\ln x_1 + \ln x_2$ without loss of generality. Differentiating with respect to $x_1$ and $x_2$ gives us

$$\text{MU}_1 = \frac{2}{x_1} \quad \text{and} \quad \text{MU}_2 = \frac{1}{x_2}$$

Hence the marginal rate of substitution, defined as the (negative) ratio of two marginal utilities, is given by $\text{MRS} = -\frac{2/x_1}{1/x_2} = -\frac{2x_2}{x_1}$. At the bundle $(1,2)$, it takes $-\frac{2\cdot 2}{1} = -4$. Since its absolute value is greater than 1, the port wine is more valuable from his perspective.

(d) (10pts) Two secrets of happiness state

$$(i) \;\; p_1x_1 + p_2x_2 = m$$
$$(ii) \;\; MRS\left(= -\frac{2x_2}{x_1}\right) = -\frac{p_1}{p_2}$$

The first condition indicates Jim's budget line; he has to utilize all his monetary resource to maximize utility. And the second condition requires the MRS coincide with the relative price. It implies that the consumer's subjective exchange ratio(MRS) must be the same as the market's objective exchange ratio at the optimal bundle. Figure 2 illustrates it in the graph. For a bundle to be optimal, it must be on the budget line and the indifference curve through it must be tangent to the budget line.

Note that solving (ii) for $x_1$ gives $x_1 = 2\frac{p_2}{p_1}x_2$. Plugging it in (i) for $x_1$ gives

$$p_1\left(2\frac{p_2}{p_1}x_2\right) + p_2x_2 = m \rightarrow x_2 = \frac{m}{3p_2}$$

Consequently, $x_1 = 2\frac{p_2}{p_1}\frac{m}{3p_2} = \frac{2m}{3p_1}$.

(e) (15pts) When $p_1 = 40$, $p_2 = 20$, $m = 240$,

$$x_1 = \frac{2 \cdot 240}{3 \cdot 40} = 4 \quad \text{and} \quad x_2 = \frac{240}{3 \cdot 20} = 4.$$

When $p_1$ falls down to $p_1 = 20$,

$$x_1 = \frac{2 \cdot 240}{3 \cdot 20} = 8 \quad \text{and} \quad x_2 = \frac{240}{3 \cdot 20} = 4.$$

The total change in $x_1$ is $8 - 4 = 4$. Note that the demand for Port wine is increasing as $p_1$ decreases. It is an ordinary good. Figure 3 illustrate the total change in $x_1$.

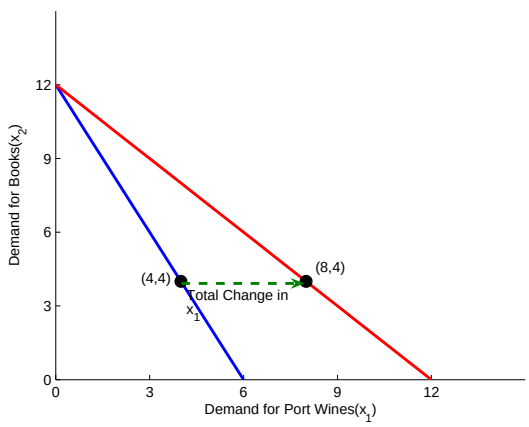


Figure 3: Total Change in $x_1$

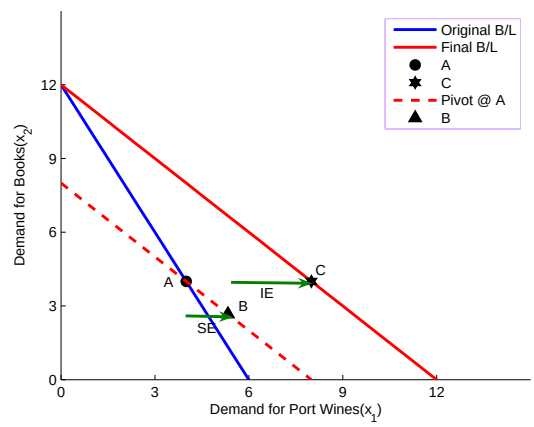


Figure 4: Substitution and Income Effect

(f) (10pts) In order to decompose it, we have to adjust money income $m'$, which represents the amount of money income that is necessary to make the original bundle(i.e. before $p_1$ changes it was (4,4)) affordable at the new price.

$$m' = 4 \cdot 20 + 4 \cdot 20 = 160.$$

After $p_1$ changes, Jim needs $m' = 160$ to purchase the original commodity bundle. Then we need figure out what is the optimal choice for Port wine at the new price and this $m'$. Plugging $p_1 = 20$ and $m = 160$ into the formula $x_1 = \frac{2m}{3p_1}$ gives us $x_1 = \frac{16}{3}$. The substitution effect is given by the difference between the new demand $\frac{16}{3}$ and the original one 4

$$\text{SE} = \frac{16}{3} - 4 = \frac{4}{3}$$

The associated income effect immediately follows from Slutsky's equation, which tells us that the total change can be decomposed into the substitution and income effect.

$$\text{IE} = \text{Total Change} - \text{SE} = 4 - \frac{4}{3} = \frac{8}{3}$$

In figure 4, I denote by $A$,$B$ and $C$ the original optimal choice, the final optimal choice, and the intermediate optimal choice we obtained from $m'$, respectively. The substitution effect is the gap between $A$ and $C$ on the horizontal axis while the income effect is between $C$ and $B$.

## Problem 2 (20pts)

(a) (2pts) $\text{PV} = 80 + \dfrac{240}{1+r} = 80 + \dfrac{240}{3} = 160$

(b) (3pts) The budget constraint is written by

$$C_1 + \frac{1}{3}C_2 = 160$$

To wit briefly, the left-hand side is the present value of consumption stream $(C_1, C_2)$. Observe that I discounted $C_2$ by $\frac{1}{1+r} = \frac{1}{3}$ to express it in terms of present value. The right-hand side simply represents the present value of his income we got in part (a). The budget line tells us that they should be balanced. See figure 5.

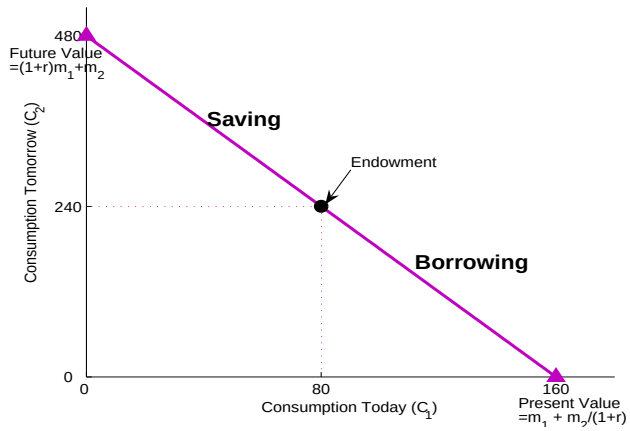


Figure 5: Intertemporal Choice Model

(c) (7pts) For $\delta = 2$, observe that the utility function comes down to

$$U(C_1, C_2) = \ln C_1 + \frac{1}{3}\ln C_2$$

which is a Cobb-Douglas function. Using its magic formula, we obtain

$$C_1 = \frac{a}{a+b} \cdot \frac{m}{p_1} = \frac{1}{1+\frac{1}{3}} \cdot \frac{160}{1} = 120$$

$$C_2 = \frac{b}{a+b} \cdot \frac{m}{p_2} = \frac{\frac{1}{3}}{1+\frac{1}{3}} \cdot \frac{160}{\frac{1}{3}} = 120.$$

To consume 120 today, he must save $80 - 120 = -40$, that is, he must borrow 40. He does smooth his consumption plan because $C_1 = C_2 = 120$.

(d) (3pts) $\text{PV} = \dfrac{x}{r} = \dfrac{200}{0.1} = 2,000.$

(e) (5pts) The characterization equation is given by

$$\frac{200,000}{0.01}\left[1 - \frac{1}{(1+0.01)^{40}}\right] = \frac{C}{0.01}\left[1 - \frac{1}{(1+0.01)^{70}}\right]$$

The left-hand side and the right-hand side represents the present value of earnings for next 40 years and the present value of consumptions for next 80 years, respectively. And the constant level of consumption $C$ comes from the equation in which these two values are balanced.

## Problem 3 (15pts)

(a) (2pts) $U(x_1, x_2) = \min\{x_1, 3x_2\}$

(b) (3pts) See figure 6.

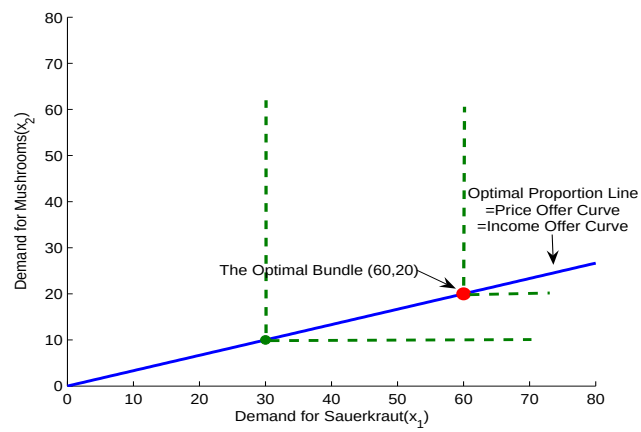


Figure 6: Perfect Complements

(c) (6pts) The associated "two secrets of happiness" with perfect complements is

$$(i)\ \ 2x_1 + 2x_2 = 160$$
$$(ii)\ \ x_1 = 3x_2.$$

The first condition indicates the consumer's balanced budget, and its economic intuition is the same as before: All income must be exhausted. The second condition represents the optimal proportion $(3:1)$ between quantities demanded for sauerkraut and mushrooms.

To figure out the optimal choice, we plug in $3x_2$ for $x_1$ in the budget line and obtain

$$2(3x_2) + 2x_2 = 8x_2 = 160 \Rightarrow x_2 = 20.$$

Substituting into $x_1 = 3x_2$ we get $x_1 = 60$. The solution is interior as $x_1$ and $x_2$ are strictly positive.

(d) (4pts) Since the two goods are perfect complements, both the price offer curve and the income offer curve would coincide with the optimal proportion line. Refer Figure 6.

## Problem 4 (15pts)

(a) (4pts) $x_1 + x_2 = 300$. The value of endowment, the total resources available for consumption, is $p_1\omega_1 + p_2\omega_2 = 100 + 200 = 300$. See Figure 7.

(b) (4pts) Since $p_1 = p_2 = 1$ and the two goods are perfect substitutes, Sam is willing to consume the good with more marginal utilities. In this example, it follows from the utility function that apples(good 1) yield more marginal utilities. Therefore, $x_1 = 300$ and $x_2 = 0$. Since $x_2 = 0$, the solution is not interior but it is at the corner.

(c) (4pts) The net demand for apples is $x_1 - \omega_1 = 300 - 100 = 200$, which is positive. It means that Sam is a net buyer of apples. The net demand for oranges, on the other hand, is $x_2 - \omega_2 = 0 - 200 = -200 < 0$. Hence he is a net seller for oranges.

(d) (3pts) It is because apples and oranges, in this example, are perfectly substitutable. Hence the consumer would choose either one good only, depending on the marginal utility from a dollar.

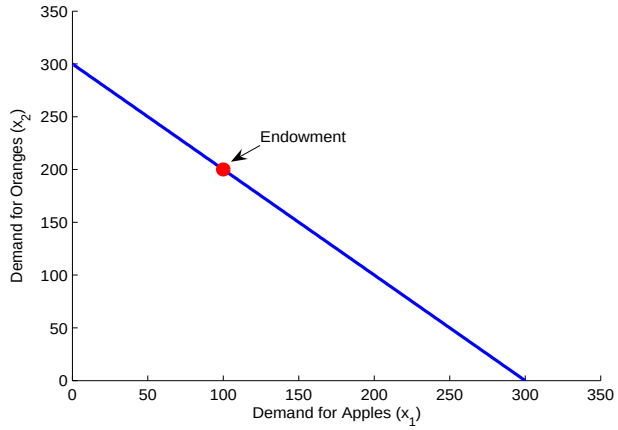


Figure 7: Budget Line and Endowment Point

# Bonus Problem

## Perpetuity

A perpetuity in an annuity that has no definite end, or a stream of cash payments, say $x$, that continues indefinitely. With the (risk-free) interest rate $r$, we can write its present value as

$$\begin{aligned}
\text{PV} &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots \\
&= \frac{x}{(1+r)} + \left[\frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots\right] \\
&= \frac{x}{(1+r)} + \frac{1}{(1+r)} \underbrace{\left[\frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \cdots\right]}_{=\text{PV}}
\end{aligned}$$

In the second step, I just partitioned the sum into the first term and the remainder using the parenthesis. The key step is the next one, factoring $\frac{1}{1+r}$ out of the remainder. Then we come up with the same sequence as the original PV. It leads to the following simple equation

$$\text{PV} = \frac{x}{(1+r)} + \frac{1}{(1+r)} \cdot \text{PV}$$

Solving for PV gives us the desired formula, $\text{PV} = \dfrac{x}{r}$.

## Annuity

The annuity is an asset that promises a terminating stream of fixed payments over a prespecified period of time. Its value is closely linked to *"No Arbitrage Principle"*, the most fundamental principle in finance. It states that "two assets with identical cash flows must trade at the same price." With this principle in hand, consider one annuity which guarantees to annually give us payment $x$ over $T$ years. Now it is the key idea that with the two perpetuities we can replicate the identical cash flows as the annuity.

- Perpetuity 1 - It promises $x$ payment forever from now on.
- Perpetuity 2 - It promises $x$ payment forever but $T+1$ years from now.

Observe that the cash flow of the annuity is the same as the cash flow of "Perpetuity 1 − Perpetuity 2". By no arbitrage condition, then, it must be the case that

$$\begin{aligned}
\text{PV(Annuity)} &= \text{PV(Perpetuity} \quad 1) - \text{PV(Perpetuity} \quad 2) \\
&= \frac{x}{r} - \frac{x}{r} \cdot \frac{1}{(1+r)^T} \\
&= \frac{x}{r}\left[1 - \frac{1}{(1+r)^T}\right].
\end{aligned}$$

**Econ 301**
**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (10 extra "e" points). Make sure you answer the first four questions before working on the bonus one!

**Problem 1.** (50 points) To reenergize for Econ 301 class, in the morning, Tony always drinks Mountain Dew $(x_1)$ and eats Burritos $(x_2)$.

a) Suppose Mountain Dew costs $p_1 = \$2$, burrito costs $p_2 = \$10$, and his daily budget is $m = \$40$. Show graphically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set is affected by inflation of 100% that affects prices of both commodities but does not affect income (so his income stays the same $m = \$40$)?

b) Tony's preferences are given by the following utility function

$$U(x_1, x_2) = x_1^7 x_2^7.$$

Find Tony's marginal rate of substitution (MRS) as a function of $x_1, x_2$ (give a formula for MRS).

- What is the value of MRS at consumption bundle $(2, 1)$ (give a number)?

- Which of the two goods is more valuable, soda or burrito, if Tony drinks two Mountain Dews and consumes one burrito?

- Depict his indifference curve map in a commodity space. Mark the slope of the indifference curve at the bundle $(2, 1)$.

c) In the commodity space $(x_1, x_2)$, find (geometrically) Tony's optimal choice,.assuming pre-inflation prices $p_1 = \$2$, and $p_2 = \$10$. Describe how the two properties of the optimal bundle, known as two "secrets of happiness" (two short sentences) can be seen in the graph.

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).

- Derive the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation).

- What fraction of income is spent on burritos (give the percentage)?

- Find analytically and geometrically the demand curve for Mountain Dew (given $p_2 = \$10$ and $m = \$40$) and Engel curve (given $p_1 = \$2$, and $p_2 = \$10$)

- Are they Giffen goods? Why? (yes/no answer + one sentence).

e) Using your formula from d) find the optimal consumption levels of both types of commodities $(x_1, x_2)$ for:

- $p_1 = \$2, p_2 = \$10$ and $m = \$40$ (give two numbers).

and after the price of Mountain Dew decreased:

- for $p_1 = \$1, p_2 = \$10$ and $m = \$40$ (give two numbers).

What is the total change in consumption of Mountain Dew? (give a number). Illustrate the change on the graph.

f) decompose the total change in consumption of $x_1$ from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

**Problem 2.** (20 points) Bill is a wild-animal lover. From his recent trip to Galapagos Islands he brought a small Iguana. His new pet has only three legs: one left and two right. (Iguanas use magma heated soil to warm their eggs and his favorite pet lost one left leg during the last volcano eruption). To survive the famous Madisonian winter, the iguana has to wear shoes,.left $(x_1)$ and right ones $(x_2)$.

a) Write down Bill's utility function representing his preferences over right and left shoe (function $U(x_1, x_2)$).

b) In the commodity space $(x_1, x_2)$, carefully depict Bill's indifference curves.

c) Find analytically Bill's demand for shoes if $p_1 = \$6$ and $p_2 = \$2$ and Bill's budget for iguana shoes is $m = \$40$. Is the solution interior? (give two numbers and a yes/no answer).

d) Illustrate Bill's optimal choice on the graph including the indifference curves and the budget set.

e) Suppose the price of left shoe goes down to $p_1 = \$1$. Find Bill's new demand for shoes. What can you say about the substitution effect? How about the income effect? (Answer the latter question without any calculations, using only a graph).

**Problem 3.** (15 points) Ramon decides about his new collections of postage stamps. He is interested in two themes: "the birds of the world", $x_1$, (measures the number of stamps in the subcollection with birds) and "the famous mathematicians", $x_2$. The utility derived from the collection is given by

$$U(x_1, x_2) = x_1 + 100 \times \ln x_2.$$

a) What is the optimal collection of stamps if the prices are $p_1 = 1$ and $p_2 = 1$ and $m = 200$. (find two numbers $x_1$ and $x_2$). Is your solution interior, or corner?

b) Find the optimal collection if the prices are still $p_1 = 1$ and $p_2 = 1$, but the income is only $m = 50$. Depict Ramon's optimal choice in the commodity space.

**Problem 4.** (15 points) Jacob can use his 24h for leisure $R$, or work. The hourly wage rate is $w = 10$. Jacob spends all his money on cheese curds $C$.

a) Draw Jacob's budget set, given the price of cheese is $p_c = \$5$ (mark the endowment point).

Let the utility function be given by

$$U(x_1, x_2) = R + C.$$

b) Are leisure and cheese curds perfect complements, perfect substitutes or none of them?

c) What is the optimal choice of leisure, cheese curds and labor supply? (Find geometrically and give three numbers).

d) Harder: Plot a graph with labor supply (horizontal axis) and wage rate (vertical one) assuming $p_c = \$5$ (hint: for what value of $w$ do we go from one "bang" to the other "bang" solution?)

**Bonus Problem.** (extra 10 points) Let $U(x_1, x_2) = x_1^3 x_2$, and $V(x_1, x_2) = 3 \ln x_1 + \ln x_2$ be two utility functions.

a) Show that $U(\cdot)$ is a monotone transformation of $V(\cdot)$, and hence they define the same preferences.

b) Derive MRS for each of the two functions. Using the two formulas for MRS, argue that the functions define the same indifference curve maps.

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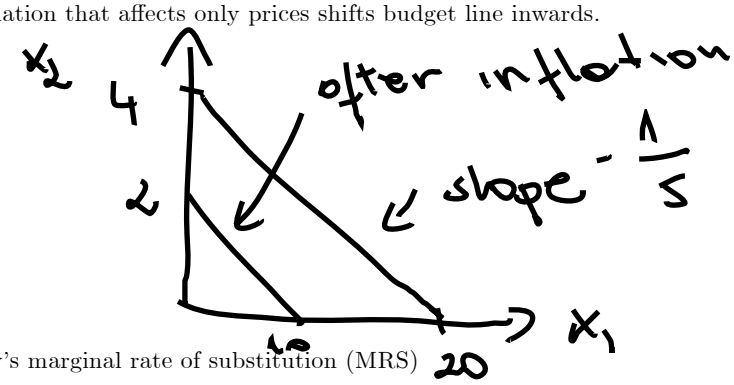
**Solutions to midterm 1 (Group A)**

Problem 1. (50 points)

a) With $p_1 = \$2$, $p_2 = \$10$ and $m = \$40$ the budget set is (two extreme consumption bundles are 20 and 4). Inflation that affects only prices shifts budget line inwards.




b) Tony's marginal rate of substitution (MRS)

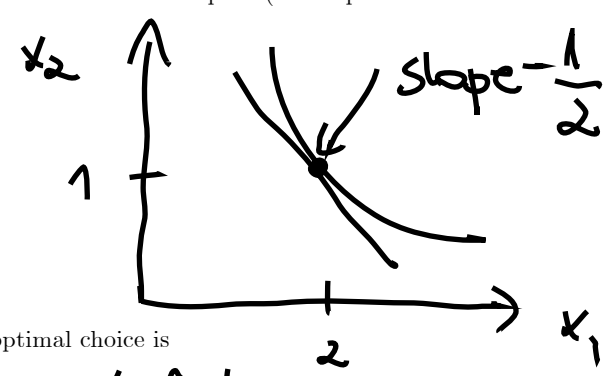
$$MRS = -\frac{MU_1}{MU_2} = -\frac{x_2}{x_1}$$

- The value of MRS at consumption bundle $(2, 1)$ is

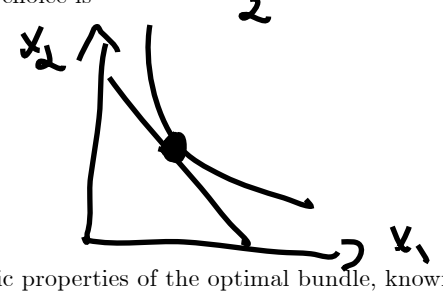
$$|MRS| = |-\frac{1}{2}| = \frac{1}{2}$$

- Burrito $(x_2)$ is more valuable than Mountain Dew $(x_1)$

- Tony's indifference curve map is. (the slope of her indifferent curve that passes through bundle $(2, 1)$ is $-\frac{1}{2}$.




c) Tony's optimal choice is




- the two geometric properties of the optimal bundle, known as two "secrets of happiness" are:

1. At the optimal bundle, the indifference curve is tangent to a budget set
2. The optimal bundle is located on budget line

d) mathematically the two secrets of happiness, are

$$\begin{cases} MRS = -\frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 = m \end{cases}$$

- the economic intuition behind the two conditions is:
The individual value of $x_1$ in terms of $x_2$ coincides with the market value
The income of a consumer is exhausted
- the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ can be found as follows
From the MRS condition

$$MRS = -\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

hence

$$x_2 = \frac{p_1}{p_2} x_1$$

plugging in budget constraint

$$p_1 x_1 + p_2 \left( \frac{p_1}{p_2} x_1 \right) = m$$

Solving for $x_1$ gives

$$x_1 = \frac{1}{2} \frac{m}{p_1}$$

Plugging in

$$x_2 = \frac{p_1}{p_2} \left( \frac{1}{2} \frac{m}{p_1} \right) = \frac{1}{2} \frac{m}{p_2}$$

- the fraction of income spent on burritos is

$$\frac{p_1 x_1}{m} = \frac{1}{2} = 50\%$$

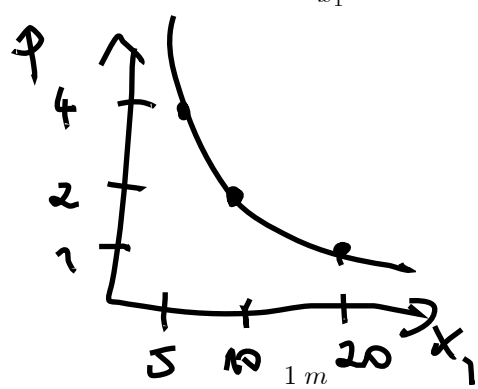
- and the demand curve for burritos book (given $p_2 = \$10$, and $m = \$40$) and Engel curve (given $p_1 = \$2$, and $p_2 = \$10$)
Demand curve

$$x_1 = \frac{1}{2} \frac{m}{p_1} = \frac{1}{2} \frac{40}{p_1} = \frac{20}{p_1}$$

and hence inverse demand is

$$p_1 (x_1) = \frac{20}{x_1}$$

Geometrically




Engel curve: Since

$$x_1 = \frac{1}{2} \frac{m}{p_1}$$

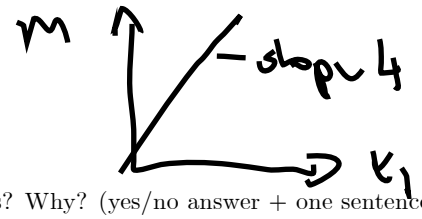
at $p_1 = \$2$

$$x_1 = \frac{1}{2} \frac{m}{2} = \frac{1}{4} m$$

hence

$$m (x_1) = 4 x_1$$

Geometrically




- are they Giffen goods? Why? (yes/no answer + one sentence).
No, because the demand curve is downwardslopping on the whole domain.
e) The optimal consumption levels for $(x_1, x_2)$.
- at $p_1 = \$2, p_2 = \$10$ and $m = \$40$

$$x_1 = \frac{1}{2}\frac{m}{p_1} = \frac{1}{2}\frac{40}{2} = 10$$

and

$$x_2 = \frac{1}{2}\frac{m}{p_2} = \frac{1}{2}\frac{40}{10} = 2$$

and after the price of science-fiction book decreased, for $p_1 = \$1, p_2 = \$10$ and $m = \$40$

$$x_1 = \frac{1}{2}\frac{m}{p_1} = 20$$

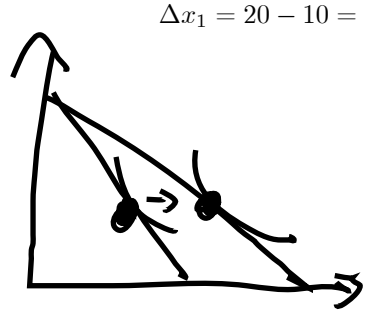
and

$$x_2 = \frac{1}{2}\frac{m}{p_2} = \frac{1}{2}\frac{40}{10} = 2$$

Hence the total change in consumption of $x_1$ is

$$\Delta x_1 = 20 - 10 = 10$$

Geometrically



f) Substitution effect: auxiliary budget

$$m' = 10 \times 1 + 10 \times 2 = 30$$

and hence

$$x_1 = \frac{1}{2}\frac{30}{1} = 15$$

so $SE$ is equal to

$$SE = 15 - 10 = 5$$

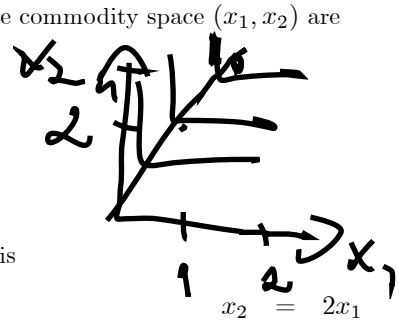
and income effect is

$$IE = 10 - 5 = 5$$

Problem 2. .
a) Bill's utility function is

$$U(x_1, x_2) = \min(2x_1, x_2)$$

b) indifference curves.in the commodity space $(x_1, x_2)$ are




c) Bill's demand for shoes is

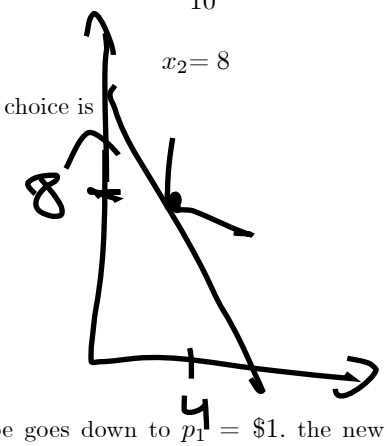
$$\begin{aligned} x_2 &= 2x_1 \\ 6x_1 + 2x_2 &= 40 \end{aligned}$$

$$6x_1 + 2(2x_1) = 40$$

$$x_1 = \frac{40}{10} = 4$$

$$x_2 = 8$$

d) geometrically Bills's optimal choice is




e) when the price of a left shoe goes down to $p_1 = \$1$. the new demand is given by the system of equations

$$\begin{aligned} x_2 &= 2x_1 \\ x_1 + 2x_2 &= 40 \end{aligned}$$

and hence demand is

$$\begin{aligned} x_1 &= 8 \\ x_2 &= 16 \end{aligned}$$

The substitution effect is zero (perfect complements) and the income effect is 4.

Problem 3.

a) the two secrets of happiness are

$$\begin{aligned} -\frac{x_2}{100} &= -1 \\ x_1 + x_2 &= 200 \end{aligned}$$

and hence $x_2 = 100$ and $x_1 = 100$. Since both are positive, this is interior solution.

b) the two secrets of happiness are

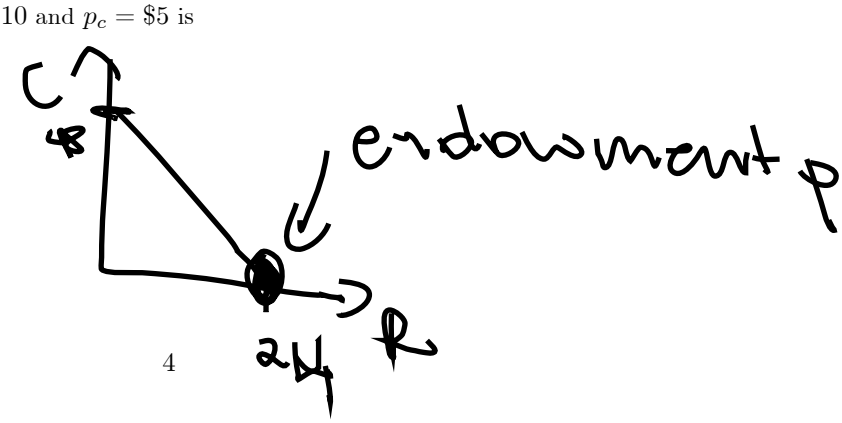
$$\begin{aligned} -\frac{x_2}{100} &= -1 \\ x_1 + x_2 &= 50 \end{aligned}$$

and hence secrets of happiness give $x_2 = 100$ and $x_1 = -50$. Since consumption must be non-negative the optimal consumption is $x_1 = 0$ and $x_2 = 50$, which is a cornet solution.

Problem 4.

a) Jacob's budget set, with $w = \$10$ and $p_c = \$5$ is

Income is $m = 10 \times 24 = \$240$

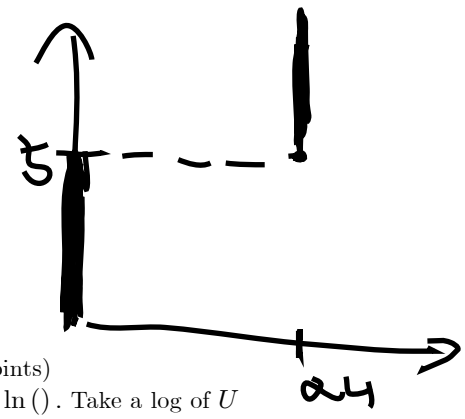



b) They are perfect substitutes

c) $|MRS| = 1 < \frac{w}{p_c} = 2$ which implies that Jacob cares less about leisure than consumption,. therefore he will spend the whole day at work

$$R = 0, LS = 24 \text{ and } C = 24\frac{10}{5} = 48$$

d)




Bonus Problem. (extra 10 points)

a) Monotone transformation $\ln()$. Take a log of $U$

$$\ln U() = \ln x_1^3 x_2 = \ln x_1^3 + \ln x_2 = 3\ln x_1 + \ln x_2 = V()$$

where we used two properties of ln function.

b) For $U()$, marginal rate of substitution is

$$MRS = -\frac{MU_1}{MU_2} = -\frac{3x_1^2 x_2}{x_1^3} = -\frac{3x_2}{x_1}$$

and for $V()$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{3/x_1}{1/x_2} = -\frac{3x_2}{x_1}$$

and hence MRS coincides for all $(x_1, x_2)$.It follows that the slopes of indifference curves are the same at any point and hence they must be the same.

**Econ 301**
**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 3 questions (60+25+15=100 points) + bonus (10 "e" points). Make sure you answer the first three questions before working on the bonus one!

**Problem 1.** (60 points) Maggie likes to read science fiction $(x_1)$ and romance $(x_2)$ novels. Her preferences over the two types of books are represented by a utility function

$$U(x_1, x_2) = (x_1)^{10} (x_2)^{20}$$

a) Find Maggie's marginal rate of substitution (MRS) as a function of $x_1, x_2$ (give a formula).

- what is the value of MRS at consumption bundle $(2, 2)$ (give a number).

- complete the sentence: "The Marginal Rate of Substitution is a (marginal) value of a ... in terms of ..."

- how much one must compensate Maggie in terms of romance books, after taking away 0.00001 of a science-fiction book, in order to keep her indifferent? (give a number, assume she consumes bundle $(2, 2)$).

- depict her indifference curve map in a commodity space. Mark the slope of her indifferent curve that passes through bundle $(2, 2)$.

b) Suppose the price of a science-fiction book is $p_1 = \$10$, a romance book costs $p_2 = \$5$ and her total monthly spending on books is $m = \$300$. Show graphically her budget constraint in the commodity space. Mark the two extreme consumption bundles (give values). On the same graph, show how the budget set would be affected by the introduction of an ad valorem tax on romance books $(x_2)$ at rate 100%?

c) In the commodity space $(x_1, x_2)$, find (geometrically) Maggie's optimal choice.

- describe the two properties of the optimal bundle, known as two "secrets of happiness" (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values).

- provide some economic intuition behind the two conditions (ca. two sentences for each).

- derive the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation).

- what fraction of income is spent on science - fiction novels (give the percentage).

- find analytically and geometrically the demand curve for science finction book (given $p_2 = \$5$, and $m = \$300$) and Engel curve (given $p_1 = \$10$, and $p_2 = \$5$)

- are science-fiction books normal goods? Why? (yes/no answer + one sentence).

- are they Giffen goods? Why? (yes/no answer + one sentence).

e) Using your formula from d) find the optimal consumption levels for both types of books $(x_1, x_2)$.

- for $p_1 = \$10, p_2 = \$5$ and $m = \$300$ (give two numbers).

and after the price of science-fiction book decreased:

- for $p_1 = \$5, p_2 = \$5$ and $m = \$300$ (give two numbers).

What is the total change in consumption of $x_1$? (give one number). Illustrate this change on the graph.

f) decompose the total change in $x_1$ from f) into a substitution and income effect. (Calculate the two numbers, show how you found them on the graph.) Complete the two sentences:

"The substitution effect is attributed to the pure change in .... induced by the decrease of nominal price $p_1$"

"The income effect can be attribute to the pure change in .... induced by the decrease of nominal price $p_1$"

g) which of the following alternative utility functions represents Maggie's preferences (there are two such

functions)?

$$\begin{aligned} V(x_1, x_2) &= 30\left(x_1^{10} \times x_2^{20}\right) + 3 \\ V(x_1, x_2) &= 10x_1 \times 20x_2 \\ V(x_1, x_2) &= 10\ln x_1 \times 20\ln x_2 + 2 \\ V(x_1, x_2) &= 10\ln x_1 + 20\ln x_2 + 7 \end{aligned}$$

Explain why the utility functions you have selected represent Maggie's preferences (one sentence). Suggest the transformation of $U()$ function that makes the two $V()$ functions equivalent.

**Problem 2.** (20 points) Jimmy's favorite hobby is slot car racing. He assembles slot cars from parts, by adding four wheels $(x_1)$ to an engine $(x_2)$ (these are supertrucks, with five wheels on each side). He purchases the parts on the market.

a) write down Jimmy's utility function representing his preferences over wheels and engines (function $U(x_1, x_2)$).

b) in the commodity space $(x_1, x_2)$, carefully depict Jimmy's indifference curves.

c) find analytically Jimmy's demand for parts if one wheel costs $p_1 = \$50$, an engine is $p_2 = \$100$ and Jimmy's budget for slot cars is $m = \$600$. Is the solution interior (give two numbers and yes/no answer).

d) illustrate Jimmy's optimal choice on the graph including the indifference curves and the budget set.

e) suppose the price of one wheel goes down to $p_1 = \$25$. Find Jimmy's new demand for the parts. What can you say about the substitution effect? How about the income effect? (you can answer the last question without any calculations, using only a graph).

**Problem 3.** (20 points) Ramon Gonzales M. Panetelas is a specialist in Habanos cigars (famous Cuban cigars). Cuban cigars are sold either in 10 cigar packs $(x_1)$, or in singles $(x_2)$. Ramon has no income. Instead he is initially endowed with $\omega_1 = 5$ packs and $\omega_2 = 50$ cigars.

a) draw Ramon's budget set, given the price of a pack is equal to $p_1 = \$5$ and a single cigar is $p_2 = \$1$ (mark the endowment point).

b) Illustrate geometrically Ramon's optimal demand for packs and single cigars, given his utility function

$$U(x_1, x_2) = 10x_1 + x_2$$

(Give two numbers $(x_1, x_2)$, and mark them on the graph, including budget set and the indifference curves.)

c) What is your answer to b) when prices are $p_1 = 20$ and $p_2 = 1$. (Give two numbers $(x_1, x_2)$, and plot the graph.)

d) Harder: Give the formula for the demands $x_1, x_2$ as a function of $p_1, p_2$ and endowments $\omega_1$ and $\omega_2$. Show the demand curve for $x_1$ on the graph, assuming $p_2 = 1$, $\omega_1 = 5$ and $\omega_2 = 50$.

**Bonus Problem.** (extra 10 points) Depict a map of indifference curves that is consistent with

a) inferior goods

b) Giffen goods

(Make sure you explain why these graphs represent the respective preferences.)

**Econ 301**
**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Solutions to midterm 1 (Group A)

Problem 1. (60 points)
a) Maggie's marginal rate of substitution (MRS)

$$MRS = -\frac{MU_1}{MU_2} = -\frac{1}{2}\frac{x_2}{x_1}$$

- the value of MRS at consumption bundle $(2,2)$ is

$$MRS = -\frac{1}{2}$$

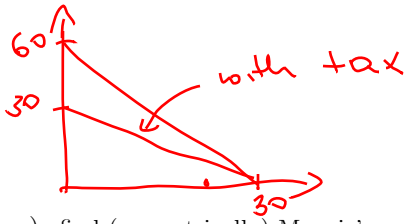
- "The Marginal Rate of Substitution is a (marginal) value of a science finction books in terms of romance novels"

- after taking away 0.00001 of a science-fiction book, to keep her indifferent one must compensate Maggie in terms of romance books

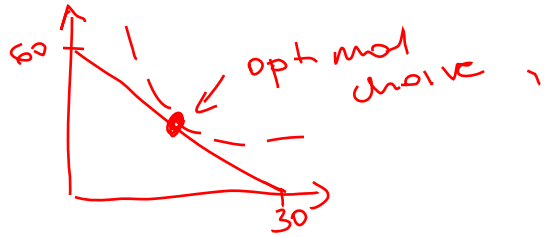
$$0.00001 \times \frac{1}{2} = 0.000005$$

- her indifference curve map is. (the slope of her indifferent curve that passes through bundle $(2,2)$ is $-\frac{1}{2}$.


b) With $p_1 = \$10$, $p_2 = \$5$ and $m = \$300$ the budget set is (two extreme consumption bundles are 30 and 60). The budget set with ad valorem tax shifts inwards.




c) In the commodity space $(x_1, x_2)$, find (geometrically) Maggie's optimal choice.




- the two geometric properties of the optimal bundle, known as two "secrets of happiness" are:
1. At the optimal bundle, the indifference curve is tangent to a budget set
2. The oprimal bundle is locaded on budget line
d) mathematically the two secrets of happiness, are

$$\begin{cases} MRS = -\frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 = m \end{cases}$$

- the economic intuition behind the two conditions is:

The individual value of $x_1$ in temrms of $x_2$ coincides with the market value

The income of a consumer is exhausted

- the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ can be found as follows

From the MRS condition

$$MRS = -\frac{1}{2}\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

hence

$$x_2 = 2\frac{p_1}{p_2}x_1$$

plugging in budget constraint

$$p_1 x_1 + p_2\left(2\frac{p_1}{p_2}x_1\right) = m$$

Solving for $x_1$ gives

$$x_1 = \frac{1}{3}\frac{m}{p_1}$$

Pluging in

$$x_2 = 2\frac{p_1}{p_2}\left(\frac{1}{3}\frac{m}{p_1}\right) = \frac{2}{3}\frac{m}{p_2}$$

- the fraction of income spent on science - fiction novels is

$$\frac{p_1 x_1}{m} = \frac{1}{3} = 33\%$$

- and the demand curve for science finction book (given $p_2 = \$5$, and $m = \$300$) and Engel curve (given $p_1 = \$10$, and $p_2 = \$5$)

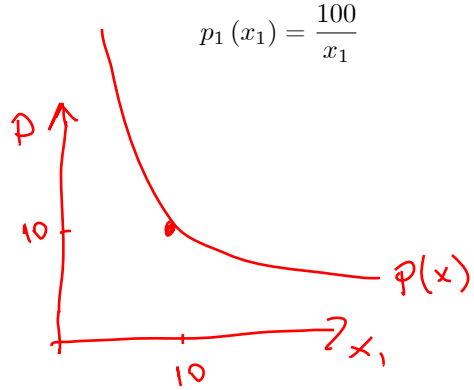
Demand curve

$$x_1 = \frac{1}{3}\frac{m}{p_1} = \frac{100}{p_1}$$

and hence invese demand is

$$p_1(x_1) = \frac{100}{x_1}$$

Geometrically




Engel curve: Since

$$x_1 = \frac{1}{3}\frac{m}{p_1}$$

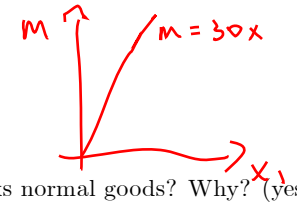
at $p_1 = \$10$

$$x_1 = \frac{1}{3}\frac{m}{10} = \frac{1}{30}m$$

hence

$$m(x_1) = 30x_1$$

Geomettrically




- are science-fiction books normal goods? Why? (yes/no answer + one sentence).

Yes, because their demand increases in income

- are they Giffen goods? Why? (yes/no answer + one sentence).

No, becase the demand curve is downwardslopping on the whole domain.

e) The optimal consumption levels for both types of books $(x_1, x_2)$.

- for $p_1 = \$10, p_2 = \$5$ and $m = \$300$

$$x_1 = \frac{1}{3}\frac{m}{p_1} = \frac{1}{3}\frac{300}{10} = 10$$

and

$$x_2 = \frac{2}{3}\frac{m}{p_2} = \frac{2}{3}\frac{300}{5} = 40$$

and after the price of science-fiction book decreased, for $p_1 = \$5, p_2 = \$5$ and $m = \$300$

$$x_1 = \frac{1}{3}\frac{m}{p_1} = \frac{1}{3}\frac{300}{5} = 20$$

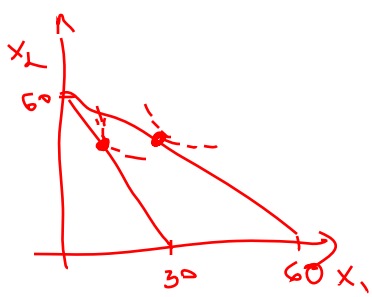
and

$$x_2 = \frac{2}{3}\frac{m}{p_2} = \frac{2}{3}\frac{300}{5} = 40$$

Hence the total change in consumption of $x_1$ is

$$\Delta x_1 = 20 - 10 = 10$$

Geometrically




f) Substitution effect: auxiliary budget

$$m' = 10 \times 5 + 5 \times 40 = 250$$

and hence

$$x_1 = \frac{1}{3}\frac{250}{5} = \frac{50}{3} = 16\frac{2}{3}$$

so $SE$ is equal to

$$SE = 16\frac{2}{3} - 10 = 6\frac{2}{3}$$

and income effect is

$$IE = 10 - 6\frac{2}{3} = 3\frac{1}{3}$$

"The substitution effect is attributed to the pure change in relative price induced by the decrease of nominal price $p_1$"

"The income effect can be attribute to the pure change in real income induced by the decrease of nominal price $p_1$"

g) From the functions

$$\begin{aligned}
V(x_1, x_2) &= 30\left(x_1^{10} \times x_2^{20}\right) + 3 \\
V(x_1, x_2) &= 10x_1 \times 20x_2 \\
V(x_1, x_2) &= 10\ln x_1 \times 20\ln x_2 + 2 \\
V(x_1, x_2) &= 10\ln x_1 + 20\ln x_2 + 7
\end{aligned}$$

the first and last represent Maggie's preferences - they are monotone transformations of $U()$. The first transformation is

$$f(U) = 30U + 3$$

and the second is

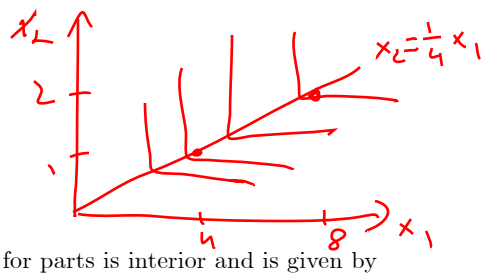
$$f(U) = \ln U + 7$$

Problem 2. .

a) Jimmy's utility function is

$$U(x_1, x_2) = \min(x_1, 4x_2)$$

b) in the commodity space $(x_1, x_2)$, carefully depict Jimmy's indifference curves.




c) Jimmy's demand for parts is interior and is given by

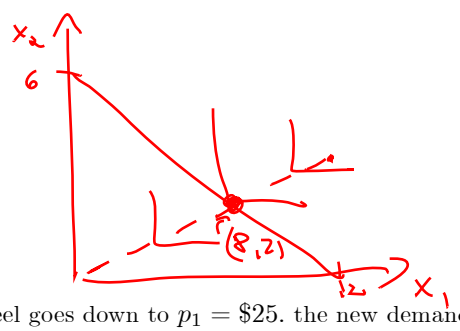
$$\begin{aligned} x_2 &= \frac{1}{4}x_1 \\ 50x_1 + 100x_2 &= 600 \end{aligned}$$

$$50x_1 + 100\frac{1}{4}x_1 = 600$$

$$x_1 = \frac{600}{75} = 8$$

$$x_2 = \frac{1}{4} \times 8 = 2$$

d) geometrically Jimmy's optimal choice is



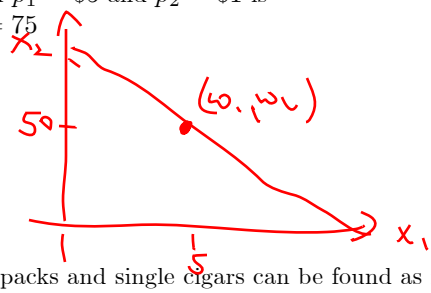

e) when the price of a wheel goes down to $p_1 = \$25$. the new demand is

$$\begin{aligned} x_2 &= \frac{1}{4}x_1 \\ 25x_1 + 100x_2 &= 600 \end{aligned}$$

$$\begin{aligned} x_1 &= 12 \\ x_2 &= 3 \end{aligned}$$

Thte substitution effec is zero (perfect complements) and the income effect is 4 wheels.

Problem 3.
a) Ramon's budget set, with $p_1 = \$5$ and $p_2 = \$1$ is
Budget set: $m = 25 + 50 = 75$

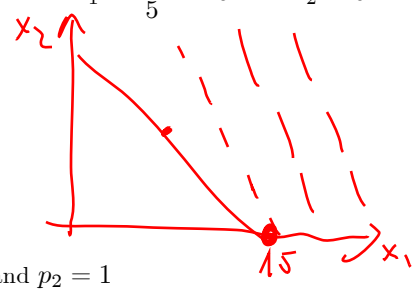



b) The optimal demand for packs and single cigars can be found as follows. Since

$$-MRS = 10 > \frac{p_1}{p_2} = 5$$

therefore the total income will be invested in packs

$$x_1 = \frac{75}{5} = 15 \text{ and } x_2 = 0$$




c) When prices are $p_1 = 20$ and $p_2 = 1$

$$-MRS = 10 < \frac{p_1}{p_2} = 20$$

and hence the total income will be invested in $x_2$. At such prices demands are

$$x_1 = 0 \text{ and } x_2 = \frac{5 \times 20 + 50}{1} = 150$$

d) The demands are
If $\frac{p_1}{p_2} < 10$ then

$$\begin{aligned} x_1 &= \frac{p_1\omega_1 + p_2\omega_2}{p_1} \\ x_2 &= 0 \end{aligned}$$

if $\frac{p_1}{p_2} > 10$ then

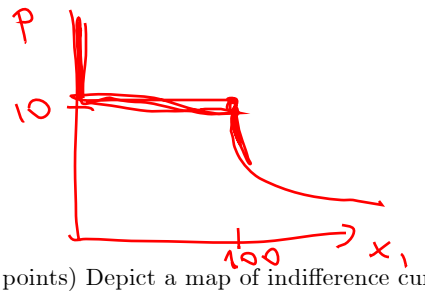
$$\begin{aligned} x_1 &= 0 \\ x_2 &= \frac{p_1\omega_1 + p_2\omega_2}{p_2} \end{aligned}$$

and if $\frac{p_1}{p_2} = 10$

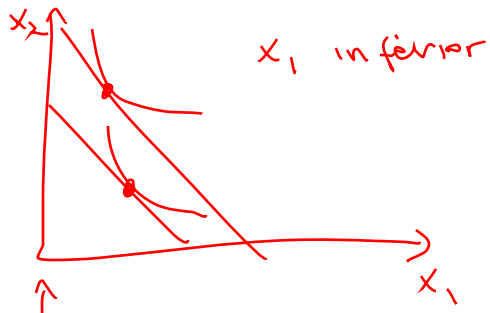
$$\begin{aligned} x_1 &= \alpha\frac{p_1\omega_1 + p_2\omega_2}{p} \\ x_2 &= (1-\alpha)\frac{p_1\omega_1 + p_2\omega_2}{p_2} \end{aligned}$$

for $\alpha \in (0, 1)$

The demand curve for $x_1$ is ($p_2 = 1$, $\omega_1 = 5$ and $\omega_2 = 50$).




Bonus Problem. (extra 10 points) Depict a map of indifference curves that is consistent with
a) inferior goods




b) Giffen goods


(Make sure you explain why these graphs represent the respective preferences.)

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## Midterm 1 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

**Problem 1.** (50 points)
Patrick spends his income on books $(x_1)$ and CD $(x_2)$.

a) Suppose the price of a book is $p_1 = \$10$, the price of a CD is $p_2 = \$5$, and Patrick's daily budget is $m = \$40$. Show graphically Patrick's budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 2 CDs (assume that he can always dispose the gift).

b) Patrick's preferences are given by the following utility function

$$U(x_1, x_2) = x_1 + 2\ln x_2.$$

Find Patrick's marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give the formula for MRS).

- What is the value of MRS at consumption bundle $(5, 8)$ (give a number)?

- Suppose Patrick "consumes" 5 books and 8 CDs and one takes away 0.0001 of a book. What is compensation in terms of CDs is sufficient to make Patrick indifferent?

- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle $(5, 8)$.

c) From now on assume no gift. In the commodity space $(x_1, x_2)$, find (geometrically) Patrick's optimal choice. Describe how the two "secrets of happiness" can be seen geometrically in the graph (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).

e) Using the two conditions from d) find the optimal consumption levels of both types of commodities $(x_1, x_2)$ for:

- $p_1 = \$10, p_2 = \$5$ and $m = \$40$ (give two numbers).

and after the price of a book increases:

- for $p_1 = \$30, p_2 = \$5$ and $m = \$40$ (give two numbers).

Is each of the solutions interior? Illustrate the change on the graph.

f) Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

**Problem 2.** (25 points) Michael *always* consumes three hamburgers $x_1$ along with one Coke $x_2$ (this is the only healthy combination of the two products!) .

a) Propose Michael's utility function that represents his preferences over hamburgers and Coke (function $U(x_1, x_2)$).

b) In the commodity space $(x_1, x_2)$, carefully depict Michael's indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters $p_1$ and $p_2$ and $m$). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels's optimal choice of $x_1$ and $x_2$ as a function of ($p_1$, $p_2$ and $m$). Is the choice (solution) interior for any price and income? (Give formulas $x_1(p_1, p_2, m)$ and a yes-no answer.)

e) Using $x_1(p_1, p_2, m)$ derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption $TCH$? (You do not have to give any number. Just relate two effects to $TCH$.)

**Problem 3.** (15 points) Adam spends all his income on food $(x_1)$ and clothing $x_2$).He is a fairly sophisticated fellow and his utility function is quite complicated

$$U(x_1, x_2) = \left[700 \times \sqrt{\ln\left[(2\ln x_1 + \ln x_2)^2\right]} + 10\right]^{800}.$$

a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of $x_1$ and $x_2$if the prices are $p_1 = 4$ and $p_2 = 4$ and $m = 1200$ (find two numbers $x_1$ and $x_2$). Is your solution interior, or corner?

c) Assume $p_2 = 4$ and $m = 1200$. Find analitically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

**Problem 4.** (10 points) Frank can use his 24h for leisure $R$ or work. The hourly wage rate is $w = 20$. Frank is a committed skier and uses all his income on ski passes in Devil's Head Resort $C$.

a) Draw Frank's budget set, given that the price of one ski pass is $p_c = \$10$ (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by

$$U(x_1, x_2) = RC^5.$$

b) What is the real wage? (formula + nubmer) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).

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## Midterm 1 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

**Problem 1.** (50 points)
Patrick spends his income on books $(x_1)$ and CD $(x_2)$.

a) Suppose the price of a book is $p_1 = \$10$, the price of a CD is $p_2 = \$10$, and Patrick's daily budget is $m = \$50$. Show graphically Patrick's budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 2 CDs (assume that he can always dispose the gift).

b) Patrick's preferences are given by the following utility function

$$U(x_1, x_2) = 2x_1 + 6\ln x_2.$$

Find Patrick's marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give the formula for MRS).

- What is the value of MRS at consumption bundle $(3, 6)$ (give a number)?

- Suppose Patrick "consumes" 3 books and 6 CDs and one takes away 0.0001 of a CD. What is compensation in terms of CDs is sufficient to make Patrick indifferent?

- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle $(3, 6)$.

c) From now on assume no gift. In the commodity space $(x_1, x_2)$, find (geometrically) Patrick's optimal choice. Describe how the two "secrets of happiness" can be seen geometrically in the graph (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).

e) Using the two conditions from d) find the optimal consumption levels of both types of commodities $(x_1, x_2)$ for:

- for $p_1 = \$10, p_2 = \$10$ and $m = \$50$ (give two numbers).

and after the price of a book increases:

- for $p_1 = \$20, p_2 = \$10$ and $m = \$50$ (give two numbers).

Is each of the solutions interior? Illustrate the change on the graph.

f) Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

**Problem 2.** (25 points) Michael *always* consumes five hamburgers $x_1$ along with one Coke $x_2$ (this is the only healthy combination of the two products!) .

a) Propose Michael's utility function that represents his preferences over hamburgers and Coke (function $U(x_1, x_2)$.

b) In the commodity space $(x_1, x_2)$, carefully depict Michael's indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters $p_1$ and $p_2$ and $m$). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels's optimal choice of $x_1$ and $x_2$ as a function of ($p_1, p_2$ and $m$). Is the choice (solution) interior for any price and income? (Give formulas $x_1(p_1, p_2, m)$ and a yes-no answer.)

e) Using $x_1(p_1, p_2, m)$ derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption $TCH$? (You do not have to give any number. Just relate two effects to $TCH$.)

**Problem 3.** (15 points) Adam spends all his income on food $(x_1)$ and clothing $(x_2)$ .He is a fairly sophisticated fellow and his utility function is quite complicated

$$U(x_1, x_2) = \left[12 \times \sqrt{\ln\left[(6\ln x_1 + 2\ln x_2)^2\right]} + 3\right]^{300}.$$

a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of $x_1$ and $x_2$if the prices are $p_1 = 5$ and $p_2 = 10$ and $m = 80$ (find two numbers $x_1$ and $x_2$). Is your solution interior, or corner?

c) Assume $p_2 = 10$ and $m = 80$. Find analytically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

**Problem 4.** (10 points) Frank can use his 24h for leisure $R$ or work. The hourly wage rate is $w = 20$. Frank is a committed skier and uses all his income on ski passes in Devil's Head Resort $C$.

a) Draw Frank's budget set, given that the price of one ski pass is $p_c = \$10$ (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by

$$U(x_1, x_2) = R^{17} C^{34}.$$

b) What is the real wage? (formula + number) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).

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## Midterm 1 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

**Problem 1.** (50 points)
Patrick spends his income on books $(x_1)$ and CD $(x_2)$.

a) Suppose the price of a book is $p_1 = \$6$, the price of a CD is $p_2 = \$2$, and Patrick's daily budget is $m = \$18$. Show graphically Patrick's budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 2 CDs (assume that he can always dispose the gift).

b) Patrick's preferences are given by the following utility function

$$U(x_1, x_2) = 4x_1 + 4\ln x_2.$$

Find Patrick's marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give the formula for MRS).

- What is the value of MRS at consumption bundle $(3, 8)$ (give a number)?

- Suppose Patrick "consumes" 3 books and 8 CDs and one takes away 0.0001 of a CD. What is compensation in terms of CDs is sufficient to make Patrick indifferent?

- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle $(3, 8)$.

c) From now on assume no gift. In the commodity space $(x_1, x_2)$, find (geometrically) Patrick's optimal choice. Describe how the two "secrets of happiness" can be seen geometrically in the graph (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).

e) Using the two conditions from d) find the optimal consumption levels of both types of commodities $(x_1, x_2)$ for:

- for $p_1 = \$6, p_2 = \$2$ and $m = \$18$ (give two numbers).

and after the price of a book increases:

- for $p_1 = \$20, p_2 = \$2$ and $m = \$18$ (give two numbers).

Is each of the solutions interior? Illustrate the change on the graph.

f) Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

**Problem 2.** (25 points) Michael *always* consumes one hamburger $x_1$ along with two Cokes $x_2$ (this is the only healthy combination of the two products!) .

a) Propose Michael's utility function that represents his preferences over hamburgers and Coke (function $U(x_1, x_2)$.

b) In the commodity space $(x_1, x_2)$, carefully depict Michael's indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters $p_1$ and $p_2$ and $m$). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels's optimal choice of $x_1$ and $x_2$ as a function of ($p_1, p_2$ and $m$). Is the choice (solution) interior for any price and income? (Give formulas $x_1(p_1, p_2, m)$ and a yes-no answer.)

e) Using $x_1(p_1, p_2, m)$ derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption $TCH$? (You do not have to give any number. Just relate two effects to $TCH$.)

**Problem 3.** (15 points) Adam spends all his income on food $(x_1)$ and clothing $(x_2)$ .He is a fairly sophisticated fellow and his utility function is quite complicated

$$U(x_1, x_2) = \ln\left[0.5 \times \sqrt{\left[(10\ln x_1 + 2\ln x_2)^2\right]} + 3\right]^{300}.$$

a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of $x_1$ and $x_2$if the prices are $p_1 = 2$ and $p_2 = 2$ and $m = 120$ (find two numbers $x_1$ and $x_2$). Is your solution interior, or corner?

c) Assume $p_2 = 2$ and $m = 100$. Find analytically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

**Problem 4.** (10 points) Frank can use his 24h for leisure $R$ or work. The hourly wage rate is $w = \$120$. Frank is a committed skier and uses all his income on ski passes in Devil's Head Resort $C$.

a) Draw Frank's budget set, given that the price of one ski pass is $p_c = \$30$ (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by

$$U(x_1, x_2) = R^3 C.$$

b) What is the real wage? (formula + number) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).

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## Midterm 1 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

**Problem 1.** (50 points)
Patrick spends his income on books $(x_1)$ and CD $(x_2)$.

a) Suppose the price of a book is $p_1 = \$10$, the price of a CD is $p_2 = \$2$, and Patrick's daily budget is $m = \$30$. Show graphically Patrick's budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 1 CDs (assume that he can always dispose the gift).

b) Patrick's preferences are given by the following utility function

$$U(x_1, x_2) = 8x_1 + 8\ln x_2.$$

Find Patrick's marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give the formula for MRS).

- What is the value of MRS at consumption bundle $(4, 8)$ (give a number)?

- Suppose Patrick "consumes" 4 books and 8 CDs and one takes away 0.0001 of a CD. What is compensation in terms of CDs is sufficient to make Patrick indifferent?

- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle $(4, 8)$.

c) From now on assume no gift. In the commodity space $(x_1, x_2)$, find (geometrically) Patrick's optimal choice. Describe how the two "secrets of happiness" can be seen geometrically in the graph (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).

e) Using the two conditions from d) find the optimal consumption levels of both types of commodities $(x_1, x_2)$ for:

- for $p_1 = \$10, p_2 = \$2$ and $m = \$30$ (give two numbers).

and after the price of a book increases:

- for $p_1 = \$40, p_2 = \$2$ and $m = \$30$ (give two numbers).

Is each of the solutions interior? Illustrate the change on the graph.

f) Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

**Problem 2.** (25 points) Michael *always* consumes one hamburger $x_1$ along with four Cokes $x_2$ (this is the only healthy combination of the two products!) .

a) Propose Michael's utility function that represents his preferences over hamburgers and Coke (function $U(x_1, x_2)$.

b) In the commodity space $(x_1, x_2)$, carefully depict Michael's indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters $p_1$ and $p_2$ and $m$). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels's optimal choice of $x_1$ and $x_2$ as a function of ($p_1, p_2$ and $m$). Is the choice (solution) interior for any price and income? (Give formulas $x_1(p_1, p_2, m)$ and a yes-no answer.)

e) Using $x_1(p_1, p_2, m)$ derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption $TCH$? (You do not have to give any number. Just relate two effects to $TCH$.)

**Problem 3.** (15 points) Adam spends all his income on food $(x_1)$ and clothing $(x_2)$ .He is a fairly sophisticated fellow and his utility function is quite complicated

$$U(x_1, x_2) = \ln\left[0.5 \times \sqrt{\left[(12\ln x_1 + 6\ln x_2)^2\right]} + 17 * 21 - 7\right]^{300}.$$

a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of $x_1$ and $x_2$if the prices are $p_1 = 1$ and $p_2 = 1$ and $m = 60$ (find two numbers $x_1$ and $x_2$). Is your solution interior, or corner?

c) Assume $p_2 = 1$ and $m = 60$. Find analytically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

**Problem 4.** (10 points) Frank can use his 24h for leisure $R$ or work. The hourly wage rate is $w = \$120$. Frank is a committed skier and uses all his income on ski passes in Devil's Head Resort $C$.

a) Draw Frank's budget set, given that the price of one ski pass is $p_c = \$30$ (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by

$$U(x_1, x_2) = R^3 C.$$

b) What is the real wage? (formula + number) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).

# 1 Problem 1

(a) His budget line without the CD gifts is straightforward: $10x_1 + 5x_2 = 40$ and $x_1$-intercept $\frac{40}{10} = 4$ and $x_2$-intercept $\frac{40}{5} = 8$ represent his real incomes in terms of books and CDs, respectively. The red line in the figure below exhibits his budget line after those CD gifts.

(b) Since $\text{MU}_1 = \frac{\partial U}{\partial x_1} = 1$ and $\text{MU}_2 = \frac{\partial U}{\partial x_2} = \frac{2}{x_2}$, we can write his MRS as

$$\text{MRS}_{1,2} = -\frac{1}{2/x_2} = -\frac{x_2}{2}. \tag{1.1}$$

At $(5, 8)$, its value is $-4$(or just its absolute value 4). Note that it does not rely upon $x_1$, the quantity demanded for books. And he has to be compensated $0.0001 \times 4$ CDs in order to preserve his current utility level. The indifference is described below, and the slope at (5,8) should be the same as the value of MRS, 4.

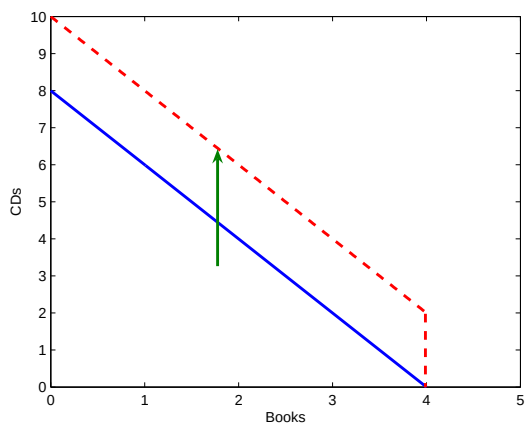


Figure 1: Budget Line

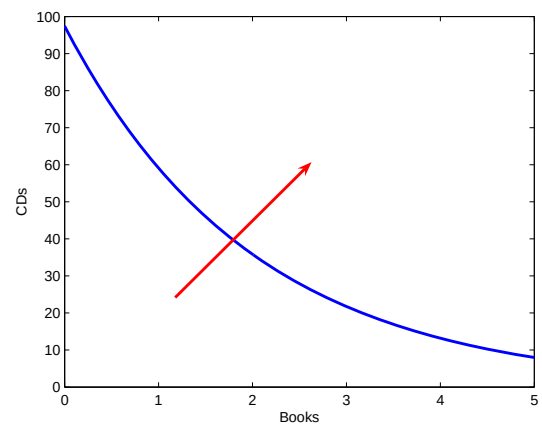


Figure 2: Indifference Curve

(c) Two things are required to be an optimal; first, the tangency condition: the budget line should be a tangent line to the indifference curve. Second, the optimal point should be located on the budget line.

(d) The first Secret of Happiness(SOH) says that the MRS should coincide with (negative) the relative price at equilibrium: $\text{MRS} = -\frac{p_1}{p_2}$. It means that rate of exchange at which the consumer is willing to stay put(MRS) must be equal to the price ratio. The second SOH is $p_1x_1 + p_2x_2 = m$, which simply means that all the money should be exhausted by consuming both of the goods.

(e) Since MRS$= -\frac{x_2}{2}$, $x_2 = 4$ immediately follows from the first SOH. Note that he can afford $x_2 = 4$. The extra money $\$40 - \$5 \times 4 = \$20$ must be spent on good 1 by the second SOH

and thus $x_1 = 2$. Now if $p_1$ increased to \$30, the first SOH says $\frac{x_2}{2} = \frac{30}{5} = 6$, that is, $x_2 = 12$ but it is not affordable since he needs \$60 for 12 CDs. Plugging $x_2 = 12$ into the budget line

$$30x_1 + 5 \times 12 = 40 \rightarrow x_1 = -\frac{2}{3} < 0. \tag{1.2}$$

Hence $x_1 = 0$ and $x_2 = \frac{m}{p_2} = \frac{40}{5} = 8$. While the first bundle $(x_1, x_2) = (2, 4)$ is interior, the second bundle $(x_1, x_2) = (0, 8)$ is at the corner.

(f) As depicted below, $(2, 4)$ is in the interior. Hence the marginal utilities from a dollar for books and for CDs at $(2, 4)$ must be the same. You can easily verify that

$$\text{when } (p_1, p_2) = (\$10, \$5), \quad \frac{\text{MU}_1}{p_1} = \frac{1}{10} = \frac{2/4}{5} = \frac{2/x_2}{5} = \frac{\text{MU}_2}{p_2}. \tag{1.3}$$

At $(x_1, x_2) = (0, 8)$, on the other hand, their marginal utilities from a dollar would not be the same. Observe that

$$\text{when } (p_1, p_2) = (\$30, \$5), \quad \frac{\text{MU}_1}{p_1} = \frac{1}{30} \text{ but } \frac{\text{MU}_2}{p_2} = \frac{2/x_2}{p_2} = \frac{2/8}{5} = \frac{1}{20} \tag{1.4}$$

## 2 Problem 2

(a) Since hamburgers and cokes are perfect *complements* for Michael, his utility function over two goods must take a form of

$$U(x_1, x_2) = \min\{x_1, 3x_2\}. \tag{2.1}$$

(b) Recall that the indifference curve has a "L" shape in case of perfect complements. And the optimal preference line should be $x_1 = 3x_2$.

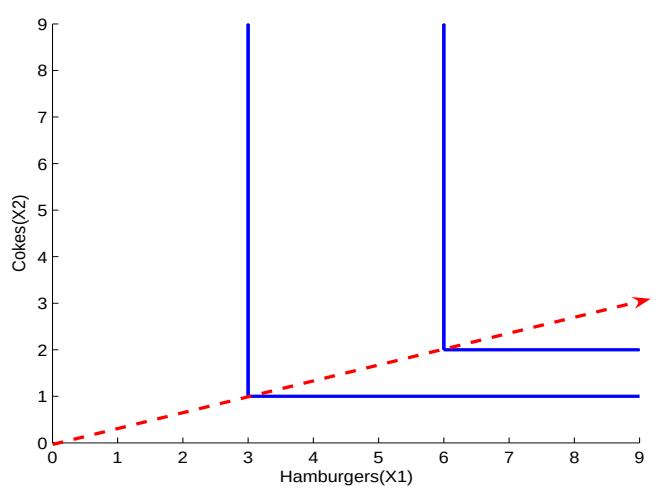


Figure 3: Indifference Curve and Optimal Proportion Line

(c) Two secrets of Happiness in case of perfect complements are as follows:

$$(i) \quad x_1 = 3x_2 \tag{2.2}$$

$$(ii) \quad p_1x_1 + p_2x_2 = m. \tag{2.3}$$

The condition $(i)$ provides us with the economic intuition that he is willing to consume two goods in the same proportion $2 : 1$. The second condition says that his income must be exhausted.

(d) Plugging (2.2) into (2.3) gives us the equation of $x_1$ only;

$$p_1(3x_2) + p_2x_2 = x_2(3p_1 + p_2) = m. \tag{2.4}$$

Hence $x_2 = \frac{m}{3p_1+p_2}$ and $x_1 = \frac{3m}{3p_1+p_2}$. Since $x_1, x_2 > 0$(without loss of generality, his income $m > 0$.) they are interior.

(e) They are *ordinary* since as $x_1$ and $x_2$ are downward sloping. They are *normal* since as $m$ goes up, so do $x_1$ and $x_2$. Finally, they are *gross complements* since as the price of one good goes up the quantity of the other good will decrease.

(f) By Slutsky equation we can decompose the total change in demand into two effects; *substitution* and *income* effects. However, its substitution effects would be zero because they are perfect complements. Its main intuition follows from the fact that he is willing to consume in the same proportion so his optimal choice will be determined by the set of kinks in his indifference curve.

# 3 Problem 3

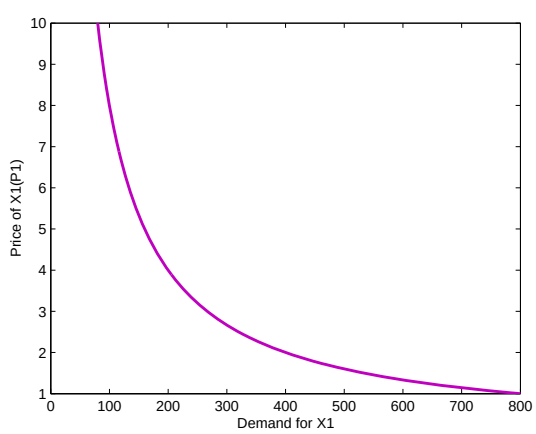


Figure 4: Demand Curve for $x_1$

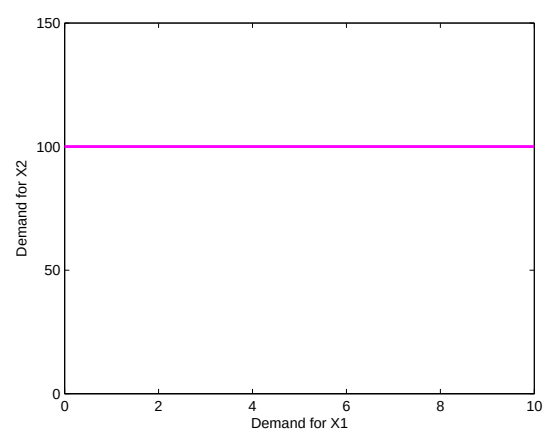


Figure 5: Price($p_1$)-Offer Curve

(a) Let a function $V(x_1, x_2) = 2\log x_1 + 1\log x_2$. You can easily see that Adam's complicated utility function is just a monotone transformation of $V$. In fact, we shall rewrite his original utility function in terms of $V$ as

$$U(x_1, x_2) = \left[700 \times \sqrt{\log V^2} + 10\right]^{800}. \tag{3.1}$$

(b) Part (a) enables us to analyze his consumption behavior using the simple function $V(x_1, x_2)$. Since it is a Cobb-Douglas utility, the magic formulae lead us to his demand function:

$$x_1 = \frac{a}{a+b}\frac{m}{p_1} = \frac{2}{2+1}\frac{1200}{4} = 200$$
$$x_2 = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{2+1}\frac{1200}{4} = 100.$$

(c) Assume that $p_2 = 4$ and $m = 1200$. Now we can think of $x_1$'s demand function which displays how the quantity of $x_1$ changes as $p_1$ varies, as well as the $p_1$-offer curve which displays how the set of optimal bundles $(x_1, x_2)$ changes as $p_1$ varies. When $p_2 = 4$ and $m = 1200$, the above demand function comes down to

$$x_1 = \frac{a}{a+b}\frac{m}{p_1} = \frac{2}{2+1}\frac{1200}{p_1} = \frac{800}{p_1} \tag{3.2}$$
$$x_2 = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{2+1}\frac{1200}{4} = 100. \tag{3.3}$$

(3.2) addresses his demand function which is depicted below. With both of them (3.2) and (3.3), you can draw $p_1$-offer curve which must be flat.

# 4 Problem 4

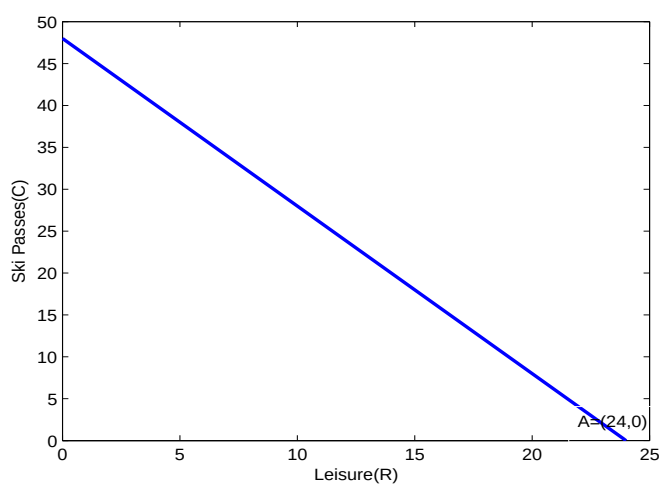


Figure 6: Frank's Budget Line and His Endowments

(a) If Frank enjoys leisure during $R$ hours, his labor income would be $w(24 - R) = 20 \times (24 - R)$. Those money will be totally spent on ski passes($C$). Denoting its price by $p_C$, we can write his budget line as

$$p_C C = w(24 - R). \tag{4.1}$$

Hence you end up with $10C + 20R = 20 \times 24$ when $p_C = 10$ and $w = 20$. Note that he is just endowed with his daily hours, 24 hours. Point $A = (24, 0)$ in the figure indicates his endowment.

(b) The real wage is simply $\frac{w}{p_C} = 2$. It is the slope of his budget line.

(c) Since $U(R, C) = RC^5$ is Cobb-Douglas, his demand for $R$ and $C$ are

$$R = \frac{a}{a+b}\frac{m}{w} = \frac{1}{1+5}\frac{480}{20} = 4$$
$$\text{and} C = \frac{b}{a+b}\frac{m}{p_C} = \frac{5}{1+5}\frac{480}{10} = 40,$$

respectively. From $R = 4$, his labor supply is immediate; $24 - 4 = 20$ hours.

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## Midterm 1 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (10 extra "e" points). Make sure you answer the first four questions before working on the bonus one!

**Problem 1.** (50 points) To reenergize for Econ 301 class, in the morning, Tony always drinks Mountain Dew $(x_1)$ and eats Burritos $(x_2)$.

a) Suppose Mountain Dew costs $p_1 = \$2$, burrito costs $p_2 = \$10$, and his daily budget is $m = \$40$. Show graphically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set is affected by inflation of 100% that affects prices of both commodities but does not affect income (so his income stays the same $m = \$40$)?

b) Tony's preferences are given by the following utility function

$$U(x_1, x_2) = x_1^3 x_2^3.$$

Find Tony's marginal rate of substitution (MRS) as a function of $x_1, x_2$ (give a formula for MRS).

- What is the value of MRS at consumption bundle $(1, 2)$ (give a number)?

- Which of the two goods is more valuable, soda or burrito, if Tony drinks one Mountain Dew and consumes two burritos?

- Depict his indifference curve map in a commodity space. Mark the slope of the indifference curve at the bundle $(1, 2)$.

c) In the commodity space $(x_1, x_2)$, find (geometrically) Tony's optimal choice,.assuming pre-inflation prices $p_1 = \$2$, and $p_2 = \$10$. Describe how the two properties of the optimal bundle, known as two "secrets of happiness" (two short sentences) can be seen in the graph.

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).

- Derive the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation).

- What fraction of income is spent on burritos (give the percentage)?

- Find analytically and geometrically the demand curve for Mountain Dew (given $p_2 = \$10$ and $m = \$40$) and Engel curve (given $p_1 = \$2$, and $p_2 = \$10$)

- Are they Giffen goods? Why? (yes/no answer + one sentence).

e) Using your formula from d) find the optimal consumption levels of both types of commodities $(x_1, x_2)$ for:

- $p_1 = \$2, p_2 = \$10$ and $m = \$40$ (give two numbers).

and after the price of Mountain Dew decreased:

- for $p_1 = \$1, p_2 = \$10$ and $m = \$40$ (give two numbers).

What is the total change in consumption of Mountain Dew? (give a number). Illustrate the change on the graph.

f) decompose the total change in consumption of $x_1$ from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

**Problem 2.** (20 points) Bill is a wild-animal lover. From his recent trip to Galapagos Islands he brought a small Iguana. His new pet has only three legs: two left and one right. (Iguanas use magma heated soil to warm their eggs and his favorite pet lost one right leg during the last volcano eruption). To survive the famous Madisonian winter, the iguana has to wear shoes, left $(x_1)$ and right ones $(x_2)$.

a) Write down Bill's utility function representing his preferences over right and left shoe (function $U(x_1, x_2)$).

b) In the commodity space $(x_1, x_2)$, carefully depict Bill's indifference curves.

c) Find analytically Bill's demand for shoes if $p_1 = \$2$ and $p_2 = \$1$ and Bill's budget for iguana shoes is $m = \$15$. Is the solution interior? (give two numbers and a yes/no answer).

d) Illustrate Bill's optimal choice on the graph including the indifference curves and the budget set.

e) Suppose the price of left shoe goes down to $p_1 = \$1$. Find Bill's new demand for shoes. What can you say about the substitution effect? How about the income effect? (Answer the latter question without any calculations, using only a graph).

**Problem 3.** (15 points) Ramon decides about his new collections of postage stamps. He is interested in two themes: "the birds of the world", $x_1$, (measures the number of stamps in the subcollection with birds) and "the famous mathematicians", $x_2$. The utility derived from the collection is given by

$$U(x_1, x_2) = x_1 + 10 \times \ln x_2.$$

a) What is the optimal collection of stamps if the prices are $p_1 = 1$ and $p_2 = 2$ and $m = 15$. (find two numbers $x_1$ and $x_2$). Is your solution interior, or corner?

b) Find the optimal collection if the prices are still $p_1 = 1$ and $p_2 = 2$, but the income is only $m = 8$. Depict Ramon's optimal choice in the commodity space.

**Problem 4.** (15 points) Jacob can use his 24h for leisure $R$, or work. The hourly wage rate is $w = 5$. Jacob spends all his money on cheese curds $C$.

a) Draw Jacob's budget set, given the price of cheese is $p_c = \$10$ (mark the endowment point).

Let the utility function be given by

$$U(x_1, x_2) = R + C.$$

b) Are leisure and cheese curds perfect complements, perfect substitutes or none of them?

c) What is the optimal choice of leisure, cheese curds and labor supply? (Find geometrically and give three numbers).

d) Harder: Plot a graph with labor supply (horizontal axis) and wage rate (vertical one) assuming $p_c = \$10$ (hint: for what value of $w$ do we go from one "bang" to the other "bang" solution?)

**Bonus Problem.** (extra 10 points) Let $U(x_1, x_2) = x_1 x_2^2$, and $V(x_1, x_2) = \ln x_1 + 2 \ln x_2$ be two utility functions.

a) Show that $U(\cdot)$ is a monotone transformation of $V(\cdot)$, and hence they define the same preferences.

b) Derive MRS for each of the two functions. Using the two formulas for MRS, argue that the functions define the same indifference curve maps.

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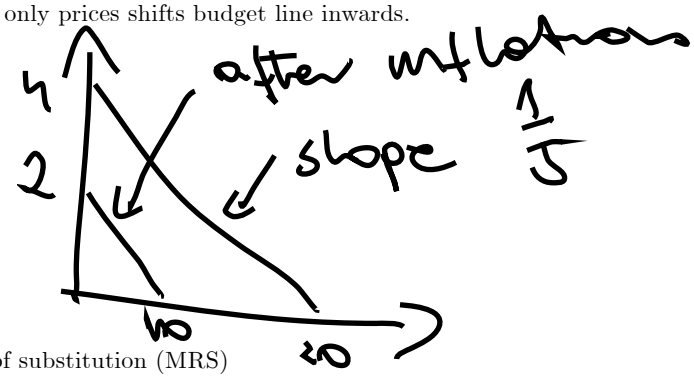
**Solutions to midterm 1 (Group B)**

Problem 1.

a) With $p_1 = \$2$, $p_2 = \$10$ and $m = \$40$ the budget set is (two extreme consumption bundles are 20 and 4). Inflation that affects only prices shifts budget line inwards.




b) Tony's marginal rate of substitution (MRS)

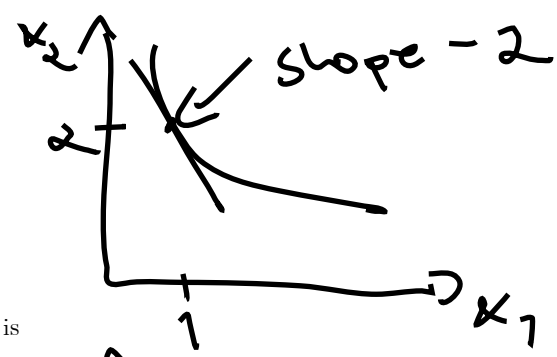
$$MRS = -\frac{MU_1}{MU_2} = -\frac{x_2}{x_1}$$

- The value of MRS at consumption bundle $(1, 2)$ is

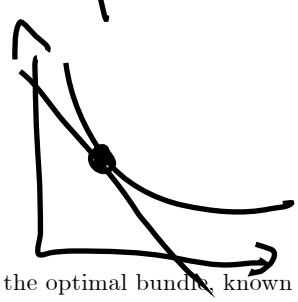
$$|MRS| = |-\frac{2}{1}| = 2$$

- Burrito $(x_2)$ is less valuable than Mountain Dew $(x_1)$

- Tony's indifference curve map is. (the slope of her indifferent curve that passes through bundle $(1, 2)$ is $-2$.




c) Tony's optimal choice is



- the two geometric properties of the optimal bundle, known as two "secrets of happiness" are:

1. At the optimal bundle, the indifference curve is tangent to a budget set
2. The optimal bundle is located on budget line

d) mathematically the two secrets of happiness, are

$$\begin{cases} MRS = -\frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 = m \end{cases}$$

- the economic intuition behind the two conditions is:
The individual value of $x_1$ in terms of $x_2$ coincides with the market value
The income of a consumer is exhausted
- the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ can be found as follows
From the MRS condition

$$MRS = -\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

hence

$$x_2 = \frac{p_1}{p_2} x_1$$

plugging in budget constraint

$$p_1 x_1 + p_2 \left( \frac{p_1}{p_2} x_1 \right) = m$$

Solving for $x_1$ gives

$$x_1 = \frac{1}{2} \frac{m}{p_1}$$

Plugging in

$$x_2 = \frac{p_1}{p_2} \left( \frac{1}{2} \frac{m}{p_1} \right) = \frac{1}{2} \frac{m}{p_2}$$

- the fraction of income spent on burritos is

$$\frac{p_1 x_1}{m} = \frac{1}{2} = 50\%$$

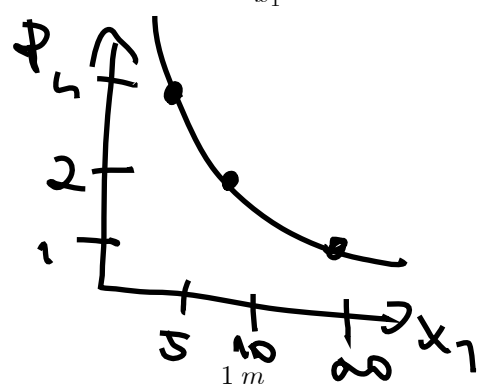
- and the demand curve for burritos book (given $p_2 = \$10$, and $m = \$40$) and Engel curve (given $p_1 = \$2$, and $p_2 = \$10$)
Demand curve

$$x_1 = \frac{1}{2} \frac{m}{p_1} = \frac{1}{2} \frac{40}{p_1} = \frac{20}{p_1}$$

and hence inverse demand is

$$p_1 (x_1) = \frac{20}{x_1}$$

Geometrically




Engel curve: Since

$$x_1 = \frac{1}{2} \frac{m}{p_1}$$

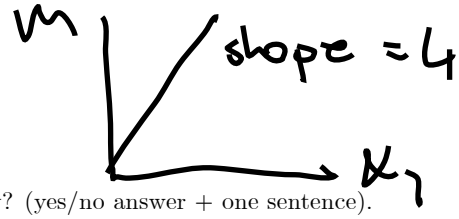
at $p_1 = \$2$

$$x_1 = \frac{1}{2} \frac{m}{2} = \frac{1}{4} m$$

hence

$$m (x_1) = 4 x_1$$

Geometrically




- are they Giffen goods? Why? (yes/no answer + one sentence).
No, because the demand curve is downwardslopping on the whole domain.
e) The optimal consumption levels for $(x_1, x_2)$.
- at $p_1 = \$2, p_2 = \$10$ and $m = \$40$

$$x_1 = \frac{1}{2}\frac{m}{p_1} = \frac{1}{2}\frac{40}{2} = 10$$

and

$$x_2 = \frac{1}{2}\frac{m}{p_2} = \frac{1}{2}\frac{40}{10} = 2$$

and after the price of science-fiction book decreased, for $p_1 = \$1, p_2 = \$10$ and $m = \$40$

$$x_1 = \frac{1}{2}\frac{m}{p_1} = 20$$

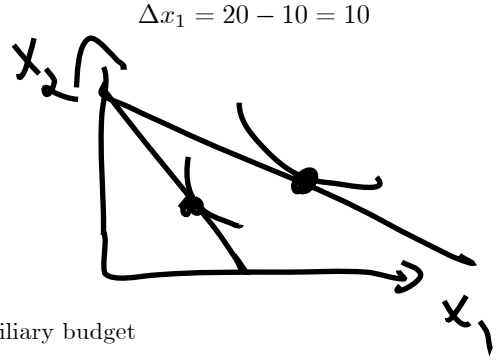
and

$$x_2 = \frac{1}{2}\frac{m}{p_2} = \frac{1}{2}\frac{40}{10} = 2$$

Hence the total change in consumption of $x_1$ is

$$\Delta x_1 = 20 - 10 = 10$$

Geometrically




f) Substitution effect: auxiliary budget

$$m' = 10 \times 1 + 10 \times 2 = 30$$

and hence

$$x_1 = \frac{1}{2}\frac{30}{1} = 15$$

so $SE$ is equal to

$$SE = 15 - 10 = 5$$

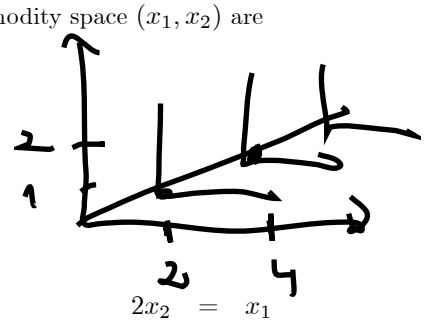
and income effect is

$$IE = 10 - 5 = 5$$

Problem 2. .
a) Bill's utility function is

$$U(x_1, x_2) = \min(x_1, 2x_2)$$

b) indifference curves.in the commodity space $(x_1, x_2)$ are



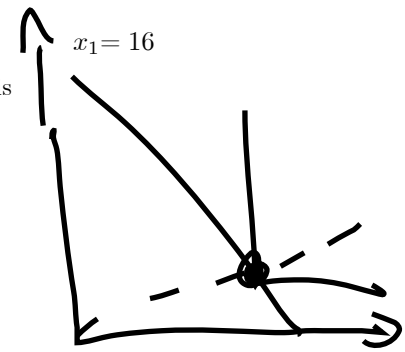

c) Bill's demand for shoes is

$$\begin{aligned} 2x_2 &= x_1 \\ 2x_1 + x_2 &= 40 \end{aligned}$$

$$4x_2 + x_2 = 40$$

$$x_2 = \frac{40}{5} = 8$$

d) geometrically Bills's optimal choice is




e) when the price of a left shoe goes down to $p_1 = \$1$. the new demand is given by the system of equations

$$\begin{aligned} 2x_2 &= x_1 \\ x_1 + x_2 &= 40 \end{aligned}$$

and hence demand is

$$\begin{aligned} x_1 &= \frac{80}{3} = 26\frac{2}{3} \\ x_2 &= \frac{40}{3} = 13\frac{1}{3} \end{aligned}$$

The substitution effect is zero (perfect complements) and the income effect is $4.10\frac{2}{3}$

Problem 3.

a) the two secrets of happiness are

$$\begin{aligned} -\frac{x_2}{10} &= -\frac{1}{2} \\ x_1 + 2x_2 &= 15 \end{aligned}$$

and hence $x_2 = 5$ and $x_1 = 5$. Since both are positive, this is interior solution.

b) the two secrets of happiness are

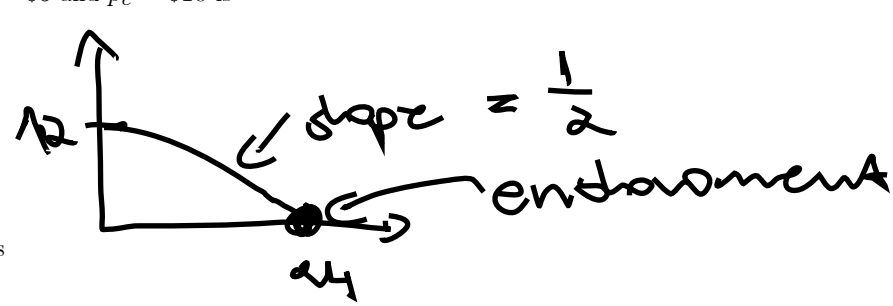
$$\begin{aligned} -\frac{x_2}{10} &= -\frac{1}{2} \\ x_1 + 2x_2 &= 8 \end{aligned}$$

and hence secrets of happiness give $x_2 = 5$ and $x_1 = -5$. Since consumption must be non-negative the optimal consumption is $x_1 = 0$ and $x_2 = 4$, which is a cornet solution.

Problem 4.

a) Jacob's budget set, with $w = \$5$ and $p_c = \$10$ is

Income is $m = 5 \times 24 = \$120$

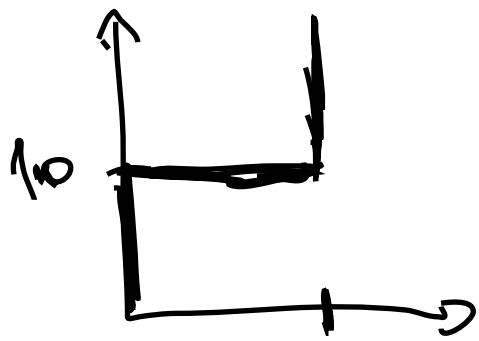



b) They are perfect substitutes

c) $|MRS| = 1 > \frac{w}{p_c} = \frac{1}{2}$ which implies that Jacob cares more about leisure than consumption,.therefore he will spend the whole day at home

$$R = 24, LS = 0 \text{ and } C = 0$$

d)




Bonus Problem. (extra 10 points)

a) Monotone transformation $\ln()$. Take a log of $U$

$$\ln U() = \ln x_1 x_2^2 = \ln x_1 + \ln x_2^2 = \ln x_1 + 2\ln x_2 = V()$$

where we used two properties of ln function.

b) For $U()$, marginal rate of substitution is

$$MRS = -\frac{MU_1}{MU_2} = -\frac{x_2^2}{2x_1x_2} = -\frac{x_2}{2x_1}$$

and for $V()$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{1/x_1}{2/x_2} = -\frac{x_2}{2x_1}$$

and hence MRS coincides for all $(x_1, x_2)$.It follows that the slopes of indifference curves are the same at any point and hence they must be the same.

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## Midterm 1 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 3 questions (60+25+15=100 points) + bonus (10 points). Make sure you answer the first three questions before working on the bonus one!

**Problem 1.** (60 points) Maggie likes to read science fiction $(x_1)$ and romance $(x_2)$ novels. Her preferences over the two types of books are represented by a utility function

$$U(x_1, x_2) = (x_1)^6 (x_2)^3$$

a) Find Maggie's marginal rate of substitution (MRS) as a function of $x_1, x_2$ (give a formula).

- what is the value of MRS at consumption bundle $(2, 2)$ (give a number).

- complete the sentence: "The Marginal Rate of Substitution is a (marginal) value of a ... in terms of ..."

- how much one must compensate Maggie in terms of romance books, after taking away 0.00001 of a science-fiction book, in order to keep her indifferent? (give a number, assume she consumes bundle $(2, 2)$).

- depict her indifference curve map in a commodity space. Mark the slope of her indifferent curve that passes through bundle $(2, 2)$.

b) Suppose the price of a science-fiction book is $p_1 = \$4$, a romance book costs $p_2 = \$10$ and her total monthly spending on books is $m = \$600$. Show graphically her budget constraint in the commodity space. Mark the two extreme consumption bundles (give values). On the same graph, show how the budget set would be affected by the introduction of an ad valorem tax on science fiction books $(x_2)$ at rate 100%?

c) In the commodity space $(x_1, x_2)$, find (geometrically) Maggie's optimal choice.

- describe the two properties of the optimal bundle, known as two "secrets of happiness" (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (and not concrete values).

- provide some economic intuition behind the two conditions (ca. two sentences for each).

- derive the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation).

- what fraction of income is spent on science - fiction novels (give the percentage).

- find analytically and geometrically the demand curve for science finction book (given $p_2 = \$10$, and $m = \$600$) and Engel curve (given $p_1 = \$4$, and $p_2 = \$10$ )

- are science-fiction books normal goods? Why? (yes/no answer + one sentence).

- are they Giffen goods? Why? (yes/no answer + one sentence).

e) Using your formula from d) find the optimal consumption levels for both types of books $(x_1, x_2)$.

- for $p_1 = \$4, p_2 = \$10$ and $m = \$600$ (give two numbers).

and after the price of science-fiction book decreased:

- for $p_1 = \$2, p_2 = \$10$ and $m = \$600$ (give two numbers).

What is the total change in consumption of $x_1$? (give one number). Illustrate this change on the graph.

f) decompose the total change in $x_1$ from f) into a substitution and income effect. (Calculate the two numbers, show how you found them on the graph.) Complete the two sentences:

"The substitution effect is attributed to the pure change in .... induced by the decrease of nominal price $p_1$"

"The income effect can be attribute to the pure change in .... induced by the decrease of nominal price $p_1$"

g) which of the following alternative utility functions represents Maggie's preferences (there are two such

functions)?

$$\begin{aligned} V(x_1, x_2) &= 6x_1 \times 3x_2 + 3 \\ V(x_1, x_2) &= 30\left(x_1^6 \times x_2^3\right) + 3 \\ V(x_1, x_2) &= 6\ln x_1 \times 3\ln x_2 \\ V(x_1, x_2) &= 6\ln x_1 + 3\ln x_2 + 7 \end{aligned}$$

Explain why the utility functions you have selected represent Maggie's preferences (one sentence). Suggest the transformation of $U()$ function that makes the two $V()$ functions equivalent.

**Problem 2.** (20 points) Jimmy's favorite hobby is slot car racing. He assembles slot trucks from parts, by adding ten wheels $(x_1)$ to an engine $(x_2)$ (these are supertrucks, with five wheels on each side). He purchases the parts on the market.

a) write down Jimmy's utility function representing his preferences over wheels and engines (function $U(x_1, x_2)$).

b) in the commodity space $(x_1, x_2)$, carefully depict Jimmy's indifference curves.

c) find analytically Jimmy's demand for parts if one wheel costs $p_1 = \$2$, an engine is $p_2 = \$10$ and Jimmy's budget for slot cars is $m = \$120$. Is the solution interior (give two numbers and yes/no answer).

d) illustrate Jimmy's optimal choice on the graph including the indifference curves and the budget set.

e) suppose the price of one wheel goes down to $p_1 = \$1$. Find Jimmy's new demand for the parts. What can you say about the substitution effect? How about the income effect? (you can answer the last question without any calculations, using only a graph).

**Problem 3.** (20 points) Ramon Gonzales M. Panetelas is a specialist in Habanos cigars (famous Cuban cigars). Cuban cigars are sold either in 5 cigar packs $(x_1)$, or in singles $(x_2)$. Ramon has no income. Instead he is initially endowed with $\omega_1 = 10$ packs and $\omega_2 = 50$ cigars.

a) draw Ramon's budget set, given the price of a pack is equal to $p_1 = \$5$ and a single cigar is $p_2 = \$1$ (mark the endowment point).

b) Illustrate geometrically Ramon's optimal demand for packs and single cigars, given his utility function

$$U(x_1, x_2) = 5x_1 + x_2$$

(Give two numbers $(x_1, x_2)$, and mark them on the graph, including budget set and the indifference curves.)

c) What is your answer to b) when prices are $p_1 = \$10$ and $p_2 = \$1$. (Give two numbers $(x_1, x_2)$, and plot the graph.)

d) Harder: Give the formula for the demands $x_1, x_2$ as a function of $p_1, p_2$ and endowments $\omega_1$ and $\omega_2$. Show the demand on the graph, assuming $p_2 = \$1$, $\omega_1 = 10$ and $\omega_2 = 50$.

**Bonus Problem.** (extra 10 points) Depict a map of indifference curves that is consistent with

a) inferior goods

b) Giffen goods

(Make sure you explain why these graphs represent the respective preferences.)

**Econ 301**
**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Solutions to midterm 1 (Group B)

Problem 1. (60 points)
a) Maggie's marginal rate of substitution (MRS)

$$MRS = -\frac{MU_1}{MU_2} = -\frac{2}{1}\frac{x_2}{x_1}$$

- the value of MRS at consumption bundle $(2, 2)$ is

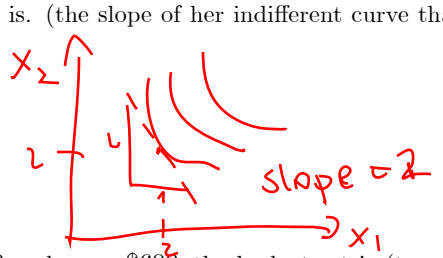
$$MRS = -2$$

- "The Marginal Rate of Substitution is a (marginal) value of a science finction books in terms of romance novels"

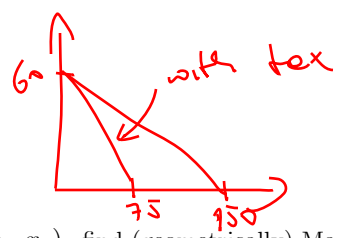
- after taking away 0.00001 of a science-fiction book, to keep her indifferent one must compensate Maggie in terms of romance books

$$0.00001 \times 2 = 0.000002$$

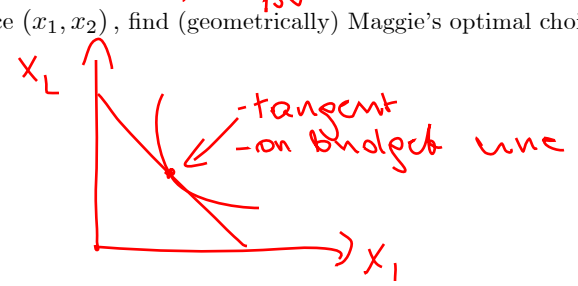
- her indifference curve map is. (the slope of her indifferent curve that passes through bundle $(2, 2)$ is $-2$.




b) With $p_1 = \$4$, $p_2 = \$10$ and $m = \$600$ the budget set is (two extreme consumption bundles are $(150, 0)$ and $(0, 60)$. The budget set with ad valorem tax shifts inwards.




c) In the commodity space $(x_1, x_2)$, find (geometrically) Maggie's optimal choice.




- the two geometric properties of the optimal bundle, known as two "secrets of happiness" are:
1. At the optimal bundle, the indifference curve is tangent to a budget set
2. The oprimal bundle is locaded on budget line
d) mathematically the two secrets of happiness, are

$$\begin{cases} MRS = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

- the economic intuition behind the two conditions is:
The individual value of $x_1$ in temrms of $x_2$ coincides with the market value

The income of a consumer is exchausted

- the optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ can be found as follows

From the MRS condition

$$MRS = -2\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

hence

$$x_2 = \frac{1}{2}\frac{p_1}{p_2}x_1$$

plugging in budget constraint

$$p_1x_1 + p_2\left(\frac{1}{2}\frac{p_1}{p_2}x_1\right) = m$$

Solving for $x_1$ gives

$$x_1 = \frac{2}{3}\frac{m}{p_1}$$

Pluging in

$$x_2 = \frac{1}{2}\frac{p_1}{p_2}\left(\frac{2}{3}\frac{m}{p_1}\right) = \frac{1}{3}\frac{m}{p_2}$$

- the fraction of income spent on science - fiction novels is

$$\frac{p_1x_1}{m} = \frac{2}{3} = 66\%$$

- and the demand curve for science finction book (given $p_2 = \$10$, and $m = \$600$) and Engel curve (given $p_1 = \$4$, and $p_2 = \$10$)

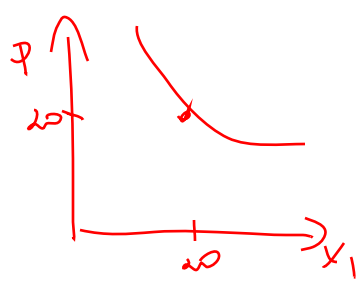
Demand curve

$$x_1 = \frac{2}{3}\frac{m}{p_1} = \frac{400}{p_1}$$

and hence invese demand is

$$p_1(x_1) = \frac{400}{x_1}$$

Geometrically




Engel curve: Since

$$x_1 = \frac{2}{3}\frac{m}{p_1}$$

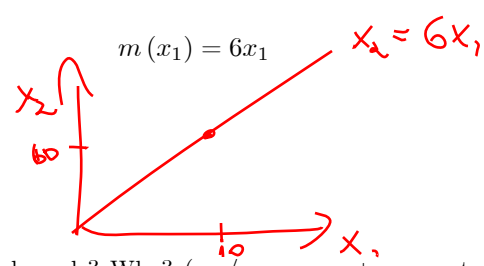
at $p_1 = \$4$

$$x_1 = \frac{2}{3}\frac{m}{4} = \frac{1}{6}m$$

hence

$$m(x_1) = 6x_1$$

Geomettrically




- are science-fiction books normal goods? Why? (yes/no answer + one sentence).

Yes, because their demand increases in income

- are they Giffen goods? Why? (yes/no answer + one sentence).

No, becase the demand curve is downwardslopping on the whole domain.

e) The optimal consumption levels for both types of books $(x_1, x_2)$.

- for $p_1 = \$4, p_2 = \$10$ and $m = \$600$

$$x_1 = \frac{2}{3}\frac{m}{p_1} = \frac{2}{3}\frac{600}{4} = 100$$

and

$$x_2 = \frac{2}{3}\frac{m}{p_2} = \frac{1}{3}\frac{600}{10} = 20$$

and after the price of science-fiction book decreased, for $p_1 = \$2, p_2 = \$10$ and $m = \$600$

$$x_1 = \frac{2}{3}\frac{m}{p_1} = \frac{2}{3}\frac{600}{2} = 200$$

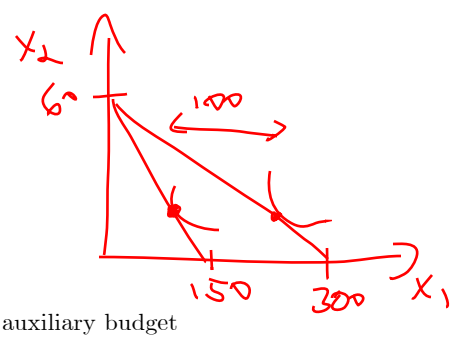
and

$$x_2 = \frac{2}{3}\frac{m}{p_2} = \frac{1}{3}\frac{600}{10} = 20$$

Hence the total change in consumption of $x_1$ is

$$\Delta x_1 = 200 - 100 = 100$$

Geometrically




f) Substitution effect: auxiliary budget

$$m' = 100 \times 2 + 20 \times 10 = 400$$

and hence

$$x_1 = \frac{2}{3}\frac{400}{2} = 133\frac{1}{3}$$

so $SE$ is equal to

$$SE = 133\frac{1}{3} - 100 = 33\frac{1}{3}$$

and income effect is

$$IE = 100 - 33\frac{1}{3} = 66\frac{2}{3}$$

"The substitution effect is attributed to the pure change in relative price induced by the decrease of nominal price $p_1$"

"The income effect can be attribute to the pure change in real income induced by the decrease of nominal price $p_1$"

g) From the functions

$$\begin{aligned}
V(x_1, x_2) &= 6x_1 \times 3x_2 + 3 \\
V(x_1, x_2) &= 30\left(x_1^6 \times x_2^3\right) + 3 \\
V(x_1, x_2) &= 6\ln x_1 \times 3\ln x_2 \\
V(x_1, x_2) &= 6\ln x_1 + 3\ln x_2 + 7
\end{aligned}$$

the second and last represent Maggie's preferences - they are monotone transformations of $U()$. The first transformation is

$$f(U) = 30U + 3$$

and the second is

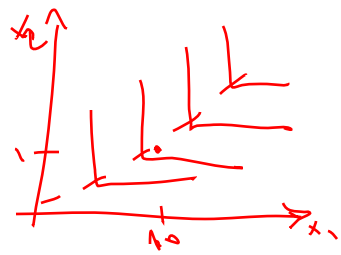
$$f(U) = \ln U + 7$$

Problem 2. .

a) Jimmy's utility function is

$$U(x_1, x_2) = \min(x_1, 10x_2)$$

b) in the commodity space $(x_1, x_2)$, carefully depict Jimmy's indifference curves.




c) Jimmy's demand for parts is interior and is given by

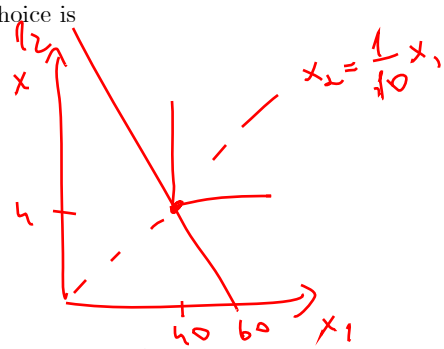
$$\begin{aligned} x_2 &= \frac{1}{10}x_1 \\ 2x_1 + 10x_2 &= 120 \end{aligned}$$

$$2x_1 + 10\frac{1}{10}x_1 = 120$$

$$x_1 = \frac{120}{3} = 40$$

$$x_2 = \frac{1}{10} \times 40 = 4$$

d) geometrically Jimmy's optimal choice is



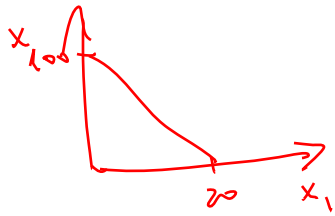

e) when the price of a wheel goes down to $p_1 = \$25$. the new demand is

$$\begin{aligned} x_2 &= \frac{1}{10}x_1 \\ x_1 + 10x_2 &= 120 \end{aligned}$$

$$\begin{aligned} x_1 &= 60 \\ x_2 &= 6 \end{aligned}$$

Thte substitution effec is zero (perfect complements) and the income effect is 20 wheels.

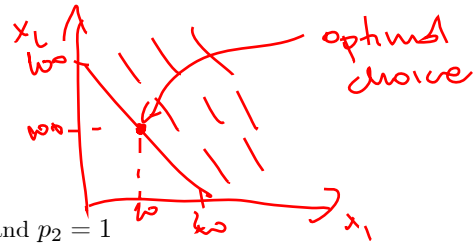
Problem 3.
a) Ramon's budget set, with $p_1 = \$5$ and $p_2 = \$1$ is
Budget set: $m = 10 \times 5 + 50 \times 1 = 100$




b) The optimal demand for packs and single cigars can be found as follows. Since

$$-MRS = 5 = \frac{p_1}{p_2} = 5$$

therefore it does not matter how Ramon allocates his income. For example his endowment point is optimal




c) When prices are $p_1 = 10$ and $p_2 = 1$

$$-MRS = 5 < \frac{p_1}{p_2} = 10$$

and hence the total income will be invested in $x_2$. At such prices demands are

$$x_1 = 0 \text{ and } x_2 = \frac{10 \times 10 + 50}{1} = 150$$

d) The demands are
If $\frac{p_1}{p_2} < 5$ then

$$\begin{aligned} x_1 &= \frac{p_1\omega_1 + p_2\omega_2}{p_1} \\ x_2 &= 0 \end{aligned}$$

if $\frac{p_1}{p_2} > 5$ then

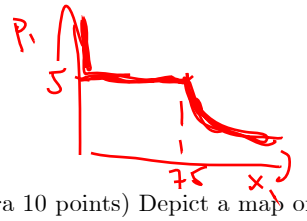
$$\begin{aligned} x_1 &= 0 \\ x_2 &= \frac{p_1\omega_1 + p_2\omega_2}{p_2} \end{aligned}$$

and if $\frac{p_1}{p_2} = 5$

$$\begin{aligned} x_1 &= \alpha\frac{p_1\omega_1 + p_2\omega_2}{p} \\ x_2 &= (1-\alpha)\frac{p_1\omega_1 + p_2\omega_2}{p_2} \end{aligned}$$

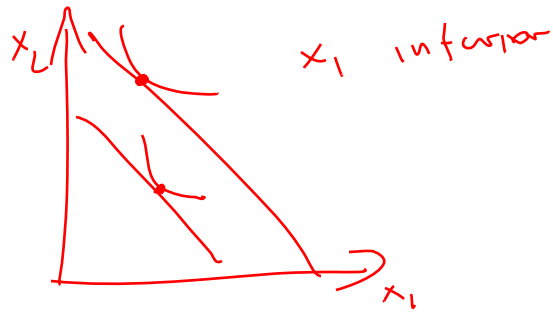
for $\alpha \in (0, 1)$

The demand curve for $x_1$ is ($p_2 = 1$, $\omega_1 = 5$ and $\omega_2 = 50$).

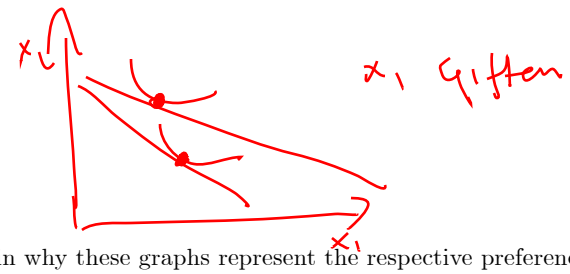



Bonus Problem. (extra 10 points) Depict a map of indifference curves that is consistent with

a) inferior goods




b) Giffen goods




(Make sure you explain why these graphs represent the respective preferences.)

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**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (55+15+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

**Problem 1 (55p)** (Well-behaved preferences)

Ava is a big fun of delicious gourmet steaks: their robust and hearty beef flavors is what she likes the most. Her two most favorite steaks are marinated ribeye $(x_1)$ and top sirloin $(x_2)$.

a) The price of a marinated ribeye steak is $p_1 = \$50$, the price of top sirloin is $p_2 = \$25$, and her income (spent entirely on steaks) is $m = \$500$. Show geometrically Ava's budget constraint. Mark the two extreme consumption bundles (give concrete values). Show on the graph how her budget set would be affected by inflation that increased the prices of both steaks by $100\%$, leaving $m$ unchanged (draw a new budget line).

b) Ava's utility function over both types of steaks is $U(x_1, x_2) = x_1^5 x_2^5$. Find her marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give a formula for MRS). What is the value of MRS at bundle $(2, 4)$ (one number)? Which of the two steak types is more valuable to Ava?

d) Write down two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (two equations).

- Provide economic intuition behind the two conditions (two sentences for each).

- Derive optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation of magic formulas).

- Using magic formulas determine whether the solution is interior for all values of $p_1, p_2, m$ (one sentence).

- What fraction of total income is spent on top sirloin?

e) Find the optimal consumption levels of two types of steak $(x_1, x_2)$ for:

- $p_1 = \$50, p_2 = \$25$ and $m = \$500$ (give two numbers).

and after the price of ribeye decreased:

- for $p_1 = \$25, p_2 = \$25$ and $m = \$500$ (give two numbers).

What is the total change in consumption of marinated ribeye? (give a number). Illustrate the change on the graph. Is marinated ribeye an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of $x_1$ from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

**Problem 2 (15p)** (Perfect complements)

Alex is a hotdog lover. In each hotdog bun $(x_1)$ Alex always inserts two hotdogs $(x_2)$: he hates hotdogs with ratios of buns to hotdogs different from 1:2.

a) Propose a utility function that captures Alex's preferences over $(x_1)$ and $(x_2)$ (function $U(x_1, x_2)$).

b) In the commodity space, carefully depict Alex's indifference curves (marking the optimal proportion line).

c) Assume prices $p_1 = 4$ and $p_2 = 3$ and income $m = 40$. Write down two secrets of happiness (two equations) and determine the optimal choice (two numbers). Is your solution interior (yes or no)?

d) Suppose price of a hotdog bun goes down to $p_1 = 2$, while price $p_2 = 3$ income $m = 40$ are unchanged. Find optimal choice (two numbers). Without calculations, how much of this change can be attributed to a substitution effect (number + one sentence explaining the number).

**Problem 3 (15p)** (Perfect substitutes)

Pepsi $x_1$ and Coke $x_2$ both are goods that are indistinguishable.

a) Propose a utility function (the "craziest" function you can imagine), which captures preferences for such perfect substitutes.

b) Plot the budget line, assuming $p_1 = \$2$, $p_2 = \$3$ and $m = \$12$. In the same graph plot indifference curves.

c) Find the optimal bundle for $U(x_1, x_2) = x_1 + x_2$. (give two numbers). Is your solution interior or corner? (one sentence)

d) Plot income offer curve and Engel curve given fixed prices $p_1 = \$2, p_2 = \$3$. (two graphs). Is Pepsi normal or inferior (choose one + one sentence)

**Problem 4 (15p)** (Quasilinear Preferences, Labor Supply)

Your best friend Aiden asked you to help him to determine how much time he should spend at work. His total available time (each day) is 24h and his only source of income is labor income given wage rate $w = \$10$. His spending on consumption good $C$ is equal to what he earns. The price of such good is $p_c = 2$

a) What is his real wage rate? (number+interpretation, one sentence )

b) Plot his budget set on the graph.

c) Aiden has quasilinear utility function $U(R, C) = \ln R + C$ where $R$ is leisure (or relaxation time) and $C$ is consumption of other goods. Write down two secrets of happiness that determine optimal choice of $R, C$ for arbitrary $w, p_c$. Find optimal $R$ as a function $w, p_c$ (you can assume interior solution).

d) What is Aiden's labor supply $L$ if $w = \$1$ and $p_c = \$10$ (one number). How about $w = \$2$ and $p_c = \$10$ (one number). Is labor supply increasing in real wage rate, when preferences are quasilinear? (yes no answer)

**Bonus question** (Just for fun)

Let $MRS^V$ and $MRS^U$ be marginal rates of substitution of utility functions $V(x_1, x_2)$ and $U(x_1, x_2)$ respectively. Moreover, assume that $V(x_1, x_2) = f[U(x_1, x_2)]$ where $f[\cdot]$ is a strictly monotone transformation. Show that $MRS^V = MRS^U$. (Hint: Use formula for the derivative of a composite function)

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## Midterm 1 (B)

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**Problem 1 (55p)** (Well-behaved preferences)

Ava is a big fun of delicious gourmet steaks: their robust and hearty beef flavors is what she likes the most. Her two most favorite steaks are marinated ribeye $(x_1)$ and top sirloin $(x_2)$.

a) The price of a marinated ribeye steak is $p_1 = \$10$, the price of top sirloin is $p_2 = \$5$, and her income (spent entirely on steaks) is $m = \$100$. Show geometrically Ava's budget constraint. Mark the two extreme consumption bundles (give concrete values). Show on the graph how her budget set would be affected by inflation that increased the prices of both steaks by 200%, leaving $m$ unchanged (draw a new budget line).

b) Ava's utility function over both types of steaks is $U(x_1, x_2) = x_1^3 x_2^3$. Find her marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give a formula for MRS). What is the value of MRS at bundle $(3, 6)$ (one number)? Which of the two steak types is more valuable to Ava?

d) Write down two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (two equations).

- Provide economic intuition behind the two conditions (two sentences for each).

- Derive optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation of magic formulas).

- Using magic formulas determine whether the solution is interior for all values of $p_1, p_2, m$ (one sentence).

- What fraction of total income is spent on top sirloin?

e) Find the optimal consumption levels of two types of steak $(x_1, x_2)$ for:

- $p_1 = \$10, p_2 = \$5$ and $m = \$100$ (give two numbers).

and after the price of ribeye decreased:

- for $p_1 = \$5, p_2 = \$5$ and $m = \$100$ (give two numbers).

What is the total change in consumption of marinated ribeye? (give a number). Illustrate the change on the graph. Is marinated ribeye an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of $x_1$ from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

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b) In the commodity space, carefully depict Alex's indifference curves (marking the optimal proportion line).

c) Assume prices $p_1 = 4$ and $p_2 = 2$ and income $m = 40$. Write down two secrets of happiness (two equations) and determine the optimal choice (two numbers). Is your solution interior (yes or no)?

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Pepsi $x_1$ and Coke $x_2$ both are goods that are indistinguishable.

a) Propose a utility function (the "craziest" function you can imagine), which captures preferences for such perfect substitutes.

b) Plot the budget line, assuming $p_1 = \$4$, $p_2 = \$2$ and $m = \$12$. In the same graph plot indifference curves.

c) Find the optimal bundle for $U(x_1, x_2) = x_1 + x_2$. (give two numbers). Is your solution interior or corner? (one sentence)

d) Plot income offer curve and Engel curve given fixed prices $p_1 = \$4, p_2 = \$2$. (two graphs). Is Pepsi normal or inferior (choose one + one sentence)

**Problem 4 (15p)** (Quasilinear Preferences, Labor Supply)

Your best friend Aiden asked you to help him to determine how much time he should spend at work. His total available time (each day) is 24h and his only source of income is labor income given wage rate $w = \$10$. His spending on consumption good $C$ is equal to what he earns. The price of such good is $p_c = 2$

a) What is his real wage rate? (number+interpretation, one sentence )

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d) What is Aiden's labor supply $L$ if $w = \$1$ and $p_c = \$10$ (one number). How about $w = \$2$ and $p_c = \$10$ (one number). Is labor supply increasing in real wage rate, when preferences are quasilinear? (yes no answer)

**Bonus question** (Just for fun)

Let $MRS^V$ and $MRS^U$ be marginal rates of substitution of utility functions $V(x_1, x_2)$ and $U(x_1, x_2)$ respectively. Moreover, assume that $V(x_1, x_2) = f[U(x_1, x_2)]$ where $f[\cdot]$ is a strictly monotone transformation. Show that $MRS^V = MRS^U$. (Hint: Use formula for the derivative of a composite function)

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## Midterm 1 (C)

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**Problem 1 (55p)** (Well-behaved preferences)

Ava is a big fun of delicious gourmet steaks: their robust and hearty beef flavors is what she likes the most. Her two most favorite steaks are marinated ribeye $(x_1)$ and top sirloin $(x_2)$.

a) The price of a marinated ribeye steak is $p_1 = \$10$, the price of top sirloin is $p_2 = \$5$, and her income (spent entirely on steaks) is $m = \$200$. Show geometrically Ava's budget constraint. Mark the two extreme consumption bundles (give concrete values). Show on the graph how her budget set would be affected by inflation that increased the prices of both steaks by $100\%$, leaving $m$ unchanged (draw a new budget line).

b) Ava's utility function over both types of steaks is $U(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. Find her marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give a formula for MRS). What is the value of MRS at bundle $(3, 6)$ (one number)? Which of the two steak types is more valuable to Ava?

d) Write down two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (two equations).

- Provide economic intuition behind the two conditions (two sentences for each).

- Derive optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation of magic formulas).

- Using magic formulas determine whether the solution is interior for all values of $p_1, p_2, m$ (one sentence).

- What fraction of total income is spent on top sirloin?

e) Find the optimal consumption levels of two types of steak $(x_1, x_2)$ for:

- $p_1 = \$10, p_2 = \$5$ and $m = \$200$ (give two numbers).

and after the price of ribeye decreased:

- for $p_1 = \$5, p_2 = \$5$ and $m = \$200$ (give two numbers).

What is the total change in consumption of marinated ribeye? (give a number). Illustrate the change on the graph. Is marinated ribeye an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of $x_1$ from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

**Problem 2 (15p)** (Perfect complements)

Alex is a hotdog lover. In each hotdog bun $(x_1)$ Alex always inserts four hotdogs $(x_2)$: he hates hotdogs with ratios of buns to hotdogs different from 1:4.

a) Propose a utility function that captures Alex's preferences over $(x_1)$ and $(x_2)$ (function $U(x_1, x_2)$).

b) In the commodity space, carefully depict Alex's indifference curves (marking the optimal proportion line).

c) Assume prices $p_1 = 6$ and $p_2 = 1$ and income $m = 40$. Write down two secrets of happiness (two equations) and determine the optimal choice (two numbers). Is your solution interior (yes or no)?

d) Suppose price of a hotdog bun goes down to $p_1 = 4$, while price $p_2 = 1$ income $m = 40$ are unchanged. Find optimal choice (two numbers). Without calculations, how much of this change can be attributed to a substitution effect (number + one sentence explaining the number).

**Problem 3 (15p)** (Perfect substitutes)

Pepsi $x_1$ and Coke $x_2$ both are goods that are indistinguishable.

a) Propose a utility function (the "craziest" function you can imagine), which captures preferences for such perfect substitutes.

b) Plot the budget line, assuming $p_1 = \$6$, $p_2 = \$2$ and $m = \$12$. In the same graph plot indifference curves.

c) Find the optimal bundle for $U(x_1, x_2) = x_1 + x_2$. (give two numbers). Is your solution interior or corner? (one sentence)

d) Plot income offer curve and Engel curve given fixed prices $p_1 = \$6, p_2 = \$2$. (two graphs). Is Pepsi normal or inferior (choose one + one sentence)

**Problem 4 (15p)** (Quasilinear Preferences, Labor Supply)

Your best friend Aiden asked you to help him to determine how much time he should spend at work. His total available time (each day) is 24h and his only source of income is labor income given wage rate $w = \$10$. His spending on consumption good $C$ is equal to what he earns. The price of such good is $p_c = 2$

a) What is his real wage rate? (number+interpretation, one sentence )

b) Plot his budget set on the graph.

c) Aiden has quasilinear utility function $U(R, C) = \ln R + C$ where $R$ is leisure (or relaxation time) and $C$ is consumption of other goods. Write down two secrets of happiness that determine optimal choice of $R, C$ for arbitrary $w, p_c$. Find optimal $R$ as a function $w, p_c$ (you can assume interior solution).

d) What is Aiden's labor supply $L$ if $w = \$1$ and $p_c = \$10$ (one number). How about $w = \$2$ and $p_c = \$10$ (one number). Is labor supply increasing in real wage rate, when preferences are quasilinear? (yes no answer)

**Bonus question** (Just for fun)

Let $MRS^V$ and $MRS^U$ be marginal rates of substitution of utility functions $V(x_1, x_2)$ and $U(x_1, x_2)$ respectively. Moreover, assume that $V(x_1, x_2) = f[U(x_1, x_2)]$ where $f[\cdot]$ is a strictly monotone transformation. Show that $MRS^V = MRS^U$. (Hint: Use formula for the derivative of a composite function)

**Econ 301**
**Intermediate Microeconomics**
**Prof. Marek Weretka**

## Midterm 1 (D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (55+15+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

**Problem 1 (55p)** (Well-behaved preferences)

Ava is a big fun of delicious gourmet steaks: their robust and hearty beef flavors is what she likes the most. Her two most favorite steaks are marinated ribeye $(x_1)$ and top sirloin $(x_2)$.

a) The price of a marinated ribeye steak is $p_1 = \$4$, the price of top sirloin is $p_2 = \$2$, and her income (spent entirely on steaks) is $m = \$80$. Show geometrically Ava's budget constraint. Mark the two extreme consumption bundles (give concrete values). Show on the graph how her budget set would be affected by inflation that increased the prices of both steaks by $100\%$, leaving $m$ unchanged (draw a new budget line).

b) Ava's utility function over both types of steaks is $U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$. Find her marginal rate of substitution (MRS) for any bundle $(x_1, x_2)$ (give a formula for MRS). What is the value of MRS at bundle $(3, 6)$ (one number)? Which of the two steak types is more valuable to Ava?

d) Write down two secrets of happiness, assuming that $p_1, p_2, m$ are parameters (two equations).

- Provide economic intuition behind the two conditions (two sentences for each).

- Derive optimal consumption of $x_1$ and $x_2$ as a function of $p_1, p_2, m$ (show the derivation of magic formulas).

- Using magic formulas determine whether the solution is interior for all values of $p_1, p_2, m$ (one sentence).

- What fraction of total income is spent on top sirloin?

e) Find the optimal consumption levels of two types of steak $(x_1, x_2)$ for:

- $p_1 = \$4, p_2 = \$2$ and $m = \$80$ (give two numbers).

and after the price of ribeye decreased:

- for $p_1 = \$2, p_2 = \$2$ and $m = \$80$ (give two numbers).

What is the total change in consumption of marinated ribeye? (give a number). Illustrate the change on the graph. Is marinated ribeye an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of $x_1$ from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

**Problem 2 (15p)** (Perfect complements)

Alex is a hotdog lover. In each hotdog bun $(x_1)$ Alex always inserts two hotdogs $(x_2)$: he hates hotdogs with ratios of buns to hotdogs different from 1:2.

a) Propose a utility function that captures Alex's preferences over $(x_1)$ and $(x_2)$ (function $U(x_1, x_2)$).

b) In the commodity space, carefully depict Alex's indifference curves (marking the optimal proportion line).

c) Assume prices $p_1 = 2$ and $p_2 = 2$ and income $m = 30$. Write down two secrets of happiness (two equations) and determine the optimal choice (two numbers). Is your solution interior (yes or no)?

d) Suppose price of a hotdog bun goes down to $p_1 = 1$, while price $p_2 = 2$ income $m = 30$ are unchanged. Find optimal choice (two numbers). Without calculations, how much of this change can be attributed to a substitution effect (number + one sentence explaining the number).

**Problem 3 (15p)** (Perfect substitutes)

Pepsi $x_1$ and Coke $x_2$ both are goods that are indistinguishable.

a) Propose a utility function (the "craziest" function you can imagine), which captures preferences for such perfect substitutes.

b) Plot the budget line, assuming $p_1 = \$2$, $p_2 = \$6$ and $m = \$12$. In the same graph plot indifference curves.

c) Find the optimal bundle for $U(x_1, x_2) = x_1 + x_2$. (give two numbers). Is your solution interior or corner? (one sentence)

d) Plot income offer curve and Engel curve given fixed prices $p_1 = \$2, p_2 = \$6$. (two graphs). Is Pepsi normal or inferior (choose one + one sentence)

**Problem 4 (15p)** (Quasilinear Preferences, Labor Supply)

Your best friend Aiden asked you to help him to determine how much time he should spend at work. His total available time (each day) is 24h and his only source of income is labor income given wage rate $w = \$10$. His spending on consumption good $C$ is equal to what he earns. The price of such good is $p_c = 2$

a) What is his real wage rate? (number+interpretation, one sentence )

b) Plot his budget set on the graph.

c) Aiden has quasilinear utility function $U(R, C) = \ln R + C$ where $R$ is leisure (or relaxation time) and $C$ is consumption of other goods. Write down two secrets of happiness that determine optimal choice of $R, C$ for arbitrary $w, p_c$. Find optimal $R$ as a function $w, p_c$ (you can assume interior solution).

d) What is Aiden's labor supply $L$ if $w = \$1$ and $p_c = \$10$ (one number). How about $w = \$2$ and $p_c = \$10$ (one number). Is labor supply increasing in real wage rate, when preferences are quasilinear? (yes no answer)

**Bonus question** (Just for fun)

Let $MRS^V$ and $MRS^U$ be marginal rates of substitution of utility functions $V(x_1, x_2)$ and $U(x_1, x_2)$ respectively. Moreover, assume that $V(x_1, x_2) = f[U(x_1, x_2)]$ where $f[\cdot]$ is a strictly monotone transformation. Show that $MRS^V = MRS^U$. (Hint: Use formula for the derivative of a composite function)

**Midterm Exam 1 Solutions**
**(A) The Pink**

**Q1)** *(55 points)*

**a)** Check figure 1 for the budget set *(3 p)*
If there is 100% inflation, the prices double.
The new budget set is also shown on Figure 1. *(3 p)*

**b)** $U(X_1, X_2) = X_1^5 X_2^5$

$MRS^{12} = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2) = ((5X_1^4 X_2^5)/(5X_1^5 X_2^4)) = X_2/X_1 = MRS^{12}$
*(4 p)*
at the point $x_1 = 2$ & $x_2 = 4$: $MRS^{12} = (4/2) = 2$ *(2 p)*

which means:
$MU_1 > MU_2$ So 1st good, ribeye, is more valuable than top sirleon to Ava at the consumption level of
$x_1 = 2$ & $x_2 = 4$ *(4 p)*

**d)**

secret 1 : $p_1x_1 + p_2x_2 = m$ *(3p)*
so $50x_1 + 25x_2 = 500$
This means "spend all of your money". If the consumer does not spend all of his/her money s/he is wasting his/her opportunity to increase his/her utility since money does not have an effect on utility itself. *(2p)*

secret 2 : $MRS^{12} = (MU_1)/(MU_2) = P_1/P_2$ *(3p)*
$\rightarrow \quad X_2/X_1 = P_1/P_2$

"the last spent on each good should give the same utility" OR "marginal utility of a \$ spent on each good should be equal " . If this condition does not hold, lets say last \$ spent on good 1 brings more utility than the last \$ spent on good 2 , then the consumer should buy less of the second good and buy more of the first good to increase his/her utility. *(2p)*

start with secret 2 : $X_2/X_1 = P_1/P_2 \rightarrow X_2 = ((X_1P_1)/(P_2))$ *(1p)*
now plug this in secret 1 :
$p_1x_1 + p_2x_2 = m = P_1x_1 + P_2(X_1P_1)/(P_2) = m = 2P_1x_1$ *(2p)*
$\rightarrow$ Then $X_1 = (m/(2p_1)) = (1/2)(m/p_1)$
Since $X_2 = ((X_1P_1)/(P_2)) = ((mP_1)/(2P_1P_2)) = (m/(2p_2))$
$= (1/2)(m/(p_2)) = X_2$

$X_1 = (m/(2p_1))$

$X_2 = (m/(2p_2))$ *(2p)*

The solution is interior since none of good's optimal consumption level is zero. Furthermore in a Cobb-Douglas case the solution has to be interior as long as income level (m) is larger than zero ! (we also assume positive prices) *(1p)*

Using the magic formula $X_2 = (m/(2p_2)) \rightarrow X_2 p_2 = m/2$
note that the left hand side is the total money spent on good 2 (sirleon). So half of the income is spent on sirleon. *(2p)*

**e)** $X_1 = (m/(2p_1)) = ((500)/(2 * 50)) = 5$ *(1p)*

$X_2 = (m/(2p_2)) = ((500)/(2 * 25)) = 10$ *(1p)*

$X_1^{'} = (m/(2p_1^{'})) = ((500)/(2 * 25)) = 10$ *(1p)*

$X_2^{'} = (m/(2p_2)) = ((500)/(2 * 25)) = 10$ *(1p)*

Total change in consumption of ribeye is $X_1^{'} - X_1 = 10 - 5 = 5$ *(1p)*

Check figure 2 for the illustration of the change. *(3p)*

Ribeye is an ordinary good since it's consumption increases as it's price decrease. We can also see this fact from the magic formula . Since the price of ribeye is in the denominator in optimal consumption formula of ribeye, as the price goes down the consumption of it will increase ! *(2p)*

**f)** How much money does Ava need to consume the original bundle with the new prices ?

$p_1^{'} x_1 + p_2 x_2 = m^{'} = (\$25 * 5) + (\$25 * 10) = \$375$ is enough *(3p)*

Now calculating the optimal bundle with this imaginary income :
$X_1^s = (m^{'}/(2p_1^{'})) = ((375)/(2 * 25)) = 15/2$ *(2p)*

$X_2^s = (m^{'}/(2p_2)) = ((375)/(2 * 25)) = 15/2$
So Substitution Effect (S.E.) is : $15/2 - 5 = 5/2$ *(2p)*

$T.E = S.E. + I.E.$ then : $I.E. = 5 - (5/2) = 5/2$ *(1p)*

Check figure 2 for the illustration. *(3p)*

**Q 2)** *(15 points)*
**a)** the bundle : $(1x_1 + 2x_2)$
A suitable utility function would be : $min(2x_1, 1x_2)$ *(1p)*

**b)** Check figure 3 *(2p)*
**c)** Two secrets are : *(1.5p + 1.5p)*

$$\begin{aligned} 2x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= m \end{aligned}$$

hence

$$4x_1 + 3(2x_1) = 40$$

$$x_1^* = \frac{m}{p_1 + 2p_2} \text{ and } x_2^* = 2\frac{m}{p_1 + 2p_2}$$

$$\begin{aligned} x_1^* &= \frac{40}{4 + (2 * 3)} = 4 \\ \text{and } x_2^* &= 2\frac{40}{4 + (2 * 3)} = 8 \end{aligned}$$

(1.5p + 1.5p)
The solution is interior since none of the optimal consumption levels are zero. (1p)

**d)** Using the magic formulas :

$$\begin{aligned} x_1^* &= \frac{40}{2 + (2 * 3)} = 5 \\ \text{and } x_2^* &= 2\frac{40}{2 + (2 * 3)} = 10 \end{aligned}$$

*(1.5p + 1.5p)*
S.E. is zero. In a perfect complements case there is no substitution effect related to a price change. *(2p)*

**Q.3)** *(15 points)*
**a)** $U(X_1, X_2) = 2X_1 + 2X_2$ can be a utility function for this kind of preferences. *(2p)*
**b)** Check the figure 4 *(2p)*
**c)** $MRS^{12} = ((MU_1)/(MU_2)) = 1/1 = 1$ given the utility function. *(2p)*
comparing MRS to the price ratio : $1 > \frac{2}{3} = \frac{P_1}{P_2}$

So only good 1 is consumed : $X_1^* = m/P_1 = 12/2 = 6$ and $X_2^* = 0$ *(3p)*

The solution is a corner one. *(1p)*

**d)** check figure 5 for the income-offer curve and the engel curve. *(2p+2p)*

Pepsi is normal as the consumption of it increases as the income increases. *(1p)*

**Q.4)**

**a)** $\frac{10}{2} = 5$ is her real wage. *(1p)*

This number is her purchasing power in terms of consumption good. *(1p)*

**b)** check figure 6 *(3p)*

**c)**

secret 1: $w * R + p_c C = 24 * w$ *(2p)*

or $10R + 2C = 24 * 10 = 240$

secret 2 : $MRS^{RC} = ((MU_R)/(MU_C)) = w/P_c \rightarrow 1/R = w/P_c$ *(2p)*

$\rightarrow \quad R = \frac{P_c}{w}$ *(2p)*

**d)** If $w = \$1$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 10hr$ *(1p)*

If $w = \$2$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 5hr$ *(1p)*

The labor supply (24-R) is increasing in real wage ! *(2p)*

Bonus Question

$V(x_1, x_2) = f[U(x_1, x_2)]$

$MRS^U = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2)$

$MRS^V = ((MV_1)/(MV_2)) = (\partial V/\partial X_1)/(\partial V/\partial X_2)$

$= \frac{\partial f[U(x_1,x_2)]/\partial X_1}{\partial f[U(x_1,x_2)]/\partial X_2}$ by chain rule :

$= \frac{f^{'}(.)*\partial U(x_1,x_2)/\partial X_1}{f^{'}(.)*\partial U(x_1,x_2)/\partial X_2} = \frac{\partial U(x_1,x_2)/\partial X_1}{\partial U(x_1,x_2)/\partial X_2} = MRS^U$

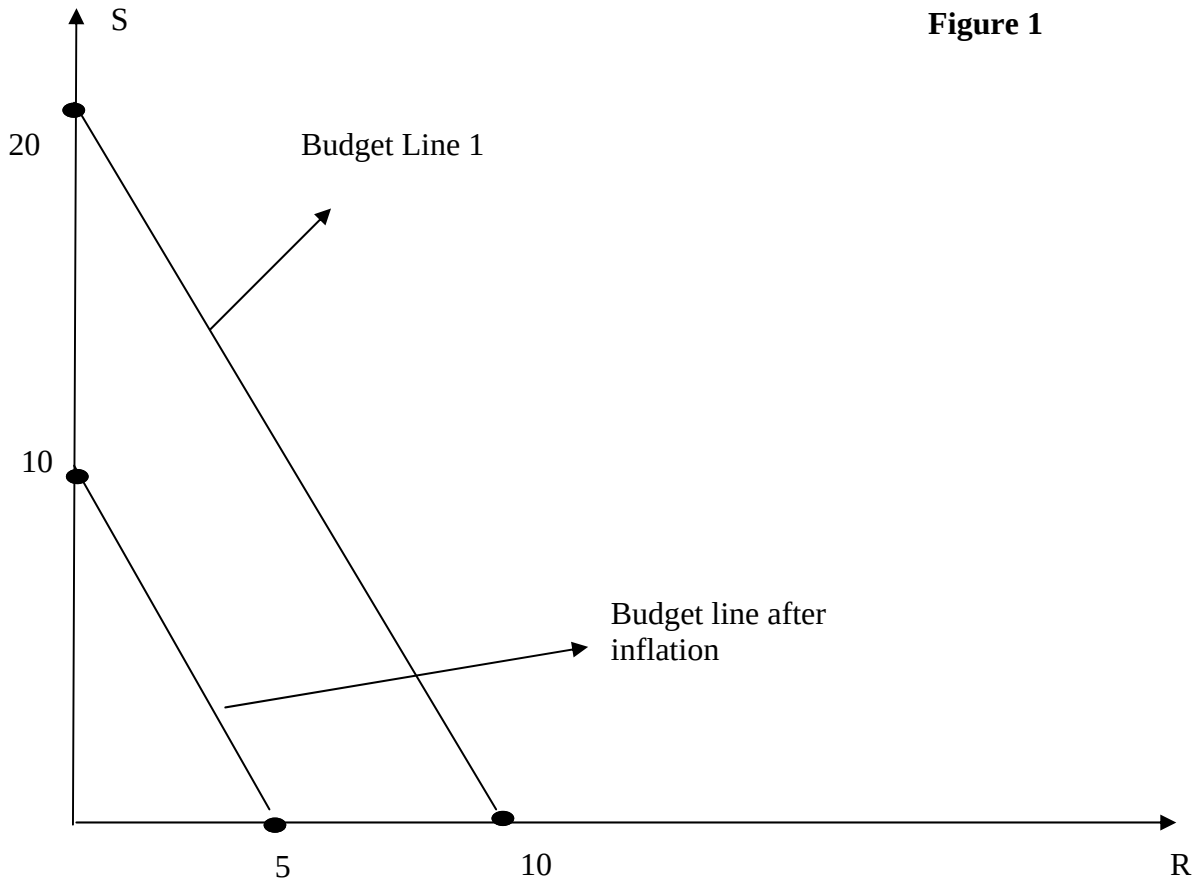



**Figure 1**

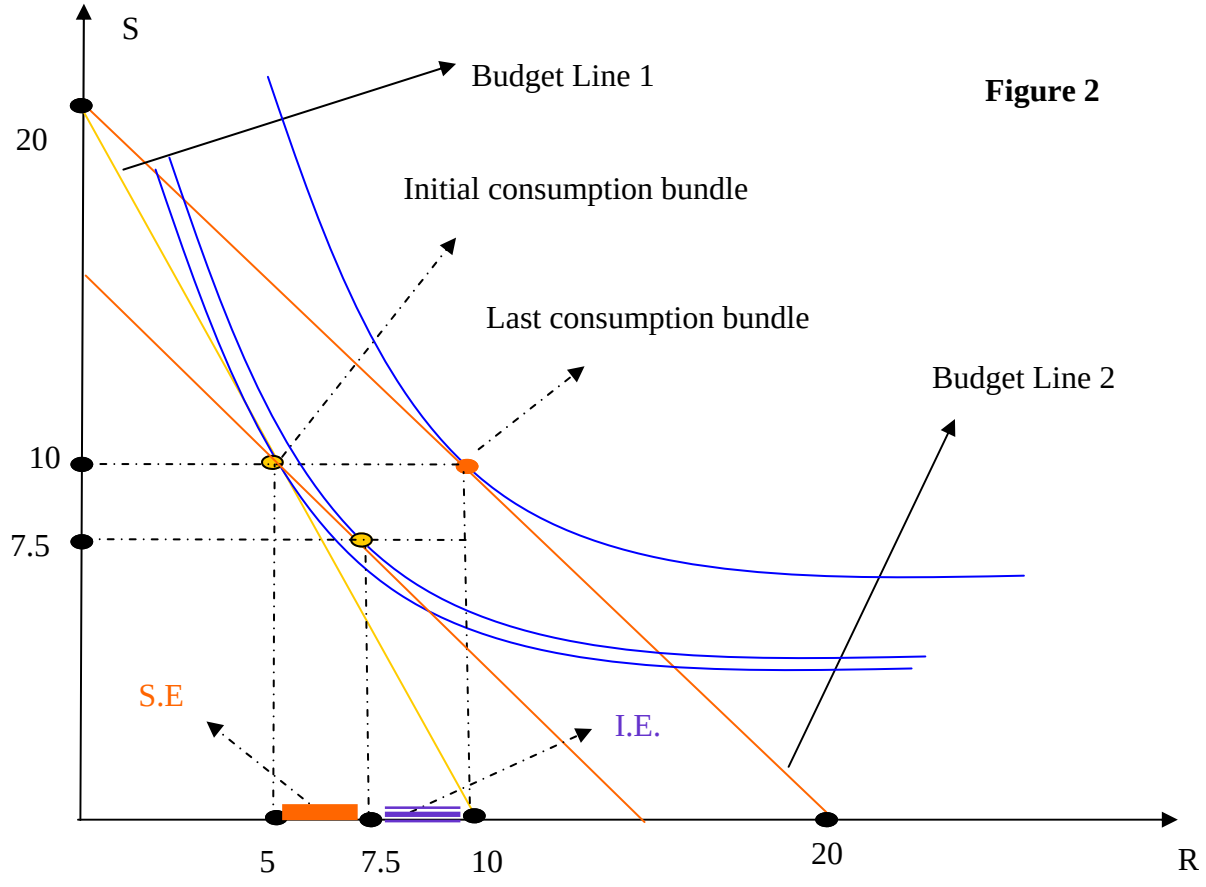



**Figure 2**

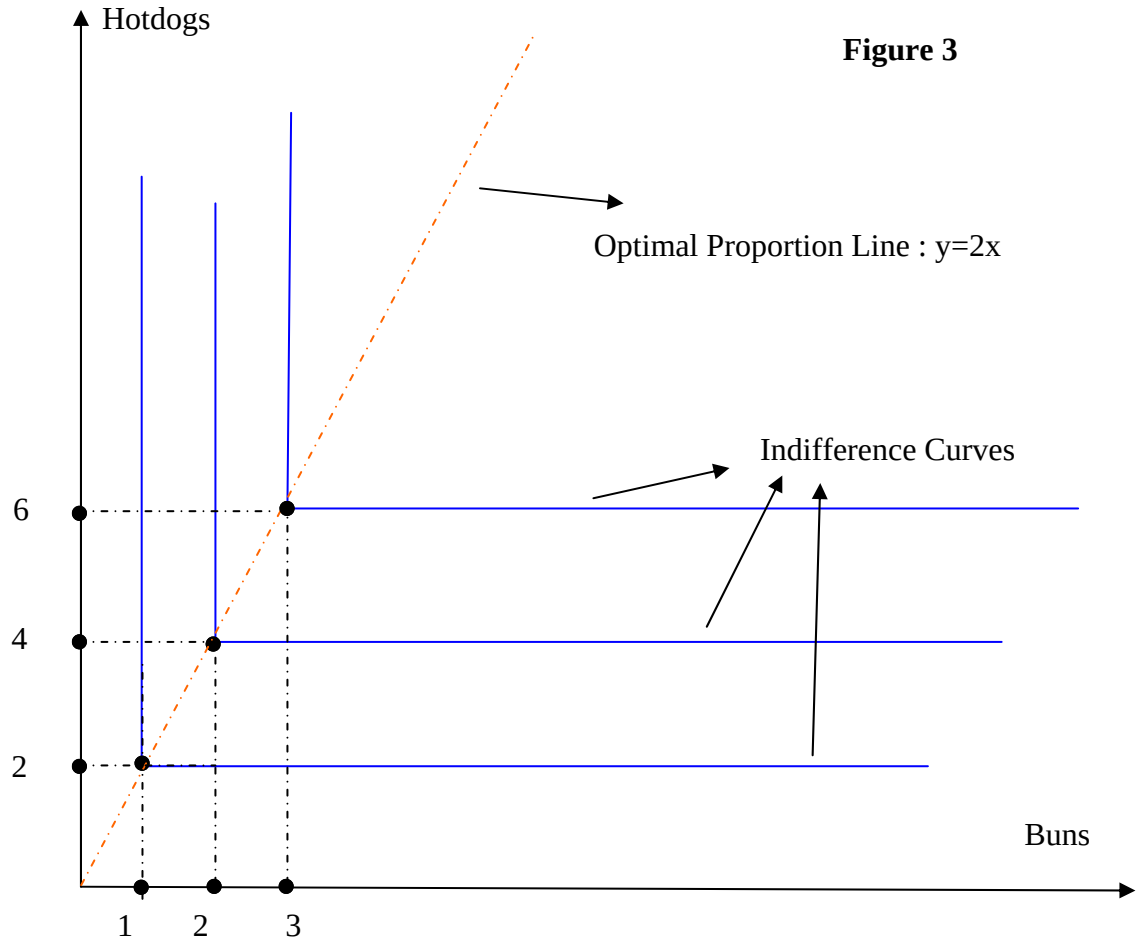



**Figure 3**

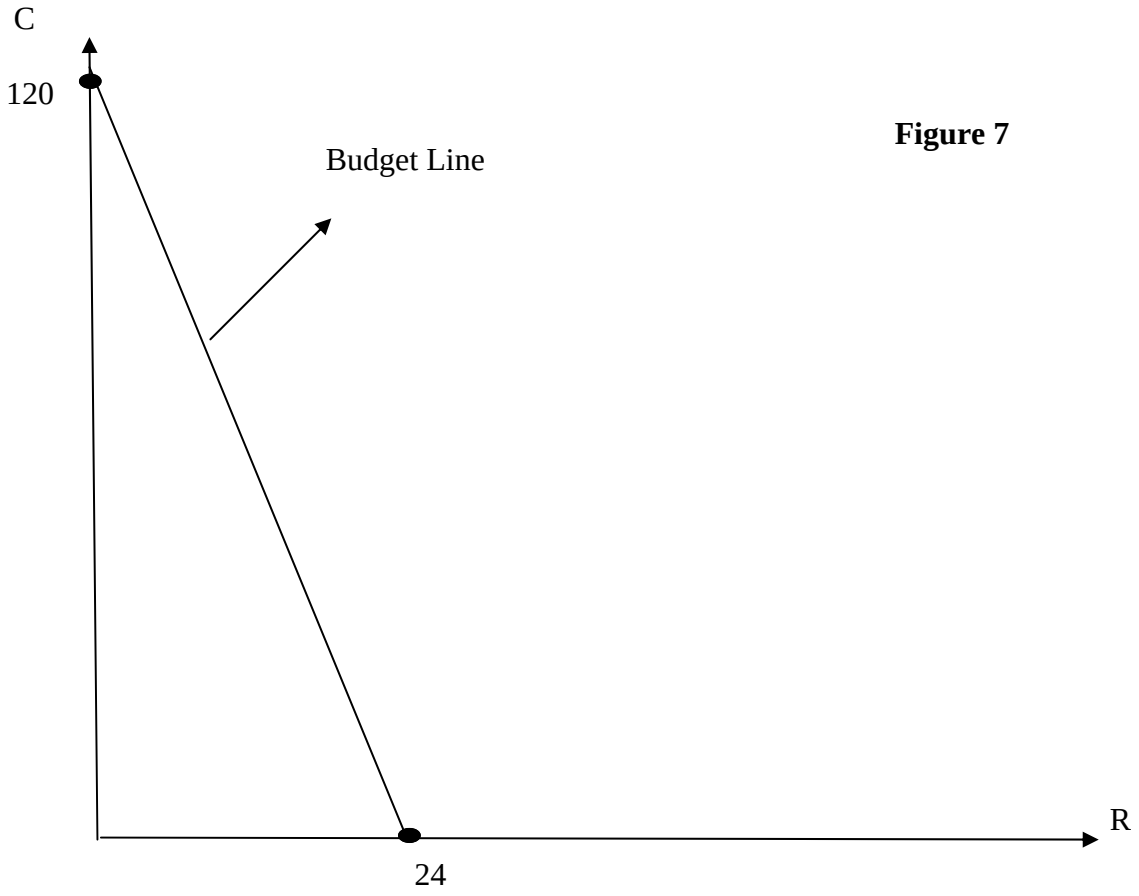



**Figure 7**

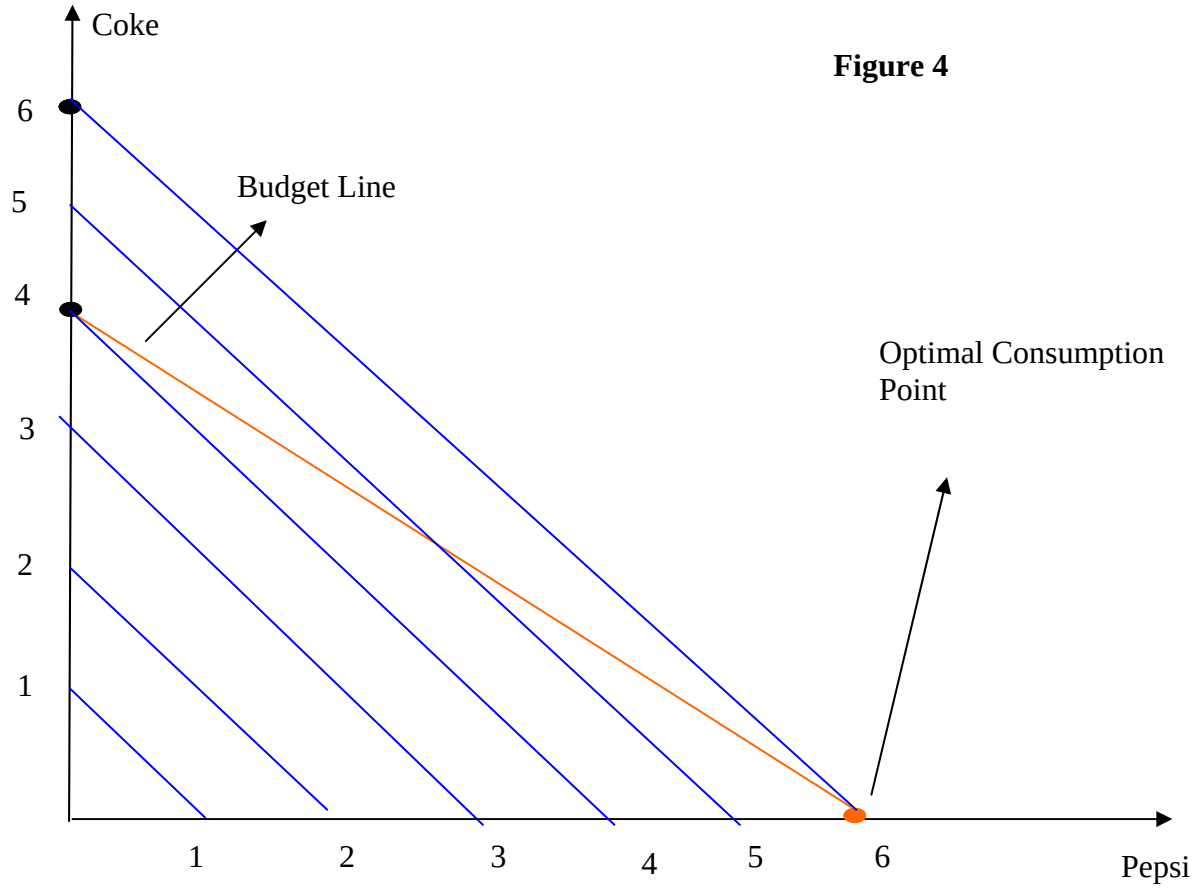



**Figure 4**

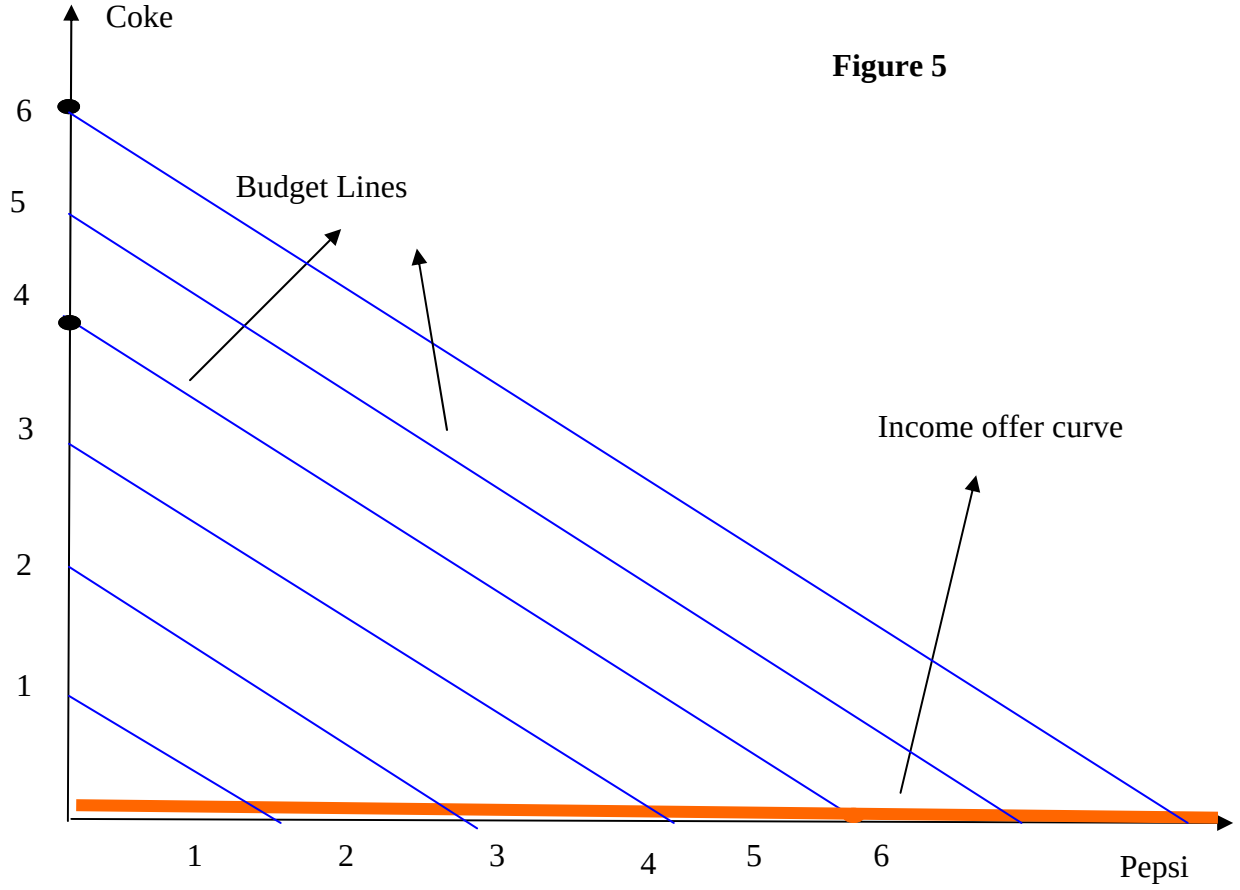
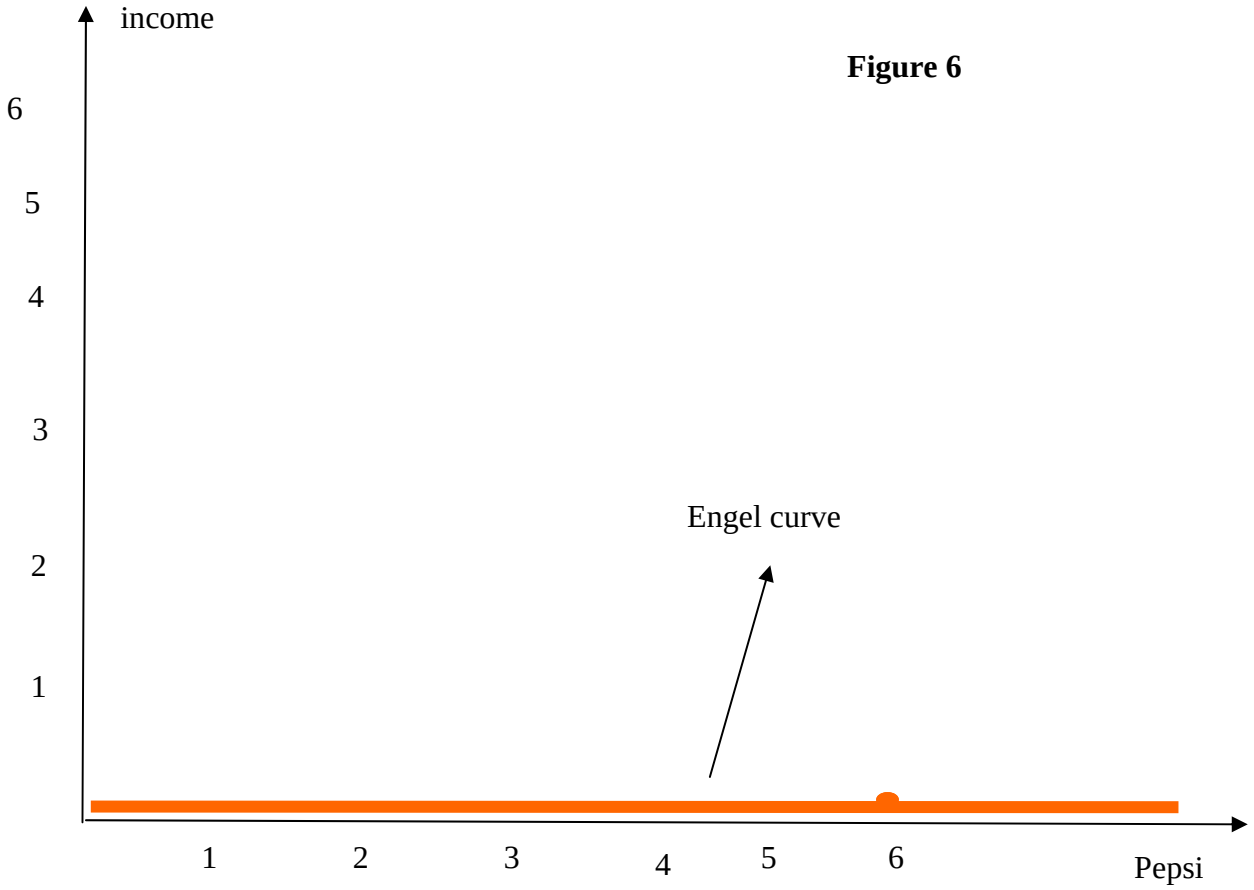



**Figure 5**

**Figure 6**



**Midterm Exam 1 Solutions**
**(B) The Green**

**Q1)** *(55 points)*

**a)** Check figure 1 for the budget set *(3 p)*
If there is 100% inflation, the prices double.
The new budget set is also shown on Figure 1. *(3 p)*

**b)** $U(X_1, X_2) = X_1^3 X_2^3$

$MRS^{12} = ((MU_1)/(MU_2)) = -(\partial U/\partial X_1)/(\partial U/\partial X_2) = (-(3X_1^2X_2^3)/(3X_1^3X_2^2)) = -X_2/X_1 = MRS^{12}$
*(4 p)*
at the point $x_1 = 3$ & $x_2 = 6$: $MRS^{12} = (-6/3) = -2$ *(2 p)*

which means:
$MU_1 > MU_2$ So 1st good, ribeye, is more valuable than top sirleon to Ava at the consumption level of
$x_1 = 3$ & $x_2 = 6$ *(4 p)*

**d)**

secret 1 : $p_1x_1 + p_2x_2 = m$ *(3p)*
so $10x_1 + 5x_2 = 100$
This means "spend all of your money". If the consumer does not spend all of his/her money s/he is wasting his/her opportunity to increase his/her utility since money does not have an effect on utility itself. *(2p)*

secret 2 : $MRS^{12} = (MU_1)/(MU_2) = P_1/P_2$ *(3p)*
$\rightarrow \quad X_2/X_1 = P_1/P_2$

"the last spent on each good should give the same utility" OR "marginal utility of a \$ spent on each good should be equal " . If this condition does not hold, lets say last \$ spent on good 1 brings more utility than the last \$ spent on good 2 , then the consumer should buy less of the second good and buy more of the first good to increase his/her utility. *(2p)*

start with secret 2 : $X_2/X_1 = P_1/P_2 \rightarrow X_2 = ((X_1P_1)/(P_2))$ *(1p)*
now plug this in secret 1 :
$p_1x_1 + p_2x_2 = m = P_1x_1 + P_2(X_1P_1)/(P_2) = m = 2P_1x_1$ *(2p)*
$\rightarrow$ Then $X_1 = (m/(2p_1)) = (1/2)(m/p_1)$
Since $X_2 = ((X_1P_1)/(P_2)) = ((mP_1)/(2P_1P_2)) = (m/(2p_2))$
$= (1/2)(m/(p_2)) = X_2$

$X_1 = (m/(2p_1))$

$X_2 = (m/(2p_2))$ *(2p)*

The solution is interior since none of good's optimal consumption level is zero. Furthermore in a Cobb-Douglas case the solution has to be interior as long as income level (m) is larger than zero ! (we also assume positive prices) *(1p)*

Using the magic formula $X_2 = (m/(2p_2)) \rightarrow X_2 p_2 = m/2$
note that the left hand side is the total money spent on good 2 (sirleon). So half of the income is spent on sirleon. *(2p)*

**e)** $X_1 = (m/(2p_1)) = ((100)/(2*10)) = 5$ *(1p)*

$X_2 = (m/(2p_2)) = ((100)/(2*5)) = 10$ *(1p)*

$X_1^{'} = (m/(2p_1^{'})) = ((100)/(2*5)) = 10$ *(1p)*

$X_2^{'} = (m/(2p_2)) = ((100)/(2*5)) = 10$ *(1p)*

Total change in consumption of ribeye is $X_1^{'} - X_1 = 10 - 5 = 5$ *(1p)*

Check figure 2 for the illustration of the change. *(3p)*

Ribeye is an ordinary good since it's consumption increases as it's price decrease. We can also see this fact from the magic formula . Since the price of ribeye is in the denominator in optimal consumption formula of ribeye, as the price goes down the consumption of it will increase ! *(2p)*

**f)** How much money does Ava need to consume the original bundle with the new prices ?

$p_1^{'} x_1 + p_2 x_2 = m^{'} = (\$5 * 5) + (\$5 * 10) = \$75$ is enough *(3p)*

Now calculating the optimal bundle with this imaginary income :
$X_1^s = (m^{'}/(2p_1^{'})) = ((75)/(2*5)) = 15/2$ *(2p)*

$X_2^s = (m^{'}/(2p_2)) = ((75)/(2*5)) = 15/2$
So Substitution Effect (S.E.) is : $15/2 - 5 = 5/2$ *(2p)*

$T.E = S.E. + I.E.$ then : $I.E. = 5 - (5/2) = 5/2$ *(1p)*

Check figure 2 for the illustration. *(3p)*

**Q 2)** *(15 points)*
**a)** the bundle : $(1x_1 + 3x_2)$
A suitable utility function would be : $min(3x_1, 1x_2)$ *(1p)*

**b)** Check figure 3 *(2p)*
**c)** Two secrets are : *(1.5p + 1.5p)*

$$\begin{aligned} 3x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= m \end{aligned}$$

hence

$$4x_1 + 2(3x_1) = 40$$

$$x_1^* = \frac{m}{p_1 + 3p_2} \text{ and } x_2^* = 3\frac{m}{p_1 + 3p_2}$$

$$\begin{aligned} x_1^* &= \frac{40}{4 + (2 * 3)} = 4 \\ \text{and } x_2^* &= 3\frac{40}{4 + (2 * 3)} = 12 \end{aligned}$$

(1.5p + 1.5p)
The solution is interior since none of the optimal consumption levels are zero. (1p)

**d)** Using the magic formulas :

$$\begin{aligned} x_1^* &= \frac{40}{2 + (2 * 3)} = 5 \\ \text{and } x_2^* &= 3\frac{40}{2 + (2 * 3)} = 15 \end{aligned}$$

*(1.5p + 1.5p)*
S.E. is zero. In a perfect complements case there is no substitution effect related to a price change. *(2p)*

**Q.3)** *(15 points)*
**a)** $U(X_1, X_2) = 2X_1 + 2X_2$ can be a utility function for this kind of preferences. *(2p)*
**b)** Check the figure 4 *(2p)*
**c)** $MRS^{12} = ((MU_1)/(MU_2)) = 1/1 = 1$ given the utility function. *(2p)*
comparing MRS to the price ratio : $1 < \frac{4}{2} = \frac{P_1}{P_2}$

So only good 2 is consumed : $X_2^* = m/P_2 = 12/2 = 6$ and $X_1^* = 0$ *(3p)*

The solution is a corner one. *(1p)*

**d)** check figure 5 & 6 for the income-offer curve and the engel curve. *(2p+2p)*

Pepsi is normal as the consumption of it does not decrease as the income increases. *(1p)*

(Answers of "inferior" and "ambigous" are also accepted IF NECESSARY EXPLANATIONS ARE DONE!)

**Q.4)**

**a)** $\frac{10}{2} = 5$ is her real wage. *(1p)*

This number is her purchasing power in terms of consumption good. *(1p)*

**b)** check figure 7 *(3p)*

**c)**

secret 1: $w * R + p_c C = 24 * w$ *(2p)*

or $10R + 2C = 24 * 10 = 240$

secret 2 : $MRS^{RC} = ((MU_R)/(MU_C)) = w/P_c \rightarrow 1/R = w/P_c$ *(2p)*

$\rightarrow \quad R = \frac{P_c}{w}$ *(2p)*

**d)** If $w = \$1$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 10hr$ *(1p)*

If $w = \$2$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 5hr$ *(1p)*

The labor supply (24-R) is increasing in real wage ! *(2p)*

Bonus Question

$V(x_1, x_2) = f[U(x_1, x_2)]$

$MRS^U = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2)$

$MRS^V = ((MV_1)/(MV_2)) = (\partial V/\partial X_1)/(\partial V/\partial X_2)$

$= \frac{\partial f[U(x_1,x_2)]/\partial X_1}{\partial f[U(x_1,x_2)]/\partial X_2}$ by chain rule :

$= \frac{f'(.)*\partial U(x_1,x_2)/\partial X_1}{f'(.)*\partial U(x_1,x_2)/\partial X_2} = \frac{\partial U(x_1,x_2)/\partial X_1}{\partial U(x_1,x_2)/\partial X_2} = MRS^U$

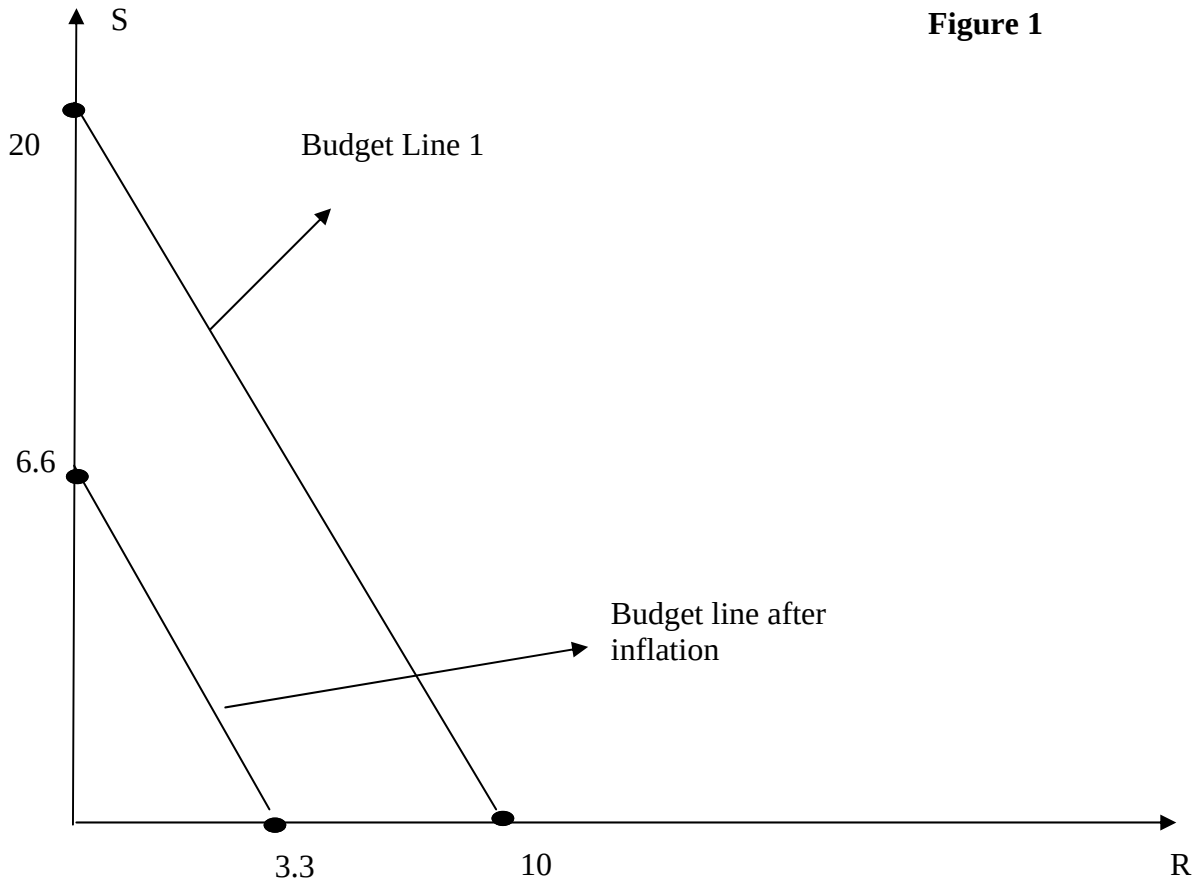



**Figure 1**

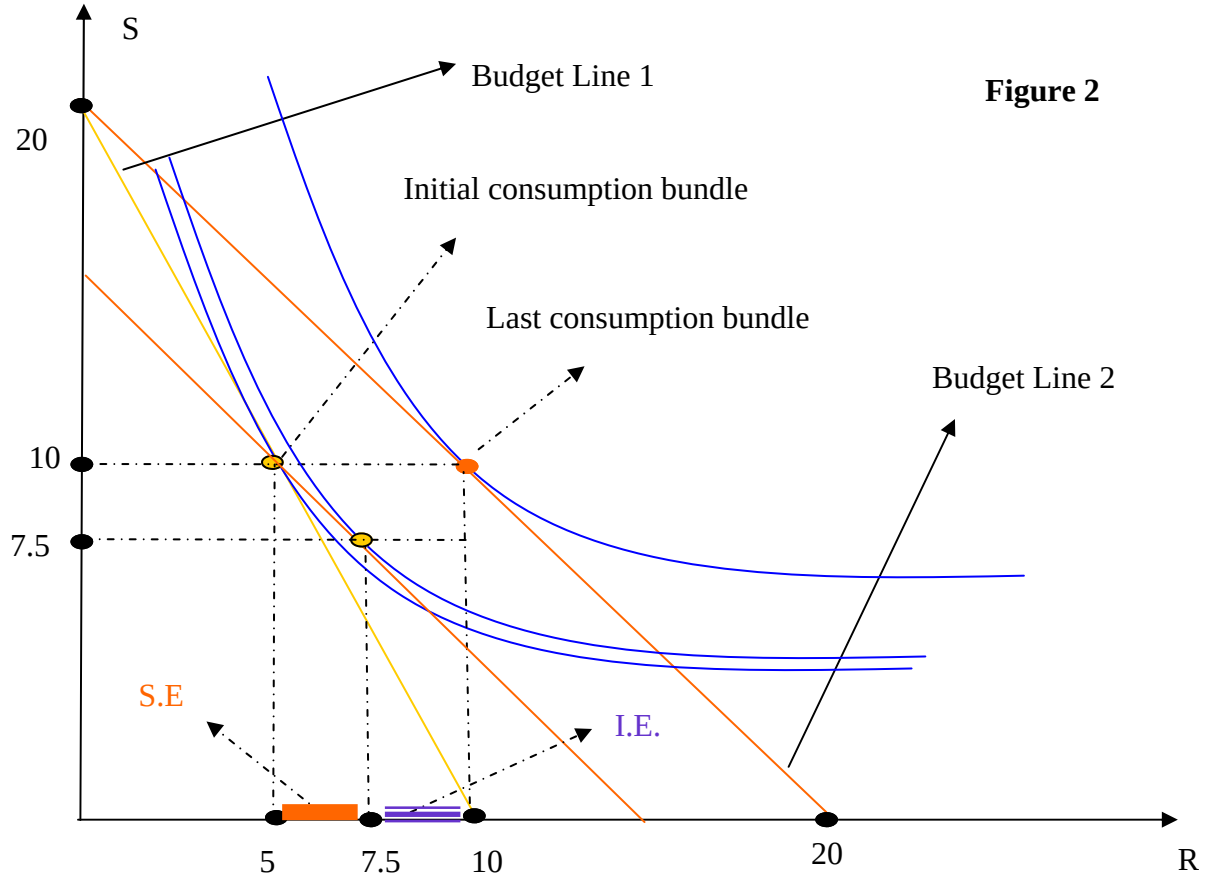



**Figure 2**

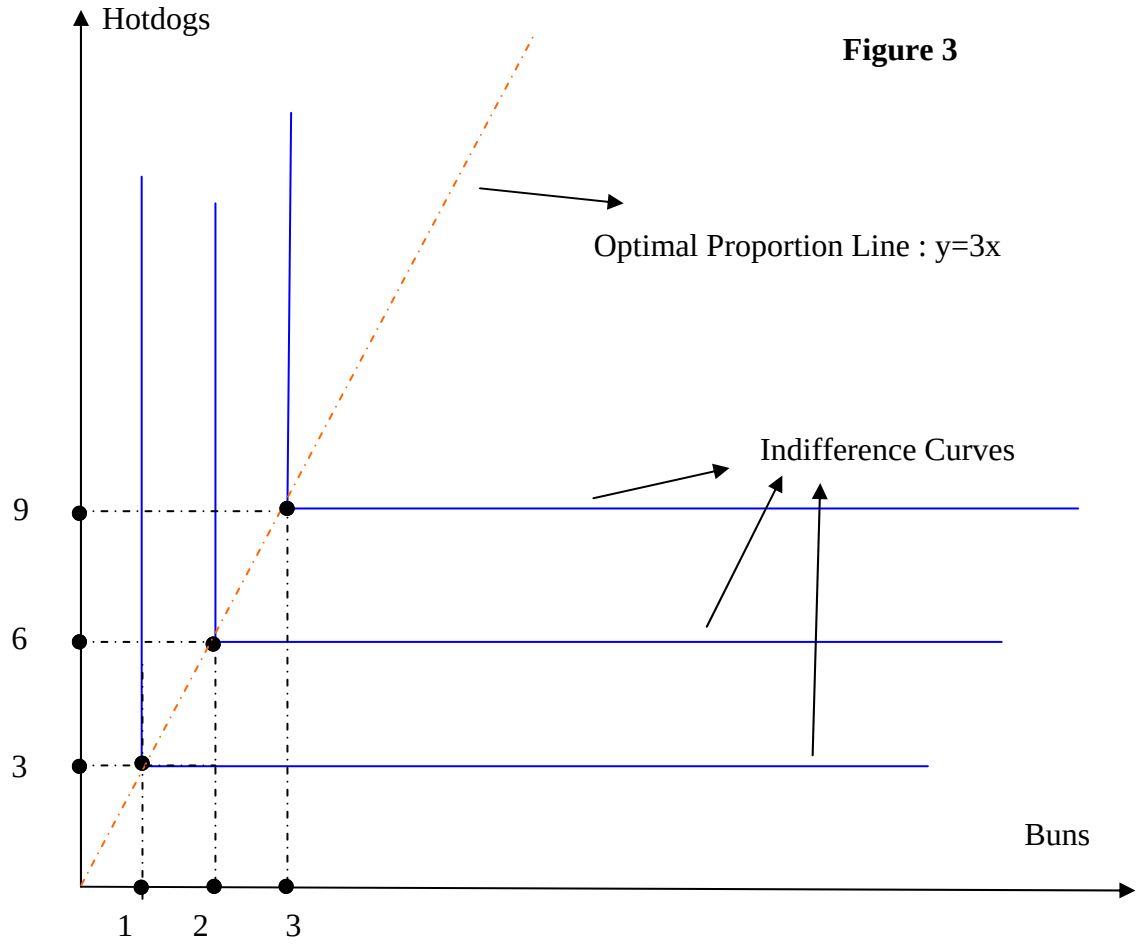



**Figure 3**

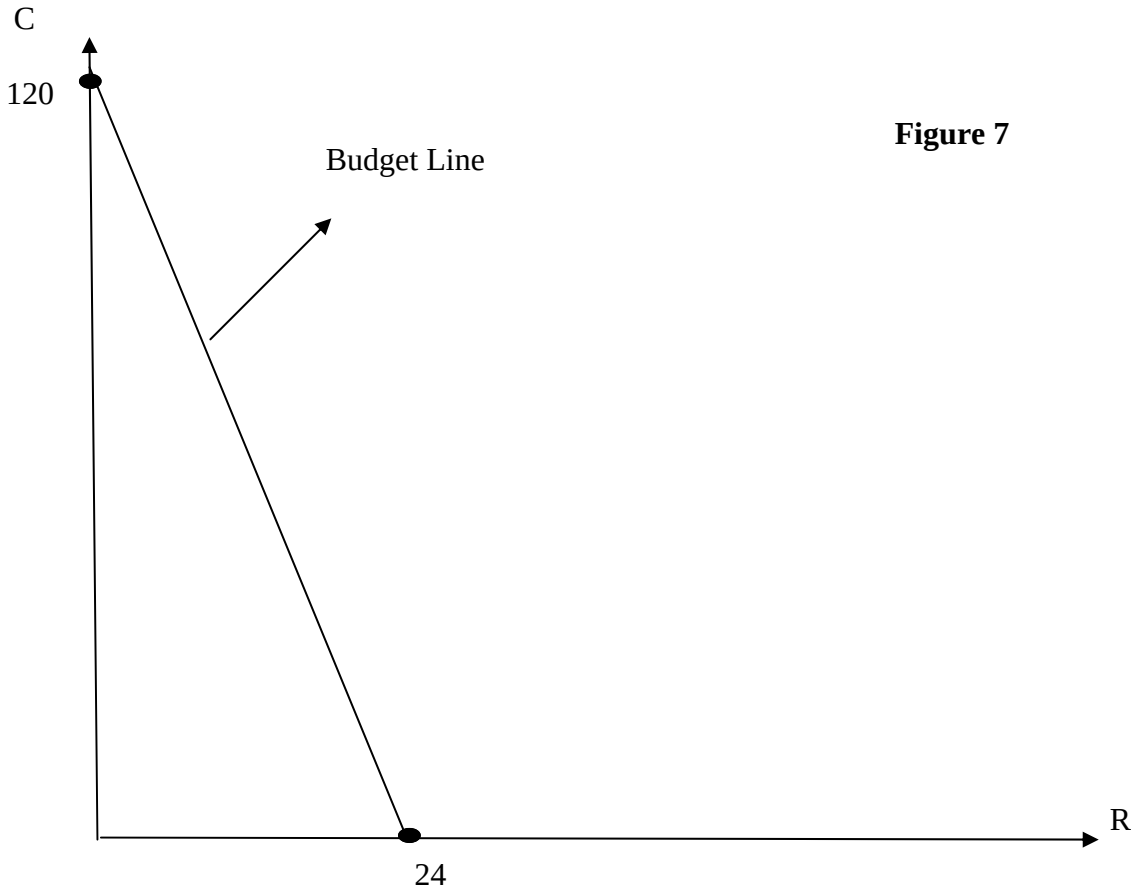



**Figure 7**

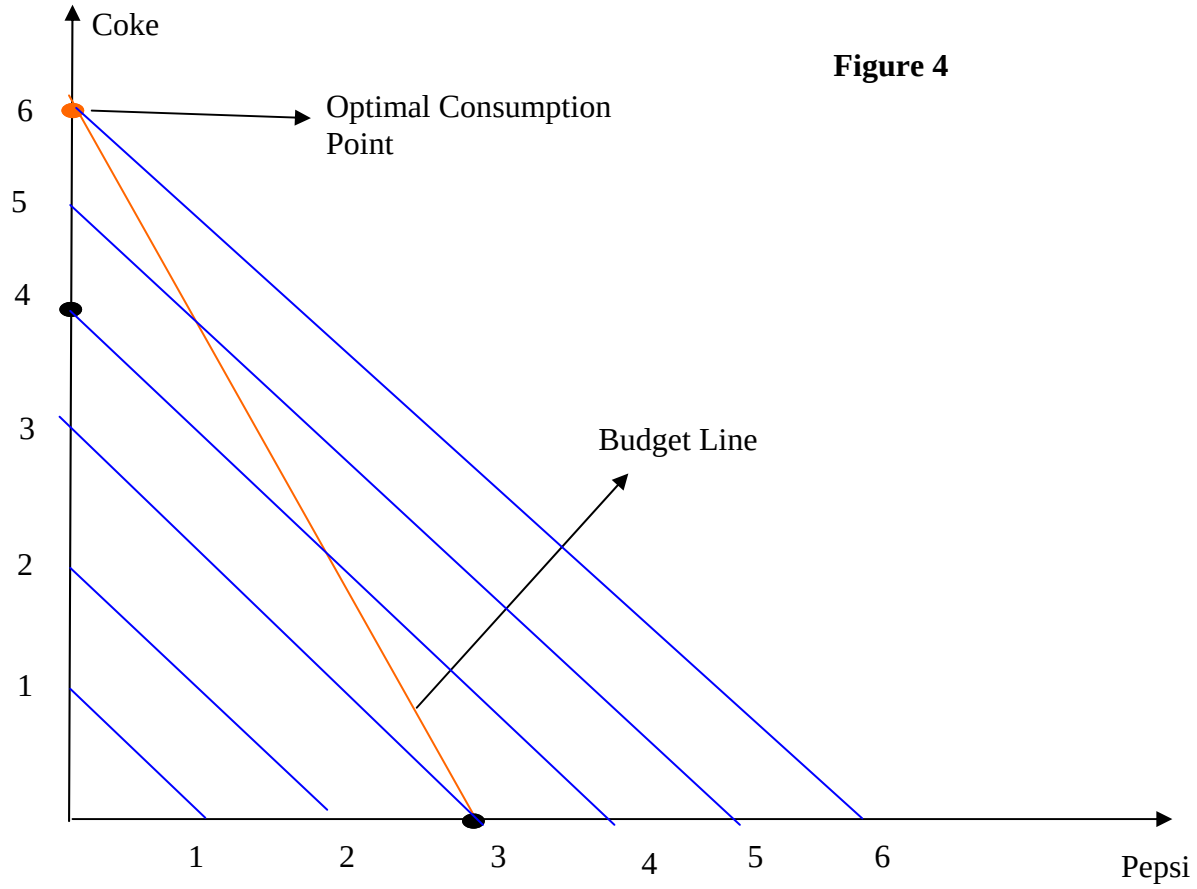



**Figure 4**

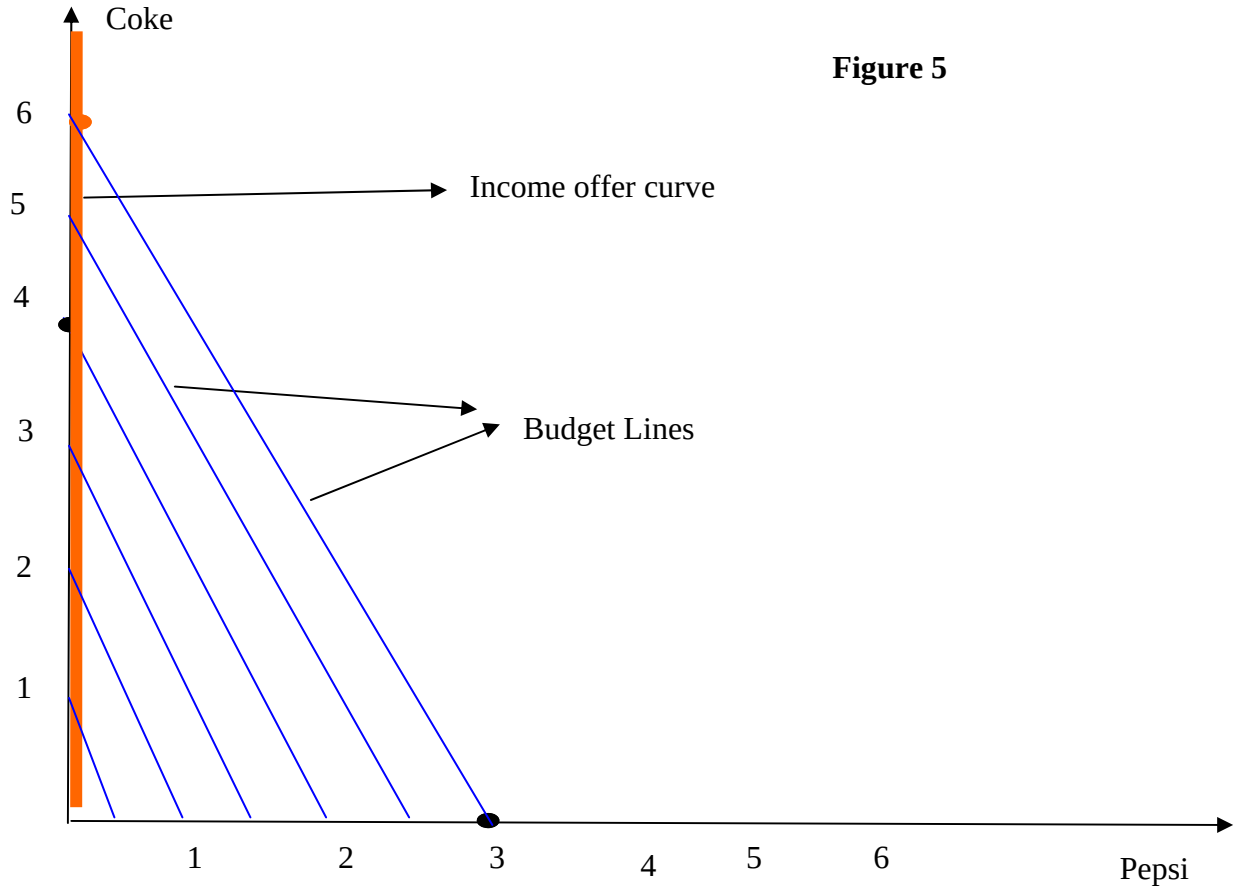
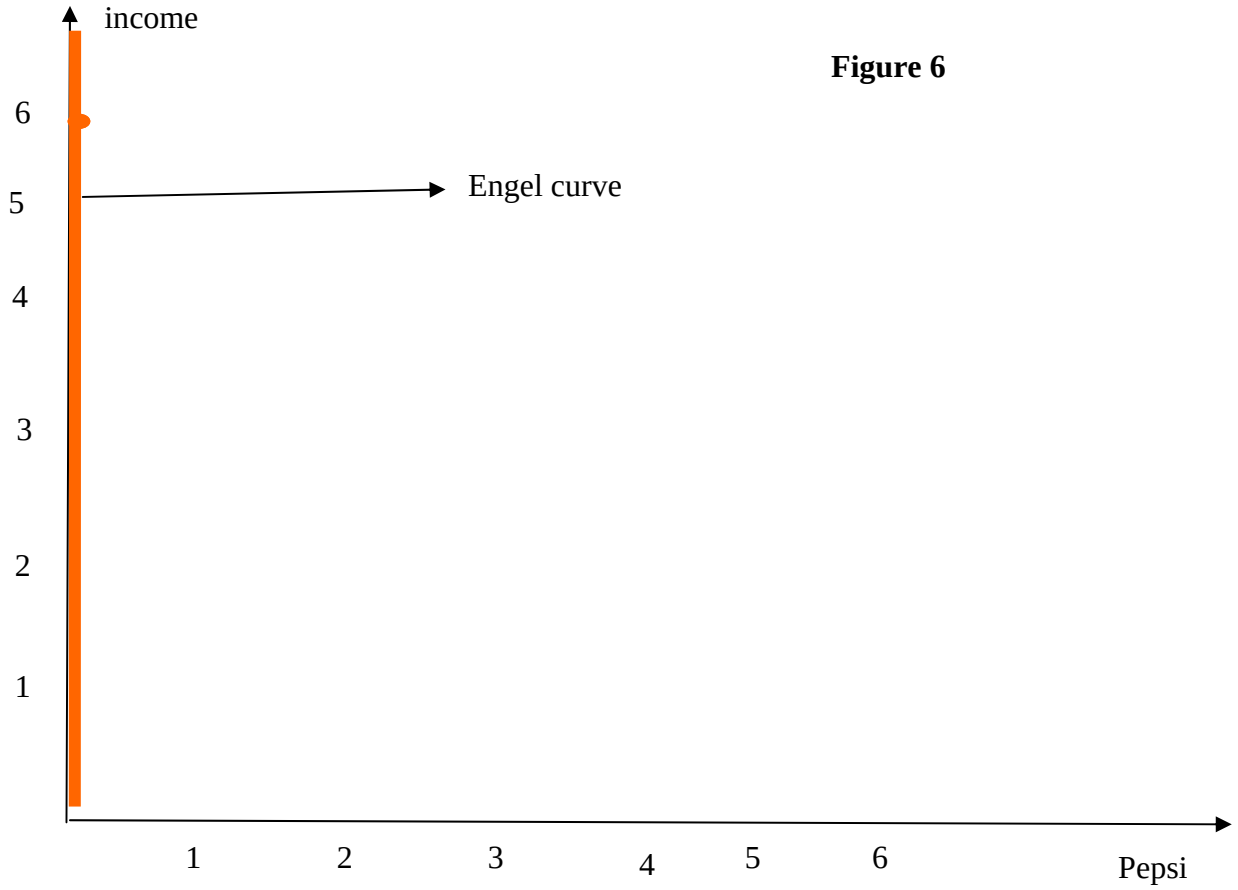



**Figure 5**

**Figure 6**



**Midterm Exam 1 Solutions**
**(C) The Yellow**

**Q1)** *(55 points)*

**a)** Check figure 1 for the budget set *(3 p)*
If there is 100% inflation, the prices double.
The new budget set is also shown on Figure 1. *(3 p)*

**b)** $U(X_1, X_2) = X_1^{1/2} X_2^{1/2}$

$MRS^{12} = ((MU_1)/(MU_2)) = -(\partial U/\partial X_1)/(\partial U/\partial X_2) = (-(\frac{1}{2}X_1^{-1/2}X_2^{1/2})/(\frac{1}{2}X_1^{1/2}X_2^{-1/2})) = -X_2/X_1 = MRS^{12}$
*(4 p)*
at the point $x_1 = 3$ & $x_2 = 6$: $MRS^{12} = (-6/3) = -2$ *(2 p)*

which means:
$MU_1 > MU_2$ So 1st good, ribeye, is more valuable than top sirleon to Ava at the consumption level of
$x_1 = 3$ & $x_2 = 6$ *(4 p)*

**d)**

secret 1 : $p_1x_1 + p_2x_2 = m$ *(3p)*
so $10x_1 + 5x_2 = 200$
This means "spend all of your money". If the consumer does not spend all of his/her money s/he is wasting his/her opportunity to increase his/her utility since money does not have an effect on utility itself. *(2p)*

secret 2 : $MRS^{12} = (MU_1)/(MU_2) = P_1/P_2$ *(3p)*
$\rightarrow \quad X_2/X_1 = P_1/P_2$

"the last spent on each good should give the same utility" OR "marginal utility of a \$ spent on each good should be equal " . If this condition does not hold, lets say last \$ spent on good 1 brings more utility than the last \$ spent on good 2 , then the consumer should buy less of the second good and buy more of the first good to increase his/her utility. *(2p)*

start with secret 2 : $X_2/X_1 = P_1/P_2 \rightarrow X_2 = ((X_1P_1)/(P_2))$ *(1p)*
now plug this in secret 1 :
$p_1x_1 + p_2x_2 = m = P_1x_1 + P_2(X_1P_1)/(P_2) = m = 2P_1x_1$ *(2p)*
$\rightarrow$ Then $X_1 = (m/(2p_1)) = (1/2)(m/p_1)$
Since $X_2 = ((X_1P_1)/(P_2)) = ((mP_1)/(2P_1P_2)) = (m/(2p_2))$
$= (1/2)(m/(p_2)) = X_2$

$X_1 = (m/(2p_1))$

$X_2 = (m/(2p_2))$ *(2p)*

The solution is interior since none of good's optimal consumption level is zero. Furthermore in a Cobb-Douglas case the solution has to be interior as long as income level (m) is larger than zero ! (we also assume positive prices) *(1p)*

Using the magic formula $X_2 = (m/(2p_2)) \rightarrow X_2 p_2 = m/2$
note that the left hand side is the total money spent on good 2 (sirleon). So half of the income is spent on sirleon. *(2p)*

**e)** $X_1 = (m/(2p_1)) = ((200)/(2*10)) = 10$ *(1p)*

$X_2 = (m/(2p_2)) = ((200)/(2*5)) = 20$ *(1p)*

$X_1^{'} = (m/(2p_1^{'})) = ((200)/(2*5)) = 20$ *(1p)*

$X_2^{'} = (m/(2p_2)) = ((200)/(2*5)) = 20$ *(1p)*

Total change in consumption of ribeye is $X_1^{'} - X_1 = 20 - 10 = 10$ *(1p)*

Check figure 2 for the illustration of the change. *(3p)*

Ribeye is an ordinary good since it's consumption increases as it's price decrease. We can also see this fact from the magic formula . Since the price of ribeye is in the denominator in optimal consumption formula of ribeye, as the price goes down the consumption of it will increase ! *(2p)*

**f)** How much money does Ava need to consume the original bundle with the new prices ?

$p_1^{'} x_1 + p_2 x_2 = m^{'} = (\$5 * 10) + (\$5 * 20) = \$150$ is enough *(3p)*

Now calculating the optimal bundle with this imaginary income :
$X_1^s = (m^{'}/(2p_1^{'})) = ((150)/(2*5)) = 15$ *(2p)*

$X_2^s = (m^{'}/(2p_2)) = ((150)/(2*5)) = 15$
So Substitution Effect (S.E.) is : $15 - 10 = 5$ *(2p)*

$T.E = S.E. + I.E.$ then : $I.E. = 10 - 5 = 5$ *(1p)*

Check figure 2 for the illustration. *(3p)*

**Q 2)** *(15 points)*
**a)** the bundle : $(1x_1 + 4x_2)$
A suitable utility function would be : $min(4x_1, 1x_2)$ *(1p)*

**b)** Check figure 3 *(2p)*
**c)** Two secrets are : *(1.5p + 1.5p)*

$$\begin{aligned} 4x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= m \end{aligned}$$

hence

$$6x_1 + 1(4x_1) = 40$$

$$x_1^* = \frac{m}{p_1 + 4p_2} \text{ and } x_2^* = 4\frac{m}{p_1 + 4p_2}$$

$$\begin{aligned} x_1^* &= \frac{40}{6 + (1 * 4)} = 4 \\ \text{and } x_2^* &= 4\frac{40}{6 + (1 * 4)} = 16 \end{aligned}$$

(1.5p + 1.5p)
The solution is interior since none of the optimal consumption levels are zero. (1p)

**d)** Using the magic formulas :

$$\begin{aligned} x_1^* &= \frac{40}{4 + (1 * 4)} = 5 \\ \text{and } x_2^* &= 4\frac{40}{4 + (1 * 4)} = 20 \end{aligned}$$

*(1.5p + 1.5p)*
S.E. is zero. In a perfect complements case there is no substitution effect related to a price change. *(2p)*

**Q.3)** *(15 points)*
**a)** $U(X_1, X_2) = 2X_1 + 2X_2$ can be a utility function for this kind of preferences. *(2p)*
**b)** Check the figure 4 *(2p)*
**c)** $MRS^{12} = ((MU_1)/(MU_2)) = 1/1 = 1$ given the utility function. *(2p)*
comparing MRS to the price ratio : $1 < \frac{6}{2} = \frac{P_1}{P_2}$

So only good 2 is consumed : $X_2^* = m/P_2 = 12/2 = 6$ and $X_1^* = 0$ *(3p)*

The solution is a corner one. *(1p)*

**d)** check figure 5 & 6 for the income-offer curve and the engel curve. *(2p+2p)*

Pepsi is normal as the consumption of it does not decrease as the income increases. *(1p)*

(Answers of "inferior" and "ambigous" are also accepted IF NECESSARY EXPLANATIONS ARE DONE!)

**Q.4)**

**a)** $\frac{10}{2} = 5$ is her real wage. *(1p)*

This number is her purchasing power in terms of consumption good. *(1p)*

**b)** check figure 7 *(3p)*

**c)**

secret 1: $w * R + p_c C = 24 * w$ *(2p)*

or $10R + 2C = 24 * 10 = 240$

secret 2 : $MRS^{RC} = ((MU_R)/(MU_C)) = w/P_c \rightarrow 1/R = w/P_c$ *(2p)*

$\rightarrow \quad R = \frac{P_c}{w}$ *(2p)*

**d)** If $w = \$1$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 10hr$ *(1p)*

If $w = \$2$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 5hr$ *(1p)*

The labor supply (24-R) is increasing in real wage ! *(2p)*

Bonus Question

$V(x_1, x_2) = f[U(x_1, x_2)]$

$MRS^U = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2)$

$MRS^V = ((MV_1)/(MV_2)) = (\partial V/\partial X_1)/(\partial V/\partial X_2)$

$= \frac{\partial f[U(x_1,x_2)]/\partial X_1}{\partial f[U(x_1,x_2)]/\partial X_2}$ by chain rule :

$= \frac{f'(.)*\partial U(x_1,x_2)/\partial X_1}{f'(.)*\partial U(x_1,x_2)/\partial X_2} = \frac{\partial U(x_1,x_2)/\partial X_1}{\partial U(x_1,x_2)/\partial X_2} = MRS^U$

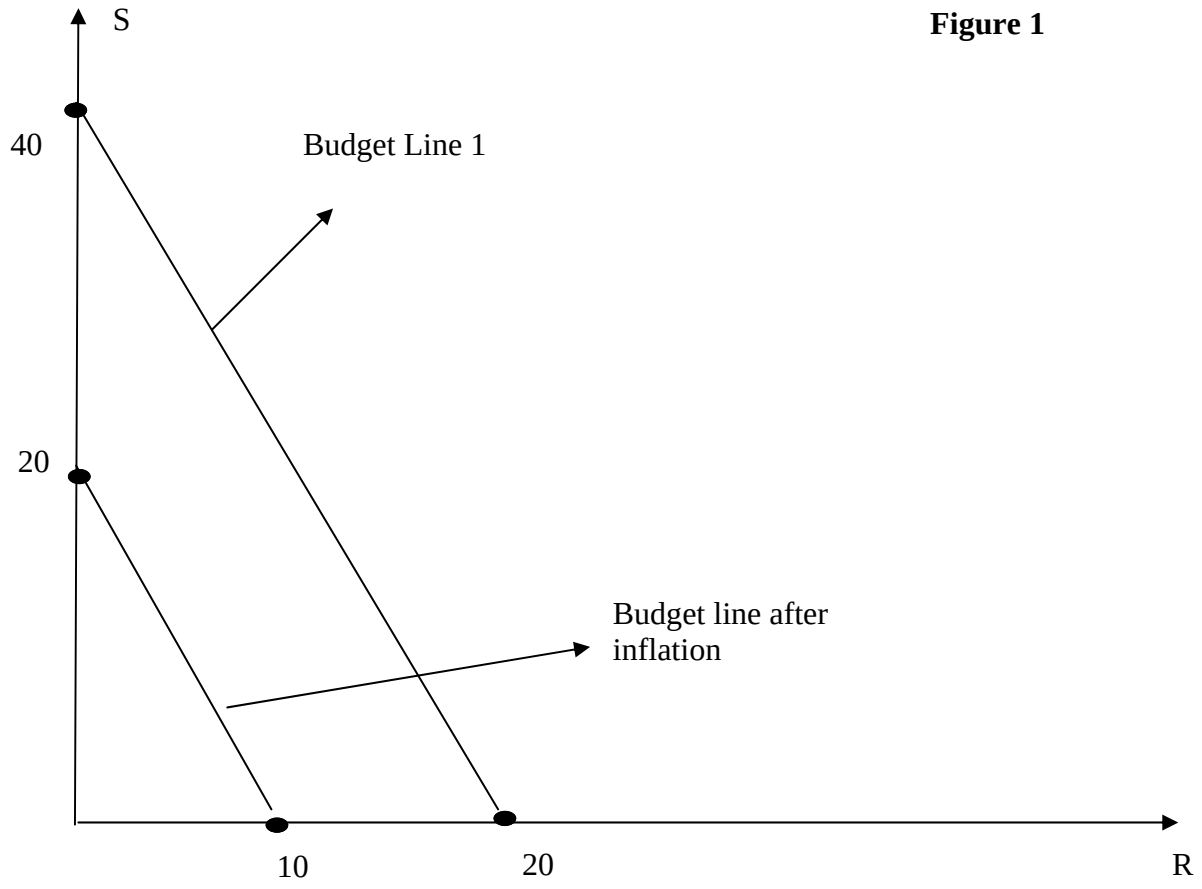



**Figure 1**

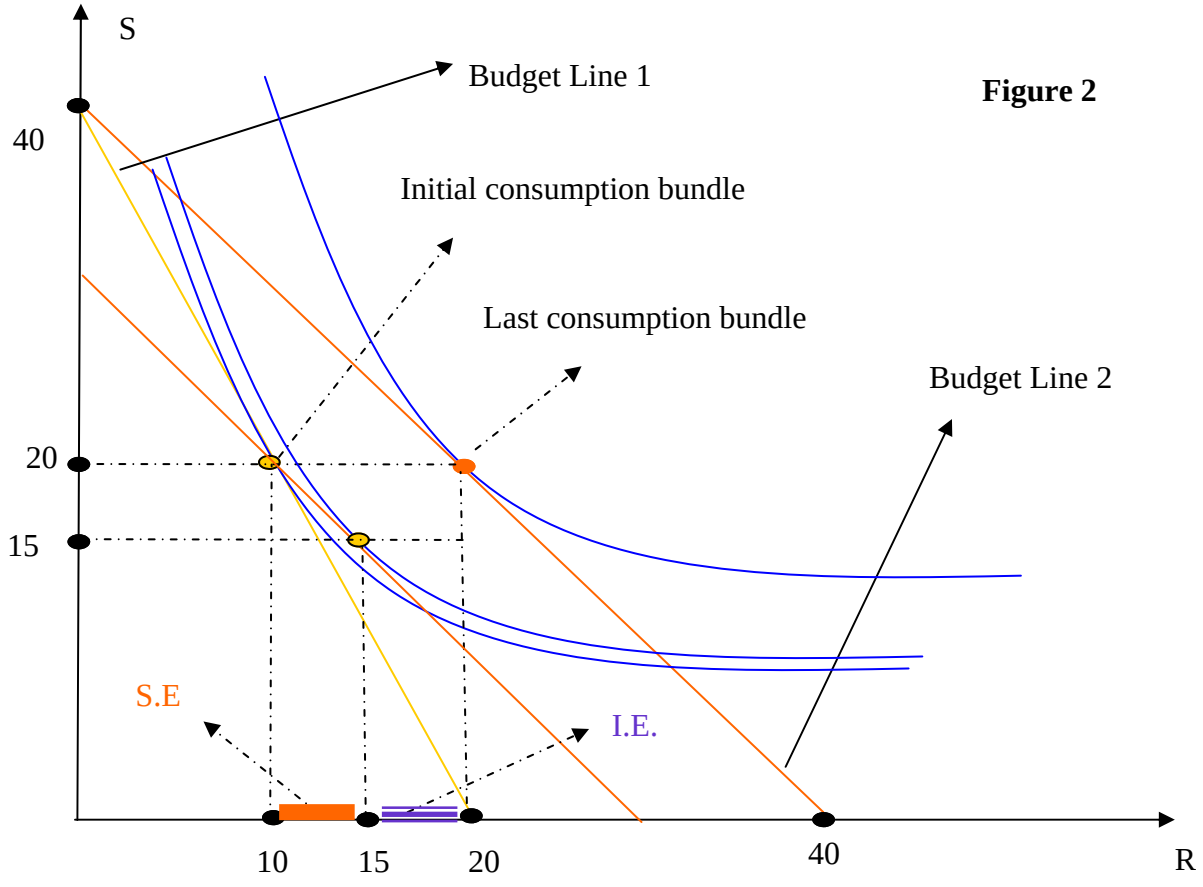



**Figure 2**

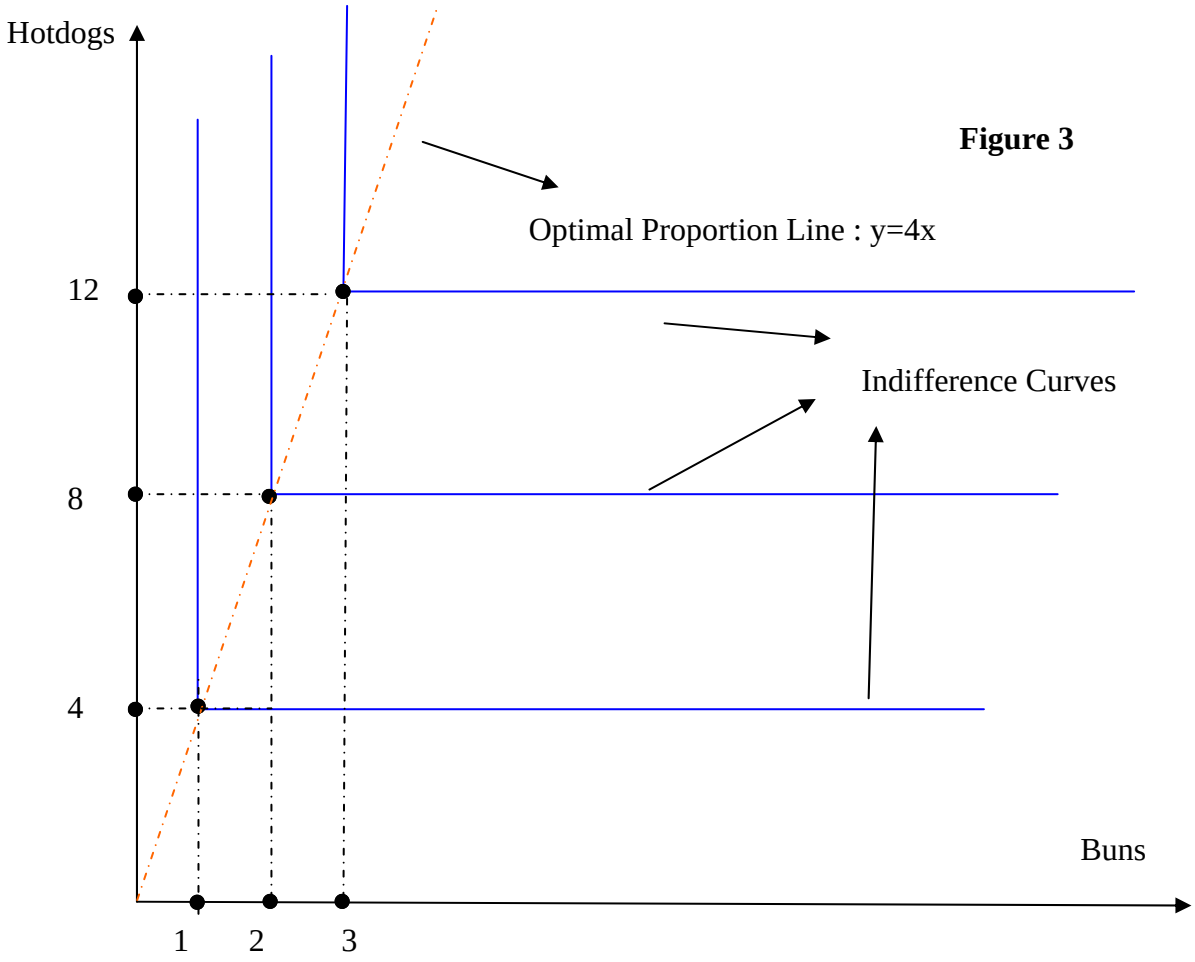



**Figure 3**

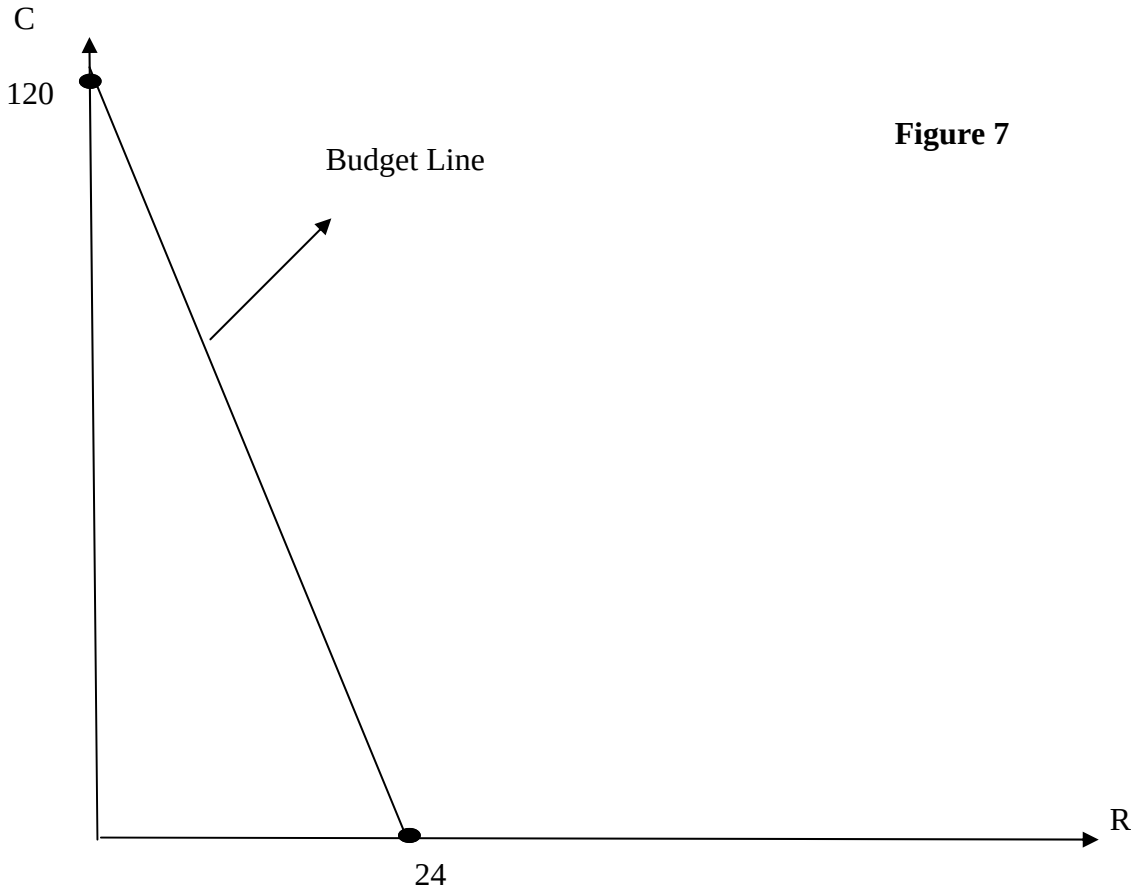



**Figure 7**

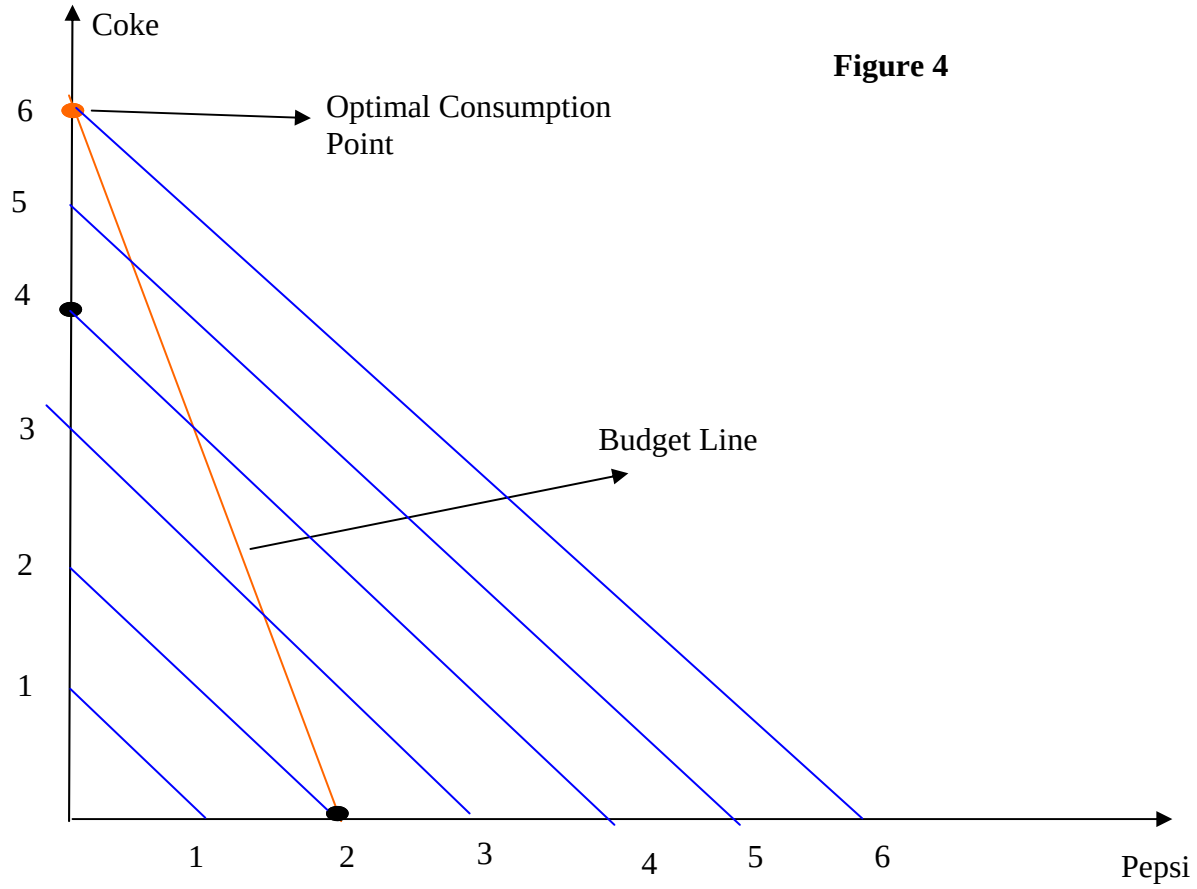



**Figure 4**

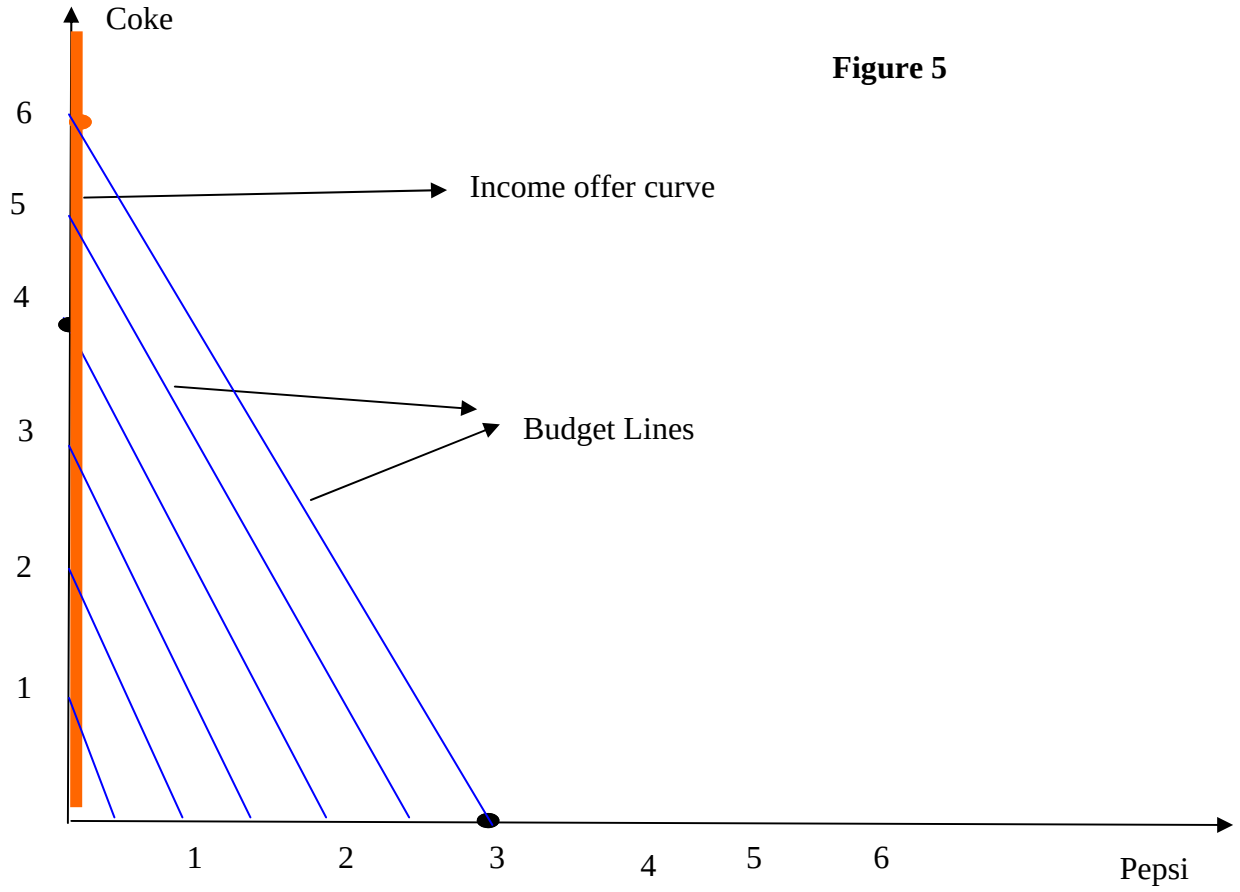
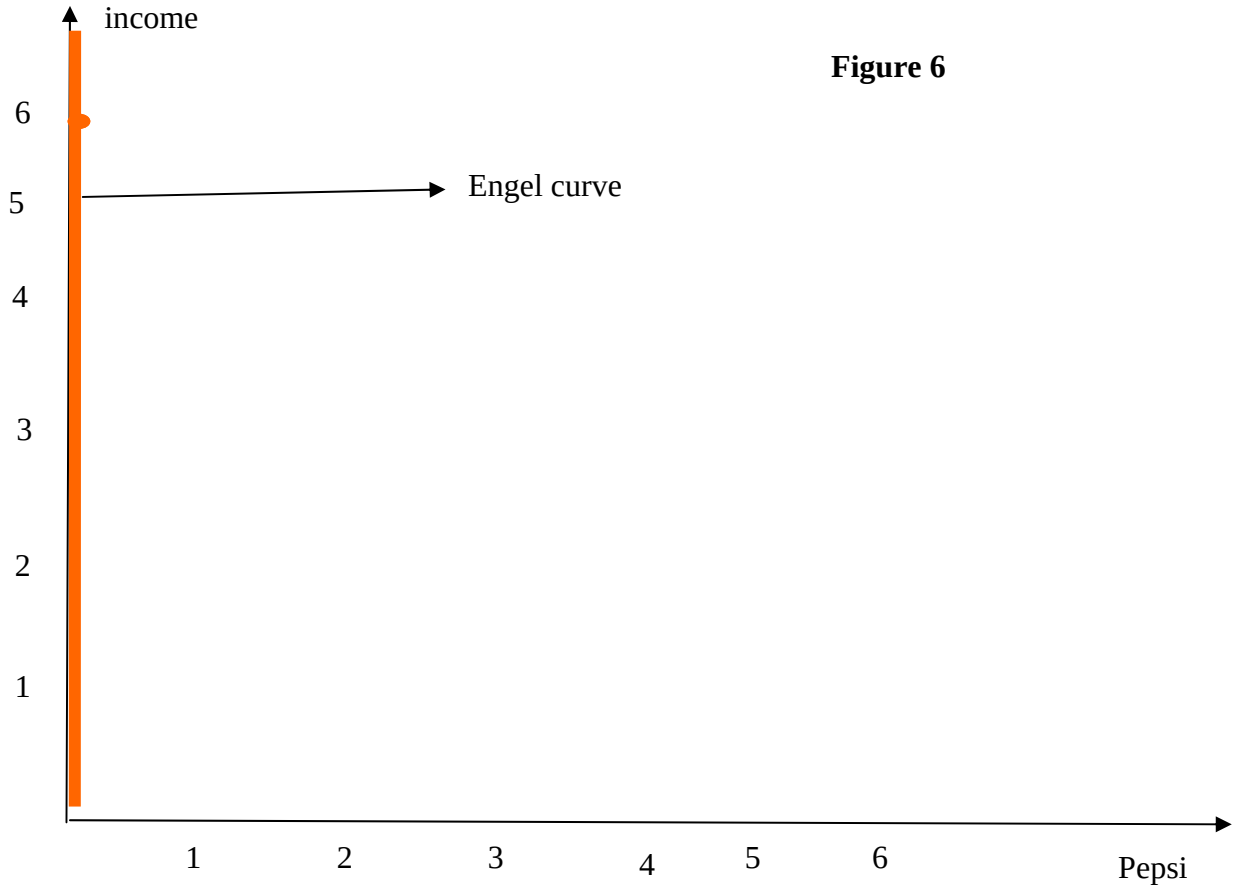



**Figure 5**

**Figure 6**



**Midterm Exam 1 Solutions**
**(D) The Blue**

**Q1)** *(55 points)*

**a)** Check figure 1 for the budget set *(3 p)*
If there is 100% inflation, the prices double.
The new budget set is also shown on Figure 1. *(3 p)*

**b)** $U(X_1, X_2) = X_1^{1/3} X_2^{1/3}$

$MRS^{12} = ((MU_1)/(MU_2)) = -(\partial U/\partial X_1)/(\partial U/\partial X_2) = (-(\frac{1}{3}X_1^{-1/3}X_2^{1/3})/(\frac{1}{3}X_1^{1/3}X_2^{-1/3})) = -X_2/X_1 = MRS^{12}$
*(4 p)*
at the point $x_1 = 3$ & $x_2 = 6$: $MRS^{12} = (-6/3) = -2$ *(2 p)*

which means:
$MU_1 > MU_2$ So 1st good, ribeye, is more valuable than top sirleon to Ava at the consumption level of
$x_1 = 3$ & $x_2 = 6$ *(4 p)*

**d)**

secret 1 : $p_1x_1 + p_2x_2 = m$ *(3p)*
so $4x_1 + 2x_2 = 80$
This means "spend all of your money". If the consumer does not spend all of his/her money s/he is wasting his/her opportunity to increase his/her utility since money does not have an effect on utility itself. *(2p)*

secret 2 : $MRS^{12} = (MU_1)/(MU_2) = P_1/P_2$ *(3p)*
$\rightarrow \quad X_2/X_1 = P_1/P_2$

"the last spent on each good should give the same utility" OR "marginal utility of a \$ spent on each good should be equal " . If this condition does not hold, lets say last \$ spent on good 1 brings more utility than the last \$ spent on good 2 , then the consumer should buy less of the second good and buy more of the first good to increase his/her utility. *(2p)*

start with secret 2 : $X_2/X_1 = P_1/P_2 \rightarrow X_2 = ((X_1P_1)/(P_2))$ *(1p)*
now plug this in secret 1 :
$p_1x_1 + p_2x_2 = m = P_1x_1 + P_2(X_1P_1)/(P_2) = m = 2P_1x_1$ *(2p)*
$\rightarrow$ Then $X_1 = (m/(2p_1)) = (1/2)(m/p_1)$
Since $X_2 = ((X_1P_1)/(P_2)) = ((mP_1)/(2P_1P_2)) = (m/(2p_2))$
$= (1/2)(m/(p_2)) = X_2$

$X_1 = (m/(2p_1))$

$X_2 = (m/(2p_2))$ *(2p)*

The solution is interior since none of good's optimal consumption level is zero. Furthermore in a Cobb-Douglas case the solution has to be interior as long as income level (m) is larger than zero ! (we also assume positive prices) *(1p)*

Using the magic formula $X_2 = (m/(2p_2)) \rightarrow X_2 p_2 = m/2$
note that the left hand side is the total money spent on good 2 (sirleon). So half of the income is spent on sirleon. *(2p)*

**e)** $X_1 = (m/(2p_1)) = ((80)/(2*4)) = 10$ *(1p)*

$X_2 = (m/(2p_2)) = ((80)/(2*2)) = 20$ *(1p)*

$X_1^{'} = (m/(2p_1^{'})) = ((80)/(2*2)) = 20$ *(1p)*

$X_2^{'} = (m/(2p_2)) = ((80)/(2*2)) = 20$ *(1p)*

Total change in consumption of ribeye is $X_1^{'} - X_1 = 20 - 10 = 10$ *(1p)*

Check figure 2 for the illustration of the change. *(3p)*

Ribeye is an ordinary good since it's consumption increases as it's price decrease. We can also see this fact from the magic formula . Since the price of ribeye is in the denominator in optimal consumption formula of ribeye, as the price goes down the consumption of it will increase ! *(2p)*

**f)** How much money does Ava need to consume the original bundle with the new prices ?

$p_1^{'} x_1 + p_2 x_2 = m^{'} = (\$2 * 10) + (\$2 * 20) = \$60$ is enough *(3p)*

Now calculating the optimal bundle with this imaginary income :
$X_1^s = (m^{'}/(2p_1^{'})) = ((60)/(2*2)) = 15$ *(2p)*

$X_2^s = (m^{'}/(2p_2)) = ((60)/(2*2)) = 15$
So Substitution Effect (S.E.) is : $15 - 10 = 5$ *(2p)*

$T.E = S.E. + I.E.$ then : $I.E. = 10 - 5 = 5$ *(1p)*

Check figure 2 for the illustration. *(3p)*

**Q 2)** *(15 points)*
**a)** the bundle : $(1x_1 + 2x_2)$
A suitable utility function would be : $min(2x_1, 1x_2)$ *(1p)*

**b)** Check figure 3 *(2p)*
**c)** Two secrets are : *(1.5p + 1.5p)*

$$\begin{aligned} 2x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= m \end{aligned}$$

hence

$$2x_1 + 2(2x_1) = 30$$

$$x_1^* = \frac{m}{p_1 + 2p_2} \text{ and } x_2^* = 2\frac{m}{p_1 + 2p_2}$$

$$\begin{aligned} x_1^* &= \frac{30}{2 + (2 * 2)} = 5 \\ \text{and } x_2^* &= 2\frac{30}{2 + (2 * 2)} = 10 \end{aligned}$$

(1.5p + 1.5p)
The solution is interior since none of the optimal consumption levels are zero. (1p)

**d)** Using the magic formulas :

$$\begin{aligned} x_1^* &= \frac{30}{1 + (2 * 2)} = 6 \\ \text{and } x_2^* &= 2\frac{30}{1 + (2 * 2)} = 12 \end{aligned}$$

*(1.5p + 1.5p)*
S.E. is zero. In a perfect complements case there is no substitution effect related to a price change. *(2p)*

**Q.3)** *(15 points)*
**a)** $U(X_1, X_2) = 2X_1 + 2X_2$ can be a utility function for this kind of preferences. *(2p)*
**b)** Check the figure 4 *(2p)*
**c)** $MRS^{12} = ((MU_1)/(MU_2)) = 1/1 = 1$ given the utility function. *(2p)*
comparing MRS to the price ratio : $1 > \frac{2}{6} = \frac{P_1}{P_2}$

So only good 1 is consumed : $X_1^* = m/P_1 = 12/2 = 6$ and $X_2^* = 0$ *(3p)*

The solution is a corner one. *(1p)*

**d)** check figure 5 & 6 for the income-offer curve and the engel curve. *(2p+2p)*

Pepsi is normal as the consumption of it increases as the income increases. *(1p)*

**Q.4)**

**a)** $\frac{10}{2} = 5$ is her real wage. *(1p)*

This number is her purchasing power in terms of consumption good. *(1p)*

**b)** check figure 7 *(3p)*

**c)**

secret 1: $w * R + p_c C = 24 * w$ *(2p)*

or $10R + 2C = 24 * 10 = 240$

secret 2 : $MRS^{RC} = ((MU_R)/(MU_C)) = w/P_c \rightarrow 1/R = w/P_c$ *(2p)*

$\rightarrow \quad R = \frac{P_c}{w}$ *(2p)*

**d)** If $w = \$1$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 10hr$ *(1p)*

If $w = \$2$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 5hr$ *(1p)*

The labor supply (24-R) is increasing in real wage ! *(2p)*

Bonus Question

$V(x_1, x_2) = f[U(x_1, x_2)]$

$MRS^U = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2)$

$MRS^V = ((MV_1)/(MV_2)) = (\partial V/\partial X_1)/(\partial V/\partial X_2)$

$= \frac{\partial f[U(x_1,x_2)]/\partial X_1}{\partial f[U(x_1,x_2)]/\partial X_2}$ by chain rule :

$= \frac{f^{'}(.)*\partial U(x_1,x_2)/\partial X_1}{f^{'}(.)*\partial U(x_1,x_2)/\partial X_2} = \frac{\partial U(x_1,x_2)/\partial X_1}{\partial U(x_1,x_2)/\partial X_2} = MRS^U$

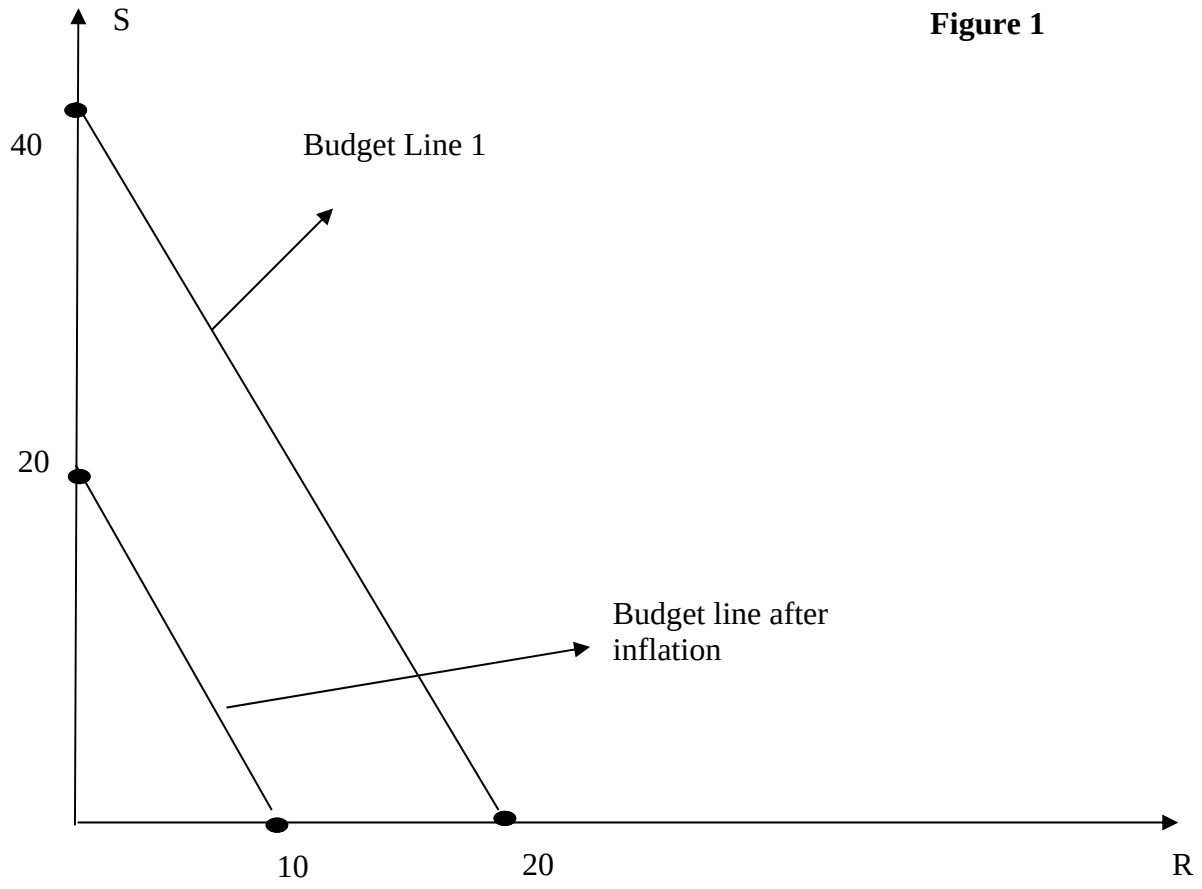



**Figure 1**

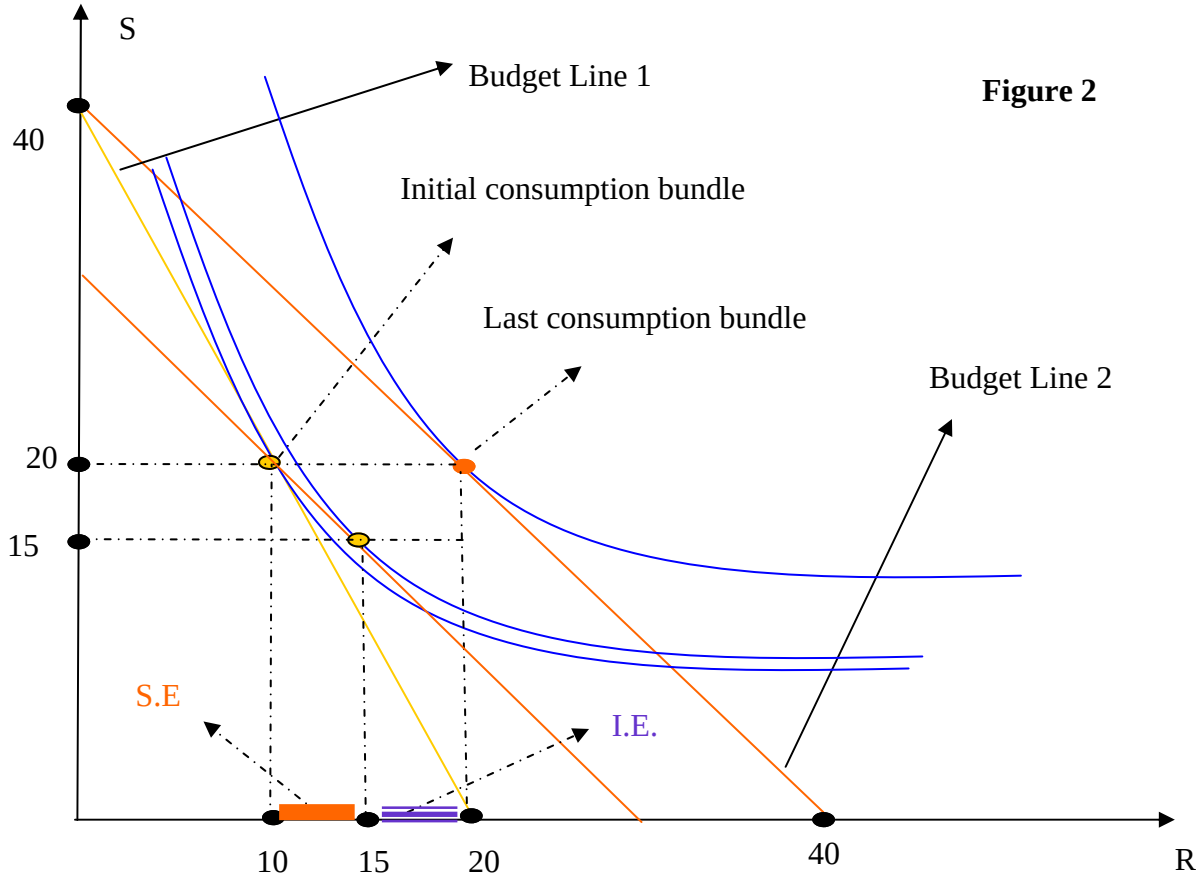



**Figure 2**

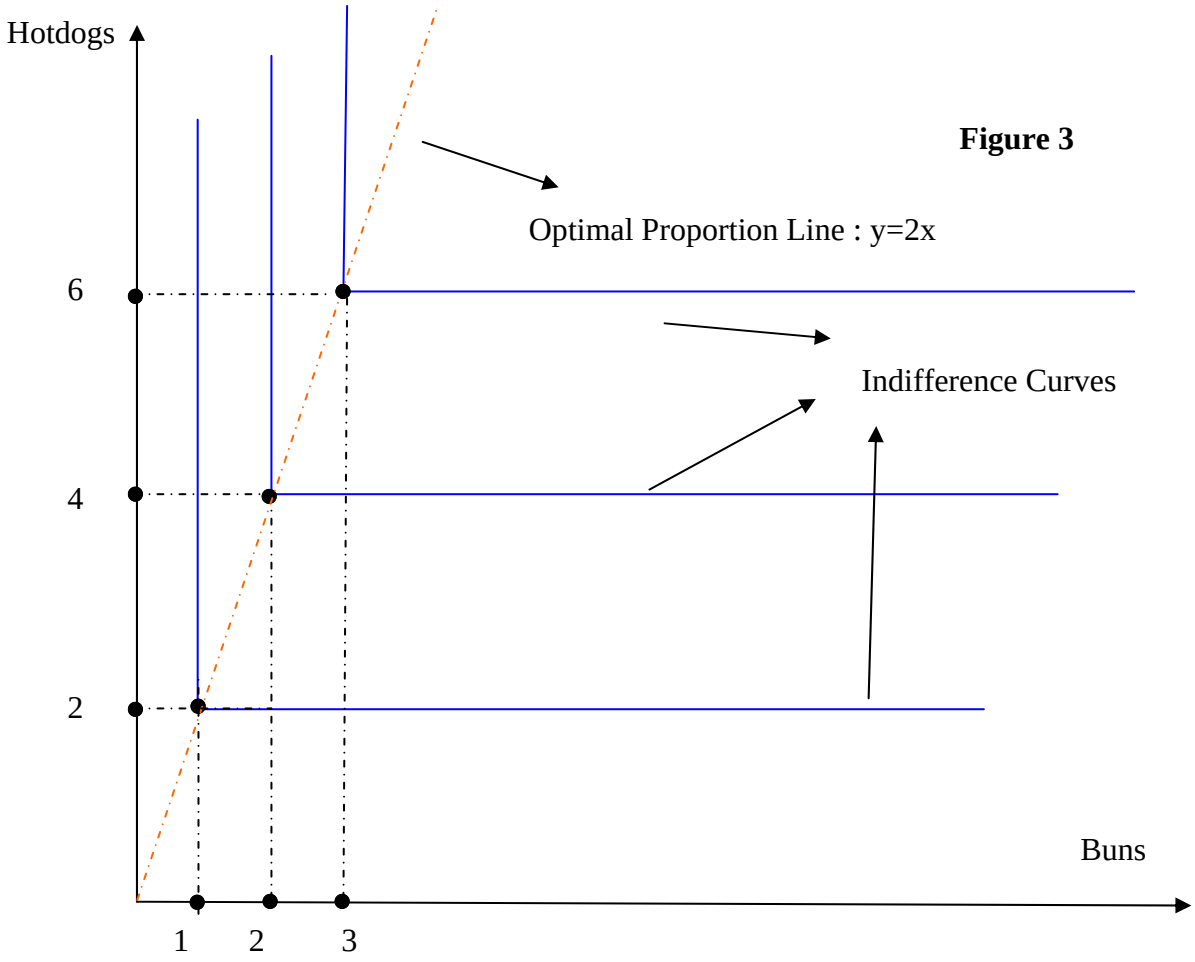



**Figure 3**

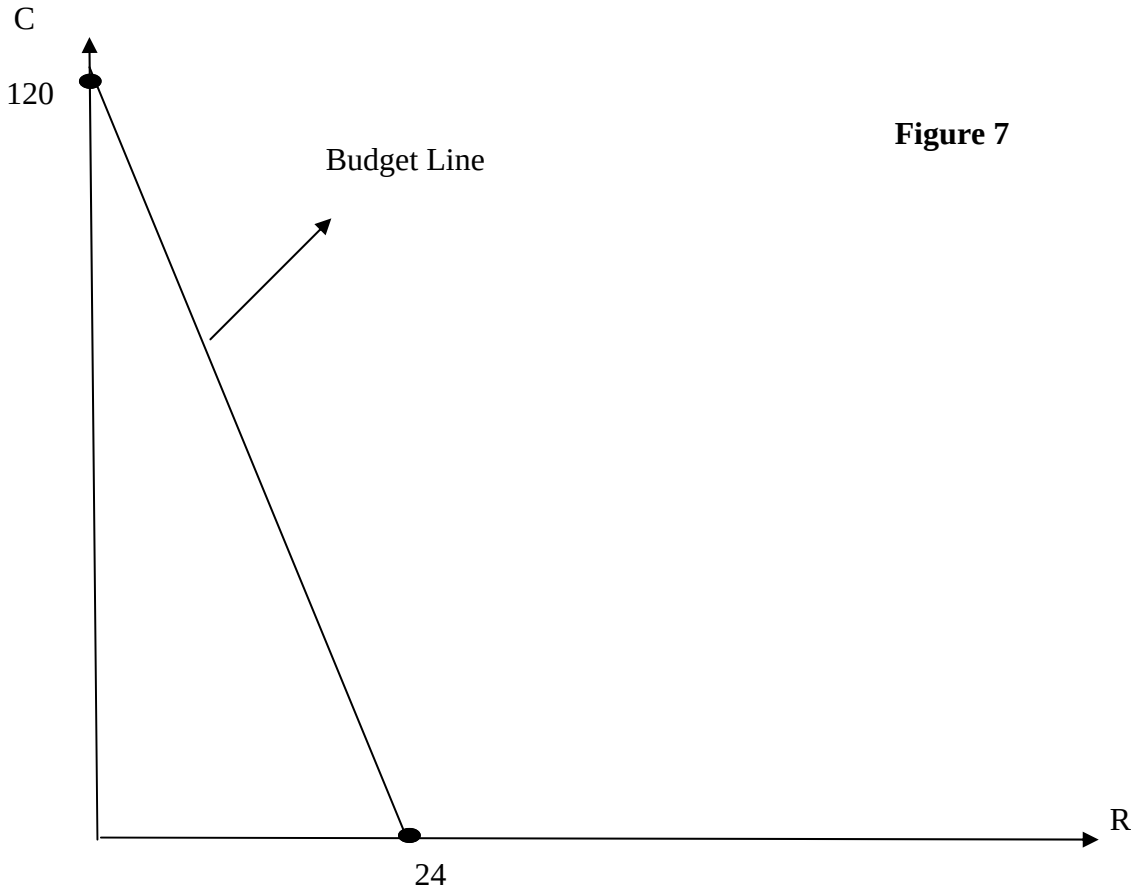



**Figure 7**

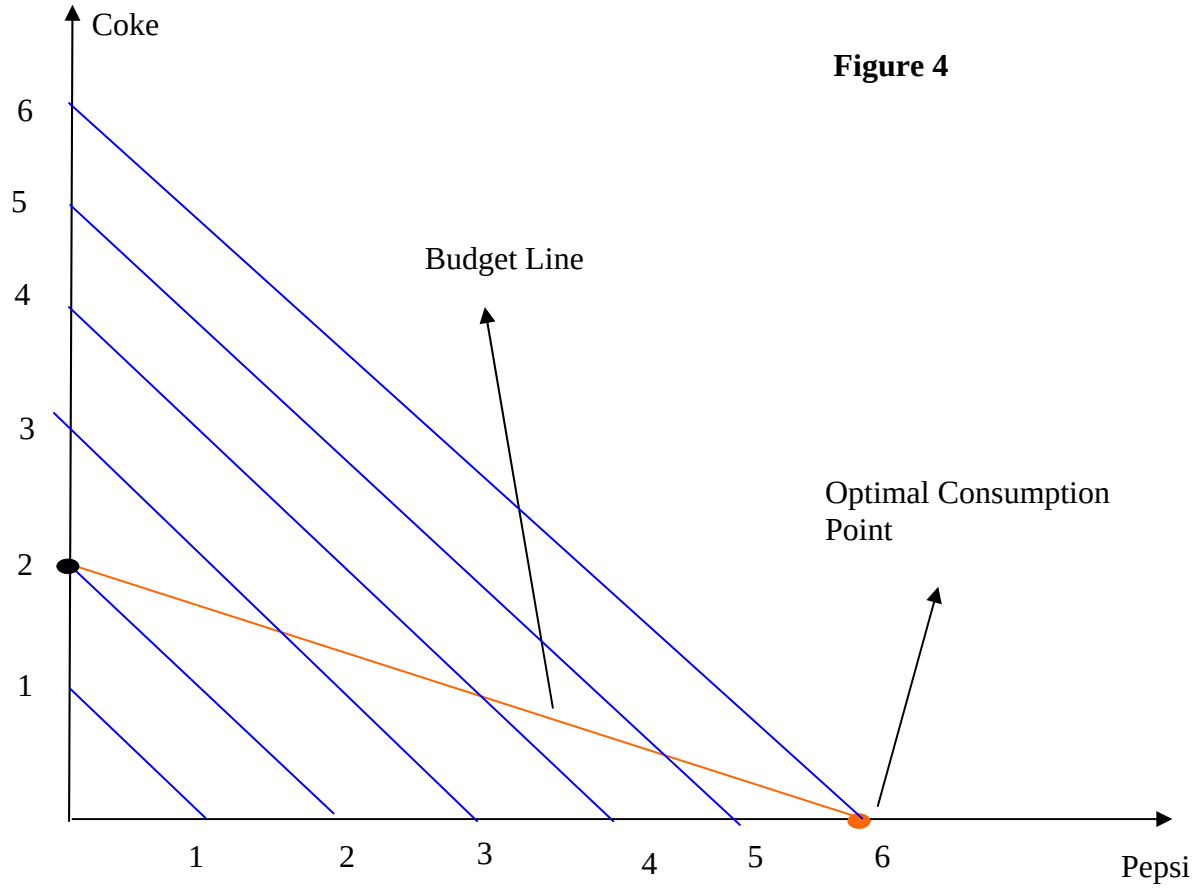



**Figure 4**

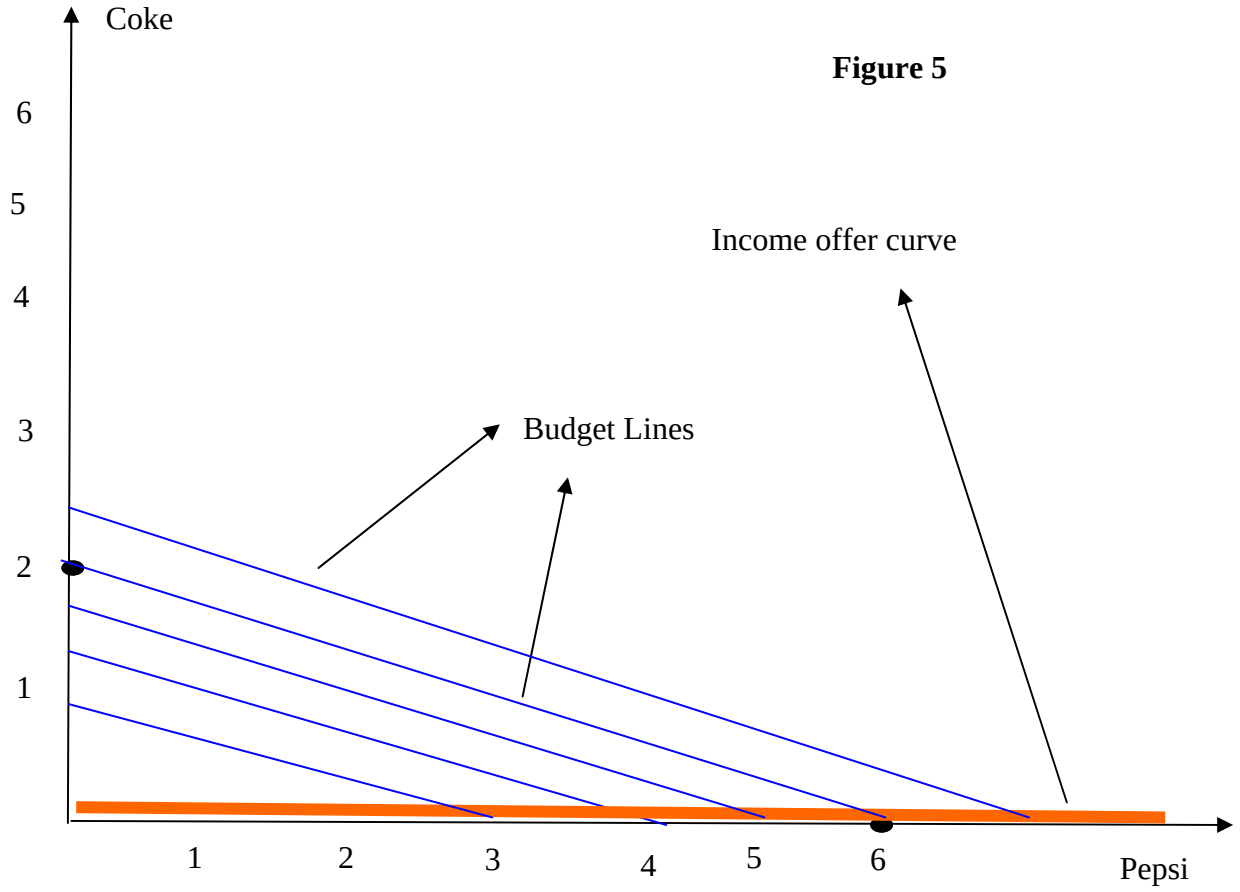



**Figure 5**

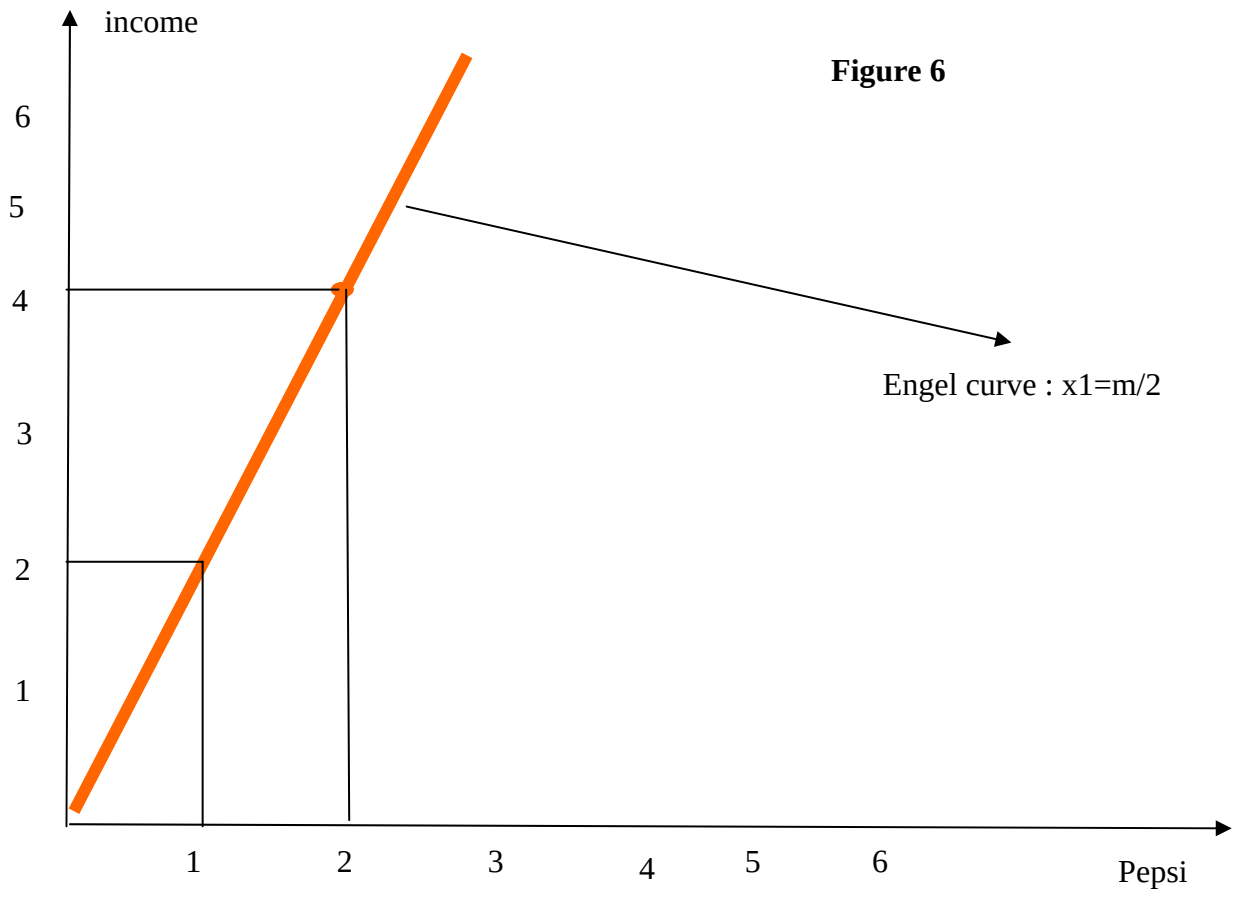



**Figure 6**