Problem 1 (50p) (Well-behaved preferences)

Jackson Balasco spends all his income on two types of commodities: cars \( x_1 \) and clothing, \( x_2 \). His utility function is

\[
U(x_1, x_2) = x_1^a x_2^b
\]

Suppose the prices are \( p_1 = 10 \) and \( p_2 = 2 \) and income is \( m = 60 \).

a) Show geometrically Jackson’s budget set. Mark all bundles that cost precisely $60. Find relative price of a car in terms of clothing (number). Give economic interpretation of the relative price (one sentence).

b) Jackson receives 10 garments (items of clothing) as a christmas gift. Plot the new budget constraint that takes into account the gift.

In the following points ignore the gift from point b).

c) Find Marginal Rate of Substitution (MRS) as a function of \( x_1, x_2 \) and parameters \( a, b \) (formula). For values \( a = 2, b = 2 \) find the value of MRS at the consumption bundle (3, 1) (number). What is the geometric interpretation of MRS? (sentence, depict in the graph) Which of the two goods is more precious to Jackson at this bundle? (one sentence)

d) Write down two secrets of happiness that determine optimal choice as a function of parameters \( a, b, p_1, p_2 \) and \( m \). Explain economic intuition behind the two conditions (two sentences for each condition). Solve for optimal choice \( x_1 \) and \( x_2 \) as the functions of five parameters (show each step of the derivation).

e) Assume again \( a = 2, b = 2 \) and \( p_1 = 10, p_2 = 2 \). Find geometrically and determine analytically income offer curve and Engel curve (two functions). Is \( x_1 \) a normal or inferior good? (choose one). Why? (one sentence)

f) Propose some other utility function that represents the same preferences as \( U(x_1, x_2) \) (formula).

Problem 2 (20p) (Perfect complements)

A motorbike is essentially a combination of one engine \( x_1 \) and two wheels, \( x_2 \), and thus these two commodities are always consumed in constant proportion 1:2. Antonio Gottardi is a motorbike enthusiast.

a) Propose a utility function that represents Antonio’s preferences over the bundles of engines and wheels (formula). Depict Antonio’s indifference curve map in a commodity space.

b) Write down two secrets of happiness that give Antonio’s optimal choice. Solve for the optimal choice in terms of prices \( p_1, p_2 \) and income \( m \) (two formulas).

c) Using (magic) formulas derived in point b) find optimal choice for \( p_1 = 8, p_2 = 1 \) and \( m = 20 \). Suppose next that the price of an engine goes down to \( p_1 = 3 \). Find the total change in consumption of engines, \( x_1 \), resulting from the price drop (number). What part of this change can be attributed to the substitution and which to the income effect? (two numbers).

d) Using (magic) formulas from point b) determine whether the two commodities are gross substitutes or gross complements (choose one). Explain why. (sentence)
Problem 3 (20p) (Perfects substitutes)

Anya Keys is a flower connoisseur. Her favorite flowers are roses, $x_1$, and daffodils, $x_2$. Her utility function over the two flower types is given by

$$U(x_1, x_2) = 3x_1 + x_2$$

Anya is initially endowed with $(\omega_1, \omega_2) = (3, 7)$ of roses and daffodils, respectively. The prices of the two flower types are $p_1 = 1$ and $p_2 = 1$.

a) Plot Anya’s budget set in a commodity space. Mark the initial endowment point. On the budget line mark all the bundles, for which net demand for roses is positive while for daffodils negative.

b) Find Anya’s MRS (formula). Sketch Anya’s indifference curve map.

c) Find optimal bundle $x_1, x_2$ (two numbers). Is the optimal choice corner or interior? (choose one) What is her net demand for the two commodities? (two numbers)

Problem 4 (10p) (Quasilinear Preferences)

You are a governor of Wisconsin, planning state budget for 2017. Two major expenses include education $x_1$ (a subsidy to UW Madison) and parks $x_2$. Your utility function over the two commodities is given by

$$U(x_1, x_2) = x_1 + \ln x_2.$$ 

a) Find optimal choice given prices $p_1 = 10, p_2 = 2$ and income $m = 30$ (two numbers). Is the solution interior? Is marginal utility of a dollar equalized in optimum? (two numbers and yes-no answer)

b) Suppose the income goes down, so that $p_1 = 10$ and $p_2 = 2$ and $m = 5$. Find optimal choice (two numbers). Is the solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)
Problem 1 (50p) (Well-behaved preferences)

Jackson Balasco spends all his income on two types of commodities: cars $x_1$ and clothing, $x_2$. His utility function is

$$U(x_1, x_2) = x_1^a x_2^b$$

Suppose the prices are $p_1 = 5$ and $p_2 = 1$ and income is $m = 30$.

a) Show geometrically Jackson’s budget set. Mark all bundles that cost precisely $30. Find relative price of a car in terms of clothing (number). Give economic interpretation of the relative price (one sentence).

b) Jackson receives 10 garments (items of clothing) as a christmas gift. Plot the new budget constraint that takes into account the gift.

In the following points ignore the gift from point b).

c) Find Marginal Rate of Substitution (MRS) as a function of $x_1, x_2$ and parameters $a, b$ (formula). For values $a = 2, b = 2$ find the value of MRS at the consumption bundle $(3, 1)$ (number). What is the geometric interpretation of MRS? (sentence, depict in the graph) Which of the two goods is more precious to Jackson at this bundle? (one sentence)

d) Write down two secrets of happiness that determine optimal choice as a function of parameters $a, b, p_1, p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each condition). Solve for optimal choice $x_1$ and $x_2$ as the functions of five parameters (show each step of the derivation).

e) Assume again $a = 2, b = 2$ and $p_1 = 5, p_2 = 1$. Find geometrically and determine analytically income offer curve and Engel curve (two functions). Is $x_1$ a normal or inferior good? (choose one). Why? (one sentence)

f) Propose some other utility function that represents the same preferences as $U(x_1, x_2)$ (formula).

Problem 2 (20p) (Perfect complements)

A motorbike is essentially a combination of one engine $x_1$ and two wheels, $x_2$, and thus these two commodities are always consumed in constant proportion 1:2. Antonio Gottardi is a motorbike enthusiast.

a) Propose a utility function that represents Antonio’s preferences over the bundles of engines and wheels (formula). Depict Antonio’s indifference curve map in a commodity space.

b) Write down two secrets of happiness that give Antonio’s optimal choice. Solve for the optimal choice in terms of prices $p_1, p_2$ and income $m$ (two formulas).

c) Using (magic) formulas derived in point b) find optimal choice for $p_1 = 8, p_2 = 2$ and $m = 24$. Suppose next that the price of an engine goes down to $p_1 = 4$. Find the total change in consumption of engines, $x_1$, resulting from the price drop (number). What part of this change can be attributed to the substitution and which to the income effect? (two numbers).

d) Using (magic) formulas from point b) determine whether the two commodities are gross substitutes or gross complements (choose one). Explain why. (sentence)
**Problem 3 (20p) (Perfect substitutes)**

Anya Keys is a flower connoisseur. Her favorite flowers are roses, $x_1$, and daffodils, $x_2$. Her utility function over the two flower types is given by

$$U(x_1, x_2) = x_1 + 3x_2$$

Anya is initially endowed with $(\omega_1, \omega_2) = (3, 7)$ of roses and daffodils, respectively. The prices of the two flower types are $p_1 = 1$ and $p_2 = 1$.

a) Plot Anya’s budget set in a commodity space. Mark the initial endowment point. On the budget line mark all the bundles, for which net demand for roses is positive while for daffodils negative.

b) Find Anya’s MRS (formula). Sketch Anya’s indifference curve map.

c) Find optimal bundle $x_1, x_2$ (two numbers). Is the optimal choice corner or interior? (choose one) What is her net demand for the two commodities? (two numbers)

**Problem 4 (10p) (Quasilinear Preferences)**

You are a governor of Wisconsin, planning state budget for 2017. Two major expenses include education $x_1$ (a subsidy to UW Madison) and parks $x_2$. Your utility function over the two commodities is given by

$$U(x_1, x_2) = x_1 + \ln x_2.$$ 

a) Find optimal choice given prices $p_1 = 5, p_2 = 1$ and income $m = 30$ (two numbers). Is the solution interior? Is marginal utility of a dollar equalized in optimum? (two numbers and yes-no answer)

b) Suppose the income goes down, so that $p_1 = 10$ and $p_2 = 2$ and $m = 4$. Find optimal choice (two numbers). Is the solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)
Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+20+10=100 points)

Problem 1 (50p) (Well-behaved preferences)
Jackson Balasco spends all his income on two types of commodities: cars $x_1$ and clothing, $x_2$. His utility function is

$$U(x_1, x_2) = x_1^a x_2^b$$

Suppose the prices are $p_1 = 10$ and $p_2 = 2$ and income is $m = 60$.

a) Show geometrically Jackson’s budget set. Mark all bundles that cost precisely $60. Find relative price of a car in terms of clothing (number). Give economic interpretation of the relative price (one sentence).

b) Jackson receives 10 garments (items of clothing) as a christmas gift. Plot the new budget constraint that takes into account the gift.

In the following points ignore the gift from point b).

c) Find Marginal Rate of Substitution (MRS) as a function of $x_1, x_2$ and parameters $a, b$ (formula). For values $a = 3, b = 3$ find the value of MRS at the consumption bundle $(3, 1)$ (number). What is the geometric interpretation of MRS? (sentence, depict in the graph) Which of the two goods is more precious to Jackson at this bundle? (one sentence)

d) Write down two secrets of happiness that determine optimal choice as a function of parameters $a, b, p_1, p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each condition). Solve for optimal choice $x_1$ and $x_2$ as the functions of five parameters (show each step of the derivation).

e) Assume again $a = 3, b = 3$ and $p_1 = 10, p_2 = 2$. Find geometrically and determine analytically income offer curve and Engel curve (two functions). Is $x_1$ a normal or inferior good? (choose one). Why? (one sentence)

f) Propose some other utility function that represents the same preferences as $U(x_1, x_2)$ (formula).

Problem 2 (20p) (Perfect complements)
A motorbike is essentially a combination of one engine $x_1$ and two wheels, $x_2$, and thus these two commodities are always consumed in constant proportion 1:2. Antonio Gottardi is a motorbike enthusiast.

a) Propose a utility function that represents Antonio’s preferences over the bundles of engines and wheels (formula). Depict Antonio’s indifference curve map in a commodity space.

b) Write down two secrets of happiness that give Antonio’s optimal choice. Solve for the optimal choice in terms of prices $p_1, p_2$ and income $m$ (two formulas).

c) Using (magic) formulas derived in point b) find optimal choice for $p_1 = 8, p_2 = 1$ and $m = 40$. Suppose next that the price of an engine goes down to $p_1 = 3$. Find the total change in consumption of engines, $x_1$, resulting from the price drop (number). What part of this change can be attributed to the substitution and which to the income effect? (two numbers).

d) Using (magic) formulas from point b) determine whether the two commodities are gross substitutes or gross complements (choose one). Explain why. (sentence)
Problem 3 (20p) (Perfects substitutes)

Anya Keys is a flower connoisseur. Her favorite flowers are roses, \( x_1 \), and daffodils, \( x_2 \). Her utility function over the two flower types is given by

\[ U(x_1, x_2) = \frac{1}{3} x_1 + x_2 \]

Anya is initially endowed with \((\omega_1, \omega_2) = (3, 7)\) of roses and daffodils, respectively. The prices of the two flower types are \( p_1 = 1 \) and \( p_2 = 1 \).

a) Plot Anya’s budget set in a commodity space. Mark the initial endowment point. On the budget line mark all the bundles, for which net demand for roses is positive while for daffodils negative.

b) Find Anya’s MRS (formula). Sketch Anya’s indifference curve map.

c) Find optimal bundle \( x_1, x_2 \) (two numbers). Is the optimal choice corner or interior? (choose one) What is her net demand for the two commodities? (two numbers)

Problem 4 (10p) (Quasilinear Preferences)

You are a governor of Wisconsin, planning state budget for 2017. Two major expenses include education \( x_1 \) (a subsidy to UW Madison) and parks \( x_2 \). Your utility function over the two commodities is given by

\[ U(x_1, x_2) = 2x_1 + 2 \ln x_2. \]

a) Find optimal choice given prices \( p_1 = 10, p_2 = 2 \) and income \( m = 30 \) (two numbers). Is the solution interior? Is marginal utility of a dollar equalized in optimum? (two numbers and yes-no answer)

b) Suppose the income goes down, so that \( p_1 = 10 \) and \( p_2 = 2 \) and \( m = 5 \). Find optimal choice (two numbers). Is the solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)
You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+20+10=100 points)

Problem 1 (50p) (Well-behaved preferences)

Jackson Balasco spends all his income on two types of commodities: cars $x_1$ and clothing, $x_2$. His utility function is

$$U(x_1, x_2) = x_1^a x_2^b$$

Suppose the prices are $p_1 = 5$ and $p_2 = 1$ and income is $m = 30$.

a) Show geometrically Jackson’s budget set. Mark all bundles that cost precisely $30. Find relative price of a car in terms of clothing (number). Give economic interpretation of the relative price (one sentence).

b) Jackson receives 10 garments (items of clothing) as a christmas gift. Plot the new budget constraint that takes into account the gift.

In the following points ignore the gift from point b).

c) Find Marginal Rate of Substitution (MRS) as a function of $x_1, x_2$ and parameters $a, b$ (formula). For values $a = 3, b = 3$ find the value of MRS at the consumption bundle $(3, 1)$ (number). What is the geometric interpretation of MRS? (sentence, depict in the graph) Which of the two goods is more precious to Jackson at this bundle? (one sentence)

d) Write down two secrets of happiness that determine optimal choice as a function of parameters $a, b, p_1, p_2$ and $m$. Explain economic intuition behind the two conditions (two sentences for each condition). Solve for optimal choice $x_1$ and $x_2$ as the functions of five parameters (show each step of the derivation).

e) Assume again $a = 3, b = 3$ and $p_1 = 5, p_2 = 1$. Find geometrically and determine analytically income offer curve and Engel curve (two functions). Is $x_1$ a normal or inferior good? (choose one). Why? (one sentence)

f) Propose some other utility function that represents the same preferences as $U(x_1, x_2)$ (formula).

Problem 2 (20p) (Perfect complements)

A motorbike is essentially a combination of one engine $x_1$ and two wheels, $x_2$, and thus these two commodities are always consumed in constant proportion 1:2. Antonio Gottardi is a motorbike enthusiast.

a) Propose a utility function that represents Antonio’s preferences over the bundles of engines and wheels (formula). Depict Antonio’s indifference curve map in a commodity space.

b) Write down two secrets of happiness that give Antonio’s optimal choice. Solve for the optimal choice in terms of prices $p_1, p_2$ and income $m$ (two formulas).

c) Using (magic) formulas derived in point b) find optimal choice for $p_1 = 8, p_2 = 2$ and $m = 48$. Suppose next that the price of an engine goes down to $p_1 = 4$. Find the total change in consumption of engines, $x_1$, resulting from the price drop (number). What part of this change can be attributed to the substitution and which to the income effect? (two numbers).

d) Using (magic) formulas from point b) determine whether the two commodities are gross substitutes or gross complements (choose one). Explain why. (sentence)
Problem 3 (20p) (Perfects substitutes)

Anya Keys is a flower connoisseur. Her favorite flowers are roses, $x_1$, and daffodils, $x_2$. Her utility function over the two flower types is given by

$$U(x_1, x_2) = x_1 + \frac{1}{3}x_2$$

Anya is initially endowed with $(\omega_1, \omega_2) = (3, 7)$ of roses and daffodils, respectively. The prices of the two flower types are $p_1 = 1$ and $p_2 = 1$.

a) Plot Anya’s budget set in a commodity space. Mark the initial endowment point. On the budget line mark all the bundles, for which net demand for roses is positive while for daffodils negative.

b) Find Anya’s MRS (formula). Sketch Anya’s indifference curve map.

c) Find optimal bundle $x_1, x_2$ (two numbers). Is the optimal choice corner or interior? (choose one) What is her net demand for the two commodities? (two numbers)

Problem 4 (10p) (Quasilinear Preferences)

You are a governor of Wisconsin, planning state budget for 2017. Two major expenses include education $x_1$ (a subsidy to UW Madison) and parks $x_2$. Your utility function over the two commodities is given by

$$U(x_1, x_2) = 2x_1 + 2 \ln x_2.$$  

a) Find optimal choice given prices $p_1 = 5, p_2 = 1$ and income $m = 30$ (two numbers). Is the solution interior? Is marginal utility of a dollar equalized in optimum? (two numbers and yes-no answer)

b) Suppose the income goes down, so that $p_1 = 10$ and $p_2 = 2$ and $m = 4$. Find optimal choice (two numbers). Is the solution interior? (yes-no answer). Is marginal utility of a dollar equalized? (give two numbers and yes-no answer)
Problem 1

(a) [9]

The budget set is every points below the straight line. All bundles that cost exact 60 dollars are the points on that straight line.

The relative price of car in terms of clothing is \( \frac{p_1}{p_2} = 5 \). It means how many garments you can get in return for the trade of one car.

(b) [8]
(c) [10] (two secrets of happiness):

\[ MRS_{12}(x_1, x_2) = \frac{MU_1}{MU_2} = \frac{ax_2}{bx_1} \]

\[ MRS_{12}(3, 1) = \frac{1}{3}. \] It means the slope of indifference curve at (3, 1). It also means locally Jackson wish to trade away one unit of \( x_1 \) for \( \frac{1}{3} \) unit of \( x_2 \). Clearly \( x_2 \) (clothing) is more precious than \( x_1 \) (car).

(d) [10]

\[
\begin{cases}
\frac{ax_2}{bx_1} = \frac{p_1}{p_2} & \text{MRS} \\
p_1x_1 + p_2x_2 = m & \text{budget constrain}
\end{cases}
\]

\[
\begin{cases}
x_1 = \frac{am}{(a+b)p_1} \\
x_2 = \frac{bm}{(a+b)p_2}
\end{cases}
\]

(e) [10] Income offer curve: \( x_2 = \frac{km}{ap_2}x_1 \), \( x_2 = 5x_1 \) which is ray from the origin.
Engel curve: \( m = \frac{x_1 p_1 (a+b)}{a} = 20x_1 \). Notice Engel curve is a upward sloping curve, \( x_1 \) is a normal good.

(f) \([3]\) \( U(x_1, x_2) = a \log x_1 + b \log x_2 \).

**Problem 2**

(a) \([5]\) \( U(x_1, x_2) = \min\{2x_1, x_2\} \). The indifference curve can be shown as follows,
(b) [5]

\[
\begin{align*}
2x_1 &= x_2 \\
p_1x_1 + p_2x_2 &= m
\end{align*}
\]

\[x_1 = \frac{m}{p_1 + 2p_2}; x_2 = \frac{2m}{p_1 + 2p_2}\]  \hspace{1cm} (1)

(c) [7] Optimal choice: \( x_1 = 2; x_2 = 4 \).
Under \( p_1 = 4 \), the new optimal bundle is \( x'_1 = 4 \) \( x'_2 = 8 \). Because two goods are perfect complements, there is no substitute effect. Thus the income effect is \( x'_1 - x_1 = 2 \).

(d) [3] As we can see from (1) or the example above, a decrease in \( p_1 \) will lead to a increase in \( x_2 \). \( x_1 \) and \( x_2 \) are gross complements.

**Problem 3**

a) [10] The budget line will be,

\[p_1x_1 + p_2x_2 = \omega_1p_1 + \omega_2p_2\]

Notice \((\omega_1, \omega_2) = (3, 7)\), plug in and we get the budget line:

\[p_1x_1 + p_2x_2 = 3 \times 1 + 7 \times 1 = 10\]
For the bundles on the green line there’s positive net demand for roses, while for the bundles on the red line, there’s negative net demand for daffodils.

b) \([4] MRS_{12} = \frac{3}{1}\), while the indifference curves are as follows,

c) \([6]\) Let’s compare \(\frac{MU_1}{p_1} = 1\) and \(\frac{MU_2}{p_2} = 3\). This implies daffodils is relatively more cheap than roses, or it’s more efficient to consume daffodils. The optimal bundle is \(x_1 = 0, x_2 = 10\) and it’s a **corner solution**. Her **net demand** for rose will be \(x_1 - \omega_1 = -3\) and \(x_2 - \omega_2 = 3\).

**Problem 4**

[20]
a) [5] Two secrets of happiness:

\[
\begin{align*}
5 \times x_1 + 1 \times x_2 &= 30 \\
\frac{1}{x_2} &= 5 \\
\end{align*}
\]

The optimal choice is then \( x_1 = 5, \ x_2 = 5 \). Since each good is consumed with a positive amount, it is an interior solution. Also, the marginal utility of a dollar are equal for \( x_1 \) and \( x_2 \).

b) [5] Now the secrets of happiness are

\[
\begin{align*}
10 \times x_1 + 2 \times x_2 &= 4 \\
\frac{1}{x_2} &= 10 \\
\end{align*}
\]

Notice the last equation yields \( x_2 = 5 \), it is not achievable for \( m = 4 \) and \( p_2 = 2 \). Now let’s check marginal utility per dollar for each good.

\[
\frac{MU_1}{p_1} = \frac{1}{10}, \quad \frac{MU_2}{p_2} = \frac{1}{2x_2}
\]

the two terms equal to each other when \( x_2 = 5 \). Yet for \( x_2 < 5 \), \( \frac{MU_2}{p_2} > \frac{MU_1}{p_1} \), which means consuming \( x_2 \) is more efficient with a relative some income. Thus, you want to spend all your income purchasing \( x_2 \), the optimal choice is then \( x_1 = 0, \ x_2 = \frac{4}{2} = 2 \). It’s a corner solution, and the marginal utility of a dollar is not equalized.
Problem 1

(a) [9]

The budget set is every points below the straight line. All bundles that cost exact 60 dollars are the points on that straight line.
The relative price of car in terms of clothing is $\frac{p_1}{p_2} = 5$. It means how many garments you can get in return for the trade of one car.

(b) [8]
(c) [10] (two secrets of happiness):

\[ MRS_{12}(x_1, x_2) = \frac{MU_1}{MU_2} = \frac{ax_2}{bx_1} \]

\[ MRS_{12}(3, 1) = \frac{1}{3}. \] It means the slope of indifference curve at \((3, 1)\). It also means locally Jackson wish to trade away one unit of \(x_1\) for \(\frac{1}{3}\) unit of \(x_2\). Clearly \(x_2\) (clothing) is more precious than \(x_1\) (car).

(d) [10]

\[
\begin{align*}
\frac{ax_2}{bx_1} &= \frac{p_1}{p_2} & \text{MRS} \\
p_1 x_1 + p_2 x_2 &= m & \text{budget constrant}
\end{align*}
\]

\[
\begin{align*}
x_1 &= \frac{am}{(a + b)p_1} \\
x_2 &= \frac{bm}{(a + b)p_2}
\end{align*}
\]

(e) [10] Income offer curve: \(x_2 = \frac{bm}{ap_2} x_1, \ x_2 = 5x_1\) which is ray from the origin.
Engel curve: \( m = \frac{x_1 p_1 (a+b)}{a} = 10x_1 \). Notice Engel curve is a upward sloping curve, \( x_1 \) is a normal good.

(f) \[ U(x_1, x_2) = a \log x_1 + b \log x_2. \]

**Problem 2**

(a) \[ U(x_1, x_2) = \min\{2x_1, x_2\} \]. The indifference curve can be shown as follows,
(b) \[5\]

\[
\begin{align*}
2x_1 &= x_2 \\
p_1x_1 + p_2x_2 &= m \\
x_1 &= \frac{m}{p_1 + 2p_2}; x_2 = \frac{2m}{p_1 + 2p_2}
\end{align*}
\] (2)

(c) \[7\] Optimal choice: \(x_1 = 2; x_2 = 4\).
Under \(p_1 = 4\), the new optimal bundle is \(x'_1 = 3, x'_2 = 6\). Because two goods are perfect complements, there is no substitute effect. Thus the income effect is \(x'_1 - x_1 = 1\).

(d) \[3\] As we can see from (1) or the example above, a decrease in \(p_1\) will lead to an increase in \(x_2\). \(x_1\) and \(x_2\) are gross complements.

**Problem 3**

a) \[10\] The budget line will be,

\[
p_1x_1 + p_2x_2 = \omega_1p_1 + \omega_2p_2
\]

Notice \((\omega_1, \omega_2) = (3, 7)\), plug in and we get the budget line:

\[
p_1x_1 + p_2x_2 = 3 \times 1 + 7 \times 1 = 10
\]
For the bundles on the green line there’s positive net demand for roses, while for the bundles on the red line, there’s negative net demand for daffodils.

b) \[ MRS_{12} = \frac{1}{3}, \] while the indifference curves are as follows,

c) \[ \text{Let’s compare } \frac{MU_1}{p_1} = 1 \text{ and } \frac{MU_2}{p_2} = 3. \] This implies rose is relatively more cheap than daffodil, or it’s more efficient to consume rose. The optimal bundle is \( x_1 = 0, x_2 = 10 \) and it’s a corner solution. Her net demand for rose will be \( x_1 - \omega_1 = -7 \) and \( x_2 - \omega_2 = 7. \)

**Problem 4**

[20]
a) [5] Two secrets of happiness:

\[
\begin{align*}
5 \times x_1 + 1 \times x_2 &= 30 \\
\frac{1}{x_2} &= \frac{5}{1}
\end{align*}
\]

The optimal choice is then \(x_1 = 5, x_2 = 5\). Since each good is consumed with a positive amount, it is an interior solution. Also, the marginal utility of a dollar are equal for \(x_1\) and \(x_2\).

b) [5] Now the secrets of happiness are

\[
\begin{align*}
10 \times x_1 + 2 \times x_2 &= 4 \\
\frac{1}{x_2} &= \frac{10}{2}
\end{align*}
\]

Notice the last equation yields \(x_2 = 5\), it is not achievable for \(m = 4\) and \(p_2 = 2\). Now let’s check marginal utility per dollar for each good.

\[
\frac{MU_1}{p_1} = \frac{1}{10}, \quad \frac{MU_2}{p_2} = \frac{1}{2x_2}
\]

the two terms equal to each other when \(x_2 = 5\). Yet for \(x_2 < 5\), \(\frac{MU_2}{p_2} > \frac{MU_1}{p_1}\), which means consuming \(x_2\) is more efficient with a relative some income. Thus, you want to spend all your income purchasing \(x_2\), the optimal choice is then \(x_1 = 0, x_2 = \frac{4}{2} = 2\). It’s a corner solution, and the marginal utility of a dollar is not equalized.
The budget set is every points below the straight line. All bundles that cost exact 60 dollars are the points on that straight line.
The relative price of car in terms of clothing is $p_1/p_2 = 5$. It means how many garments you can get in return for the trade of one car.
(c) [10] (two secrets of happiness):

$$MRS_{12}(x_1, x_2) = \frac{MU_1}{MU_2} = \frac{ax_2}{bx_1}$$

$MRS_{12}(3, 1) = \frac{1}{3}$. It means the slope of indifference curve at $(3, 1)$. It also means locally Jackson wish to trade away one unit of $x_1$ for $\frac{1}{3}$ unit of $x_2$. Clearly $x_2$ (clothing) is more precious than $x_1$ (car).

(d) [10]

$$\begin{cases} \frac{ax_2}{bx_1} = \frac{p_1}{p_2} & \text{MRS} \\ p_1x_1 + p_2x_2 = m & \text{budget constrain} \end{cases}$$

$$\begin{cases} x_1 = \frac{am}{(a+b)p_1} \\ x_2 = \frac{bm}{(a+b)p_2} \end{cases}$$

(e) [10] Income offer curve: $x_2 = \frac{bx_1}{ap_2}$, $x_1 = 5x_2$ which is ray from the origin.
Engel curve: \( m = \frac{x_1 p_1 (a+b)}{a} = 20 x_1 \). Notice Engel curve is a upward sloping curve, \( x_1 \) is a normal good.

(f) \[ U(x_1, x_2) = \min\{2x_1, x_2\} \]

Problem 2

(a) \[ U(x_1, x_2) = \min\{2x_1, x_2\} \]. The indifference curve can be shown as follows,
(b) \[5\]

\[
\begin{aligned}
2x_1 &= x_2 \\
p_1x_1 + p_2x_2 &= m
\end{aligned}
\]

\[
x_1 = \frac{m}{p_1 + 2p_2}; x_2 = \frac{2m}{p_1 + 2p_2}
\]

(c) \[7\] Optimal choice: \(x_1 = 4; x_2 = 8\).
Under \(p_1 = 3\), the new optimal bundle is \(x'_1 = 8\) \(x'_2 = 16\). Because two goods are perfect complements, there is no substitute effect. Thus the income effect is \(x'_1 - x_1 = 4\).

(d) \[3\] As we can see from (1) or the example above, a decrease in \(p_1\) will lead to an increase in \(x_2\). \(x_1\) and \(x_2\) are gross complements.

**Problem 3**

a) \[10\] The budget line will be,

\[
p_1x_1 + p_2x_2 = \omega_1p_1 + \omega_2p_2
\]

Notice \((\omega_1, \omega_2) = (3, 7)\), plug in and we get the budget line:

\[
p_1x_1 + p_2x_2 = 3 \times 1 + 7 \times 1 = 10
\]
For the bundles on the green line there’s positive net demand for roses, while for the bundles on the red line, there’s negative net demand for daffodils.

b) \( MRS_{12} = \frac{1}{3} \), while the indifference curves are as follows,

c) \( \text{Let’s compare } \frac{MU_1}{p_1} = 1 \text{ and } \frac{MU_2}{p_2} = 3. \) This implies rose is relatively more cheap than daffodil, or it’s more efficient to consume rose. The optimal bundle is \( x_1 = 0, \ x_2 = 10 \) and it’s a corner solution. Her net demand for rose will be \( x_1 - \omega_1 = -7 \) and \( x_2 - \omega_2 = 7. \)

**Problem 4**
a) [5] Two secrets of happiness:

\[
\begin{align*}
10x_1 + 2x_2 &= 30 \\
\frac{1}{x_2} &= \frac{10}{2}
\end{align*}
\]

The optimal choice is then \(x_1 = 2, x_2 = 5\). Since each good is consumed with a positive amount, it is an interior solution. Also, the marginal utility of a dollar are equal for \(x_1\) and \(x_2\).

b) [5] Now the secrets of happiness are

\[
\begin{align*}
10x_1 + 2x_2 &= 4 \\
\frac{1}{x_2} &= \frac{10}{2}
\end{align*}
\]

Notice the last equation yields \(x_2 = 5\), it is not achievable for \(m = 4\) and \(p_2 = 2\). Now let’s check marginal utility per dollar for each good.

\[
\frac{MU_1}{p_1} = \frac{1}{10}, \quad \frac{MU_2}{p_2} = \frac{1}{2x_2}
\]

the two terms equal to each other when \(x_2 = 5\). Yet for \(x_2 < 5\), \(\frac{MU_2}{p_2} > \frac{MU_1}{p_1}\), which means consuming \(x_2\) is more efficient with a relative some income. Thus, you want to spend all your income purchasing \(x_2\), the optimal choice is then \(x_1 = 0, x_2 = \frac{4}{2} = 2\). It’s a corner solution, and the marginal utility of a dollar is not equalized.
Problem 1

(a) [9]

The budget set is every points below the straight line. All bundles that cost exact 60 dollars are the points on that straight line.

The relative price of car in terms of clothing is \( \frac{p_1}{p_2} = 5 \). It means how many garments you can get in return for the trade of one car.

(b) [8]
(c) [10] (two secrets of happiness):

\[ MRS_{12}(x_1, x_2) = \frac{MU_1}{MU_2} = \frac{ax_2}{bx_1} \]

\( MRS_{12}(3, 1) = \frac{1}{3} \). It means the slope of indifference curve at (3, 1). It also means locally Jackson wish to trade away one unit of \( x_1 \) for \( \frac{1}{3} \) unit of \( x_2 \). Clearly \( x_2 \) (clothing) is more precious than \( x_1 \) (car).

(d) [10]

\[
\begin{cases}
\frac{ax_2}{bx_1} = \frac{p_1}{p_2} & \text{MRS} \\
p_1x_1 + p_2x_2 = m & \text{budget constain} \\
\end{cases}
\]

\[
\begin{cases}
x_1 = \frac{am}{(a + b)p_1} \\
x_2 = \frac{bm}{(a + b)p_2} \\
\end{cases}
\]

(e) [10] Income offer curve: \( x_2 = \frac{km}{ap_2} x_1, \ x_2 = 5x_1 \) which is ray from the origin.
Engel curve: \( m = \frac{x_1 p_1 (a+b)}{a} = 10x_1 \). Notice Engel curve is an upward sloping curve, \( x_1 \) is a normal good.

Problem 2

(a) \( U(x_1, x_2) = \min\{2x_1, x_2\} \). The indifference curve can be shown as follows,
(b) \[5\]

\[
\begin{align*}
2x_1 &= x_2 \\
p_1 x_1 + p_2 x_2 &= m
\end{align*}
\]

\[
x_1 = \frac{m}{p_1 + 2p_2}; \quad x_2 = \frac{2m}{p_1 + 2p_2}
\]  

(4)

(c) \[7\] Optimal choice: \(x_1 = 4; x_2 = 8\).
Under \(p_1 = 4\), the new optimal bundle is \(x'_1 = 8 \quad x'_2 = 16\). Because two goods are perfect complements, there is no substitute effect. Thus the income effect is \(x'_1 - x_1 = 4\).

(d) \[3\] As we can see from (1) or the example above, a decrease in \(p_1\) will lead to an increase in \(x_2\). \(x_1\) and \(x_2\) are gross complements.

**Problem 3**

a) \[10\] The budget line will be,

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p_1 x_1 + p_2 x_2 = \omega_1 p_1 + \omega_2 p_2
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Notice \((\omega_1, \omega_2) = (3, 7)\), plug in and we get the budget line:

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For the bundles on the green line there’s positive net demand for roses, while for the bundles on the red line, there’s negative net demand for daffodils.

b) \[ MRS_{12} = \frac{3}{1} \], while the indifference curves are as follows,

c) \[ \frac{MU_1}{p_1} = 1 \text{ and } \frac{MU_2}{p_2} = 3 \]. This implies daffodils is relatively more cheap than roses, or it’s more efficient to consume daffodils. The optimal bundle is \( x_1 = 0, x_2 = 10 \) and it’s a \textbf{corner solution}. Her \textbf{net demand} for rose will be \( x_1 - \omega_1 = -3 \) and \( x_2 - \omega_2 = 3 \).

\textbf{Problem 4}

[20]
a) [5] Two secrets of happiness:

\[
\begin{aligned}
5 \times x_1 + 1 \times x_2 &= 30 \\
\frac{1}{x_2} &= 5 \\
\end{aligned}
\]

The optimal choice is then \(x_1 = 5, x_2 = 5\). Since each good is consumed with a positive amount, it is an interior solution. Also, the marginal utility of a dollar are equal for \(x_1\) and \(x_2\).

b) [5] Now the secrets of happiness are

\[
\begin{aligned}
10 \times x_1 + 2 \times x_2 &= 4 \\
\frac{1}{x_2} &= \frac{10}{2}
\end{aligned}
\]

Notice the last equation yields \(x_2 = 5\), it is not achievable for \(m = 4\) and \(p_2 = 2\). Now let’s check marginal utility per dollar for each good.

\[
\frac{MU_1}{p_1} = \frac{1}{10}, \quad \frac{MU_2}{p_2} = \frac{1}{2x_2}
\]

the two terms equal to each other when \(x_2 = 5\). Yet for \(x_2 < 5\), \(\frac{MU_2}{p_2} > \frac{MU_1}{p_1}\), which means consuming \(x_2\) is more efficient with a relative some income. Thus, you want to spend all your income purchasing \(x_2\), the optimal choice is then \(x_1 = 0, x_2 = \frac{4}{2} = 2\). It’s a corner solution, and the marginal utility of a dollar is not equalized.