Econ 301

Intermediate Microeconomics

Prof. Marek Weretka

Midterm 2 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (20,25,25 and 30 points)

Problem 1 (20p) (Labor Supply)

Sheldon is a programmer at Microsoft, and his hourly wage rate is $w = \$10$. He has 24 hours a day but does not have any other form of "wealth". He consumes sandwiches everyday, at a price $p_c = \$5$.

a) Find Sheldon's real wage rate (number) and interpret its value economically.

b) Plot his budget set, marking the two extreme values of the budget line, its slope and his endowment.

c) Assume Sheldon's utility function is $U(C, R) = \min\{R, C\}$. Find the optimal choice of consumption of sandwiches C, relaxation time R and labor supply L. (three numbers)

d) (**Modest Difficult**) Depict Sheldon's labor supply curve (Assume $p_c = \$5$).

e) (**Modest Difficult**) Suppose Sheldon is also a stockholder of Microsoft, so he can earn an extra \$20 everyday. Find his new endowment and derive the budget constraint in this case. Depict his new budget line, marking all the kink point(s) if any exists.

Problem 2 (25p) (Intertemporal choice)

Serena is a pet sitter since she graduated from high school. Her salary is \$5000 (Period 1). However, she decides to switch to another job in Melemele city next month since she enjoys the beauty of this city. Her new salary will be \$2000 (Period 2). She needs to plan her consumption during these two periods, suppose her utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$, her discount rate is $\delta = 1$ and the interest rate is $r = 100\%$.

a) Suppose Alice has the same utility function but with a discount rate $\delta' = 2$. Is Alice more patient than Serena?

b) Depict Serena's intertermporal budget set, marking its slope and Serena's endowment. Find the Present and Future Value of Serena's income (two numbers) and show them in the graph.

c) Find optimal consumption in these two periods. Is Serena smoothing her consumption during these two period? Is she a borrower or a lender in period 1? What's the amount of borrowing/lending?

(Present Value)

For part d) and e), you don't need to find the value itself.

d) You own a house that pays you \$300 every month. Write down the equation that determines the present value of the rent of this house, assuming $r = 4\%$.

e) You receive a salary of \$20,000 per year when you are 21-60 years old and have constant consumption c every year when you are 21-80 years old. Write down the equation that determines your consumption level c, assuming $r = 5\%$.

Problem 3. (25p) (Uncertainty)

Jacob Bernoulli owns a company that faced a scandal recently and his wealth are all from this company. With 1/2 chance, the stock price of this company may drop drastically and Jacob's wealth would become \$4 (in million); With 1/2 chance, the stock price is unaffected and his wealth would be \$36 (in million). Suppose his Bernoulli utility function is $u(c) = 2\sqrt{c}$.

a) Is Jacob risk-averse, risk-loving or risk-neutral?

b) Jessica would like to buy Jacob's company by paying him \$9 (in million). Will Jacob accept this offer?

c) Write down Jacob's Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_F, C_{NF})$ and compute its MRS (give two formulas).

d) Suppose that Jacob can insure his wealth by paying premium $\gamma = \frac{1}{2}$. Find Jacob's budget equation and the slope of his budget line.

e) Write down two secrets of happiness, find optimal level of wealth (C_F, C_{NF}) and optimal insurance level x. (three numbers) Does Jacob fully insure (yes-no answer)?

Problem 4. (30p) (Edgeworth box and equilibrium)

Consider a Cobb Douglass economy with two consumers, Greene and Samuelson, who have identical utility function $U^i(x_1, x_2) = 3\ln x_1 + \ln x_2$. The individual endowments of Greene and Samuelson are $\omega^G = (8, 4)$ and $\omega^S = (4, 8)$.

a) Find the total resources. Plot an Edgeworth box and mark the initial endowment point.

b) Calculate Greene and Samuelson's MRS at their endowment points. Is the endowment point Pareto Efficient?

c) Find the competitive equilibrium (six numbers). Argue that this equilibrium is Pareto Efficient.

d) Derive analytically the contract curve. Use your formula to argue that the contract curve is represented by a diagonal of an Edgeworth box.

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Midterm 2 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (20,25,25 and 30 points)

Problem 1 (20p) (Labor Supply)

Sheldon is a programmer at Microsoft, and his hourly wage rate is $w = \$2$. He has 24 hours a day but does not have any other form of "wealth". He consumes sandwiches everyday, at a price $p_c = \$1$.

a) Find Sheldon's real wage rate (number) and interpret its value economically.

b) Plot his budget set, marking the two extreme values of the budget line, its slope and his endowment.

c) Assume Sheldon's utility function is $U(C, R) = \min\{2R, C\}$. Find the optimal choice of consumption of sandwiches C, relaxation time R and labor supply L. (three numbers).

d) (**Modest Difficult**) Depict Sheldon's labor supply curve (Assume $p_c = \$1$).

e) (**Modest Difficult**) Suppose Sheldon is also a stockholder of Microsoft, so he can earn an extra \$4 everyday. Find his new endowment and derive the budget constraint in this case. Depict his new budget line, marking all the kink point(s) if any exists.

Problem 2 (25p) (Intertemporal choice)

Serena is a pet sitter since she graduated from high school. Her salary is \$3000 (Period 1). However, she decides to switch to another job in Melemele city next month since she enjoys the beauty of this city. Her new salary will be \$6000 (Period 2). She needs to plan her consumption during these two periods, suppose her utility function is $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\delta} \ln C_2$, her discount rate is $\delta = 1$ and the interest rate is $r = 100\%$.

a) Suppose Alice has the same utility function but with a discount rate $\delta' = 0.5$. Is Alice more patient than Serena?

b) Depict Serena's intertermporal budget set, marking its slope and Serena's endowment. Find the Present and Future Value of Serena's income (two numbers) and show them in the graph.

c) Find optimal consumption in these two periods. Is Serena smoothing her consumption during these two period? Is she a borrower or a lender in period 1? What's the amount of borrowing/lending?

(Present Value)

For part d) and e), you don't need to find the value itself.

d) You own a house that pays you \$400 every month. Write down the equation that determines the present value of the rent of this house, assuming $r = 5\%$.

e) You receive a salary of \$10,000 per year when you are 21-70 years old and have constant consumption c every year when you are 21-90 years old. Write down the equation that determines your consumption level c, assuming $r = 5\%$.

Problem 3. (25p) (Uncertainty)

Jacob Bernoulli owns a company that faced a scandal recently and his wealth are all from this company. With 1/2 chance, the stock price of this company may drop drastically and Jacob's wealth would become \$1 (in million); With 1/2 chance, the stock price is unaffected and his wealth would be \$25 (in million). Suppose his Bernoulli utility function is $u(c) = 2\sqrt{c}$.

a) Is Jacob risk-averse, risk-loving or risk-neutral?

b) Jessica would like to buy Jacob's company by paying him \$16 (in million). Will Jacob accept this offer?

c) Write down Jacob's Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_F, C_{NF})$ and compute its MRS (give two formulas).

d) Suppose that Jacob can insure his wealth by paying premium $\gamma = \frac{1}{2}$. Find Jacob's budget equation and the slope of his budget line.

e) Write down two secrets of happiness, find optimal level of wealth (C_F, C_{NF}) and optimal insurance level x. (three numbers) Does Jacob fully insure (yes-no answer)?

Problem 4. (30p) (Edgeworth box and equilibrium)

Consider a Cobb Douglass economy with two consumers, Greene and Samuelson, who have identical utility function $U^i(x_1, x_2) = 3\ln x_1 + \ln x_2$. The individual endowments of Greene and Samuelson are $\omega^G = (4, 8)$ and $\omega^S = (8, 4)$.

a) Find the total resources. Plot an Edgeworth box and mark the initial endowment point.

b) Calculate Greene and Samuelson's MRS at their endowment points. Is the endowment point Pareto Efficient?

c) Find the competitive equilibrium (six numbers). Argue that this equilibrium is Pareto Efficient.

d) Derive analytically the contract curve. Use your formula to argue that the contract curve is represented by a diagonal of an Edgeworth box.

Solutions for ECON 301 Midterm 2

GameFreak

March 11, 2017

Solution for Midterm 2(A)

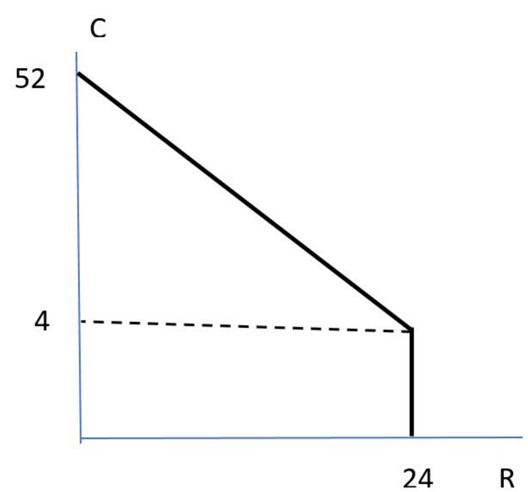
1. (a) Sheldon's real wage rate is $w/p_c = 2$. It means that Sheldon's wage for each hour worths 2 sandwiches.

   (b) Sheldon's budget constraint is $p_c C = w(24 - R)$, that is, $10R + 5C = 240$. Your can find that the intercept on C-axis is $240/5 = 48$ and intercept on R-axis is 24. The endowment point is $(24, 0)$ and slope is $-10/5 = -2$ on R-C plane.

   (c) Sheldon's utility function is perfect complement, so the optimal solution should satisfies $R = C$. Plug it to the budget equation, you can solve $R = C = 16$. Thus, $L = 24 - R = 8$.

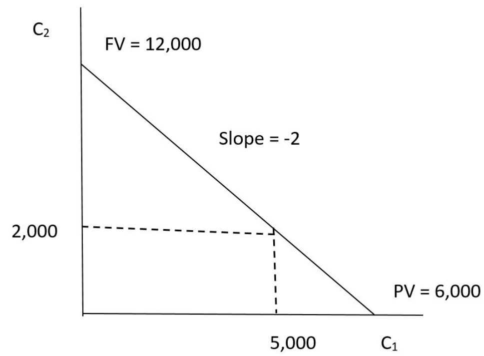
   (d) Since $5C = w(24 - R)$ and $R = C$, you can get $R = \frac{24w}{w+5}$. Therefore, $L = 24 - R = \frac{120}{w+5}$. The labor supply curve is a downward sloping curve with intercept 24 on L-axis (A graph on w-L plane).

   (e) The budget equation is determined by $p_c C = w(24 - R) + T$. Here, $T = 20$ is the stock dividend. Then we get that $10R + 5C = 260$. Since leisure cannot be larger than 24 hours, there will be a kink point. The graph is below.



2. (a) Higher δ means less patient. so the answer is NO.

   (b) Serena's budget equation can be written as $(1+r)C_1 + C_2 = (1+r)w_1 + w_2$, that is, $2C_1 + C_2 = 12,000$. [Here, we use FV form, it is also correct to write it in PV form.] The intercept on C_1-axis is $6,000$ and intercept on C_2-axis is $12,000$. Therefore, $PV_w = 6,000$ and $FV_w = 12,000$. The graph is below.



   (c) Note that the utility function is Cobb-Douglass type with $a = 1$ and $b = 1/2$. Using the magic formula, we have

$$C_1 = \frac{1}{1+0.5}\frac{12,000}{2} = 4,000$$
$$C_2 = 12,000 - 2C_1 = 4,000$$

   Since $C_1 = C_2$, Serena smooths her consumption. Her saving is $5,000 - 4,000 = 1,000 > 0$. Therefore, Serena is a lender and the amount of saving is \$1,000.

   (d) The present value of the rent is $PV_0 = \frac{300}{1.04} + \frac{300}{1.04^2} + \frac{300}{1.04^3} + \cdots = \frac{300}{0.04} = 75,000$.

   (e) The present value of salary is

$$PV_s = \frac{salary}{r}\left[1 - \frac{1}{(1+r)^{40}}\right]$$

   The present value of consumption is

$$PV_c = \frac{c}{r}\left[1 - \frac{1}{(1+r)^{60}}\right]$$

   By $PV_s = PV_c$, we get

$$c = salary\frac{1 - \frac{1}{(1+r)^{40}}}{1 - \frac{1}{(1+r)^{60}}} = 20,000 \times \frac{1 - 1.04^{-40}}{1 - 1.04^{-60}} \approx 17,498$$

3. (a) You can draw the graph of Bernoulli utility function, it is a concave function. Therefore, Jacob is risk-averse.

   (b) If Jacob reject this offer, he have to face the possible fluctuation of stock price and his expected utility is $1/2 \cdot u(4) + 1/2 \cdot u(36) = \sqrt{4} + \sqrt{36} = 8$. If Jacob accept this offer, his utility is $u(9) = 2\sqrt{9} = 6$. Therefore, Jacob will reject this offer.

(c) Jacob's expected utility function is

$$U(C_F, C_{NF}) = \frac{1}{2}u(C_f) + \frac{1}{2}u(C_{NF}) = \sqrt{C_F} + \sqrt{C_{NF}}.$$

Since $MU_F = \frac{1}{2\sqrt{C_F}}$ and $MU_{NF} = \frac{1}{2\sqrt{C_{NF}}}$, therefore,

$$MRS = -\frac{MU_F}{MU_{NF}} = -\frac{\sqrt{C_{NF}}}{\sqrt{C_F}}.$$

(d) The budget equation is

$$\frac{\gamma}{1-\gamma}C_F + C_{NF} = \frac{\gamma}{1-\gamma}w_F + w_{NF}$$

Since $\gamma = 1/2, w_F = 4$ and $w_{NF} = 36$, we get

$$C_F + C_{NF} = 40.$$

The slope of this budget is -1.

(e) The two secrets of happiness are

$$C_F + C_{NF} = 40 \tag{1}$$

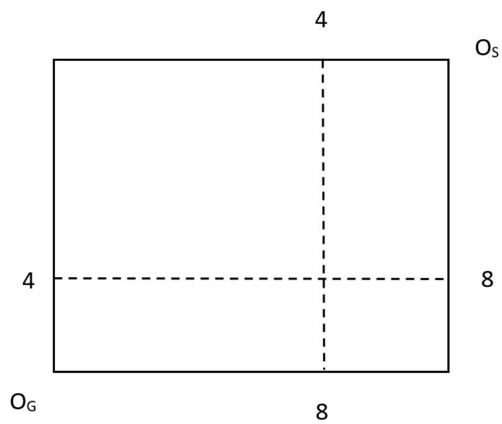
$$-\frac{\sqrt{C_{NF}}}{\sqrt{C_F}} = -1 \tag{2}$$

From (2), $C_{NF} = C_F$. Then, $C_F + C_F = 40$. We solves $C_F = C_{NF} = 20$. Therefore, Jacob fully insure his wealth. The insurance level x satisfies

$$C_{NF} = w_{NF} - \gamma x \Longrightarrow 20 = 36 - \frac{1}{2}x.$$

Therefore, $x = 32$.

4. (a) The total resources are $\omega = (12, 12)$. The Edgeworth box is below.



(b) Since $MU_1 = \frac{3}{x_1}$ and $MU_2 = \frac{1}{x_2}$, then we know

$$MRS = -\frac{MU_1}{MU_2} = -\frac{3x_2}{x_1}.$$

Therefore, Greene's MRS is $MRS^G = -3 \times 4/8 = -1.5$ and Samuelson's MRS is $MRS^S = -3 * 8/4 = -6$. Since $MRS^G \neq MRS^S$. Therefore, the endowment is not Pareto Efficient.

(c) Let $P_1/P_2 = p$. Then the two secrets of happiness for Greene are

$$px_1^G + x_2^G = 8p + 4 \tag{3}$$
$$-\frac{3x_2^G}{x_1^G} = -p \tag{4}$$

Then we solves that $x_1^G = \frac{6p+3}{p}$ and $x_2^G = 2p + 1$. Similarly, the two secret of happiness for Samuelson are

$$px_1^S + x_2^S = 4p + 8 \tag{5}$$
$$-\frac{3x_2^S}{x_1^S} = -p \tag{6}$$

Then we solves $x_1^S = \frac{6+3p}{p}$ and $x_2^S = 2 + p$. From $x_2^G + x_2^S = 12$, we solves $p = 3$. Therefore, we can get $x_1^G = x_2^G = 7$ and $x_1^S = x_2^S = 5$. Then this is the competitive equilibrium. By the secret of happiness for these two consumers, we have

$$MRS^G = -p = MRS^S$$

Therefore, this competitive equilibrium is Pareto efficient. [Alternative: You can also cite the first welfare theorem.]

(d) The contract curve are the allocations that satisfies $MRS^G = MRS^S$, that is,

$$-\frac{3x_2^G}{x_1^G} = -\frac{3x_2^S}{x_1^S} \implies \frac{x_2^G}{x_1^G} = \frac{x_2^S}{x_1^S} = \frac{12 - x_2^G}{12 - x_1^G}$$

Therefore, $x_1^G = x_2^G$. This is the diagonal of the Edgeworth box.

Solution for Midterm 2(B)

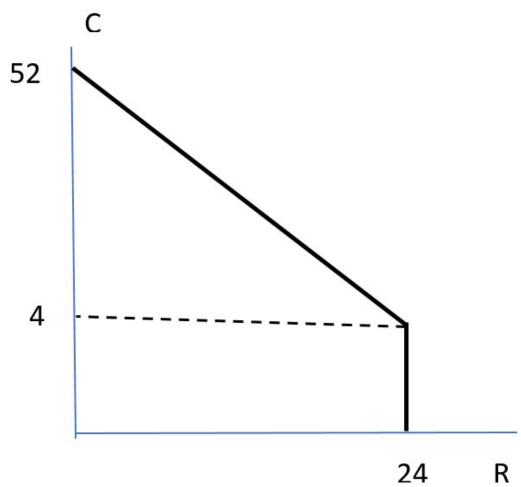
1. (a) Sheldon's real wage rate is $w/p_c = 2$. It means that Sheldon's wage for each hour worths 2 sandwiches.

   (b) Sheldon's budget constraint is $p_cC = w(24 - R)$, that is, $2R + C = 48$. Your can find that the intercept on C-axis is $240/5 = 48$ and intercept on R-axis is 24. The endowment point is $(24, 0)$ and slope is $-2/1 = -2$ on R-C plane.

   (c) Sheldon's utility function is perfect complement, so the optimal solution should satisfies $2R = C$. Plug it to the budget equation, you can solve $R = 12$ and $C = 24$. Thus, $L = 24 - R = 12$.

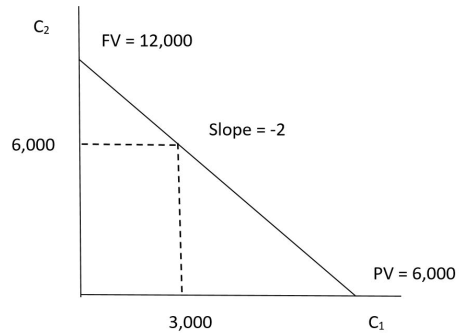
   (d) Since $C = w(24 - R)$ and $2R = C$, you can get $R = \frac{24w}{w+2}$. Therefore, $L = 24 - R = \frac{48}{w+2}$. The labor supply curve is a downward sloping curve with intercept 24 on L-axis (A graph on w-L plane).

   (e) The budget equation is determined by $p_cC = w(24 - R) + T$. Here, $T = 4$ is the stock dividend. Then we get that $2R + C = 52$. Since leisure cannot be larger than 24 hours, there will be a kink point. The graph is below.



2. (a) Higher δ means less patient. so the answer is YES.

   (b) Serena's budget equation can be written as $(1 + r)C_1 + C_2 = (1 + r)w_1 + w_2$, that is, $2C_1 + C_2 = 12,000$. [Here, we use FV form, it is also correct to write it in PV form.] The intercept on C_1-axis is $6,000$ and intercept on C_2-axis is $12,000$. Therefore, $PV_w = 6,000$ and $FV_w = 12,000$. The graph is below.



(c) Note that the utility function is Cobb-Douglass type with $a = 1$ and $b = 1/2$. Using the magic formula, we have

$$C_1 = \frac{1}{1+0.5}\frac{12,000}{2} = 4,000$$
$$C_2 = 12,000 - 2C_1 = 4,000$$

Since $C_1 = C_2$, Serena smooths her consumption. Her saving is $3,000 - 4,000 = -1,000 < 0$. Therefore, Serena is a borrower and the amount of borrowing is \$1,000.

(d) The present value of the rent is $PV_0 = \frac{400}{1.05} + \frac{400}{1.05^2} + \frac{400}{1.05^3} + \cdots = \frac{400}{0.05} = 80,000$.

(e) The present value of salary is

$$PV_s = \frac{salary}{r}\left[1 - \frac{1}{(1+r)^{50}}\right]$$

The present value of consumption is

$$PV_c = \frac{c}{r}\left[1 - \frac{1}{(1+r)^{70}}\right]$$

By $PV_s = PV_c$, we get

$$c = salary\frac{1 - \frac{1}{(1+r)^{50}}}{1 - \frac{1}{(1+r)^{70}}} = 10,000 \times \frac{1 - 1.04^{-50}}{1 - 1.04^{-70}} \approx 9,183$$

3. (a) You can draw the graph of Bernoulli utility function, it is a concave function. Therefore, Jacob is risk-averse.

(b) If Jacob reject this offer, he have to face the possible fluctuation of stock price and his expected utility is $1/2 \cdot u(1) + 1/2 \cdot u(25) = \sqrt{1} + \sqrt{25} = 6$. If Jacob accept this offer, his utility is $u(16) = 2\sqrt{16} = 8$. Therefore, Jacob will accept this offer.

(c) Jacob's expected utility function is

$$U(C_F, C_{NF}) = \frac{1}{2}u(C_f) + \frac{1}{2}u(C_{NF}) = \sqrt{C_F} + \sqrt{C_{NF}}.$$

Since $MU_F = \frac{1}{2\sqrt{C_F}}$ and $MU_{NF} = \frac{1}{2\sqrt{C_{NF}}}$, therefore,

$$MRS = -\frac{MU_F}{MU_{NF}} = -\frac{\sqrt{C_{NF}}}{\sqrt{C_F}}.$$

(d) The budget equation is

$$\frac{\gamma}{1-\gamma}C_F + C_{NF} = \frac{\gamma}{1-\gamma}w_F + w_{NF}$$

Since $\gamma = 1/2, w_F = 1$ and $w_{NF} = 25$, we get

$$C_F + C_{NF} = 26.$$

The slope of this budget is -1.

(e) The two secrets of happiness are

$$C_F + C_{NF} = 26 \tag{7}$$

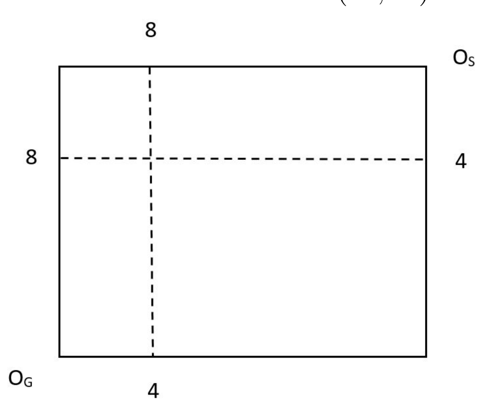
$$-\frac{\sqrt{C_{NF}}}{\sqrt{C_F}} = -1 \tag{8}$$

From (2), $C_{NF} = C_F$. Then, $C_F + C_F = 26$. We solves $C_F = C_{NF} = 13$. Therefore, Jacob fully insure his wealth. The insurance level x satisfies

$$C_{NF} = w_{NF} - \gamma x \Longrightarrow 13 = 25 - \frac{1}{2}x.$$

Therefore, $x = 24$.

4. (a) The total resources are $\omega = (12, 12)$. The Edgeworth box is below.



(b) Since $MU_1 = \frac{3}{x_1}$ and $MU_2 = \frac{1}{x_2}$, then we know

$$MRS = -\frac{MU_1}{MU_2} = -\frac{3x_2}{x_1}.$$

Therefore, Samuelson's MRS is $MRS^S = -3 \times 4/8 = -1.5$ and Greene's MRS is $MRS^G = -3 * 8/4 = -6$. Since $MRS^G \neq MRS^S$. Therefore, the endowment is not Pareto Efficient.

(c) Let $P_1/P_2 = p$. Then the two secrets of happiness for Greene are

$$px_1^G + x_2^G = 8 + 4p \tag{9}$$

$$-\frac{3x_2^G}{x_1^G} = -p \tag{10}$$

Then we solves that $x_1^G = \frac{6+3p}{p}$ and $x_2^G = 2 + p$. Similarly, the two secret of happiness for Samuelson are

$$px_1^S + x_2^S = 4 + 8p \tag{11}$$

$$-\frac{3x_2^S}{x_1^S} = -p \tag{12}$$

Then we solves $x_1^S = \frac{6p+3}{p}$ and $x_2^S = 2p+1$. From $x_2^G + x_2^S = 12$, we solves $p = 3$. Therefore, we can get $x_1^G = x_2^G = 5$ and $x_1^S = x_2^S = 7$. Then this is the competitive equilibrium. By the secret of happiness for these two consumers, we have

$$MRS^G = -p = MRS^S$$

Therefore, this competitive equilibrium is Pareto efficient. [Alternative: You can also cite the first welfare theorem.]

(d) The contract curve are the allocations that satisfies $MRS^G = MRS^S$, that is,

$$-\frac{3x_2^G}{x_1^G} = -\frac{3x_2^S}{x_1^S} \Longrightarrow \frac{x_2^G}{x_1^G} = \frac{x_2^S}{x_1^S} = \frac{12 - x_2^G}{12 - x_1^G}$$

Therefore, $x_1^G = x_2^G$. This is the diagonal of the Edgeworth box.