Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (Group A)

You have 2h to complete the exam. The final consists of 6 questions (15+10+15+25+20+15=100).

Problem 1. (Choice with Cobb-Douglas preferences)

Sara spends her income on books x_1 and food x_2. The prices of the two commodities are $p_1 = p_2 = 5$ and her income is $m = 100$. Sara's utility function is given by

$$U(x_1, x_2) = (x_1)^3 (x_2)^3 .$$

a) Find analytically Sara's MRS as a function of (x_1, x_2) (give a function) and determine its value for consumption bundle $(x_1, x_2) = (6, 2)$. Provide economic and geometric interpretation of MRS at this bundle (one sentence + graph).

b) Give two secrets of happiness that determine Sara's optimal choice (two equation). Explain why violation of any of them implies that the bundle cannot be optimal (one sentence for each condition).

c) Find Sara's optimal choice (two numbers) and mark the optimal bundle in the commodity space.

d) Using magic formulas for Cobb-Douglas preferences argue that both commodities are ordinary commodities. (formulas and one sentence)

Problem 2. (Intertemporal choice with perfect substitutes)

Josh chooses a consumption plans for two periods. His income in the two periods is $(\omega_1, \omega_2) = (\$30, \$60)$ and the utility function is

$$U(x_1, x_2) = x_1 + \frac{1}{8} x_2.$$

a) Propose some other utility function that gives a higher level of utility for any bundle (x_1, x_2), which represents the same preferences. (utility function)

b) Plot intertemporal budget set of Josh for interest rate $r = 100\%$. Find PV and FV of the endowment cash flow and depict the two values in the graph. On the budget line mark all consumption plans that involve borrowing.

c) Find optimal consumption plan $(x_1, x_2)$and show it in the graph (give two numbers). Is your solution interior?

Problem 3. (Equilibrium)

Consider an economy with two goods: clothing x_1 and food x_2. Onur's initial endowement is $\omega^O = (80, 20)$ and Janet initially has $\omega^J = (20, 30)$. Utility functions of Onur and Janet are given by

$$U^i(x_1, x_2) = \frac{1}{4} \ln(x_1) + \frac{1}{4} \ln(x_2).$$

.

a) Plot an Edgeworth box and mark the point corresponding to the initial endowments.

b) Give the definition of a Pareto efficient allocation (one sentence) and provide its equivalent characterization in terms of MRS (equation). Verify whether the endowment allocation is Pareto efficient (compare two numbers).

c) Find the prices and the allocation in the competitive equilibrium (six numbers)

d) Using MRS condition demonstrate that the competitive allocation is Pareto efficient.

Problem 4.(Short questions)

a) You are renting a home that gives you \$1000 each month in form of rent (forever). Find PV of the cashflow if the monthly interest rate is $r = 1\%$.

b) Demonstrate that production function $f(K, L) = K^{0.3}L^{0.3}$ exhibits decreasing returns to scale. (use "lambda" argument). Without any calculations sketch the cost curve associated with this production function.

c) Suppose fixed cost is $F = 2$ and variable cost is $c(y) = 2y^2$. Find ATC^{MES} and y^{MES}. Give formula for a supply function of individual firm and plot it in a graph. Find equilibrium price and aggregate output in an industry with 4 firms, assuming demand $y = 10 - p$.

d) Give a von Neumann-Morgenstern utility function over lotteries for a Bernoulli utility function is $u(c) = \ln c$ and the probability of each state is 0.5 (formula). Is a consumer with this utility function risk loving, risk averse or risk neutral? (choose one)

e) In a market for second-hand vehicles there are two types of cars: lemons (bad quality cars) and plums (good quality ones). The value of a car depends on its type and is given by

	Lemon	Plum
Seller	0	20
Buyer	10	26

Are plums going to be traded if the probability of a lemon is $\frac{1}{2}$? (compare relevant numbers).

Problem 5.(Market Power)

Consider an industry with inverse demand $p(y) = 200 - y$ and total cost $TC = 40y$.

a) What are the total gains to trade in this industry? (number). Find the HHI index of this industry with one firm, a monopoly. (one number)

b) Find the optimal level of output and the price of a monopoly assuming uniform pricing (give two numbers). Illustrate its choice in a graph. Mark a DWL.

c) Find profit and a DWL if monopoly uses the first-degree price discrimination.

d) Find the aggregate output, the price and the markup in a Cournot-Nash equilibrium with N firms (all functions of N). What is the limit of the markup function as N goes to infinity? Why?

Problem 6.(Public good)

Alfonsia and Betonia are two countries that are members of the same military alliance. Their security depends positively on joint military spending $x^A + x^B$ of the two countries. Thus, Alfonsia's "utility" net of cost of military spending is given by

$$U^A = 2\ln(x^A + x^B) - x^A$$

and the analogous function for Bretonia is

$$U^B = 4\ln(x^A + x^B) - x^B.$$

a) Find the best response functions for Alfonsia and Bretonia (two formulas) and plot them in the coordinate system (x^A, x^B).

b) Find the Nash Equilibrium (give two numbers). Is one of the two countries free riding? If yes, which one?

c) Find the efficient level of military spending of the alliance (one number). Is the efficient spending smaller or bigger than the one observed in the Nash equilibrium? Why? (one sentence)

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Final Exam (Group B)

You have 2h to complete the exam. The final consists of 6 questions (15+10+15+25+20+15=100).

Problem 1. (Choice with Cobb-Douglas preferences)

Sara spends her income on books x_1 and food x_2. The prices of the two commodities are $p_1 = p_2 = 10$ and her income is $m = 200$. Sara's utility function is given by

$$U(x_1, x_2) = (x_1)^6 (x_2)^6 .$$

a) Find analytically Sara's MRS as a function of (x_1, x_2) (give a function) and determine its value for consumption bundle $(x_1, x_2) = (2, 6)$. Provide economic and geometric interpretation of MRS at this bundle (one sentence + graph).

b) Give two secrets of happiness that determine Sara's optimal choice (two equation). Explain why violation of any of them implies that the bundle cannot be optimal (one sentence for each condition).

c) Find Sara's optimal choice (two numbers) and mark the optimal bundle in the commodity space.

d) Using magic formulas for Cobb-Douglas preferences argue that both commodities are ordinary commodities. (formulas and one sentence)

Problem 2. (Intertemporal choice with perfect substitutes)

Josh chooses a consumption plans for two periods. His income in the two periods is $(\omega_1, \omega_2) = (\$30, \$60)$ and the utility function is

$$U(x_1, x_2) = x_1 + \frac{1}{3}x_2.$$

a) Propose some other utility function that gives a higher level of utility for any bundle (x_1, x_2), which represents the same preferences. (utility function)

b) Plot intertemporal budget set of Josh for interest rate $r = 100\%$. Find PV and FV of the endowment cash flow and depict the two values in the graph. On the budget line mark all consumption plans that involve borrowing.

c) Find optimal consumption plan $(x_1, x_2)$and show it in the graph (give two numbers). Is your solution interior?

Problem 3. (Equilibrium)

Consider an economy with two goods: clothing x_1 and food x_2. Onur's initial endowement is $\omega^O = (40, 10)$ and Janet initially has $\omega^J = (10, 15)$. Utility functions of Onur and Janet are given by

$$U^i(x_1, x_2) = \frac{1}{4}\ln(x_1) + \frac{1}{4}\ln(x_2).$$

.

a) Plot an Edgeworth box and mark the point corresponding to the initial endowments.

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c) Suppose fixed cost is $F = 4$ and variable cost is $c(y) = 4y^2$. Find ATC^{MES} and y^{MES}. Give formula for a supply function of individual firm and plot it in a graph. Find equilibrium price and aggregate output in an industry with 8 firms, assuming demand $y = 20 - p$.

d) Give a von Neumann-Morgenstern utility function over lotteries for a Bernoulli utility function is $u(c) = \ln c$ and the probability of each state is 0.5 (formula). Is a consumer with this utility function risk loving, risk averse or risk neutral? (choose one)

e) In a market for second-hand vehicles there are two types of cars: lemons (bad quality cars) and plums (good quality ones). The value of a car depends on its type and is given by

	Lemon	Plum
Seller	0	20
Buyer	10	40

Are plums going to be traded if the probability of a lemon is $\frac{1}{2}$? (compare relevant numbers).

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Consider an industry with inverse demand $p(y) = 200 - y$ and total cost $TC = 40y$.

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c) Find the efficient level of military spending of the alliance (one number). Is the efficient spending smaller or bigger than the one observed in the Nash equilibrium? Why? (one sentence)

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301 Final, Spring 2015 - Solutions
(Version A)

Problem 1.

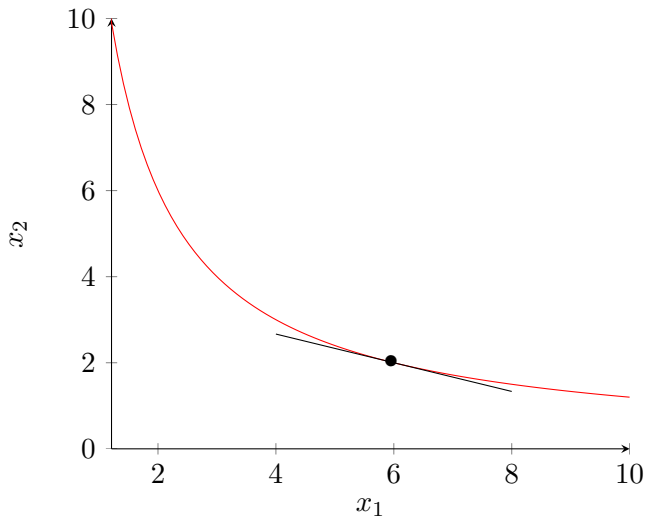
a)

$$MRS(x_1, x_2) = -\frac{(\frac{\partial U}{\partial x_1})}{(\frac{\partial U}{\partial x_2})} = -\frac{3x_1^2 x_2^3}{3x_1^3 x_2^2} = -\frac{x_2}{x_1} \tag{1}$$

$$MRS(6, 2) = -\frac{2}{6} = -\frac{1}{3} \tag{2}$$

Economic interpretation: The MRS is the rate at which Sara is willing to give up x_1 for x_2.

Geometric Interpretation: $MRS(x_1, x_2)$ is the slope of Sara's indifference curve at the point (x_1, x_2).



b) Sara's two secrets of happiness are 1) Sara spends all of her income (and no more): $p_1x_1 + p_2x_2 = w$ and 2) The rate at which Sara is willing to give up x_1 for x_2 is equal to the price of x_1 relative to the price of x_2: $MRS(x_1, x_2) = -\frac{p_1}{p_2}$. These become

$$5x_1 + 5x_2 = 100 \tag{3}$$

$$\frac{x_1}{x_2} = 1 \tag{4}$$

c) Solving these two equations, we get $x_1 = x_2 = 10$

d) Magic formulas:

$$x_1 = \frac{a}{a+b}\frac{m}{p_1} = \frac{1}{2}\frac{m}{p_1} \tag{5}$$

$$x_2 = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{2}\frac{m}{p_2} \tag{6}$$

x_1 is decreasing in p_1 and x_2 is decreasing in p_2. So, both x_1 and x_2 are ordinary commodities.

Problem 2.

a) We can take the monotone transformation $\tilde{U}(x_1, x_2) = 8(U(x_1, x_2)) = 8x_1 + x_2$. $\tilde{U}$ and U represent the same preferences since the bundles have the same utility ordering, but $\tilde{U}(x_1, x_2) > U(x_1, x_2)$ for every bundle (x_1, x_2).

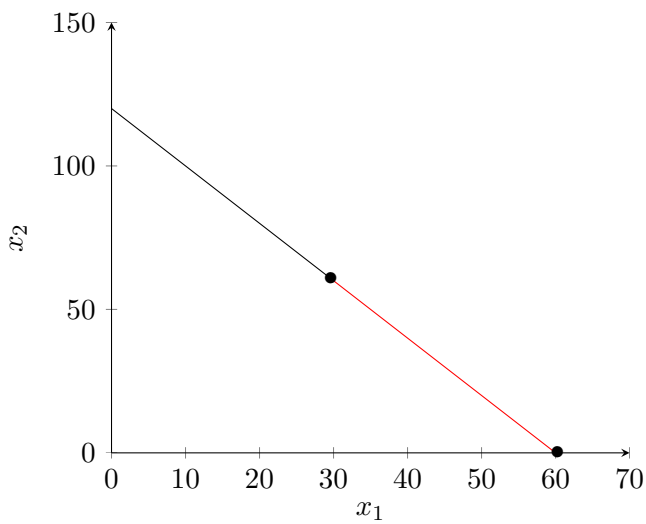
b) Since $r = 100\%$, the slope of the intertemporal budget line is $-(1 + r) = -2$.

The present and future values of the endowment are

$$PV = \omega_1 + \frac{\omega_2}{1+r} = 30 + \frac{60}{2} = 60 \tag{7}$$

$$FV = (1 + r)\omega_1 + \omega_2 = (1 + r)PV = 2(60) = 120 \tag{8}$$

The PV is where the intertemporal budget line crosses the x_1-axis, and the FV is where the intertemporal budget line crosses the x_2-axis. The red portion of the line, to the right of the endowment $(30, 60)$, are all the consumption plans that involve borrowing.



c) We can interpret the prices as $p_1 = 1 + r = 2$ and $p_2 = 1$, so $\frac{p_1}{p_2} = 2$.

Note that $|MRS(x_1, x_2)| = 8 > 2 = \frac{p_1}{p_2}$, so Josh will buy only x_1, and none of x_2. Therefore, $x_2 = 0$. The budget constraint then gives $2x_1 = 2\omega_1 + \omega_2 = 2(30) + 60$, or $x_1 = 60$.

The solution $(x_1, x_2) = (60, 0)$ is not interior. It is at the bottom right corner of the budget line.

Problem 3.

a) The total endowments are $\omega_1 = \omega_1^O + \omega_1^J = 80 + 20 = 100$ and $\omega_2 = \omega_2^O + \omega_2^J = 20 + 30 = 50$ so the Edgeworth box has dimensions 100 by 50. Put Onur on the bottom left origin and Janet on the upper right origin. The endowment is 80 units left and 20 units up from Onur's origin. This point is also 20 units left and 30 units down from Janet's origin.

b) Definition: A Pareto efficient allocation is where no other allocation can make one person off without hurting the other person.

Equivalent Characteristic: An allocation $(x_1^O, x_2^O, x_1^J, x_2^J)$ is Pareto efficient if $MRS(x_1^O, x_2^O) = MRS(x_1^J, x_2^J)$.

Since the utility is Cobb-Douglas with $a = b = \frac{1}{4}$, then $MRS(x_1, x_2) = \frac{a}{b}\frac{x_2}{x_1} = \frac{x_1}{x_2}$. We can check the Pareto efficiency of the endowment point:

$MRS(\omega_1^O, \omega_2^O) = \frac{\omega_2^O}{\omega_1^O} = 4 \neq \frac{2}{3} = \frac{\omega_2^J}{\omega_1^J} = MRS(\omega_1^J, \omega_2^J)$ so the endowment point is not Pareto optimal.

c) Cobb-Douglas magic formulas with parameters $a = b = \frac{1}{4}$ are

$$x_1^O = \frac{1}{2}\frac{m^O}{p_1} \tag{9}$$

$$x_2^O = \frac{1}{2}\frac{m^O}{p_2} \tag{10}$$

$$x_1^J = \frac{1}{2}\frac{m^J}{p_1} \tag{11}$$

$$x_2^J = \frac{1}{2}\frac{m^J}{p_2} \tag{12}$$

m^O and m^J are the total endowments of Onur and Janet, respectively, in terms of dollars.

Plugging these into the market clearing conditions $x_1^O + x_1^J = \omega_1$ and $x_2^O + x_2^J = \omega_2$ (where ω_1 and ω_2 are the aggregate endowments of goods 1 and 2, respectively), we get

$$\frac{1}{2}\frac{1}{p_1}(m^O + m^J) = \omega_1 = 80 + 20 = 100 \tag{13}$$

$$\frac{1}{2}\frac{1}{p_2}(m^O + m^J) = \omega_2 = 20 + 30 = 50 \tag{14}$$

Let us set $p_2 = 1$ and find the relative price p_1. The second equation gives $(m^O + m^J) = 100$. We can substitute this into the first equation to get $\frac{1}{2}\frac{1}{p_1}(100) = 100$, or $p_1 = \frac{1}{2}$.

Now, the dollar endowments are $m^O = \frac{1}{2}(80) + 20 = 60$ and $m^J = \frac{1}{2}(20) + 30 = 40$

Plugging all of these back into the magic formulas gives the equilibrium allocation:

$x_1^O = 60, x_2^O = 30, x_1^J = 40, x_2^J = 20.$

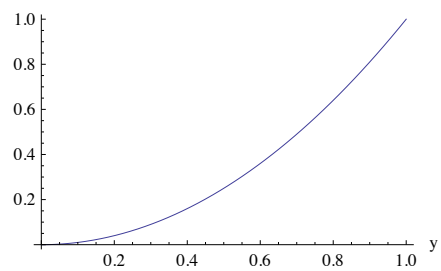
d) $MRS(x_1^O, x_2^O) = \frac{60}{30} = \frac{1}{2} = \frac{20}{40} = MRS(x_1^J, x_2^J)$, so the equilibrium allocation is indeed Pareto efficient.

Problem 4

a) $PV = \$1000/r = \$1000/0.01 = 100,000.$

b) $f(\lambda K, \lambda L) = \lambda^{0.6} K^{0.3} L^{0.3} < \lambda f(K, L)$, DRS.

Figure 1:



c) $MC = ATC \Rightarrow 4y = 2/y + 2y \Rightarrow y^{\text{MES}} = 1,\ ATC^{\text{MES}} = 4.$
$y^{\text{each firm}} = p/4,\ y^{\text{aggregate}} = p.$ Demand=Supply$\Rightarrow 10 - p = p \Rightarrow$
$p = 5,\ y^{\text{aggregate}} = 5$

d) $EU = \ln c_1/2 + \ln c_2/2$, risk averse as the Bernoulli utility is concave.

e) Expected value for the buyer if both types of sellers are in the market is $(10+26)/2 = 18$, less than the plum seller's valuation 20, thus plum won't be traded.

Problem 5

a) Total Gain is $160^2/2 = 12800$;
HHI $= 100^2 = 10,000.$

b) MR=MC $\Rightarrow 200 - 2y = 40 \Rightarrow y = 80 \Rightarrow p = 120.$

c) Profit is equal to total gain from trade found in a), 12800;
There's no DWL.

d) For a firm i, $\pi = (200 - y_i - \sum_{j \neq i} y_j)y_1 - 40y_1$, $MR = 160 - 2y_1 - \sum_{j \neq i} y_j$, $MR = 0 \Rightarrow y_i = \frac{160 - \sum_{j \neq i}^{N} y_j}{2}$, all y are equal$\Rightarrow y_i = \frac{160-(N-1)y_i}{2} \Rightarrow y_i = \frac{160}{N+1}$, $p = 200 - \frac{160N}{N+1} = \frac{40N}{N+1} + \frac{200}{N+1} \overset{N \to \infty}{=} 40$. Markup approaches 1 when N shoots to infinity.

Problem 6

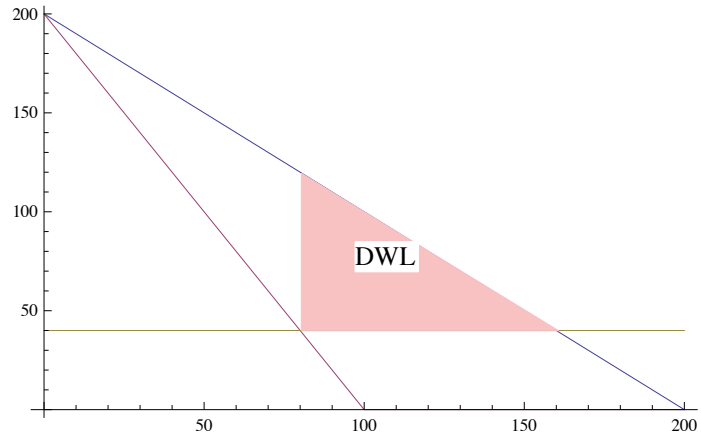


Figure 2:

a) $MU^A = 0 \Rightarrow \frac{2}{x^A + x^B} = 1 \Rightarrow x^A = 2 - x^B$ if $x^B < 2$ and $x^A = 0$ if $x^B \geq 2$. Similarly, $x^B = 4 - x^A$ if $x^A < 4$ and $x^B = 0$ if $x^A \geq 4$.

b) $x^A = 0$, $x^B = 4$. A is free riding as he values the military spending less than B.

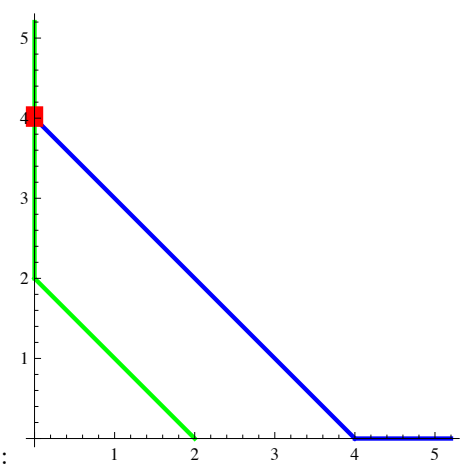


Figure 3:

c) Let $x = x^A + x^B$, $U^{\text{joint}} = 6\ln(x) - x$, $MU^{\text{joint}} = 0 \Rightarrow x = 6$, it's greater than the sum of Nash equilibrium level since the maximizing joint utility internalizes the positive externality.

301 Final, Spring 2015 - Solutions
(Version B)

Problem 1.

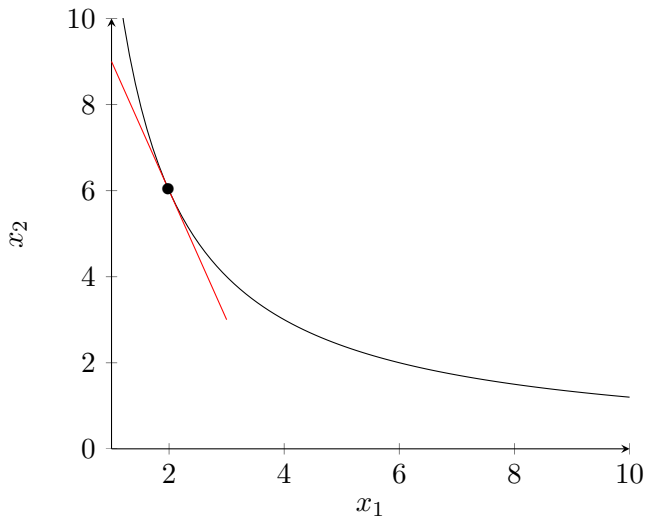
a)

$$MRS(x_1, x_2) = -\frac{(\frac{\partial U}{\partial x_1})}{(\frac{\partial U}{\partial x_2})} = -\frac{6x_1^5 x_2^6}{6x_1^6 x_2^5} = -\frac{x_2}{x_1} \tag{1}$$

$$MRS(6, 2) = -\frac{6}{2} = -3 \tag{2}$$

Economic interpretation: The MRS is the rate at which Sara is willing to give up x_1 for x_2.

Geometric Interpretation: $MRS(x_1, x_2)$ is the slope of Sara's indifference curve at the point (x_1, x_2).



b) Sara's two secrets of happiness are 1) Sara spends all of her income (and no more): $p_1x_1 + p_2x_2 = w$ and 2) The rate at which Sara is willing to give up x_1 for x_2 is equal to the price of x_1 relative to the price of x_2: $MRS(x_1, x_2) = -\frac{p_1}{p_2}$. These become

$$10x_1 + 10x_2 = 200 \tag{3}$$

$$\frac{x_1}{x_2} = 1 \tag{4}$$

c) Solving these two equations, we get $x_1 = x_2 = 10$

d) Magic formulas:

$$x_1 = \frac{a}{a+b}\frac{m}{p_1} = \frac{1}{2}\frac{m}{p_1} \tag{5}$$

$$x_2 = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{2}\frac{m}{p_2} \tag{6}$$

x_1 is decreasing in p_1 and x_2 is decreasing in p_2. So, both x_1 and x_2 are ordinary commodities.

Problem 2.

a) For example, take the monotone transformation $\tilde{U}(x_1, x_2) = 3(U(x_1, x_2)) = 3x_1 + x_2$. $\tilde{U}$ and U represent the same preferences since the bundles have the same utility ordering, but $\tilde{U}(x_1, x_2) > U(x_1, x_2)$ for every bundle (x_1, x_2).

b) Since $r = 100\%$, the slope of the intertemporal budget line is $-(1 + r) = -2$.

The present and future values of the endowment are

$$PV = \omega_1 + \frac{\omega_2}{1+r} = 30 + \frac{60}{2} = 60 \tag{7}$$

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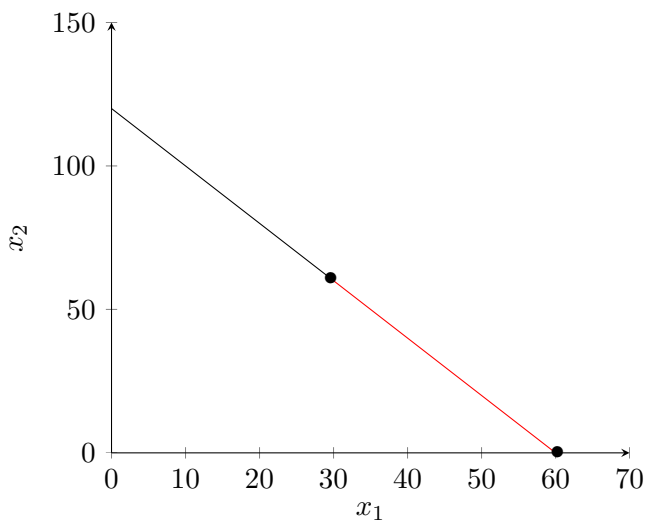
The PV is where the intertemporal budget line crosses the x_1-axis, and the FV is where the intertemporal budget line crosses the x_2-axis. The red portion of the line, to the right of the endowment $(30, 60)$, are all the consumption plans that involve borrowing.



c) We can interpret the prices as $p_1 = 1 + r = 2$ and $p_2 = 1$, so $\frac{p_1}{p_2} = 2$.

Note that $|MRS(x_1, x_2)| = 3 > 2 = \frac{p_1}{p_2}$, so Josh will buy only x_1, and none of x_2. Therefore, $x_2 = 0$. The budget constraint then gives $2x_1 = 2\omega_1 + \omega_2 = 2(30) + 60$, or $x_1 = 60$.

The solution $(x_1, x_2) = (60, 0)$ is not interior. It is at the bottom right corner of the budget line.

Problem 3.

a) The total endowments are $\omega_1 = \omega_1^O + \omega_1^J = 40 + 10 = 50$ and $\omega_2 = \omega_2^O + \omega_2^J = 10 + 15 = 25$ so the Edgeworth box has dimensions 50 by 25. Put Onur on the bottom left origin and Janet on the upper right origin. The endowment is 40 units left and 10 units up from Onur's origin. This point is also 10 units left and 15 units down from Janet's origin.

b) Definition: A Pareto efficient allocation is where no other allocation can make one person off without hurting the other person.

Equivalent Characteristic: An allocation $(x_1^O, x_2^O, x_1^J, x_2^J)$ is Pareto efficient if $MRS(x_1^O, x_2^O) = MRS(x_1^J, x_2^J)$.

Since the utility is Cobb-Douglas with $a = b = \frac{1}{4}$, then $MRS(x_1, x_2) = \frac{a}{b}\frac{x_2}{x_1} = \frac{x_1}{x_2}$. We can check the Pareto efficiency of the endowment point:

$MRS(\omega_1^O, \omega_2^O) = \frac{\omega_2^O}{\omega_1^O} = \frac{1}{4} \neq \frac{3}{2} = \frac{\omega_2^J}{\omega_1^J} = MRS(\omega_1^J, \omega_2^J)$ so the endowment point is not Pareto optimal.

c) Cobb-Douglas magic formulas with parameters $a = b = \frac{1}{4}$ are

$$x_1^O = \frac{1}{2}\frac{m^O}{p_1} \tag{9}$$

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$$x_2^J = \frac{1}{2}\frac{m^J}{p_2} \tag{12}$$

m^O and m^J are the total endowments of Onur and Janet, respectively, in terms of dollars.

Plugging these into the market clearing conditions $x_1^O + x_1^J = \omega_1$ and $x_2^O + x_2^J = \omega_2$ (where ω_1 and ω_2 are the aggregate endowments of goods 1 and 2, respectively), we get

$$\frac{1}{2}\frac{1}{p_1}(m^O + m^J) = \omega_1 = 40 + 10 = 50 \tag{13}$$

$$\frac{1}{2}\frac{1}{p_2}(m^O + m^J) = \omega_2 = 10 + 15 = 25 \tag{14}$$

Let us set $p_2 = 1$ and find the relative price p_1. The second equation gives $(m^O + m^J) = 50$. We can substitute this into the first equation to get $\frac{1}{2}\frac{1}{p_1}(50) = 50$, or $p_1 = \frac{1}{2}$.

Now, the dollar endowments are $m^O = \frac{1}{2}(40) + 10 = 30$ and $m^J = \frac{1}{2}(10) + 15 = 20$

Plugging all of these back into the magic formulas gives the equilibrium allocation:

$x_1^O = 30, x_2^O = 15, x_1^J = 20, x_2^J = 10.$

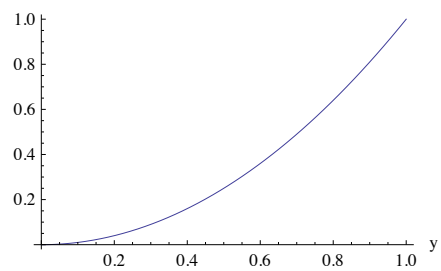
d) $MRS(x_1^O, x_2^O) = \frac{30}{15} = \frac{1}{2} = \frac{10}{20} = MRS(x_1^J, x_2^J)$, so the equilibrium allocation is indeed Pareto efficient.

Problem 4

a) $PV = \$1000/r = \$1000/0.01 = 100,000.$

b) $f(\lambda K, \lambda L) = \lambda^{0.6} K^{0.3} L^{0.3} < \lambda f(K, L)$, DRS.

Figure 1:



c) $MC = ATC \Rightarrow 8y = 4/y + 4y \Rightarrow y^{\text{MES}} = 1,\ ATC^{\text{MES}} = 8.$
$y^{\text{each firm}} = p/8,\ y^{\text{aggregate}} = p.$ Demand=Supply$\Rightarrow 20 - p = p \Rightarrow$
$p = 10,\ y^{\text{aggregate}} = 10$

d) $EU = \ln c_1/2 + \ln c_2/2$, risk averse as the Bernoulli utility is concave.

e) Expected value for the buyer if both types of sellers are in the market is $(10 + 40)/2 = 25$, greater than the plum seller's valuation 20, thus plum will be traded.

Problem 5

a) Total Gain is $160^2/2 = 12800$;
HHI $= 100^2 = 10,000.$

b) MR=MC $\Rightarrow 200 - 2y = 40 \Rightarrow y = 80 \Rightarrow p = 120.$

c) Profit is equal to total gain from trade found in a), 12800;
There's no DWL.

d) For a firm i, $\pi = (200 - y_i - \sum_{j \neq i} y_j)y_1 - 40y_1$, $MR = 160 - 2y_1 - \sum_{j \neq i} y_j$, $MR = 0 \Rightarrow y_i = \frac{160 - \sum_{j \neq i}^{N} y_j}{2}$, all y are equal$\Rightarrow y_i = \frac{160-(N-1)y_i}{2} \Rightarrow y_i = \frac{160}{N+1}$, $p = 200 - \frac{160N}{N+1} = \frac{40N}{N+1} + \frac{200}{N+1} \overset{N \to \infty}{=} 40$. Markup approaches 1 when N shoots to infinity.

Problem 6

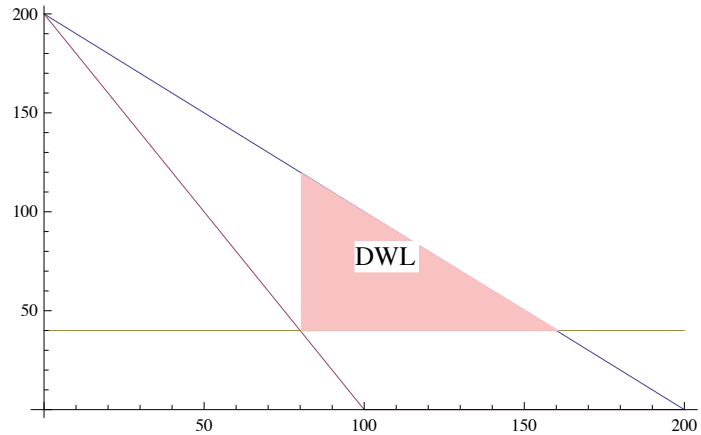


Figure 2:

a) $MU^A = 0 \Rightarrow \frac{3}{x^A + x^B} = 1 \Rightarrow x^A = 3 - x^B$ if $x^B < 3$ and $x^A = 0$ if $x^B \geq 3$. Similarly, $x^B = 4 - x^A$ if $x^A < 4$ and $x^B = 0$ if $x^A \geq 4$.

b) $x^A = 0$, $x^B = 4$. A is free riding as he values the military spending less than B.

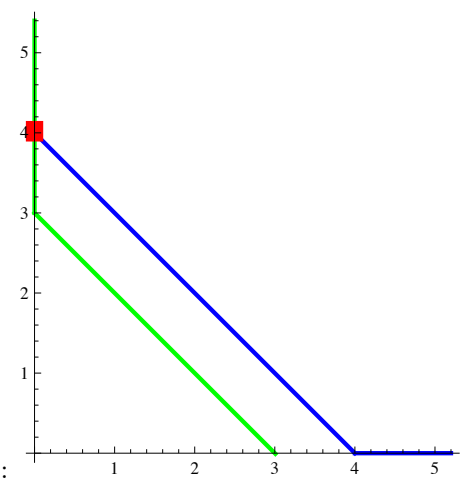


Figure 3:

c) Let $x = x^A + x^B$, $U^{\text{joint}} = 7\ln(x) - x$, $MU^{\text{joint}} = 0 \Rightarrow x = 7$, it's greater than the sum of Nash equilibrium level since the maximizing joint utility internalizes the positive externality.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (Group A)

You have 2h to complete the exam. The final consists of 6 questions (25+20+20+15+10+10=100).

Problem 1. (Quasilinaer income effect)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 4, p_2 = 4$, respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2 \ln x_1 + x_2$$

a) In the commodity space plot Mirabella's budget set. Find the slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

b) Find analytically formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of MRS (two sentences). Find the value of MRS at bundle $(x_1, x_2) = (4, 4)$ (one number). At this bundle, which of the two commodities is (locally) more valuable? (chose one)

c) Write down two secrets of happiness that determine Mirabellas's optimal choice (two equation). Provide the geometric interpretation of the conditions in the commodity space.

d) Find Mirabella's optimal choice (two numbers). Is solution interior (yes-no answer).

e) Suppose the price of a chocolate candy bar goes down to $p_1 = 2$, while other price $p_2 = 4$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good (pick one)?

f) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Problem 2. (Equilibrium)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles x_1 and flowers x_2. Adalia initial endowment of the commodities is $\omega^A = (40, 60)$ and Briana endowment is $\omega^B = (60, 40)$. Adalia and Briana utility functions are given by, $i = A, B$

$$U^i(x_1, x_2) = 4 \ln x_1 + 4 \ln x_2$$

a) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

b) Give a definition of a Pareto efficient allocation (one sentence).

c) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide geometric arguments that demonstrate the necessity and sufficiency of MRS condition for Pareto efficiency.

d) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

e) Using (one of) the secrets of happiness prove that a competitive equilibrium is Pareto efficient in any economy.

Problem 3. (Short questions)

a) Using λ argument prove that Cobb-Douglass production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

b) Now consider a firm (different from point a)) with variable cost $c(y) = 2y^2$ and fixed cost $F = 2$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expect in the industry if inverse demand is $D(p) = 10 - p$?

c) Suppose a Bernoulli utility function is $u(x) = x^2$ and two states are equally likely (probability $\frac{1}{2}$). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery $(0, 2)$ (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

d) Find Herfindahl–Hirschman Index (HHI) for industry with $N = 50$ identical firms (one number). Is the industry concentrated?

e) Derive formula for the present value of perpetuity.

Problem 4. (Market Power)

Consider an industry with inverse demand $p(y) = 8 - y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

b) Write down monopoly's profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

c) Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

d) Assuming the same demand function find the individual and the aggregate level of production and the price in the Cournot-Nash equilibrium with $N = 3$ identical firms (give three numbers). Show the deadweight loss in the graph.

Problem 5.(Externality)

Lucy is addicted to nicotine. Her utility from smoking c cigarettes (net of their cost) is given by

$$U^L(c) = 2\ln c - c$$

Her sister Taja prefers healthy lifestyle, her favorite commodity is orange juice, j. The two sisters live together and Taja is exposed to second-hand smoke and hence her utility is adversely affected by Lucy consumption of cigarettes c. In particular, her utility function (net of cost of orange juice) is given by

$$U^T(j, c) = \ln(j - c) - j.$$

a) Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers)

b) Find the Pareto efficient level of c and j. Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence) Hint: Derivative of $\ln(j - c)$ with respect to c is $-\frac{1}{j-c}$.

Problem 6. (Asymmetric information)

In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to $\frac{1}{2}$. The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
Seller	0	12
Buyer	10	18

Both parties agree on the price that is in between the value of a buyer and a seller.

a) Buyers and sellers can perfectly determine the quality of a house before transaction takes place What is expected total, buyers and sellers surplus (three numbers)

b) Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (Group B)

You have 2h to complete the exam. The final consists of 6 questions (25+20+20+15+10+10=100).

Problem 1. (Quasilinaer income effect)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 2, p_2 = 2$, respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2 \ln x_1 + x_2$$

a) In the commodity space plot Mirabella's budget set. Find the slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

b) Find analytically formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of MRS (two sentences). Find the value of MRS at bundle $(x_1, x_2) = (8, 8)$ (one number). At this bundle, which of the two commodities is (locally) more valuable? (chose one)

c) Write down two secrets of happiness that determine Mirabellas's optimal choice (two equation). Provide the geometric interpretation of the conditions in the commodity space.

d) Find Mirabella's optimal choice (two numbers). Is solution interior (yes-no answer).

e) Suppose the price of a chocolate candy bar goes down to $p_1 = 1$, while other price $p_2 = 2$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good (pick one)?

f) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Problem 2. (Equilibrium)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles x_1 and flowers x_2. Adalia initial endowment of the commodities is $\omega^A = (50, 100)$ and Briana endowment is $\omega^B = (100, 50)$. Adalia and Briana utility functions are given by, $i = A, B$

$$U^i(x_1, x_2) = 2 \ln x_1 + 2 \ln x_2$$

a) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

b) Give a definition of a Pareto efficient allocation (one sentence).

c) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide geometric arguments that demonstrate the necessity and sufficiency of MRS condition for Pareto efficiency.

d) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

e) Using (one of) the secrets of happiness prove that a competitive equilibrium is Pareto efficient in any economy.

Problem 3. (Short questions)

a) Using λ argument prove that Cobb-Douglass production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

b) Now consider a firm (different from point a)) with variable cost $c(y) = 4y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expect in the industry if inverse demand is $D(p) = 16 - p$?

c) Suppose a Bernoulli utility function is $u(x) = x^2$ and two states are equally likely (probability $\frac{1}{2}$). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery $(0, 2)$ (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

d) Find Herfindahl–Hirschman Index (HHI) for industry with $N = 100$ identical firms (one number). Is the industry concentrated?

e) Derive formula for the present value of perpetuity

Problem 4. (Market Power)

Consider an industry with inverse demand $p(y) = 12 - y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

b) Write down monopoly's profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

c) Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

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$$U^T(j, c) = \ln(j - c) - j.$$

a) Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers)

b) Find the Pareto efficient level of c and j. Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence) Hint: Derivative of $\ln(j - c)$ with respect to c is $-\frac{1}{j-c}$.

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In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to $\frac{1}{2}$. The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
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Buyer	10	22

Both parties agree on the price that is in between the value of a buyer and a seller.

a) Buyers and sellers can perfectly determine the quality of a house before transaction takes place What is expected total, buyers and sellers surplus (three numbers)

b) Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

Solutions to Spring 2014 ECON 301 Final Group A

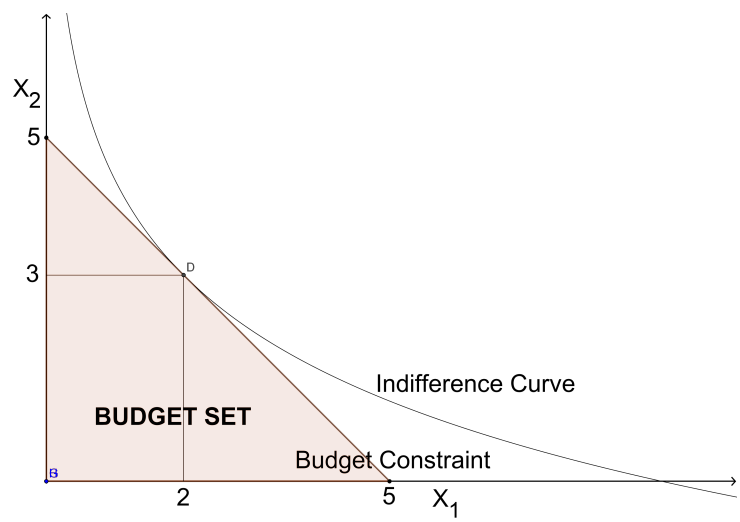
Problem 1. (Quasilinear income effect) (25 points)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 4$ and $p_2 = 4$ respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2 \ln x_1 + x_2$$

a) (4 points) In the commodity space plot Mirabella's budget set. Find slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

Solution: See figure below for Mirabella's budget set - it's the entire shaded region. The slope of the budget line is equal to $-p_1/p_2 = -4/4 = -1$. The economic interpretation of the slope is that it represents the rate at which the market prices allow Mirabella to trade between the two goods.



b) (4 points) Find analytically the formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of the MRS (two sentences). Find the value of the MRS at bundle $(x_1, x_2) = (4, 4)$ (one number). Which of the two commodities is (locally) more valuable? (choose one)

Solution: $MRS = -\frac{MUx_1}{MUx_2} = -\frac{2/x_1}{1} = -\frac{2}{x_1}$. The economic interpretation of the MRS is that it represents the rate at which Mirabella would trade-off between goods while remaining indifferent. Geometrically, it is the slope of the indifference curve at that bundle. $MRS(4, 4) = -2/4 = -1/2$. Since at that bundle, $MUx_1 = 1/2 < 1 = MUx_2$, x_2 is locally more valuable.

c) (5 points) Write down two secrets to happiness that determine Mirabella's optimal choice (two equations). Provide the geometric interpretation of the conditions in the commodity space.

Solution: The two secrets to happiness are

$$\text{(1) Budget Constraint} : x_1 p_1 + x_2 p_2 = m, \text{ i.e. } 4x_1 + 4x_2 = 20$$

$$\text{(2) Equating Bang-Per-Buck} : \frac{MUx_1}{p_1} = \frac{MUx_2}{p_2}, \text{ i.e. } \frac{2/x_1}{4} = \frac{1}{4}$$

Note that (2) above is equivalent to $MRS = -\frac{p_1}{p_2}$. Using either is fine. The geometric interpretation of (1) is that the optimal bundle is on the budget constraint. The economic interpretation of (1) is that you spend all of your money. The geometric interpretation of (2) is that the optimal bundle is

where the indifference curve and budget constraint have the same slope (i.e. the indifference curve is tangent to the budget constraint).

d) (4 points) Find Mirabella's optimal choice (two numbers). Is solution interior? (yes-no answer)

Solution: To solve, note that (2) above implied that $\frac{2/x_1}{4} = \frac{1}{4}$, which, when solved, yields $x_1^* = 2$. Plugging $x_1 = 2$ into (1) gives us that $4 \cdot 2 + 4x_2 = 20$, which implies that $x_2^* = 3$. Therefore the optimal bundle is $(2, 3)$. This is an interior solution as it includes a strictly positive quantity of each good. You can also see this in the diagram in the solution to a).

e) (4 points) Suppose the price of a chocolate candy bar goes down to $p_1 = 2$, while other price $p_2 = 4$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good? (pick one)

Solution: Now we have the following:

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \Leftrightarrow \frac{2/x_1}{2} = \frac{1}{4} \Leftrightarrow x_1^* = 4$$
$$x_1p_1 + x_2p_2 = m \Leftrightarrow 4 \cdot 2 + x_2 \cdot 4 = 20 \Leftrightarrow x_2^* = 3$$

Therefore the optimal choice is $(4, 3)$. Since demand for the chocolate candy bar x_1^* went up (from 2 to 4) when the price was reduced, the chocolate candy bar is an ordinary good.

f) (4 points) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Solution:

To decompose the change in demand, we have to consider an auxiliary/Slutsky step in which we give the agent exactly enough income to purchase the original bundle (2,3) at the new prices:

$$m' = 2 \cdot 2 + 4 \cdot 3 = 16$$

We then consider what the agent's optimal bundle is given that income level, m', and the new prices.

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \Leftrightarrow \frac{2/x_1}{2} = \frac{1}{4} \Leftrightarrow x_1^* = 4$$
$$x_1p_1 + x_2p_2 = m' \Leftrightarrow 4 \cdot 2 + x_2 \cdot 4 = 16 \Leftrightarrow x_2^* = 2$$

Then the substitution effect is the auxiliary/Slutsky x_1^* minus the original x_1^*, which equals $4-2 = 2$. The income effect is the new x_1^* minus the auxiliary/Slutsky x_1^*, which equals $4 - 4 = 0$. Therefore, $SE = 2$ and $IE = 0$.

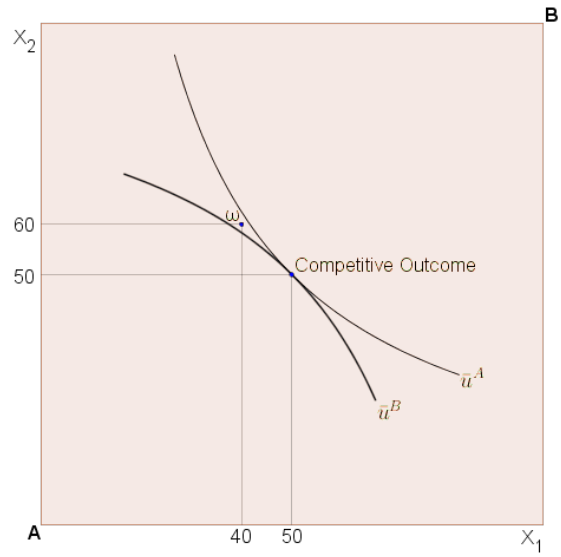
Problem 2. (Equilibrium) (20 points)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles, x_1, and flowers, x_2. Adalia's initial endowment of the commodities is $\omega^A = (40, 60)$ and Brianna's endowment is $\omega^B = (60, 40)$. Adalia and Briana's utility functions are given by, for $i = A, B$,

$$U^i(x_1, x_2) = 4 \ln x_1 + 4 \ln x_2$$

a) (3 points) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

Solution: See diagram below (which also includes solutions for later parts of this question).



b) (3 points) Give a definition of a Pareto efficient allocation (one sentence).

Solution: An allocation is Pareto efficient if it is impossible to make one person better off without making another worse off.

c) (3 points) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide arguments that demonstrate the necessity and sufficiency of the MRS condition for Pareto efficiency.

Solution: The equivalent condition is that $MRS^A = MRS^B$, i.e. all agents have the same MRS at the Pareto efficient allocation. The arguments for necessity and suffiency of the MRS condition are:

- Necessity: If MRS are not equal at a given point then the indifference curves through that point have different slopes. This implies that there exists a lens-shape region between the two indifference curves (better for both people). Any point inside of that region is a Pareto improvement over the original given point. Therefore, if MRS are not equal at a point, that point isn't Pareto optimal. Hence the MRS condition is necessary for Pareto optimality.
- Sufficiency: If the MRS of the agents are the same, then everything down and to the left of A's indifference curve is strictly worse for A. Everything up and to the right of B's indifference curve is stricty worse for B. Every point in the edgeworth box is either down and to the left of A's indifference curve or up and to the right of B's indifference curve, or both. Hence there is no feasible alternative allocation that makes one agent better off without hurting the other, and $MRS^A = MRS^B$ is a sufficient condition for Pareto efficiency.

d) (8 points) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

Solution: To solve this, we normalize $p_2 = 1$. (You could also normalize $p_1 = 1$ and solve for p_2 if you prefer.) Then, to solve for p_1, we use Cobb-Douglas formulas and clear the market for x_1 (you

could clear the market for x_2 if you prefer):

$$\begin{aligned}
\text{Demand for Good 1} &= \text{Supply for Good 1} \\
\frac{a}{a+b}\frac{m^A}{p_1} + \frac{a}{a+b}\frac{m^B}{p_1} &= 40+60 \\
\frac{4}{4+4}\frac{40p_1+60p_2}{p_1} + \frac{4}{4+4}\frac{60p_1+40p_2}{p_1} &= 100 \\
\frac{1}{2}\frac{40p_1+60}{p_1} + \frac{1}{2}\frac{60p_1+40}{p_1} &= 100 \\
40p_1+60+60p_1+40 &= 200p_1 \\
100 &= 100p_1 \\
p_1 &= 1
\end{aligned}$$

Now that we know that $p_1 = p_2 = 1$, we simply plug into the Cobb-Douglas demand formulas to solve for the demands:

$$\begin{aligned}
x_1^A &= \frac{a}{a+b}\frac{m^A}{p_1} = \frac{1}{2}\frac{40+60}{1} = 50 \\
x_2^A &= \frac{b}{a+b}\frac{m^A}{p_2} = \frac{1}{2}\frac{40+60}{1} = 50 \\
x_1^B &= \frac{a}{a+b}\frac{m^B}{p_1} = \frac{1}{2}\frac{60+40}{1} = 50 \\
x_2^B &= \frac{b}{a+b}\frac{m^B}{p_2} = \frac{1}{2}\frac{60+40}{1} = 50
\end{aligned}$$

Therefore the market outcome is $p_1 = p_2 = 1$ and $x_1^A = x_2^A = x_1^B = x_2^B = 50$. This is depicted in the diagram above in part a). At this outcome, $MRS^A = -\frac{4/50}{4/50} = -1 = MRS^B$. Therefore we have verified that the equilibrium is Pareto efficient.

e) (3 points) Harder: Using (one of) the secrets of happiness, prove that a competitive equilibrium is Pareto efficient in any economy.

Solution: The secret to happiness for A is that $MRS^A = -p_1/p_2$. The secret to happiness for B is that $MRS^B = -p_1/p_2$. Since the secrets to happiness dictate that each sets her MRS equal to $-p_1/p_2$, it follows that $MRS^A = MRS^B$.

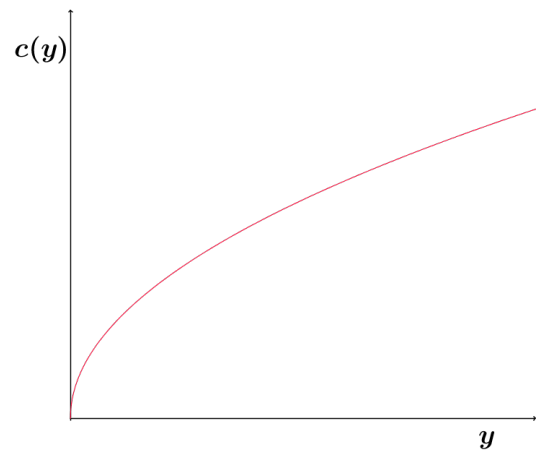
Problem 3. (Short Questions) (20 points)

a) (4 points) Using λ argument, prove that Cobb-Douglas production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

Solution: The λ argument is as follows:

$$F(\lambda K, \lambda L) = 2(\lambda K)(\lambda L) = \lambda^2 2KL = \lambda^2 F(K,L) > \lambda F(K,l) \text{ for all } \lambda > 1$$

This implies IRS. As for $c(y)$, note that you cannot actually do the calculations without the wage rates for capital and labor. However, we know that the cost function is increasing in output. We also know that, because of IRS, it should be concave. Therefore, any function you can draw that is increasing and concave is valid. One example:



b) (4 points) Now consider a firm (different from point a)) with variable cost $c(y) = 4y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expected in the industry if demand is $D(p) = 16 - p$?

Solution: $TC = 4y^2 + 4$, $MC = 8y$, $ATC = 4y + 4/y$. To solve for y^{MES}, set $ATC = MC$ (note you can also set the derivative of ATC with respect to y equal to zero to find y^{MES}).

$$ATC = MC \;\Leftrightarrow\; 4y + 4/y = 8y \;\Leftrightarrow\; y^{MES} = 1$$

Plugging that into the ATC gives us that $ATC^{MES} = 4 + 4 = 8$. In the long-run, free-entry equilibrium, we know that each firm produces y^{MES} and equilibrium price is ATC^{MES}. Therefore

$$\begin{aligned} \text{Supply} &= \text{Demand} \\ N \cdot y^{MES} &= 16 - ATC^{MES} \\ N &= 8 \end{aligned}$$

So we would expect 8 firms in this equilibrium.

c) (4 points) Suppose a Bernoulli utility function is $u(x) = x^2$, and two states are equally likely (probability 1/2). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery (0,2) (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

Solution: von Neuman-Morgenstern utility function: $U(x_g, x_b) = 1/2x_g^2 + 1/2x_b^2$.
Expected value of lottery: EV=$0 + 1/2 * 2 = 1$.
certainty equivalent, CE makes $U(CE, CE) = CE^2$ = Expected Utility $= 0 + 1/2 * 2^2 = 2$, so CE=$\sqrt{2}$.
CE > EV, because this Bernoulli utility function represents preference of a risk lover, and risk lover enjoys the lottery more than its expected value (he loves uncertainty).

d) (4 points) Find Herfindahl-Hirschman Index (HHI) for industry with N = 50 identical firms (one number). Is the industry concentrated?

Solution: Each firm occupies 2% market share. So HHI= $50 * 2^2 = 200 < 1800$. This industry is not concentrated (or the industry is competitive).

e) (4 points) Derive formula for the present value of perpetuity.

Solution: Suppose the interest rate is r and the constant payment of the asset is x.

$$\begin{aligned}
PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \dots \\
&= \frac{x}{1+r} + \frac{1}{1+r}[\frac{x}{1+r} + \frac{x}{(1+r)^2} + \dots] \\
&= \frac{x}{1+r} + \frac{1}{1+r}PV \\
\Rightarrow & PV[1 - \frac{1}{1+r}] = \frac{x}{1+r} \\
\Rightarrow & PV = \frac{x}{r}
\end{aligned}$$

Problem 4. (Market Power) (15 points)
Consider an industry with inverse demand $p(y) = 8-y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) **(2 points)** What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

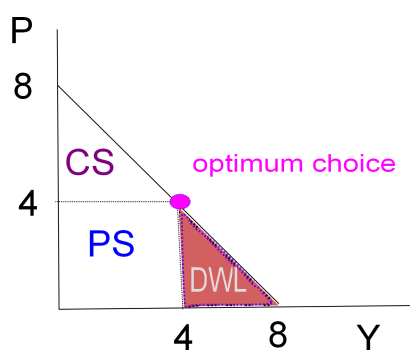
Solution: TS=$1/2 * 8 * 8 = 32$.

b) **(4 points)**Write down monopolys profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

Solution: $\pi = p(y)y - TC((y) = (8-y)y - TC(y)$.
Derive: $\frac{\partial \pi}{\partial y} = MR - MC = 8 - 2y = 0$. (Write MR=MC explicitly.)
MR is decreasing in y and MC is increasing in y. So when MR>MC, the monopoly can increase its profits by producing more until MR=MC; when MR<MC, the last unit produced has negative profit, the monopoly can increase its profits by reducing production until MR=MC.

c) **(5 points)** Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

Solution: From (b),$8 - 2y = 0 \rightarrow y^M = 4, p^M = 4$, DWL=$1/2 * 4 * 4 = 8$, elasticity at the optimum choice: $\epsilon = \frac{1}{p'(y)}\frac{p}{y} = -1$.



d) **(4 points)** Assuming the same demand function find the individual and the aggregate level of production and the price in the Cournot-Nash equilibrium with N = 3 identical firms (give three numbers). Show the deadweight loss in the graph.

Solution: We need to derive the best response function first. Suppose there are three firms, 1,2,3.

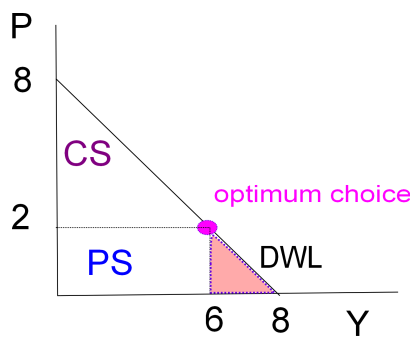
For firm 1,

$$\max_{y_1} \pi_1 = (8 - y_1 - y_2 - y_3)y_1$$
$$\Rightarrow \frac{\partial \pi_1}{\partial y_1} = 8 - 2y_1 - (y_2 + y_3) = 0$$
$$\Rightarrow y_1^* = \frac{8 - (y_2 + y_3)}{2}.$$

At the Nash equilibrium, since three firms are identical, $y_1^* = y_2^* = y_3^*$, plug this information into the best response function

$$\Rightarrow y_1^* = \frac{8 - 2y_1^*}{2} = 4 - y_1^*, \Rightarrow y_1^* = 2$$

So the aggreagate output is $3y_1^* = 6, p = 2$, DWL=2.



Problem 5.(Externality) (10 points)
Lucy is addicted to nicotine. Her utility from smoking c cigarettes (net of their cost) is given by

$$U^L(c) = 2\ln c - c$$

Her sister Taja prefers healthy lifestyle, her favorite commodity is orange juice, j. The two sisters live together and Taja is exposed to second-hand smoke and hence her utility is adversely affected by Lucy consumption of cigarettes c. In particular, her utility function (net of cost of orange juice) is given by

$$U^T(j, c) = \ln(j - c) - j$$

.

a) **(4 points)** Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers).

Solution: For Lucy,

$$\frac{\partial U^L(c)}{\partial c} = \frac{2}{c} - 1 = 0 \Rightarrow c^* = 2$$

For Taja,

$$\frac{\partial U^T(j, c)}{\partial j} = \frac{1}{j - c} - 1 = 0 \Rightarrow j^* = c + 1 = 3.$$

b) **(6 points)** Find the Pareto efficient level of c and j: Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence)

Solution: Joint utility is $U = 2\ln c + \ln(j-c) - c - j$.

$$\frac{\partial U}{\partial c} = \frac{2}{c} - \frac{1}{j-c} - 1 = 0$$
$$\frac{\partial U}{\partial j} = \frac{1}{j-c} - 1 = 0.$$

So $j - c = 1, \rightarrow \tilde{c} = 1, \tilde{j} = 2$. The social optimal $\tilde{c} = 1 < 2$. Because Lucy does not internalize the externality of her cigarettes to Taja, she smokes too much comparing to the social optimal level of c.

Problem 6. (Asymmetric information) (10 points) In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to 1/2 : The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
Seller	0	12
Buyer	10	18

Both parties agree on the price that is in between the value of a buyer and a seller.

a) **(3 points)** Buyers and sellers can perfectly determine the quality of a house before transaction takes place. What is expected total, buyers and sellers surplus (three numbers)

Solution: The *expected* gains to trade (ETS) =1/2*10+1/2*6=8.
Expected Buyer surplus (EBS)=1/2*5+1/2*3=4,
and Expected Seller surplus (ESS) =1/2*5+1/2*3=4.

b) **(7 points)** Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

Solution: With probability 1/2, the expected value of a house to a buyer is EV=1/2∗10+1/2∗18 = 14 **(1 point)**, which is larger than 12 (seller's value for plums), so both houses can be sold, and we end up with a pooling equilibrium **(1 point)**.
This pooling equilibrium is sustainable. **(1 point)**
The price is $p = \frac{12+14}{2} = 13$. **(1 point)**
Expected buyer surplus= 1/2*(10-13) + 1/2 (18-13)=1,**(1 point)**
expected seller surplus = 1/2(13-0) + 1/2(13-12)=7.**(1 point)**
So total expected gain to trade is EBS+ESS=8, which is the same as in (a) (full information), this outcome is Pareto efficient.**(1 point)**

Solutions to Spring 2014 ECON 301 Final Group B

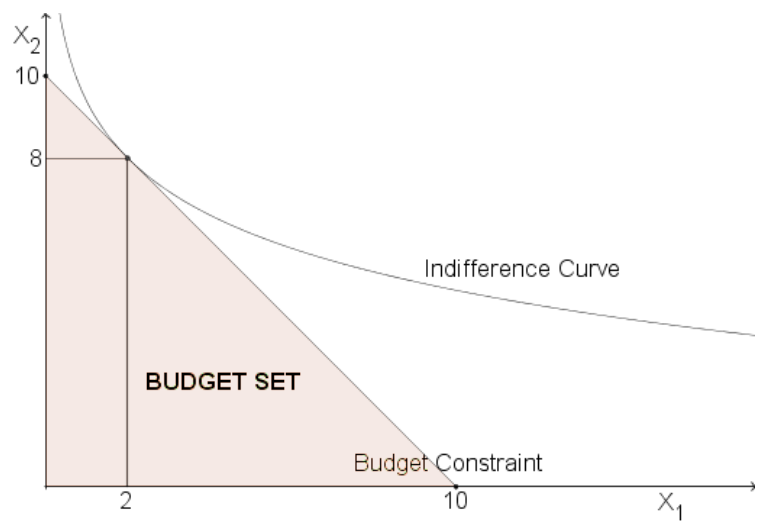
Problem 1. (Quasilinear income effect) (25 points)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 2$ and $p_2 = 2$ respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2 \ln x_1 + x_2$$

a) (4 points) In the commodity space plot Mirabella's budget set. Find slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

Solution: See figure below for Mirabella's budget set - it's the entire shaded region. The slope of the budget line is equal to $-p_1/p_2 = -2/2 = -1$. The economic interpretation of the slope is that it represents the rate at which the market prices allow Mirabella to trade between the two goods.



b) (4 points) Find analytically the formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of the MRS (two sentences). Find the value of the MRS at bundle $(x_1, x_2) = (8, 8)$ (one number). Which of the two commodities is (locally) more valuable? (choose one)

Solution: $MRS = -\frac{MUx_1}{MUx_2} = -\frac{2/x_1}{1} = -\frac{2}{x_1}$. The economic interpretation of the MRS is that it represents the rate at which Mirabella would trade-off between goods while remaining indifferent. Geometrically, it is the slope of the indifference curve at that bundle. $MRS(8, 8) = -2/8 = -1/4$. Since at that bundle, $MUx_1 = 1/4 < 1 = MUx_2$, x_2 is locally more valuable.

c) (5 points) Write down two secrets to happiness that determine Mirabella's optimal choice (two equations). Provide the geometric interpretation of the conditions in the commodity space.

Solution: The two secrets to happiness are

$$\text{(1) Budget Constraint} : x_1 p_1 + x_2 p_2 = m, \text{ i.e. } 2x_1 + 2x_2 = 20$$

$$\text{(2) Equating Bang-Per-Buck} : \frac{MUx_1}{p_1} = \frac{MUx_2}{p_2}, \text{ i.e. } \frac{2/x_1}{2} = \frac{1}{2}$$

Note that (2) above is equivalent to $MRS = -\frac{p_1}{p_2}$. Using either is fine. The geometric interpretation of (1) is that the optimal bundle is on the budget constraint. The economic interpretation of (1) is that you spend all of your money. The geometric interpretation of (2) is that the optimal bundle is

where the indifference curve and budget constraint have the same slope (i.e. the indifference curve is tangent to the budget constraint).

d) (4 points) Find Mirabella's optimal choice (two numbers). Is solution interior? (yes-no answer)

Solution: To solve, note that (2) above implied that $\frac{2/x_1}{2} = \frac{1}{2}$, which, when solved, yields $x_1^* = 2$. Plugging $x_1 = 2$ into (1) gives us that $2 \cdot 2 + 2x_2 = 20$, which implies that $x_2^* = 8$. Therefore the optimal bundle is $(2, 8)$. This is an interior solution as it includes a strictly positive quantity of each good. You can also see this in the diagram in the solution to a).

e) (4 points) Suppose the price of a chocolate candy bar goes down to $p_1 = 1$, while other price $p_2 = 2$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good? (pick one)

Solution: Now we have the following:

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \quad \Leftrightarrow \quad \frac{2/x_1}{1} = \frac{1}{2} \quad \Leftrightarrow \quad x_1^* = 4$$
$$x_1p_1 + x_2p_2 = m \quad \Leftrightarrow \quad 4 \cdot 1 + x_2 \cdot 2 = 20 \quad \Leftrightarrow \quad x_2^* = 8$$

Therefore the optimal choice is $(4, 8)$. Since demand for the chocolate candy bar x_1^* went up (from 2 to 4) when the price was reduced, the chocolate candy bar is an ordinary good.

f) (4 points) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Solution:

To decompose the change in demand, we have to consider an auxiliary/Slutsky step in which we give the agent exactly enough income to purchase the original bundle (2,8) at the new prices:

$$m' = 1 \cdot 2 + 2 \cdot 8 = 18$$

We then consider what the agent's optimal bundle is given that income level, m', and the new prices.

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \quad \Leftrightarrow \quad \frac{2/x_1}{1} = \frac{1}{2} \quad \Leftrightarrow \quad x_1^* = 4$$
$$x_1p_1 + x_2p_2 = m' \quad \Leftrightarrow \quad 4 \cdot 1 + x_2 \cdot 2 = 18 \quad \Leftrightarrow \quad x_2^* = 7$$

Then the substitution effect is the auxiliary/Slutsky x_1^* minus the original x_1^*, which equals $4-2 = 2$. The income effect is the new x_1^* minus the auxiliary/Slutsky x_1^*, which equals $4 - 4 = 0$. Therefore, $SE = 2$ and $IE = 0$.

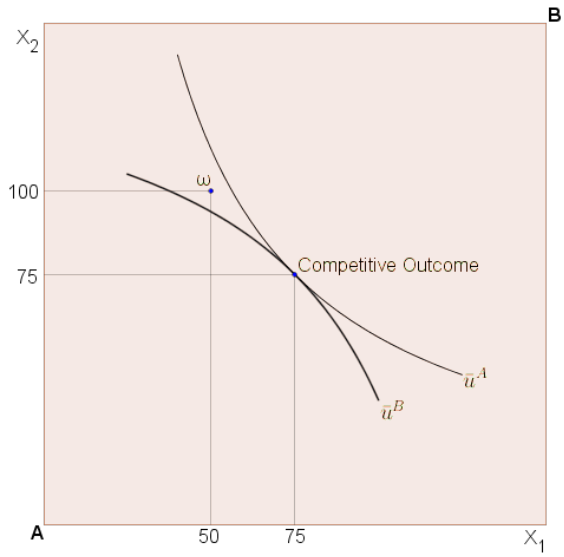
Problem 2. (Equilibrium) (20 points)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles, x_1, and flowers, x_2. Adalia's initial endowment of the commodities is $\omega^A = (50, 100)$ and Brianna's endowment is $\omega^B = (100, 50)$. Adalia and Briana's utility functions are given by, for $i = A, B$,

$$U^i(x_1, x_2) = 2 \ln x_1 + 2 \ln x_2$$

a) (3 points) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

Solution: See diagram below (which also includes solutions for later parts of this question).



b) (3 points) Give a definition of a Pareto efficient allocation (one sentence).

Solution: An allocation is Pareto efficient if it is impossible to make one person better off without making another worse off.

c) (3 points) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide arguments that demonstrate the necessity and sufficiency of the MRS condition for Pareto efficiency.

Solution: The equivalent condition is that $MRS^A = MRS^B$, i.e. all agents have the same MRS at the Pareto efficient allocation. The arguments for necessity and suffiency of the MRS condition are:

- Necessity: If MRS are not equal at a given point then the indifference curves through that point have different slopes. This implies that there exists a lens-shape region between the two indifference curves (better for both people). Any point inside of that region is a Pareto improvement over the original given point. Therefore, if MRS are not equal at a point, that point isn't Pareto optimal. Hence the MRS condition is necessary for Pareto optimality.
- Sufficiency: If the MRS of the agents are the same, then everything down and to the left of A's indifference curve is strictly worse for A. Everything up and to the right of B's indifference curve is stricty worse for B. Every point in the edgeworth box is either down and to the left of A's indifference curve or up and to the right of B's indifference curve, or both. Hence there is no feasible alternative allocation that makes one agent better off without hurting the other, and $MRS^A = MRS^B$ is a sufficient condition for Pareto efficiency.

d) (8 points) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

Solution: To solve this, we normalize $p_2 = 1$. (You could also normalize $p_1 = 1$ and solve for p_2 if you prefer.) Then, to solve for p_1, we use Cobb-Douglas formulas and clear the market for x_1 (you

could clear the market for x_2 if you prefer):

$$\begin{aligned}
\text{Demand for Good 1} &= \text{Supply for Good 1} \\
\frac{a}{a+b}\frac{m^A}{p_1} + \frac{a}{a+b}\frac{m^B}{p_1} &= 50 + 100 \\
\frac{2}{2+2}\frac{50p_1 + 100p_2}{p_1} + \frac{2}{2+2}\frac{100p_1 + 50p_2}{p_1} &= 150 \\
\frac{1}{2}\frac{50p_1 + 100}{p_1} + \frac{1}{2}\frac{100p_1 + 50}{p_1} &= 150 \\
50p_1 + 100 + 100p_1 + 50 &= 300p_1 \\
150 &= 150p_1 \\
p_1 &= 1
\end{aligned}$$

Now that we know that $p_1 = p_2 = 1$, we simply plug into the Cobb-Douglas demand formulas to solve for the demands:

$$\begin{aligned}
x_1^A &= \frac{a}{a+b}\frac{m^A}{p_1} = \frac{1}{2}\frac{50+100}{1} = 75 \\
x_2^A &= \frac{b}{a+b}\frac{m^A}{p_2} = \frac{1}{2}\frac{50+100}{1} = 75 \\
x_1^B &= \frac{a}{a+b}\frac{m^B}{p_1} = \frac{1}{2}\frac{100+50}{1} = 75 \\
x_2^B &= \frac{b}{a+b}\frac{m^B}{p_2} = \frac{1}{2}\frac{100+50}{1} = 75
\end{aligned}$$

Therefore the market outcome is $p_1 = p_2 = 1$ and $x_1^A = x_2^A = x_1^B = x_2^B = 75$. This is depicted in the diagram above in part a). At this outcome, $MRS^A = -\frac{2/75}{2/75} = -1 = MRS^B$. Therefore we have verified that the equilibrium is Pareto efficient.

e) (3 points) Harder: Using (one of) the secrets of happiness, prove that a competitive equilibrium is Pareto efficient in any economy.

Solution: The secret to happiness for A is that $MRS^A = -p_1/p_2$. The secret to happiness for B is that $MRS^B = -p_1/p_2$. Since the secrets to happiness dictate that each sets her MRS equal to $-p_1/p_2$, it follows that $MRS^A = MRS^B$.

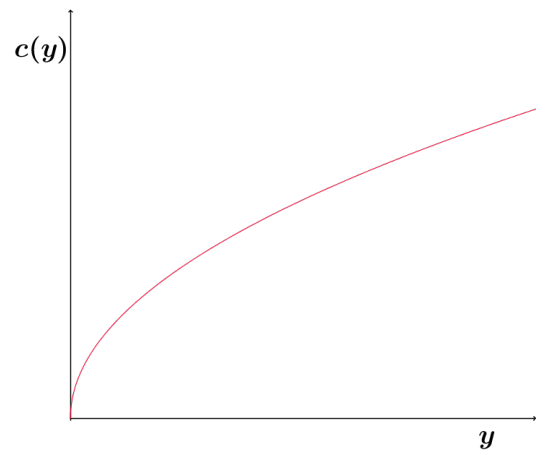
Problem 3. (Short Questions) (20 points)

a) (4 points) Using λ argument, prove that Cobb-Douglas production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

Solution: The λ argument is as follows:

$$F(\lambda K, \lambda L) = 2(\lambda K)(\lambda L) = \lambda^2 2KL = \lambda^2 F(K, L) > \lambda F(K, l) \text{ for all } \lambda > 1$$

This implies IRS. As for $c(y)$, note that you cannot actually do the calculations without the wage rates for capital and labor. However, we know that the cost function is increasing in output. We also know that, because of IRS, it should be concave. Therefore, any function you can draw that is increasing and concave is valid. One example:



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Solution: $TC = 4y^2 + 4$, $MC = 8y$, $ATC = 4y + 4/y$. To solve for y^{MES}, set $ATC = MC$ (note you can also set the derivative of ATC with respect to y equal to zero to find y^{MES}).

$$ATC = MC \;\Leftrightarrow\; 4y + 4/y = 8y \;\Leftrightarrow\; y^{MES} = 1$$

Plugging that into the ATC gives us that $ATC^{MES} = 4 + 4 = 8$. In the long-run, free-entry equilibrium, we know that each firm produces y^{MES} and equilibrium price is ATC^{MES}. Therefore

$$\begin{aligned} \text{Supply} &= \text{Demand} \\ N \cdot y^{MES} &= 16 - ATC^{MES} \\ N &= 8 \end{aligned}$$

So we would expect 8 firms in this equilibrium.

c) (4 points) Suppose a Bernoulli utility function is $u(x) = x^2$, and two states are equally likely (probability 1/2). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery (0,2) (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

Solution: von Neuman-Morgenstern utility function: $U(x_g, x_b) = 1/2x_g^2 + 1/2x_b^2$.
Expected value of lottery: EV=$0 + 1/2 * 2 = 1$.
certainty equivalent, CE makes $U(CE, CE) = CE^2$ = Expected Utility $= 0 + 1/2 * 2^2 = 2$, so CE=$\sqrt{2}$.
CE > EV, because this Bernoulli utility function represents preference of a risk lover, and risk lover enjoys the lottery more than its expected value (he loves uncertainty).

d) (4 points) Find Herfindahl-Hirschman Index (HHI) for industry with N = 100 identical firms (one number). Is the industry concentrated?

Solution: Each firm occupies 1% market share. So HHI= $100 * 1^2 = 100 < 1800$. This industry is not concentrated.

e) (4 points) Derive formula for the present value of perpetuity. *Solution:* Suppose the interest

rate is r and the constant payment of the asset is x.

$$\begin{aligned}
PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \dots \\
&= \frac{x}{1+r} + \frac{1}{1+r}[\frac{x}{1+r} + \frac{x}{(1+r)^2} + \dots] \\
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\Rightarrow & PV = \frac{x}{r}
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Problem 4. (Market Power) (15 points)
Consider an industry with inverse demand $p(y) = 12 - y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) **(2 points)** What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

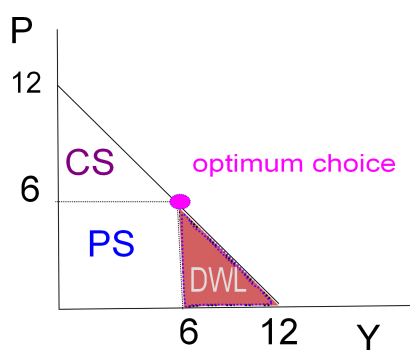
Solution: TS=$1/2 * 12 * 12 = 72$.

b) **(4 points)**Write down monopolys profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

Solution: $\pi = p(y)y - TC((y) = (12 - y)y - TC(y)$.
Derive: $\frac{\partial \pi}{\partial y} = MR - MC = 12 - 2y = 0$. (Write MR=MC explicitly.)
MR is decreasing in y and MC is increasing in y. So when MR>MC, the monopoly can increase its profits by producing more until MR=MC; when MR<MC, the last unit produced has negative profit, the monopoly can increase its profits by reducing production until MR=MC.

c) **(5 points)** Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

Solution: From (b),$12 - 2y = 0 \rightarrow y^M = 6, p^M = 6$, DWL=$1/2 * 6 * 6 = 18$, elasticity at the optimum choice: $\epsilon = \frac{1}{p'(y)}\frac{p}{y} = -1$.



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Solution: We need to derive the best response function first. Suppose there are three firms, 1,2,3.

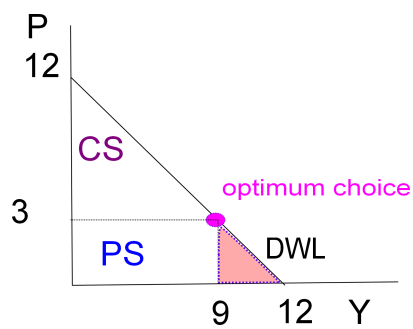
For firm 1,

$$\max_{y_1} \pi_1 = (12 - y_1 - y_2 - y_3)y_1$$
$$\Rightarrow \frac{\partial \pi_1}{\partial y_1} = 12 - 2y_1 - (y_2 + y_3) = 0$$
$$\Rightarrow y_1^* = \frac{12 - (y_2 + y_3)}{2}.$$

At the Nash equilibrium, since three firms are identical, $y_1^* = y_2^* = y_3^*$, plug this information into the best response function

$$\Rightarrow y_1^* = \frac{12 - 2y_1^*}{2} = 6 - y_1^*, \Rightarrow y_1^* = 3$$

So the aggregate output is $3y_1^* = 9, p = 3$, DWL=$1/2 * 3 * 3 = 4.5$.



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Lucy is addicted to nicotine. Her utility from smoking c cigarettes (net of their cost) is given by

$$U^L(c) = 2\ln c - c$$

Her sister Taja prefers healthy lifestyle, her favorite commodity is orange juice, j. The two sisters live together and Taja is exposed to second-hand smoke and hence her utility is adversely affected by Lucy consumption of cigarettes c. In particular, her utility function (net of cost of orange juice) is given by

$$U^T(j, c) = \ln(j - c) - j$$

.

a) **(4 points)** Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers).

Solution: For Lucy,

$$\frac{\partial U^L(c)}{\partial c} = \frac{2}{c} - 1 = 0 \Rightarrow c^* = 2$$

For Taja,

$$\frac{\partial U^T(j, c)}{\partial j} = \frac{1}{j - c} - 1 = 0 \Rightarrow j^* = c + 1 = 3.$$

b) **(6 points)** Find the Pareto efficient level of c and j: Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence)

Solution: Joint utility is $U = 2\ln c + \ln(j-c) - c - j$.

$$\frac{\partial U}{\partial c} = \frac{2}{c} - \frac{1}{j-c} - 1 = 0$$
$$\frac{\partial U}{\partial j} = \frac{1}{j-c} - 1 = 0.$$

So $j - c = 1, \rightarrow \tilde{c} = 1, \tilde{j} = 2$. The social optimal $\tilde{c} = 1 < 2$. Because Lucy does not internalize the externality of her cigarettes to Taja, she smokes too much comparing to the social optimal level of c.

Problem 6. (Asymmetric information) (10 points) In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to 1/2 : The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
Seller	6	14
Buyer	10	22

Both parties agree on the price that is in between the value of a buyer and a seller.

a) **(3 points)** Buyers and sellers can perfectly determine the quality of a house before transaction takes place. What is expected total, buyers and sellers surplus (three numbers)

Solution: Two prices are $P_L = 8, P_P = 18$. The *expected* gains to trade (ETS) =1/2*4+1/2*8=6.
Expected Buyer surplus (EBS)=1/2*2+1/2*4=3,
and Expected Seller surplus (ESS) =1/2*2+1/2*4=3.

b) **(7 points)** Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

Solution: With probability 1/2, the expected value of a house to a buyer is EV=1/2∗10+1/2∗22 = 16**(1 point)**, which is larger than 14 (seller's value for plums), so both houses can be sold, and we end up with a pooling equilibrium.**(1 point)**
This pooling equilibrium is sustainable.**(1 point)**
The price is $p = \frac{14+16}{2} = 15$.**(1 point)**
Expected buyer surplus= 1/2*(10-15) + 1/2 (22-15)=1,**(1 point)**
expected seller surplus = 1/2(15-6) + 1/2(15-14)=5.**(1 point)**
So total expected gain to trade is EBS+ESS=6, which is the same as in (a) (full information), this outcome is Pareto efficient.**(1 point)**

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (Group A)

You have 2h to complete the exam. The final consists of 6 questions (25+20+20+15+10+10=100).

Problem 1. (Quasilinaer income effect)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 4, p_2 = 4$, respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2 \ln x_1 + x_2$$

a) In the commodity space plot Mirabella's budget set. Find the slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

b) Find analytically formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of MRS (two sentences). Find the value of MRS at bundle $(x_1, x_2) = (4, 4)$ (one number). At this bundle, which of the two commodities is (locally) more valuable? (chose one)

c) Write down two secrets of happiness that determine Mirabellas's optimal choice (two equation). Provide the geometric interpretation of the conditions in the commodity space.

d) Find Mirabella's optimal choice (two numbers). Is solution interior (yes-no answer).

e) Suppose the price of a chocolate candy bar goes down to $p_1 = 2$, while other price $p_2 = 4$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good (pick one)?

f) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Problem 2. (Equilibrium)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles x_1 and flowers x_2. Adalia initial endowment of the commodities is $\omega^A = (40, 60)$ and Briana endowment is $\omega^B = (60, 40)$. Adalia and Briana utility functions are given by, $i = A, B$

$$U^i(x_1, x_2) = 4 \ln x_1 + 4 \ln x_2$$

a) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

b) Give a definition of a Pareto efficient allocation (one sentence).

c) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide geometric arguments that demonstrate the necessity and sufficiency of MRS condition for Pareto efficiency.

d) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

e) Using (one of) the secrets of happiness prove that a competitive equilibrium is Pareto efficient in any economy.

Problem 3. (Short questions)

a) Using λ argument prove that Cobb-Douglass production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

b) Now consider a firm (different from point a)) with variable cost $c(y) = 2y^2$ and fixed cost $F = 2$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expect in the industry if inverse demand is $D(p) = 10 - p$?

c) Suppose a Bernoulli utility function is $u(x) = x^2$ and two states are equally likely (probability $\frac{1}{2}$). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery $(0, 2)$ (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

d) Find Herfindahl–Hirschman Index (HHI) for industry with $N = 50$ identical firms (one number). Is the industry concentrated?

e) Derive formula for the present value of perpetuity.

Problem 4. (Market Power)

Consider an industry with inverse demand $p(y) = 8 - y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

b) Write down monopoly's profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

c) Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

d) Assuming the same demand function find the individual and the aggregate level of production and the price in the Cournot-Nash equilibrium with $N = 3$ identical firms (give three numbers). Show the deadweight loss in the graph.

Problem 5.(Externality)

Lucy is addicted to nicotine. Her utility from smoking c cigarettes (net of their cost) is given by

$$U^L(c) = 2\ln c - c$$

Her sister Taja prefers healthy lifestyle, her favorite commodity is orange juice, j. The two sisters live together and Taja is exposed to second-hand smoke and hence her utility is adversely affected by Lucy consumption of cigarettes c. In particular, her utility function (net of cost of orange juice) is given by

$$U^T(j, c) = \ln(j - c) - j.$$

a) Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers)

b) Find the Pareto efficient level of c and j. Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence) Hint: Derivative of $\ln(j - c)$ with respect to c is $-\frac{1}{j-c}$.

Problem 6. (Asymmetric information)

In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to $\frac{1}{2}$. The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
Seller	0	12
Buyer	10	18

Both parties agree on the price that is in between the value of a buyer and a seller.

a) Buyers and sellers can perfectly determine the quality of a house before transaction takes place What is expected total, buyers and sellers surplus (three numbers)

b) Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (Group B)

You have 2h to complete the exam. The final consists of 6 questions (25+20+20+15+10+10=100).

Problem 1. (Quasilinaer income effect)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 2, p_2 = 2$, respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2 \ln x_1 + x_2$$

a) In the commodity space plot Mirabella's budget set. Find the slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

b) Find analytically formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of MRS (two sentences). Find the value of MRS at bundle $(x_1, x_2) = (8, 8)$ (one number). At this bundle, which of the two commodities is (locally) more valuable? (chose one)

c) Write down two secrets of happiness that determine Mirabellas's optimal choice (two equation). Provide the geometric interpretation of the conditions in the commodity space.

d) Find Mirabella's optimal choice (two numbers). Is solution interior (yes-no answer).

e) Suppose the price of a chocolate candy bar goes down to $p_1 = 1$, while other price $p_2 = 2$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good (pick one)?

f) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Problem 2. (Equilibrium)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles x_1 and flowers x_2. Adalia initial endowment of the commodities is $\omega^A = (50, 100)$ and Briana endowment is $\omega^B = (100, 50)$. Adalia and Briana utility functions are given by, $i = A, B$

$$U^i(x_1, x_2) = 2 \ln x_1 + 2 \ln x_2$$

a) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

b) Give a definition of a Pareto efficient allocation (one sentence).

c) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide geometric arguments that demonstrate the necessity and sufficiency of MRS condition for Pareto efficiency.

d) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

e) Using (one of) the secrets of happiness prove that a competitive equilibrium is Pareto efficient in any economy.

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a) Using λ argument prove that Cobb-Douglass production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

b) Now consider a firm (different from point a)) with variable cost $c(y) = 4y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expect in the industry if inverse demand is $D(p) = 16 - p$?

c) Suppose a Bernoulli utility function is $u(x) = x^2$ and two states are equally likely (probability $\frac{1}{2}$). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery $(0, 2)$ (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

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Problem 4. (Market Power)

Consider an industry with inverse demand $p(y) = 12 - y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

b) Write down monopoly's profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

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$$U^T(j, c) = \ln(j - c) - j.$$

a) Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers)

b) Find the Pareto efficient level of c and j. Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence) Hint: Derivative of $\ln(j - c)$ with respect to c is $-\frac{1}{j-c}$.

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In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to $\frac{1}{2}$. The value of a home for the two parties depends on its type and is given by

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b) Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

Solutions to Spring 2014 ECON 301 Final Group A

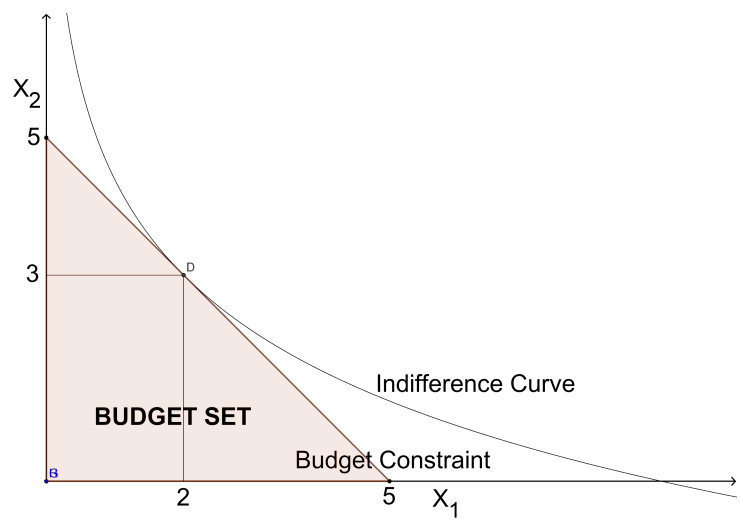
Problem 1. (Quasilinear income effect) (25 points)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 4$ and $p_2 = 4$ respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2 \ln x_1 + x_2$$

a) (4 points) In the commodity space plot Mirabella's budget set. Find slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

Solution: See figure below for Mirabella's budget set - it's the entire shaded region. The slope of the budget line is equal to $-p_1/p_2 = -4/4 = -1$. The economic interpretation of the slope is that it represents the rate at which the market prices allow Mirabella to trade between the two goods.



b) (4 points) Find analytically the formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of the MRS (two sentences). Find the value of the MRS at bundle $(x_1, x_2) = (4, 4)$ (one number). Which of the two commodities is (locally) more valuable? (choose one)

Solution: $MRS = -\frac{MUx_1}{MUx_2} = -\frac{2/x_1}{1} = -\frac{2}{x_1}$. The economic interpretation of the MRS is that it represents the rate at which Mirabella would trade-off between goods while remaining indifferent. Geometrically, it is the slope of the indifference curve at that bundle. $MRS(4, 4) = -2/4 = -1/2$. Since at that bundle, $MUx_1 = 1/2 < 1 = MUx_2$, x_2 is locally more valuable.

c) (5 points) Write down two secrets to happiness that determine Mirabella's optimal choice (two equations). Provide the geometric interpretation of the conditions in the commodity space.

Solution: The two secrets to happiness are

$$\text{(1) Budget Constraint} : x_1 p_1 + x_2 p_2 = m, \text{ i.e. } 4x_1 + 4x_2 = 20$$

$$\text{(2) Equating Bang-Per-Buck} : \frac{MUx_1}{p_1} = \frac{MUx_2}{p_2}, \text{ i.e. } \frac{2/x_1}{4} = \frac{1}{4}$$

Note that (2) above is equivalent to $MRS = -\frac{p_1}{p_2}$. Using either is fine. The geometric interpretation of (1) is that the optimal bundle is on the budget constraint. The economic interpretation of (1) is that you spend all of your money. The geometric interpretation of (2) is that the optimal bundle is

where the indifference curve and budget constraint have the same slope (i.e. the indifference curve is tangent to the budget constraint).

d) (4 points) Find Mirabella's optimal choice (two numbers). Is solution interior? (yes-no answer)

Solution: To solve, note that (2) above implied that $\frac{2/x_1}{4} = \frac{1}{4}$, which, when solved, yields $x_1^* = 2$. Plugging $x_1 = 2$ into (1) gives us that $4 \cdot 2 + 4x_2 = 20$, which implies that $x_2^* = 3$. Therefore the optimal bundle is $(2, 3)$. This is an interior solution as it includes a strictly positive quantity of each good. You can also see this in the diagram in the solution to a).

e) (4 points) Suppose the price of a chocolate candy bar goes down to $p_1 = 2$, while other price $p_2 = 4$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good? (pick one)

Solution: Now we have the following:

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \Leftrightarrow \frac{2/x_1}{2} = \frac{1}{4} \Leftrightarrow x_1^* = 4$$
$$x_1p_1 + x_2p_2 = m \Leftrightarrow 4 \cdot 2 + x_2 \cdot 4 = 20 \Leftrightarrow x_2^* = 3$$

Therefore the optimal choice is $(4, 3)$. Since demand for the chocolate candy bar x_1^* went up (from 2 to 4) when the price was reduced, the chocolate candy bar is an ordinary good.

f) (4 points) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Solution:

To decompose the change in demand, we have to consider an auxiliary/Slutsky step in which we give the agent exactly enough income to purchase the original bundle (2,3) at the new prices:

$$m' = 2 \cdot 2 + 4 \cdot 3 = 16$$

We then consider what the agent's optimal bundle is given that income level, m', and the new prices.

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \Leftrightarrow \frac{2/x_1}{2} = \frac{1}{4} \Leftrightarrow x_1^* = 4$$
$$x_1p_1 + x_2p_2 = m' \Leftrightarrow 4 \cdot 2 + x_2 \cdot 4 = 16 \Leftrightarrow x_2^* = 2$$

Then the substitution effect is the auxiliary/Slutsky x_1^* minus the original x_1^*, which equals $4-2 = 2$. The income effect is the new x_1^* minus the auxiliary/Slutsky x_1^*, which equals $4 - 4 = 0$. Therefore, $SE = 2$ and $IE = 0$.

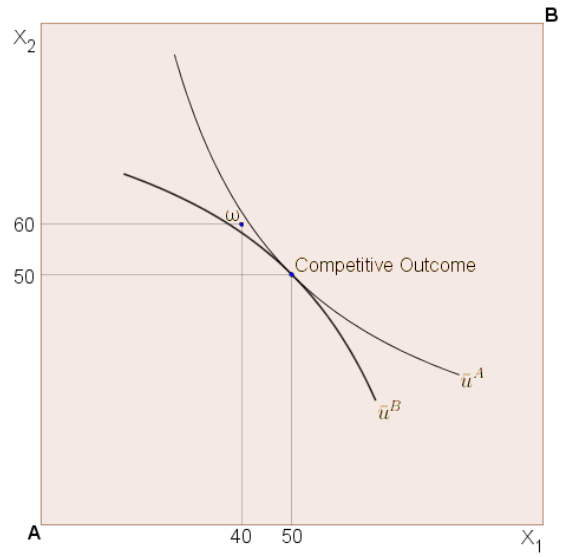
Problem 2. (Equilibrium) (20 points)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles, x_1, and flowers, x_2. Adalia's initial endowment of the commodities is $\omega^A = (40, 60)$ and Brianna's endowment is $\omega^B = (60, 40)$. Adalia and Briana's utility functions are given by, for $i = A, B$,

$$U^i(x_1, x_2) = 4 \ln x_1 + 4 \ln x_2$$

a) (3 points) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

Solution: See diagram below (which also includes solutions for later parts of this question).



b) (3 points) Give a definition of a Pareto efficient allocation (one sentence).

Solution: An allocation is Pareto efficient if it is impossible to make one person better off without making another worse off.

c) (3 points) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide arguments that demonstrate the necessity and sufficiency of the MRS condition for Pareto efficiency.

Solution: The equivalent condition is that $MRS^A = MRS^B$, i.e. all agents have the same MRS at the Pareto efficient allocation. The arguments for necessity and suffiency of the MRS condition are:

- Necessity: If MRS are not equal at a given point then the indifference curves through that point have different slopes. This implies that there exists a lens-shape region between the two indifference curves (better for both people). Any point inside of that region is a Pareto improvement over the original given point. Therefore, if MRS are not equal at a point, that point isn't Pareto optimal. Hence the MRS condition is necessary for Pareto optimality.
- Sufficiency: If the MRS of the agents are the same, then everything down and to the left of A's indifference curve is strictly worse for A. Everything up and to the right of B's indifference curve is stricty worse for B. Every point in the edgeworth box is either down and to the left of A's indifference curve or up and to the right of B's indifference curve, or both. Hence there is no feasible alternative allocation that makes one agent better off without hurting the other, and $MRS^A = MRS^B$ is a sufficient condition for Pareto efficiency.

d) (8 points) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

Solution: To solve this, we normalize $p_2 = 1$. (You could also normalize $p_1 = 1$ and solve for p_2 if you prefer.) Then, to solve for p_1, we use Cobb-Douglas formulas and clear the market for x_1 (you

could clear the market for x_2 if you prefer):

$$\begin{aligned}
\text{Demand for Good 1} &= \text{Supply for Good 1} \\
\frac{a}{a+b}\frac{m^A}{p_1} + \frac{a}{a+b}\frac{m^B}{p_1} &= 40+60 \\
\frac{4}{4+4}\frac{40p_1+60p_2}{p_1} + \frac{4}{4+4}\frac{60p_1+40p_2}{p_1} &= 100 \\
\frac{1}{2}\frac{40p_1+60}{p_1} + \frac{1}{2}\frac{60p_1+40}{p_1} &= 100 \\
40p_1+60+60p_1+40 &= 200p_1 \\
100 &= 100p_1 \\
p_1 &= 1
\end{aligned}$$

Now that we know that $p_1 = p_2 = 1$, we simply plug into the Cobb-Douglas demand formulas to solve for the demands:

$$\begin{aligned}
x_1^A &= \frac{a}{a+b}\frac{m^A}{p_1} = \frac{1}{2}\frac{40+60}{1} = 50 \\
x_2^A &= \frac{b}{a+b}\frac{m^A}{p_2} = \frac{1}{2}\frac{40+60}{1} = 50 \\
x_1^B &= \frac{a}{a+b}\frac{m^B}{p_1} = \frac{1}{2}\frac{60+40}{1} = 50 \\
x_2^B &= \frac{b}{a+b}\frac{m^B}{p_2} = \frac{1}{2}\frac{60+40}{1} = 50
\end{aligned}$$

Therefore the market outcome is $p_1 = p_2 = 1$ and $x_1^A = x_2^A = x_1^B = x_2^B = 50$. This is depicted in the diagram above in part a). At this outcome, $MRS^A = -\frac{4/50}{4/50} = -1 = MRS^B$. Therefore we have verified that the equilibrium is Pareto efficient.

e) (3 points) Harder: Using (one of) the secrets of happiness, prove that a competitive equilibrium is Pareto efficient in any economy.

Solution: The secret to happiness for A is that $MRS^A = -p_1/p_2$. The secret to happiness for B is that $MRS^B = -p_1/p_2$. Since the secrets to happiness dictate that each sets her MRS equal to $-p_1/p_2$, it follows that $MRS^A = MRS^B$.

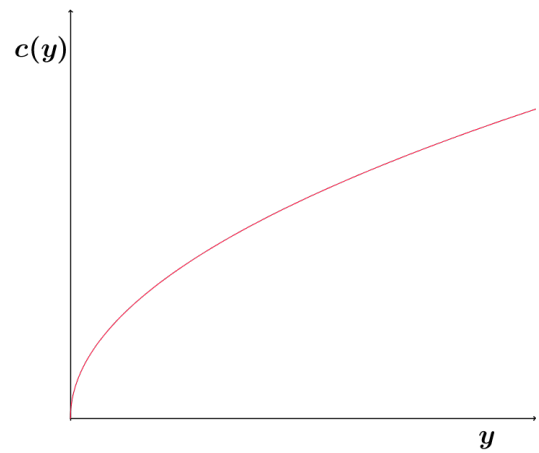
Problem 3. (Short Questions) (20 points)

a) (4 points) Using λ argument, prove that Cobb-Douglas production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

Solution: The λ argument is as follows:

$$F(\lambda K, \lambda L) = 2(\lambda K)(\lambda L) = \lambda^2 2KL = \lambda^2 F(K,L) > \lambda F(K,l) \text{ for all } \lambda > 1$$

This implies IRS. As for $c(y)$, note that you cannot actually do the calculations without the wage rates for capital and labor. However, we know that the cost function is increasing in output. We also know that, because of IRS, it should be concave. Therefore, any function you can draw that is increasing and concave is valid. One example:



b) (4 points) Now consider a firm (different from point a)) with variable cost $c(y) = 4y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expected in the industry if demand is $D(p) = 16 - p$?

Solution: $TC = 4y^2 + 4$, $MC = 8y$, $ATC = 4y + 4/y$. To solve for y^{MES}, set $ATC = MC$ (note you can also set the derivative of ATC with respect to y equal to zero to find y^{MES}).

$$ATC = MC \;\Leftrightarrow\; 4y + 4/y = 8y \;\Leftrightarrow\; y^{MES} = 1$$

Plugging that into the ATC gives us that $ATC^{MES} = 4 + 4 = 8$. In the long-run, free-entry equilibrium, we know that each firm produces y^{MES} and equilibrium price is ATC^{MES}. Therefore

$$\begin{aligned} \text{Supply} &= \text{Demand} \\ N \cdot y^{MES} &= 16 - ATC^{MES} \\ N &= 8 \end{aligned}$$

So we would expect 8 firms in this equilibrium.

c) (4 points) Suppose a Bernoulli utility function is $u(x) = x^2$, and two states are equally likely (probability 1/2). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery (0,2) (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

Solution: von Neuman-Morgenstern utility function: $U(x_g, x_b) = 1/2x_g^2 + 1/2x_b^2$.
Expected value of lottery: EV=$0 + 1/2 * 2 = 1$.
certainty equivalent, CE makes $U(CE, CE) = CE^2$ = Expected Utility $= 0 + 1/2 * 2^2 = 2$, so CE=$\sqrt{2}$.
CE > EV, because this Bernoulli utility function represents preference of a risk lover, and risk lover enjoys the lottery more than its expected value (he loves uncertainty).

d) (4 points) Find Herfindahl-Hirschman Index (HHI) for industry with N = 50 identical firms (one number). Is the industry concentrated?

Solution: Each firm occupies 2% market share. So HHI= $50 * 2^2 = 200 < 1800$. This industry is not concentrated (or the industry is competitive).

e) (4 points) Derive formula for the present value of perpetuity.

Solution: Suppose the interest rate is r and the constant payment of the asset is x.

$$\begin{aligned}
PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \dots \\
&= \frac{x}{1+r} + \frac{1}{1+r}[\frac{x}{1+r} + \frac{x}{(1+r)^2} + \dots] \\
&= \frac{x}{1+r} + \frac{1}{1+r}PV \\
\Rightarrow & PV[1 - \frac{1}{1+r}] = \frac{x}{1+r} \\
\Rightarrow & PV = \frac{x}{r}
\end{aligned}$$

Problem 4. (Market Power) (15 points)
Consider an industry with inverse demand $p(y) = 8-y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) **(2 points)** What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

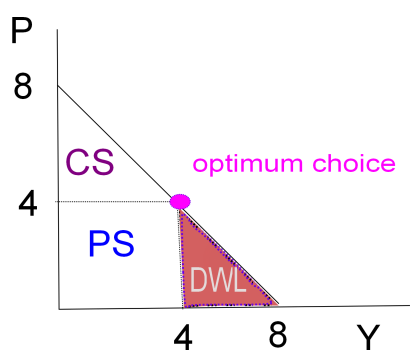
Solution: TS=$1/2 * 8 * 8 = 32$.

b) **(4 points)**Write down monopolys profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

Solution: $\pi = p(y)y - TC((y) = (8-y)y - TC(y)$.
Derive: $\frac{\partial \pi}{\partial y} = MR - MC = 8 - 2y = 0$. (Write MR=MC explicitly.)
MR is decreasing in y and MC is increasing in y. So when MR>MC, the monopoly can increase its profits by producing more until MR=MC; when MR<MC, the last unit produced has negative profit, the monopoly can increase its profits by reducing production until MR=MC.

c) **(5 points)** Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

Solution: From (b),$8 - 2y = 0 \rightarrow y^M = 4, p^M = 4$, DWL=$1/2 * 4 * 4 = 8$, elasticity at the optimum choice: $\epsilon = \frac{1}{p'(y)}\frac{p}{y} = -1$.



d) **(4 points)** Assuming the same demand function find the individual and the aggregate level of production and the price in the Cournot-Nash equilibrium with N = 3 identical firms (give three numbers). Show the deadweight loss in the graph.

Solution: We need to derive the best response function first. Suppose there are three firms, 1,2,3.

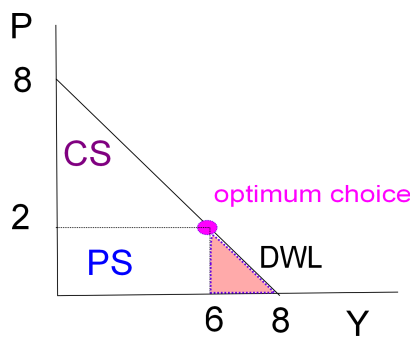
For firm 1,

$$\max_{y_1} \pi_1 = (8 - y_1 - y_2 - y_3)y_1$$
$$\Rightarrow \frac{\partial \pi_1}{\partial y_1} = 8 - 2y_1 - (y_2 + y_3) = 0$$
$$\Rightarrow y_1^* = \frac{8 - (y_2 + y_3)}{2}.$$

At the Nash equilibrium, since three firms are identical, $y_1^* = y_2^* = y_3^*$, plug this information into the best response function

$$\Rightarrow y_1^* = \frac{8 - 2y_1^*}{2} = 4 - y_1^*, \Rightarrow y_1^* = 2$$

So the aggreagate output is $3y_1^* = 6, p = 2$, DWL=2.



Problem 5.(Externality) (10 points)
Lucy is addicted to nicotine. Her utility from smoking c cigarettes (net of their cost) is given by

$$U^L(c) = 2\ln c - c$$

Her sister Taja prefers healthy lifestyle, her favorite commodity is orange juice, j. The two sisters live together and Taja is exposed to second-hand smoke and hence her utility is adversely affected by Lucy consumption of cigarettes c. In particular, her utility function (net of cost of orange juice) is given by

$$U^T(j, c) = \ln(j - c) - j$$

.

a) **(4 points)** Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers).

Solution: For Lucy,

$$\frac{\partial U^L(c)}{\partial c} = \frac{2}{c} - 1 = 0 \Rightarrow c^* = 2$$

For Taja,

$$\frac{\partial U^T(j, c)}{\partial j} = \frac{1}{j - c} - 1 = 0 \Rightarrow j^* = c + 1 = 3.$$

b) **(6 points)** Find the Pareto efficient level of c and j: Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence)

Solution: Joint utility is $U = 2\ln c + \ln(j-c) - c - j$.

$$\frac{\partial U}{\partial c} = \frac{2}{c} - \frac{1}{j-c} - 1 = 0$$
$$\frac{\partial U}{\partial j} = \frac{1}{j-c} - 1 = 0.$$

So $j - c = 1, \rightarrow \tilde{c} = 1, \tilde{j} = 2$. The social optimal $\tilde{c} = 1 < 2$. Because Lucy does not internalize the externality of her cigarettes to Taja, she smokes too much comparing to the social optimal level of c.

Problem 6. (Asymmetric information) (10 points) In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to 1/2 : The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
Seller	0	12
Buyer	10	18

Both parties agree on the price that is in between the value of a buyer and a seller.

a) **(3 points)** Buyers and sellers can perfectly determine the quality of a house before transaction takes place. What is expected total, buyers and sellers surplus (three numbers)

Solution: The *expected* gains to trade (ETS) =1/2*10+1/2*6=8.
Expected Buyer surplus (EBS)=1/2*5+1/2*3=4,
and Expected Seller surplus (ESS) =1/2*5+1/2*3=4.

b) **(7 points)** Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

Solution: With probability 1/2, the expected value of a house to a buyer is EV=1/2∗10+1/2∗18 = 14 **(1 point)**, which is larger than 12 (seller's value for plums), so both houses can be sold, and we end up with a pooling equilibrium **(1 point)**.
This pooling equilibrium is sustainable. **(1 point)**
The price is $p = \frac{12+14}{2} = 13$. **(1 point)**
Expected buyer surplus= 1/2*(10-13) + 1/2 (18-13)=1,**(1 point)**
expected seller surplus = 1/2(13-0) + 1/2(13-12)=7.**(1 point)**
So total expected gain to trade is EBS+ESS=8, which is the same as in (a) (full information), this outcome is Pareto efficient.**(1 point)**

Solutions to Spring 2014 ECON 301 Final Group B

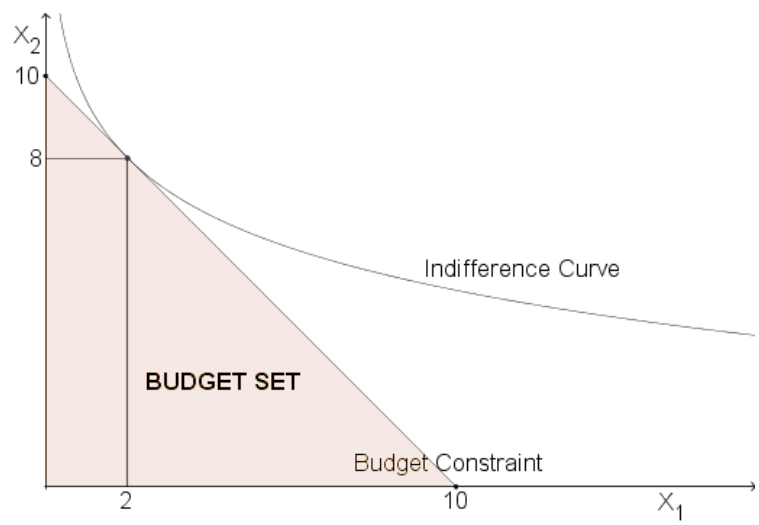
Problem 1. (Quasilinear income effect) (25 points)

Mirabella consumes chocolate candy bars x_1 and fruits x_2. The prices of the two goods are $p_1 = 2$ and $p_2 = 2$ respectively and Mirabella's income is $m = 20$. Her utility function is

$$U(x_1, x_2) = 2 \ln x_1 + x_2$$

a) (4 points) In the commodity space plot Mirabella's budget set. Find slope of budget line (one number). Provide the economic interpretation of the slope (one sentence).

Solution: See figure below for Mirabella's budget set - it's the entire shaded region. The slope of the budget line is equal to $-p_1/p_2 = -2/2 = -1$. The economic interpretation of the slope is that it represents the rate at which the market prices allow Mirabella to trade between the two goods.



b) (4 points) Find analytically the formula that gives Mirabella's MRS for any bundle (x_1, x_2) (a function). Give the economic and the geometric interpretation of the MRS (two sentences). Find the value of the MRS at bundle $(x_1, x_2) = (8, 8)$ (one number). Which of the two commodities is (locally) more valuable? (choose one)

Solution: $MRS = -\frac{MUx_1}{MUx_2} = -\frac{2/x_1}{1} = -\frac{2}{x_1}$. The economic interpretation of the MRS is that it represents the rate at which Mirabella would trade-off between goods while remaining indifferent. Geometrically, it is the slope of the indifference curve at that bundle. $MRS(8, 8) = -2/8 = -1/4$. Since at that bundle, $MUx_1 = 1/4 < 1 = MUx_2$, x_2 is locally more valuable.

c) (5 points) Write down two secrets to happiness that determine Mirabella's optimal choice (two equations). Provide the geometric interpretation of the conditions in the commodity space.

Solution: The two secrets to happiness are

$$\text{(1) Budget Constraint} : x_1 p_1 + x_2 p_2 = m, \text{ i.e. } 2x_1 + 2x_2 = 20$$

$$\text{(2) Equating Bang-Per-Buck} : \frac{MUx_1}{p_1} = \frac{MUx_2}{p_2}, \text{ i.e. } \frac{2/x_1}{2} = \frac{1}{2}$$

Note that (2) above is equivalent to $MRS = -\frac{p_1}{p_2}$. Using either is fine. The geometric interpretation of (1) is that the optimal bundle is on the budget constraint. The economic interpretation of (1) is that you spend all of your money. The geometric interpretation of (2) is that the optimal bundle is

where the indifference curve and budget constraint have the same slope (i.e. the indifference curve is tangent to the budget constraint).

d) (4 points) Find Mirabella's optimal choice (two numbers). Is solution interior? (yes-no answer)

Solution: To solve, note that (2) above implied that $\frac{2/x_1}{2} = \frac{1}{2}$, which, when solved, yields $x_1^* = 2$. Plugging $x_1 = 2$ into (1) gives us that $2 \cdot 2 + 2x_2 = 20$, which implies that $x_2^* = 8$. Therefore the optimal bundle is $(2, 8)$. This is an interior solution as it includes a strictly positive quantity of each good. You can also see this in the diagram in the solution to a).

e) (4 points) Suppose the price of a chocolate candy bar goes down to $p_1 = 1$, while other price $p_2 = 2$ and income $m = 20$ are unchanged. Find the new optimal choice (two numbers). Is a chocolate candy bar an ordinary or Giffen good? (pick one)

Solution: Now we have the following:

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \quad \Leftrightarrow \quad \frac{2/x_1}{1} = \frac{1}{2} \quad \Leftrightarrow \quad x_1^* = 4$$
$$x_1p_1 + x_2p_2 = m \quad \Leftrightarrow \quad 4 \cdot 1 + x_2 \cdot 2 = 20 \quad \Leftrightarrow \quad x_2^* = 8$$

Therefore the optimal choice is $(4, 8)$. Since demand for the chocolate candy bar x_1^* went up (from 2 to 4) when the price was reduced, the chocolate candy bar is an ordinary good.

f) (4 points) Decompose the change in demand for x_1 in points d) and e) into a substitution and income effect.

Solution:

To decompose the change in demand, we have to consider an auxiliary/Slutsky step in which we give the agent exactly enough income to purchase the original bundle (2,8) at the new prices:

$$m' = 1 \cdot 2 + 2 \cdot 8 = 18$$

We then consider what the agent's optimal bundle is given that income level, m', and the new prices.

$$\frac{MUx_1}{p_1} = \frac{MUx_2}{p_2} \quad \Leftrightarrow \quad \frac{2/x_1}{1} = \frac{1}{2} \quad \Leftrightarrow \quad x_1^* = 4$$
$$x_1p_1 + x_2p_2 = m' \quad \Leftrightarrow \quad 4 \cdot 1 + x_2 \cdot 2 = 18 \quad \Leftrightarrow \quad x_2^* = 7$$

Then the substitution effect is the auxiliary/Slutsky x_1^* minus the original x_1^*, which equals $4-2 = 2$. The income effect is the new x_1^* minus the auxiliary/Slutsky x_1^*, which equals $4 - 4 = 0$. Therefore, $SE = 2$ and $IE = 0$.

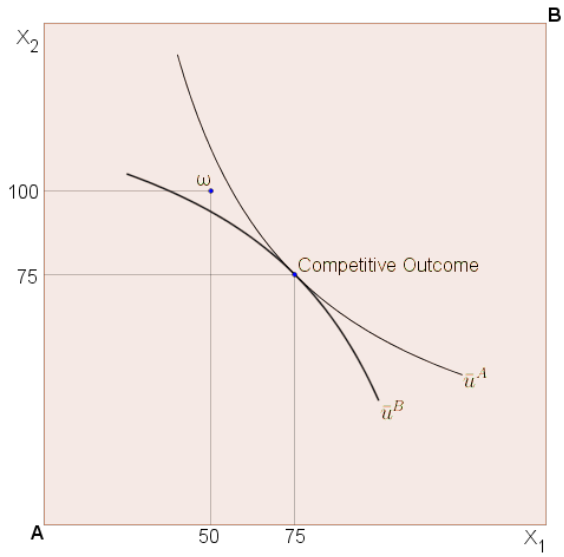
Problem 2. (Equilibrium) (20 points)

Consider an economy with two consumers, Adalia and Briana and two goods: bicycles, x_1, and flowers, x_2. Adalia's initial endowment of the commodities is $\omega^A = (50, 100)$ and Brianna's endowment is $\omega^B = (100, 50)$. Adalia and Briana's utility functions are given by, for $i = A, B$,

$$U^i(x_1, x_2) = 2 \ln x_1 + 2 \ln x_2$$

a) (3 points) Plot an Edgeworth box and mark the point that corresponds to initial endowments.

Solution: See diagram below (which also includes solutions for later parts of this question).



b) (3 points) Give a definition of a Pareto efficient allocation (one sentence).

Solution: An allocation is Pareto efficient if it is impossible to make one person better off without making another worse off.

c) (3 points) Give a (general) equivalent condition for Pareto efficiency in terms of MRS. Provide arguments that demonstrate the necessity and sufficiency of the MRS condition for Pareto efficiency.

Solution: The equivalent condition is that $MRS^A = MRS^B$, i.e. all agents have the same MRS at the Pareto efficient allocation. The arguments for necessity and suffiency of the MRS condition are:

- Necessity: If MRS are not equal at a given point then the indifference curves through that point have different slopes. This implies that there exists a lens-shape region between the two indifference curves (better for both people). Any point inside of that region is a Pareto improvement over the original given point. Therefore, if MRS are not equal at a point, that point isn't Pareto optimal. Hence the MRS condition is necessary for Pareto optimality.
- Sufficiency: If the MRS of the agents are the same, then everything down and to the left of A's indifference curve is strictly worse for A. Everything up and to the right of B's indifference curve is stricty worse for B. Every point in the edgeworth box is either down and to the left of A's indifference curve or up and to the right of B's indifference curve, or both. Hence there is no feasible alternative allocation that makes one agent better off without hurting the other, and $MRS^A = MRS^B$ is a sufficient condition for Pareto efficiency.

d) (8 points) Find competitive equilibrium (six numbers). Depict the obtained equilibrium in the Edgeworth box. Using MRS condition verify that the equilibrium is Pareto efficient.

Solution: To solve this, we normalize $p_2 = 1$. (You could also normalize $p_1 = 1$ and solve for p_2 if you prefer.) Then, to solve for p_1, we use Cobb-Douglas formulas and clear the market for x_1 (you

could clear the market for x_2 if you prefer):

$$\begin{aligned}
\text{Demand for Good 1} &= \text{Supply for Good 1} \\
\frac{a}{a+b}\frac{m^A}{p_1} + \frac{a}{a+b}\frac{m^B}{p_1} &= 50 + 100 \\
\frac{2}{2+2}\frac{50p_1 + 100p_2}{p_1} + \frac{2}{2+2}\frac{100p_1 + 50p_2}{p_1} &= 150 \\
\frac{1}{2}\frac{50p_1 + 100}{p_1} + \frac{1}{2}\frac{100p_1 + 50}{p_1} &= 150 \\
50p_1 + 100 + 100p_1 + 50 &= 300p_1 \\
150 &= 150p_1 \\
p_1 &= 1
\end{aligned}$$

Now that we know that $p_1 = p_2 = 1$, we simply plug into the Cobb-Douglas demand formulas to solve for the demands:

$$\begin{aligned}
x_1^A &= \frac{a}{a+b}\frac{m^A}{p_1} = \frac{1}{2}\frac{50+100}{1} = 75 \\
x_2^A &= \frac{b}{a+b}\frac{m^A}{p_2} = \frac{1}{2}\frac{50+100}{1} = 75 \\
x_1^B &= \frac{a}{a+b}\frac{m^B}{p_1} = \frac{1}{2}\frac{100+50}{1} = 75 \\
x_2^B &= \frac{b}{a+b}\frac{m^B}{p_2} = \frac{1}{2}\frac{100+50}{1} = 75
\end{aligned}$$

Therefore the market outcome is $p_1 = p_2 = 1$ and $x_1^A = x_2^A = x_1^B = x_2^B = 75$. This is depicted in the diagram above in part a). At this outcome, $MRS^A = -\frac{2/75}{2/75} = -1 = MRS^B$. Therefore we have verified that the equilibrium is Pareto efficient.

e) (3 points) Harder: Using (one of) the secrets of happiness, prove that a competitive equilibrium is Pareto efficient in any economy.

Solution: The secret to happiness for A is that $MRS^A = -p_1/p_2$. The secret to happiness for B is that $MRS^B = -p_1/p_2$. Since the secrets to happiness dictate that each sets her MRS equal to $-p_1/p_2$, it follows that $MRS^A = MRS^B$.

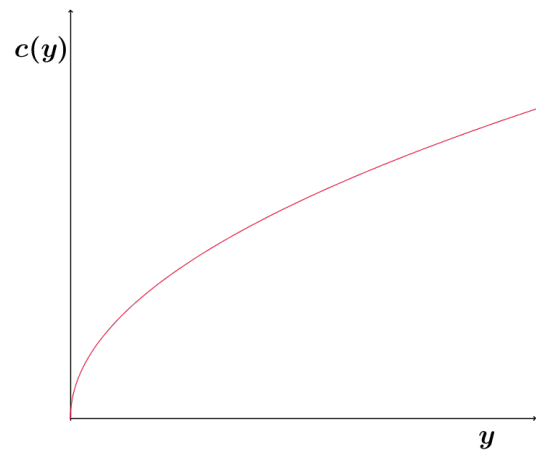
Problem 3. (Short Questions) (20 points)

a) (4 points) Using λ argument, prove that Cobb-Douglas production function $y = 2KL$ exhibits increasing returns to scale. Without any calculations, sketch total cost function $c(y)$ corresponding to the production function.

Solution: The λ argument is as follows:

$$F(\lambda K, \lambda L) = 2(\lambda K)(\lambda L) = \lambda^2 2KL = \lambda^2 F(K, L) > \lambda F(K, l) \text{ for all } \lambda > 1$$

This implies IRS. As for $c(y)$, note that you cannot actually do the calculations without the wage rates for capital and labor. However, we know that the cost function is increasing in output. We also know that, because of IRS, it should be concave. Therefore, any function you can draw that is increasing and concave is valid. One example:



b) (5 points) Now consider a firm (different from point a)) with variable cost $c(y) = 4y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). In a long-run equilibrium with free entry how many firms should be expected in the industry if demand is $D(p) = 16 - p$?

Solution: $TC = 4y^2 + 4$, $MC = 8y$, $ATC = 4y + 4/y$. To solve for y^{MES}, set $ATC = MC$ (note you can also set the derivative of ATC with respect to y equal to zero to find y^{MES}).

$$ATC = MC \;\Leftrightarrow\; 4y + 4/y = 8y \;\Leftrightarrow\; y^{MES} = 1$$

Plugging that into the ATC gives us that $ATC^{MES} = 4 + 4 = 8$. In the long-run, free-entry equilibrium, we know that each firm produces y^{MES} and equilibrium price is ATC^{MES}. Therefore

$$\begin{aligned} \text{Supply} &= \text{Demand} \\ N \cdot y^{MES} &= 16 - ATC^{MES} \\ N &= 8 \end{aligned}$$

So we would expect 8 firms in this equilibrium.

c) (4 points) Suppose a Bernoulli utility function is $u(x) = x^2$, and two states are equally likely (probability 1/2). Write down the corresponding von Neuman-Morgenstern utility function. Find the certainty equivalent and the expected value of lottery (0,2) (two numbers). Which of the two is bigger and why? (two numbers and one sentence.)

Solution: von Neuman-Morgenstern utility function: $U(x_g, x_b) = 1/2x_g^2 + 1/2x_b^2$.
Expected value of lottery: EV=$0 + 1/2 * 2 = 1$.
certainty equivalent, CE makes $U(CE, CE) = CE^2$ = Expected Utility $= 0 + 1/2 * 2^2 = 2$, so CE=$\sqrt{2}$.
CE > EV, because this Bernoulli utility function represents preference of a risk lover, and risk lover enjoys the lottery more than its expected value (he loves uncertainty).

d) (4 points) Find Herfindahl-Hirschman Index (HHI) for industry with N = 100 identical firms (one number). Is the industry concentrated?

Solution: Each firm occupies 1% market share. So HHI= $100 * 1^2 = 100 < 1800$. This industry is not concentrated.

e) (4 points) Derive formula for the present value of perpetuity. *Solution:* Suppose the interest

rate is r and the constant payment of the asset is x.

$$\begin{aligned}
PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \dots \\
&= \frac{x}{1+r} + \frac{1}{1+r}[\frac{x}{1+r} + \frac{x}{(1+r)^2} + \dots] \\
&= \frac{x}{1+r} + \frac{1}{1+r}PV \\
\Rightarrow & PV[1 - \frac{1}{1+r}] = \frac{x}{1+r} \\
\Rightarrow & PV = \frac{x}{r}
\end{aligned}$$

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Consider an industry with inverse demand $p(y) = 12 - y$, and a monopoly with cost function $TC(y) = 0$ who cannot discriminate.

a) **(2 points)** What are the total gains-to-trade (or potential total surplus) in this industry? (give one number)

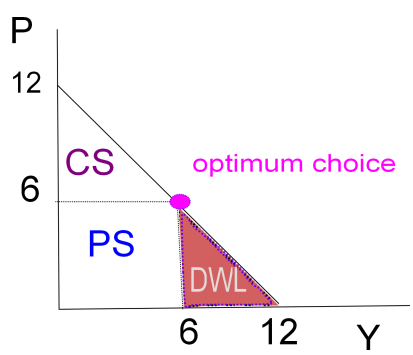
Solution: TS=$1/2 * 12 * 12 = 72$.

b) **(4 points)**Write down monopolys profit function. Derive the condition on MR and MC that gives profit maximizing level of production. Provide economic interpretation of this condition.

Solution: $\pi = p(y)y - TC((y) = (12 - y)y - TC(y)$.
Derive: $\frac{\partial \pi}{\partial y} = MR - MC = 12 - 2y = 0$. (Write MR=MC explicitly.)
MR is decreasing in y and MC is increasing in y. So when MR>MC, the monopoly can increase its profits by producing more until MR=MC; when MR<MC, the last unit produced has negative profit, the monopoly can increase its profits by reducing production until MR=MC.

c) **(5 points)** Find the level of production, the price, the deadweight loss and the elasticity of the demand at optimum (four numbers). Illustrate the choice in a graph.

Solution: From (b),$12 - 2y = 0 \rightarrow y^M = 6, p^M = 6$, DWL=$1/2 * 6 * 6 = 18$, elasticity at the optimum choice: $\epsilon = \frac{1}{p'(y)}\frac{p}{y} = -1$.



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Solution: We need to derive the best response function first. Suppose there are three firms, 1,2,3.

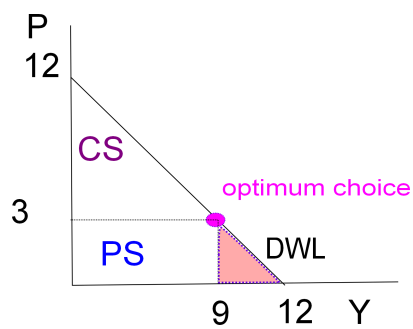
For firm 1,

$$\max_{y_1} \pi_1 = (12 - y_1 - y_2 - y_3)y_1$$
$$\Rightarrow \frac{\partial \pi_1}{\partial y_1} = 12 - 2y_1 - (y_2 + y_3) = 0$$
$$\Rightarrow y_1^* = \frac{12 - (y_2 + y_3)}{2}.$$

At the Nash equilibrium, since three firms are identical, $y_1^* = y_2^* = y_3^*$, plug this information into the best response function

$$\Rightarrow y_1^* = \frac{12 - 2y_1^*}{2} = 6 - y_1^*, \Rightarrow y_1^* = 3$$

So the aggregate output is $3y_1^* = 9, p = 3$, DWL=$1/2 * 3 * 3 = 4.5$.



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Her sister Taja prefers healthy lifestyle, her favorite commodity is orange juice, j. The two sisters live together and Taja is exposed to second-hand smoke and hence her utility is adversely affected by Lucy consumption of cigarettes c. In particular, her utility function (net of cost of orange juice) is given by

$$U^T(j, c) = \ln(j - c) - j$$

.

a) **(4 points)** Market outcome: Find consumption of cigarettes c that maximizes the utility of Lucy and the amount of orange juice chosen by Taja (assuming c is optimal for Lucy) (two numbers).

Solution: For Lucy,

$$\frac{\partial U^L(c)}{\partial c} = \frac{2}{c} - 1 = 0 \Rightarrow c^* = 2$$

For Taja,

$$\frac{\partial U^T(j, c)}{\partial j} = \frac{1}{j - c} - 1 = 0 \Rightarrow j^* = c + 1 = 3.$$

b) **(6 points)** Find the Pareto efficient level of c and j: Is the value of c higher or smaller than in a)? Why? (two numbers + one sentence)

Solution: Joint utility is $U = 2\ln c + \ln(j-c) - c - j$.

$$\frac{\partial U}{\partial c} = \frac{2}{c} - \frac{1}{j-c} - 1 = 0$$
$$\frac{\partial U}{\partial j} = \frac{1}{j-c} - 1 = 0.$$

So $j - c = 1, \rightarrow \tilde{c} = 1, \tilde{j} = 2$. The social optimal $\tilde{c} = 1 < 2$. Because Lucy does not internalize the externality of her cigarettes to Taja, she smokes too much comparing to the social optimal level of c.

Problem 6. (Asymmetric information) (10 points) In Shorewood Hills area there are two types of homes: lemons (bad quality homes) and plums (good quality ones). The fraction of lemons is equal to 1/2 : The value of a home for the two parties depends on its type and is given by

	Lemon	Plum
Seller	6	14
Buyer	10	22

Both parties agree on the price that is in between the value of a buyer and a seller.

a) **(3 points)** Buyers and sellers can perfectly determine the quality of a house before transaction takes place. What is expected total, buyers and sellers surplus (three numbers)

Solution: Two prices are $P_L = 8, P_P = 18$. The *expected* gains to trade (ETS) =1/2*4+1/2*8=6.
Expected Buyer surplus (EBS)=1/2*2+1/2*4=3,
and Expected Seller surplus (ESS) =1/2*2+1/2*4=3.

b) **(7 points)** Now assume that the buyers are not able to determine quality of a house. Find the price of a house, and the expected buyers and sellers surplus (three numbers). Is a pooling equilibrium sustainable, or will this market result in a separating equilibrium? Is outcome Pareto efficient (why or why not)?

Solution: With probability 1/2, the expected value of a house to a buyer is EV=1/2∗10+1/2∗22 = 16**(1 point)**, which is larger than 14 (seller's value for plums), so both houses can be sold, and we end up with a pooling equilibrium.**(1 point)**
This pooling equilibrium is sustainable.**(1 point)**
The price is $p = \frac{14+16}{2} = 15$.**(1 point)**
Expected buyer surplus= 1/2*(10-15) + 1/2 (22-15)=1,**(1 point)**
expected seller surplus = 1/2(15-6) + 1/2(15-14)=5.**(1 point)**
So total expected gain to trade is EBS+ESS=6, which is the same as in (a) (full information), this outcome is Pareto efficient.**(1 point)**

Econ 301

Intermediate Microeconomics

Prof. Marek Weretka

Final Exam (Group A)

You have 2h to complete the exam. The exam consists of 6 questions (20,15, 15, 15, 15 and 20 points)

Problem 1. (Consumer Choice)

Lionel watches movies x_1 while drinking beer, x_2. His utility function from consuming the two types of goods is given by

$$U(x_1, x_2) = 2\ln(x_1) + \ln(x_2)$$

a) Plot Lionel's indifference curve map (graph). Find his MRS analytically (give formula). Find the value of MRS at the consumption bundle $(2, 4)$ and depict it in the graph.

b) Using "magic formulas," find the optimal level of consumption of x_1 and x_2 if $p_1 = p_2 = 4$ and $m = 15$. Plot carefully the optimal point along with the budget line and the indifference curve passing through the optimal point (a graph + two numbers).

c) Argue that the commodities are neither gross complements nor gross substitutes (argue using the magic formula, one sentence).

For the rest of the problem suppose that Lionel's preferences change and now they are given by $U(x_1, x_2) = 3x_1 + 3x_2$.

d) In a separate graph plot his indifference curves, and find MRS for consumption bundle $(2, 1)$. (a graph +one number).

e) Find Lionel's optimal choice given prices $p_1 = p_2 = 4$ and $m = 15$ (two numbers). Is the optimal choice unique? (yes/no answer)

Problem 2. (Equilibrium with Intertemporal Choice)

Consider an intertemporal choice problem with Ambrosia and Fergus. Ambrosia is an athlete with income of 4 when young and 1 when old, i.e, $\omega^A = (4, 1)$. Fergus is a manager who earns 1 when young and 4 when old, i.e., his endowment is $\omega^F = (1, 4)$. Ambrosia and Fergus have the same utility function given by

$$U^i(x_1^i, x_2^i) = \frac{1}{2}\ln(x_1^i) + \frac{1}{4}\ln(x_2^i)$$

where $i = A, F$.

a) Plot the Edgeworth box and mark the point corresponding to endowments of Ambrosia and Fergus (graph).

b) Give a general economic definition of Pareto efficiency (one sentence). Write the equivalent condition for Pareto efficiency in terms of MRS (equation). Use your equation to verify whether the endowment is Pareto efficient. (argue formally using your condition)

c) Find all the allocations that are Pareto efficient (derive formula for the contract curve). Plot the contract curve in the Edgeworth box.

d) Find the competitive equilibrium. Calculate borrowing and savings for both agents in your equilibrium. (Hint: use that for the intertemporal choice $1 + r = \frac{p_1}{p_2}$).

Problem 3. (Technology)

Suppose a producer has access to the technology given by the Cobb-Douglass production function $y = 2K^{\frac{1}{4}}L^{\frac{1}{4}}$.

a) What can you say about the returns to scale (chose: IRS, CRS or DRS) and MPK (chose: increasing, constant, or decreasing).

b) Find the (variable) cost function $c(y)$ given that the prices of inputs are $w_K = w_L = 2$ (give a function).

For the rest of the problem suppose that in order to have access to the technology, the producer first needs to pay fixed cost $F = 9$ and hence the total cost is given by $TC = 9 + c(y)$.

c) Find the supply function of the individual competitive firm and plot it in the graph (give the formula, in the graph mark the prices for which the market will not open).

d) Assume that the producers are competitive and there is free entry. Determine the number of firms operating in the industry in the long run if the demand is $D(p) = 24 - p$ (one number).

Problem 4. (Short Questions)

a) A Bernoulli utility function is $u(x) = x^2$ and there are two states of the world which are equally likely. Find the certainty equivalent and the expected value of lottery $(1, 7)$ (two numbers). Which of the two is bigger (choose one)? Explain why (one sentence).

b) Consider an industry in which the market share of the dominant firm is 30%, while the market shares of the ten other firms is 10% each. Find HHI index for this industry. Is the industry competitive, moderately concentrated, or concentrated?

c) Suppose there are two types of managers: talented with productivity 9 and not talented with productivity 3. The types are unobservable to employers and the competitive wage is given by the expected productivity of the manager. Is an MBA diploma from a program that takes one year to complete $e = 1$ a credible signal if the cost of effort for the not talented agent is

$$c(e) = 3e$$

(yes/no + one sentence explaining why). Find minimal e for which the MBA diploma is a credible signal.

d) Externality: Give two methods through which a government can achieve market efficiency in the presence of a negative externality. (two sentences for each method).

Problem 5. (Market Power)

Consider an industry with an inverse demand $p(y) = 6 - y$ and the total cost $TC = 0$.

a) Find the level of production and the price chosen by a monopoly who is **not** allowed to price discriminate (give two numbers). Illustrate the choice using a graph. Find the consumer surplus (CS), the producer surplus (PS) and the deadweight loss (DWL) (give three numbers and mark them on the graph).

b) Which pricing strategy of a monopoly gives rise to the Pareto efficient outcome (one sentence)? Find the consumer surplus (CS), the producer surplus (PS) and the deadweight loss (DWL) under your proposed strategy (give three numbers and mark them on the graph).

c) Find the individual level of production, the price and the profit of each firm in the Cournot-Nash equilibrium if there are two identical firms in the industry with cost fucntions $TC = 0$ (give three numbers).

d) Find the joint profit of the two firms from part c) if they form a cartel. Explain the mechanism that prevents the formation of a (short-run) cartel in a Cournot-Nash equilibrium.

Problem 6. (Provision of a Public Good)

There are two countries, the USA and a country that represents "the rest of the world" (denoted by R). The national products of both countries are increasing in the **world's** spending on research, $x = x^{US} + x^R$. Thus research is a public good. The "profit" of the USA, net scientific expenses is given by

$$\pi^{US} = 12\ln(x^{US} + x^R) - x^{US}.$$

The net profit of country R is less sensitive to the scientific advancements, and is given by,

$$\pi^R = 3\ln(x^{US} + x^R) - x^R.$$

a) Find analytically the best response of the US to any level of spending x^R (derive a function) and plot it in the coordinate system x^{US}, x^R. (Make sure you show optimal choice x^{US} for $x^R > 12$).

b) Find analytically the best response function of country R and add it to the graph in part a).

c) Find the Nash equilibrium. What is the world's spending on science, x? Is the predicted outcome associated with free riding? If so by which country?

d) Find the Pareto efficient level of spending on reserch? Is it greater, smaller or equal to the one observed in markets (part c)? Explain intuitively why is it so?

Econ 301

Intermediate Microeconomics

Prof. Marek Weretka

Final Exam (Group B)

You have 2h to complete the exam. The exam consists of 6 questions (20,15, 15, 15, 15 and 20 points)

Problem 1. (Consumer Choice)

Lionel watches movies x_1 while drinking beer, x_2. His utility function from consuming the two types of goods is given by

$$U(x_1, x_2) = 5\ln(x_1) + 10\ln(x_2)$$

a) Plot Lionel's indifference curve map (graph). Find his MRS analytically (give formula). Find the value of MRS at the consumption bundle $(1, 1)$ and depict it in the graph.

b) Using "magic formulas," find the optimal level of consumption of x_1 and x_2 if $p_1 = p_2 = 5$ and $m = 45$. Plot carefully the optimal point along with the budget line and the indifference curve passing through the optimal point (a graph + two numbers).

c) Argue that the commodities are neither gross complements nor gross substitutes (argue using the magic formula, one sentence).

For the rest of the problem suppose that Lionel's preferences change and now they are given by $U(x_1, x_2) = \frac{1}{3}x_1 + \frac{1}{3}x_2$.

d) In a separate graph plot his indifference curves, and find MRS for consumption bundle $(2, 1)$. (a graph +one number).

e) Find Lionel's optimal choice given prices $p_1 = p_2 = 5$ and $m = 45$ (two numbers). Is the optimal choice unique? (yes/no answer)

Problem 2. (Equilibrium with Intertemporal Choice)

Consider an intertemporal choice problem with Ambrosia and Fergus. Ambrosia is an athlete with income of 800 when young and 200 when old, i.e, $\omega^A = (800, 200)$. Fergus is a manager who earns 200 when young and 800 when old, i.e., his endowment is $\omega^F = (200, 800)$. Ambrosia and Fergus have the same utility function given by

$$U^i(x_1^i, x_2^i) = \frac{1}{2}\ln(x_1^i) + \ln(x_2^i)$$

where $i = A, F$.

a) Plot the Edgeworth box and mark the point corresponding to endowments of Ambrosia and Fergus (graph).

b) Give a general economic definition of Pareto efficiency (one sentence). Write the equivalent condition for Pareto efficiency in terms of MRS (equation). Use your equation to verify whether the endowment is Pareto efficient. (argue formally using your condition)

c) Find all the allocations that are Pareto efficient (derive formula for the contract curve). Plot the contract curve in the Edgeworth box.

d) Find the competitive equilibrium. Calculate borrowing and savings for both agents in your equilibrium. (Hint: use that for the intertemporal choice $1 + r = \frac{p_1}{p_2}$).

Problem 3. (Technology)

Suppose a producer has access to the technology given by the Cobb-Douglass production function $y = 3K^{\frac{1}{4}}L^{\frac{1}{4}}$.

a) What can you say about the returns to scale (chose: IRS, CRS or DRS) and MPK (chose: increasing, constant, or decreasing).

b) Find the (variable) cost function $c(y)$ given that the prices of inputs are $w_K = w_L = 4.5$ (give a function).

For the rest of the problem suppose that in order to have access to the technology, the producer first needs to pay fixed cost $F = 16$ and hence the total cost is given by $TC = 16 + c(y)$.

c) Find the supply function of the individual competitive firm and plot it in the graph (give the formula, in the graph mark the prices for which the market will not open).

d) Assume that the producers are competitive and there is free entry. Determine the number of firms operating in the industry in the long run if the demand is $D(p) = 20 - p$ (one number).

Problem 4. (Short Questions)

a) A Bernoulli utility function is $u(x) = x^2$ and there are two states of the world which are equally likely. Find the certainty equivalent and the expected value of lottery $(0, 2)$ (two numbers). Which of the two is bigger (choose one)? Explain why (one sentence).

b) Consider an industry in which the market share of the dominant firm is 30%, while the market shares of the five other firms is 10% each. Find HHI index for this industry. Is the industry competitive, moderately concentrated, or concentrated?

c) Suppose there are two types of managers: talented with productivity 5 and not talented with productivity 3. The types are unobservable to employers and the competitive wage is given by the expected productivity of the manager. Is an MBA diploma from a program that takes three years to complete $e = 3$ a credible signal if the cost of effort for the not talented agent is

$$c(e) = e$$

(yes/no + one sentence explaining why). Find minimal e for which the MBA diploma is a credible signal.

d) Externality: Give two methods through which a government can achieve market efficiency in the presence of a negative externality. (two sentences for each method).

Problem 5. (Market Power)

Consider an industry with an inverse demand $p(y) = 8 - y$ and the total cost $TC = 0$.

a) Find the level of production and the price chosen by a monopoly who is **not** allowed to price discriminate (give two numbers). Illustrate the choice using a graph. Find the consumer surplus (CS), the producer surplus (PS) and the deadweight loss (DWL) (give three numbers and mark them on the graph).

b) Which pricing strategy of a monopoly gives rise to the Pareto efficient outcome (one sentence)? Find the consumer surplus (CS), the producer surplus (PS) and the deadweight loss (DWL) under your proposed strategy (give three numbers and mark them on the graph).

c) Find the individual level of production, the price and the profit of each firm in the Cournot-Nash equilibrium if there are two identical firms in the industry with cost functions $TC = 0$ (give three numbers).

d) Find the joint profit of the two firms from part c) if they form a cartel. Explain the mechanism that prevents the formation of a (short-run) cartel in a Cournot-Nash equilibrium.

Problem 6. (Provision of a Public Good)

There are two countries, the USA and a country that represents "the rest of the world" (denoted by R). The national products of both countries are increasing in the **world's** spending on research, $x = x^{US} + x^R$. Thus research is a public good. The "profit" of the USA, net scientific expenses is given by

$$\pi^{US} = 7\ln(x^{US} + x^R) - x^{US}.$$

The net profit of country R is less sensitive to the scientific advancements, and is given by,

$$\pi^R = \ln(x^{US} + x^R) - x^R.$$

a) Find analytically the best response of the US to any level of spending x^R (derive a function) and plot it in the coordinate system x^{US}, x^R. (Make sure you show optimal choice x^{US} for $x^R > 7$).

b) Find analytically the best response function of country R and add it to the graph in part a).

c) Find the Nash equilibrium. What is the world's spending on science, x? Is the predicted outcome associated with free riding? If so by which country?

d) Find the Pareto efficient level of spending on research? Is it greater, smaller or equal to the one observed in markets (part c)? Explain intuitively why is it so?

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (Group A)

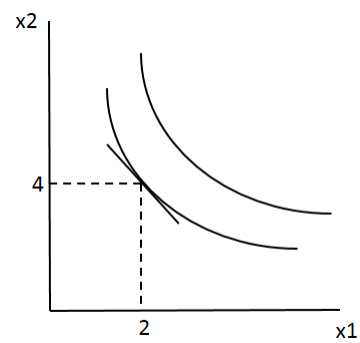
You have 2h to complete the exam. The exam consists of 6 questions (20, 15, 15, 15, 15 and 20 points)

Problem 1. (Consumer Choice)

a) Plot Lionel's indifference curve map (graph). Find his MRS analytically (give formula). Find the value of MRS at the consumption bundle $(2, 4)$ and depict it in the graph. **(4pt)**

$$MRS = -\frac{MU_1}{MU_2} = -\frac{2/x_1}{1/x_2} = -\frac{2x_2}{x_1}$$
$$MRS(2,4) = -\frac{2*4}{2} = -4$$

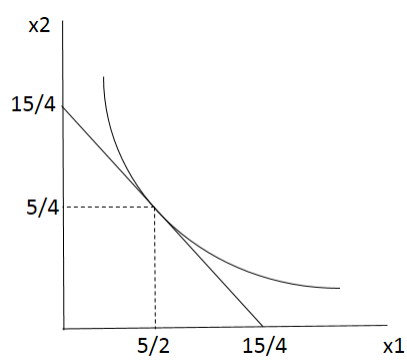
The MRS at $(2, 4)$ is the slope of the indifference curve at that point:



b) Using "magic formulas," find the optimal level of consumption of x_1 and x_2 if $p_1 = p_2 = 4$ and $m = 15$. Plot carefully the optimal point along with the budget line and the indifference curve passing through the optimal point (a graph + two numbers). **(5pt)**

$$x_1^* = \frac{a}{a+b}.\frac{m}{p_1} = \frac{2}{3}.\frac{15}{4} = \frac{5}{2}$$
$$x_2^* = \frac{b}{a+b}.\frac{m}{p_2} = \frac{1}{3}.\frac{15}{4} = \frac{5}{4}$$

The graph looks like this:



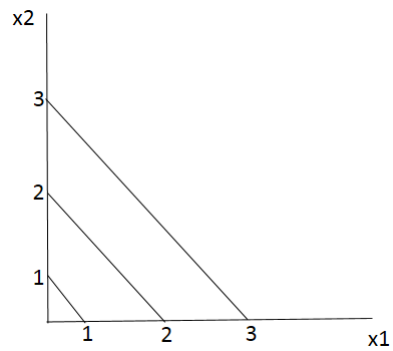
c) Argue that the commodities are neither gross complements nor gross substitutes (argue using the magic formula, one sentence). **(2pt)**

From the magic formula, $x_1^* = \frac{a}{a+b} \cdot \frac{m}{p_1}$, we can see that the optimal consumption of x_1 does not depend on the price of the other good, p_2. This implies that if p_2 goes up or down, the optimal consumption of x_1 wouldn't change. So, x_1 is neither a gross complement, nor a gross substitute for x_2. The reverse argument is identical. You can also show this by taking the derivative of the optimal x_1 with respect to p_2 (and the derivative of the optimal x_2 w.r.t. p_1) and show that these derivatives equal 0.

d) In a separate graph plot his indifference curves, and find MRS for consumption bundle $(2, 1)$. (a graph + one number). **(4pt)**

$MRS = -\frac{MU_1}{MU_2} = -\frac{3}{3} = -1$ at any bundle.

The graph looks like this:



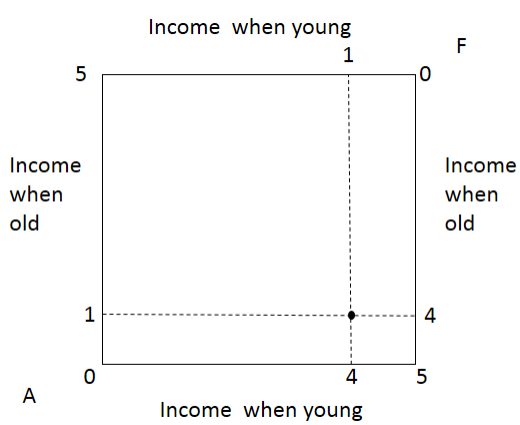
e) Find Lionel's optimal choice given prices $p_1 = p_2 = 4$ and $m = 15$ (two numbers). Is the optimal choice unique? (yes/no answer) **(5pt)**

$|MRS| = 1 = \frac{p_1}{p_2}$. Then, any bundle (x_1, x_2) that satisfies the budget constraint, i.e., $4x_1 + 4x_2 = 15$ is an optimal bundle and so the optimal bundle is not unique.

Problem 2. (Equilibrium with Intertemporal Choice)

a) Plot the Edgeworth box and mark the point corresponding to endowments of Ambrosia and Fergus (graph). **(2pt)**

The graph looks like this, where the black dot is the endowment allocation:



b) Give a general economic definition of Pareto efficiency (one sentence). Write the equivalent condition for Pareto efficiency in terms of MRS (equation). Use your equation to verify whether the endowment is Pareto efficient. (argue formally using your condition) **(4pt)**

If an allocation is Pareto efficient, neither of the two agents can be made better off with a different allocation, without making the other agent worse off.

Formally, we need $MRS^A = MRS^F$

From the utility function, we can find the MRS: $MRS = -\frac{2x_2}{x_1}$

At the endowment, $MRS^A = MRS(4,1) = -\frac{1}{2}$ and $MRS^F = MRS(1,4) = -8$, so the marginal rates of substitution are not equal for the two agents and thus the endowment allocation is not Pareto efficient.

c) Find all the allocations that are Pareto efficient (derive formula for the contract curve). Plot the contract curve in the Edgeworth box. **(4pt)**

The contract curve contains all allocations for which the MRS of the two agents are equal and which exactly exhaust the total endowment of the two goods. Formally:

$$\frac{x_2^A}{x_1^A} = \frac{x_2^F}{x_1^F} \tag{1}$$

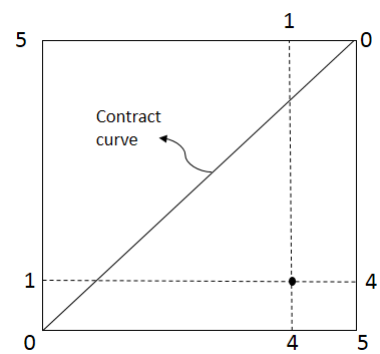
$$x_1^A + x_1^F = 5 \implies x_1^F = 5 - x_1^A \tag{2}$$

$$x_2^A + x_2^F = 5 \implies x_2^F = 5 - x_2^A \tag{3}$$

Plugging in (2) and (3) into (1), we get:

$$\frac{x_2^A}{x_1^A} = \frac{5 - x_2^A}{5 - x_1^A}$$

Cross-multiplying and solving gives: $x_1^A = x_2^A$ and $x_1^F = x_2^F$. The contract curve is the diagonal of the Edgeworth box:



d) Find the competitive equilibrium. Calculate borrowing and savings for both agents in your equilibrium. (Hint: use that for the intertemporal choice $1 + r = \frac{p_1}{p_2}$) **5pt**

First, calculate the "incomes" of the two agents based on their endowments, normalizing $p_2 = 1$:

$$m^A = 4p_1 + p_2 = 4p_1 + 1$$
$$m^F = p_1 + 4p_2 = p_1 + 4$$

Using "magic formulas", express the optimal consumption of x_1 for both agents:

$$x_1^A = \frac{a}{a+b} \cdot \frac{m^A}{p_1} = \frac{2}{3} \cdot \frac{4p_1 + 1}{p_1} \tag{4}$$

$$x_1^F = \frac{a}{a+b} \cdot \frac{m^F}{p_1} = \frac{2}{3} \cdot \frac{p_1 + 4}{p_1} \tag{5}$$

The two agents together must exhaust the total endowment of x_1, so:

$$\frac{2}{3}.\frac{4p_1+1}{p_1}+\frac{2}{3}.\frac{p_1+4}{p_1}=5 \implies p_1=2$$

Substituting $p_1 = 2$ into (4) and (5) and similarly for the optimal consumption of good 2, we get $x_1^A = x_2^A = 3$, $x_1^F = x_2^F = 2$.

Since Ambrosia is endowed with 4 when young and she only consumes $x_1^A = 3$, Ambrosia saves 1 dollar when young. Since Fergus is endowed with 1 when young and he consumes $x_1^F = 2$, Fergus borrows 1 dollar when young.

Problem 3. (Technology)

a) What can you say about the returns to scale (chose: IRS, CRS or DRS) and MPK (chose: increasing, constant, or decreasing). **(2pt)**

The technology exhibits decreasing returns to scale (DRS) and the MPK is decreasing.

b) Find the (variable) cost function $c(y)$ given that the prices of inputs are $w_K = w_L = 2$ (give a function). **(5pt)**

First, to find the optimal labor-to-capital ratio, set $\frac{MPK}{MPL} = \frac{w_K}{w_L}$

$$\frac{MPK}{MPL}=\frac{L^{\frac{3}{4}}L^{\frac{1}{4}}}{K^{\frac{3}{4}}K^{\frac{1}{4}}}=\frac{L}{K}$$
$$\frac{w_K}{w_L}=\frac{2}{2}=1$$
$$\frac{L}{K}=1 \implies L=K$$

Then, express the total cost as a function of one of the inputs by using $L = K$

$$C = w_K K + w_L L = 2K + 2L = 2K + 2K = 4K \tag{6}$$

Now, express output y as a function of that same input and use equation (6) to derive the final answer:

$$y = 2K^{\frac{1}{4}}L^{\frac{1}{4}} = 2K^{\frac{1}{4}}K^{\frac{1}{4}} = 2K^{\frac{1}{2}} \implies K=\frac{y^2}{4} \implies C(y) = 4K = y^2$$

c) Find the supply function of the individual competitive firm and plot it in the graph (give the formula, in the graph mark the prices for which the market will not open). **(4pt)**

To find its optimal output, the firm maximizes its profit function with respect to y:

$$\pi(y) = TR - TC = py - 9 - y^2$$
$$\frac{\partial \pi}{\partial y} = p - 2y = 0 \implies y=\frac{p}{2}$$

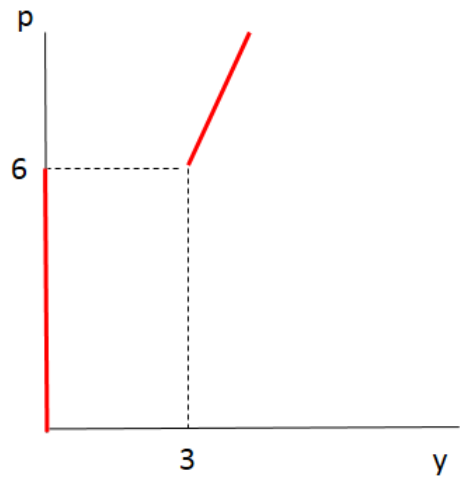
To find the prices for which the firm would not produce because it makes negative profits, we need to find y^{MES}:

$$MC = ATC$$
$$2y = \frac{9}{y} + y$$
$$\implies y^{MES} = 3 \implies ATC^{MES} = 6$$

Then, the supply function is given by:

$$y(p) = \begin{cases} \frac{p}{2} & \text{if } p \geq 6 \\ 0 & \text{if } p < 6 \end{cases}$$

The supply function is given in red:



d) Assume that the producers are competitive and there is free entry. Determine the number of firms operating in the industry in the long run if the demand is $D(p) = 24 - p$ (one number). **(4pt)**

In perfect competition, all firms make 0 profits in the long run. Thus, we know that firms operate at their minimal efficient scale, i.e., $y = y^{MES} = 3$ and $p = ATC^{MES} = 6$. Then, from the demand function, at a price of 6, the total quantity sold on the market is $Y = D(6) = 24 - 6 = 18$. The number of firms operating in the market is given by the total output Y divided by the individual firm output y and so $N = \frac{Y}{y} = \frac{18}{3} = 6$ firms.

Problem 4. (Short Questions)

a) A Bernoulli utility function is $u(x) = x^2$ and there are two states of the world which are equally likely. Find the certainty equivalent and the expected value of lottery $(1, 7)$ (two numbers). Which of the two is bigger (choose one)? Explain why (one sentence). **(4pt)**

The expected (von Neumann-Morgenstern) utility function is $U(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$.

The certainty equivalent (CE) of the lottery (1,7) solves the following equation:

$$\begin{aligned} U(CE, CE) &= U(1, 7) \\ \frac{1}{2}CE^2 + \frac{1}{2}CE^2 &= \frac{1}{2}1^2 + \frac{1}{2}7^2 \\ CE^2 &= 25 \\ CE &= 5 \end{aligned}$$

The expected value of the lottery (1,7) is: $EV = \frac{1}{2}(1) + \frac{1}{2}(7) = 4$.

The expected value is smaller than the certainty equivalent, because the agent is risk-loving. In order to make him indifferent between playing the lottery or taking the certainty equivalent, we need to compensate him not only for the expected winnings of the lottery, but also for the utility he derives from taking the risk.

b) Consider an industry in which the market share of the dominant firm is 30%, while the market shares of the ten other firms is 10% each. Find HHI index for this industry. Is the industry competitive, moderately concentrated, or concentrated? **(4pt)**

$HHI = (40)^2 + 6 \times (10)^2 = 2200 > 1800 \implies$ the industry is concentrated

c) Suppose there are two types of managers: talented with productivity 9 and not talented with productivity 3. The types are unobservable to employers and the competitive wage is given by the expected productivity of the manager. Is an MBA diploma from a program that takes one year to complete $e = 1$ a credible signal if the cost of effort for the not talented agent is

$$c(e) = 3e$$

(yes/no + one sentence explaining why). Find minimal e for which the MBA diploma is a credible signal. **(5pt)**

A not-talented manager who succeeds in convincing his employer that he is talented gains $9 - 3 = 6$ dollars. The cost to a not-talented manager of a 1-year MBA is $c(1) = 3$ dollars. Since the cost is less than the gain, this is not a credible signal to the employer. The minimal e for which the MBA diploma is a credible signal solves: $c(e) = 6 \implies 3e = 6 \implies e = 2$ years.

d) Externality: Give two methods through which a government can achieve market efficiency in the presence of a negative externality. (two sentences for each method) **(2pt)**

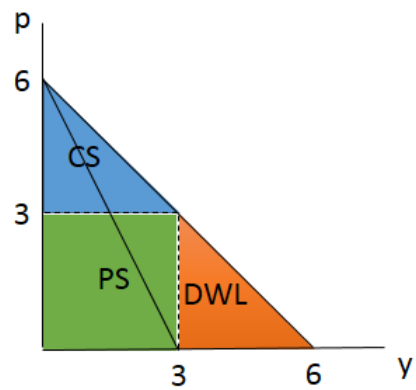
1) Taxes: The government can implement a tax on the negative externality that would make the producer of the externality feel the negative impact that his activity has on society

2) Creation of a missing market: For example, the government can create a market for pollution permits, where firms which pollute can trade permits and pollute only up to the quantity allowed by their permit.

Problem 5. (Market Power)

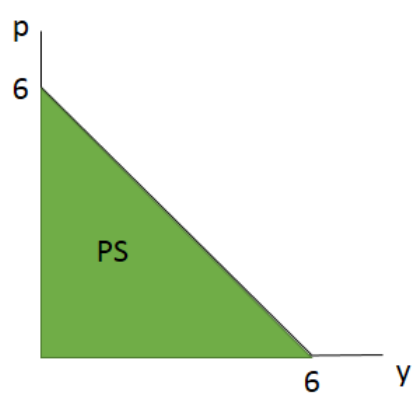
a) Find the level of production and the price chosen by a monopoly who is **not** allowed to price discriminate (give two numbers). Illustrate the choice using a graph. Find the consumer surplus (CS), the producer surplus (PS) and the deadweight loss (DWL) (give three numbers and mark them on the graph). **(3pt)**

The monopolist sets $MR = MC \implies 6 - 2y = 0 \implies y = 3$ and plugging into the demand function, the equilibrium price is $p = 6 - 3 = 3$. Welfare is given by $CS = 4.5$, $PS = 9$, $DWL = 4.5$.



b) Which pricing strategy of a monopoly gives rise to the Pareto efficient outcome (one sentence)? Find the consumer surplus (CS), the producer surplus (PS) and the deadweight loss (DWL) under your proposed strategy (give three numbers and mark them on the graph) **(4pt)**

A Pareto efficient monopoly pricing strategy is First Degree Price discrimination. Welfare under this scheme is given by $CS = DWL = 0$, $PS = 18$.



c) Find the individual level of production, the price and the profit of each firm in the Cournot-Nash equilibrium if there are two identical firms in the industry with cost fucntions $TC = 0$ (give three numbers) **(4pt)**

Let's find the best response function of firm 1 - this gives us the optimal output of firm 1 as a response to the output of firm 2.

$$\pi_1 = p(y)y_1 - TC(y_1) = (6 - y)y_1 - 0 = (6 - y_1 - y_2)y_1$$
$$\frac{\partial \pi_1}{\partial y_1} = 6 - y_2 - 2y_1 = 0$$
$$y_1 = 3 - \frac{y_2}{2}$$

Since the two firms are identical, they produce the same amount of output in equilibrium, i.e., $y_1 = y_2$. Substituting this into the best response function gives:

$$y_1 = 3 - \frac{y_1}{2} \implies y_1 = y_2 = 2$$

The total output is then $y = y_1 + y_2 = 4$ and so the equilibrium price is $p = 6 - y = 2$. Each firm earns a profit of $\pi_1 = \pi_2 = 2 \times 2 - 0 = 4$.

d) Find the joint profit of the two firms from part c) if they form a cartel. Explain the mechanism that prevents the formation of a (short-run) cartel in a Cournot-Nash equilibrium. **(4pt)**

If the two firms form a cartel, they would operate as a monopoly, so the equilibrium price and total output would be the same as in part a), $p = 3$, $y = 3$. The joint profit is then $\pi = py - TC = 9$. Since the total output is $y = 3$, then each firm produces $y_1 = y_2 = 1.5$. Suppose firm 1 decides to check what its best response is to firm 2 producing $y_2 = 1.5$. Using the best response function derived in part c), we get $y_1 = 3 - \frac{1.5}{2} = 2.25 > 1.5$. This means that firm 1 has an incentive to cheat and produce more than the agreed output of 1.5 units. This prevents the formation of short-lived cartels in a Cournot-Nash equilibrium.

Problem 6. (Provision of a Public Good)

a) Find analytically the best response of the US to any level of spending x^R (derive a function) and plot it in the coordinate system x^{US}, x^R. (Make sure you show optimal choice x^{US} for $x^R > 12$) **(5pt)**

$$\frac{\partial \pi^{US}}{\partial x_{US}} = \frac{12}{x^{US} + x^R} - 1 = 0$$
$$x^{US} = 12 - x^R$$

The best response function of the US is:

$$x^{US} = \begin{cases} 12 - x^R & \text{if } x^R \leq 12 \\ 0 & \text{if } x^R > 12 \end{cases}$$

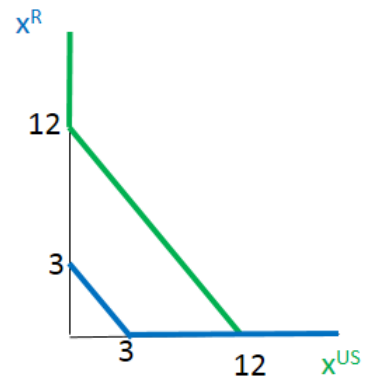
The plot is given below.

b) Find analytically the best response function of country R and add it to the graph in part a). **(5pt)**

$$\frac{\partial \pi^R}{\partial x_R} = \frac{3}{x^{US} + x^R} - 1 = 0$$
$$x^R = 3 - x^{US}$$

The best response function of the US is:

$$x^R = \begin{cases} 3 - x^R & \text{if } x^{US} \leq 3 \\ 0 & \text{if } x^{US} > 3 \end{cases}$$



c) Find the Nash equilibrium. What is the world's spending on science, x? Is the predicted outcome associated with free riding? If so by which country? **(4pt)**

The Nash equilibrium is $x^{US} = 12$, $x^R = 0$ and so the total spending on science is $x = x^{US} + x^R = 12$. Country R is free riding.

d) Find the Pareto efficient level of spending on reserch? Is it greater, smaller or equal to the one observed in markets (part c)? Explain intuitively why is it so? **(6pt)**

The Pareto efficient level of spending maximizes the world's total profit. Below, we use $x = x^{US} + x^R$

$$\pi^{TOTAL} = \pi^{US} + \pi^R = 15ln(x) - x$$
$$\frac{\partial \pi^{TOTAL}}{\partial x} = \frac{15}{x} - 1 = 0$$
$$x = 15 > 12$$

The Pareto efficient outcome is greater than the market outcome because the US values research more than the rest of the world, so country R is able to free ride.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (Group B)

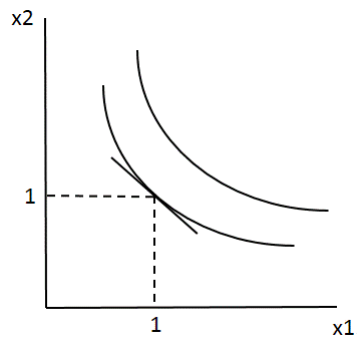
You have 2h to complete the exam. The exam consists of 6 questions (20, 15, 15, 15, 15 and 20 points)

Problem 1. (Consumer Choice)

a) Plot Lionel's indifference curve map (graph). Find his MRS analytically (give formula). Find the value of MRS at the consumption bundle $(1, 1)$ and depict it in the graph. **(4pt)**

$$MRS = -\frac{MU_1}{MU_2} = -\frac{5/x_1}{10/x_2} = -\frac{x_2}{2x_1}$$
$$MRS(1,1) = -\frac{1}{2}$$

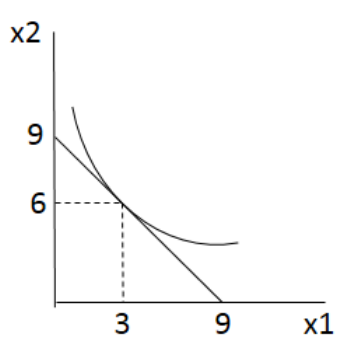
The MRS at $(1, 1)$ is the slope of the indifference curve at that point:



b) Using "magic formulas," find the optimal level of consumption of x_1 and x_2 if $p_1 = p_2 = 5$ and $m = 45$. Plot carefully the optimal point along with the budget line and the indifference curve passing through the optimal point (a graph + two numbers). **(5pt)**

$$x_1^* = \frac{a}{a+b} \cdot \frac{m}{p_1} = \frac{5}{15} \cdot \frac{45}{5} = 3$$
$$x_2^* = \frac{b}{a+b} \cdot \frac{m}{p_2} = \frac{10}{15} \cdot \frac{45}{5} = 6$$

The graph looks like this:



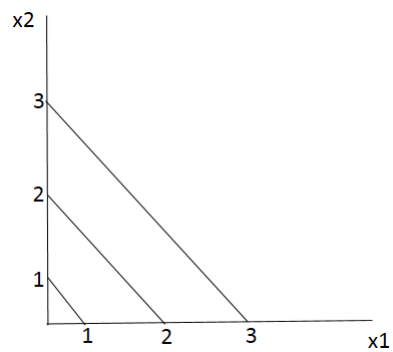
c) Argue that the commodities are neither gross complements nor gross substitutes (argue using the magic formula, one sentence). **(2pt)**

From the magic formula, $x_1^* = \frac{a}{a+b} \cdot \frac{m}{p_1}$, we can see that the optimal consumption of x_1 does not depend on the price of the other good, p_2. This implies that if p_2 goes up or down, the optimal consumption of x_1 wouldn't change. So, x_1 is neither a gross complement, nor a gross substitute for x_2. The reverse argument is identical. You can also show this by taking the derivative of the optimal x_1 with respect to p_2 (and the derivative of the optimal x_2 w.r.t. p_1) and show that these derivatives equal 0.

d) In a separate graph plot his indifference curves, and find MRS for consumption bundle $(2, 1)$. (a graph + one number). **(4pt)**

$MRS = -\frac{MU_1}{MU_2} = -\frac{\frac{1}{3}}{\frac{1}{3}} = -1$ at any bundle.

The graph looks like this:



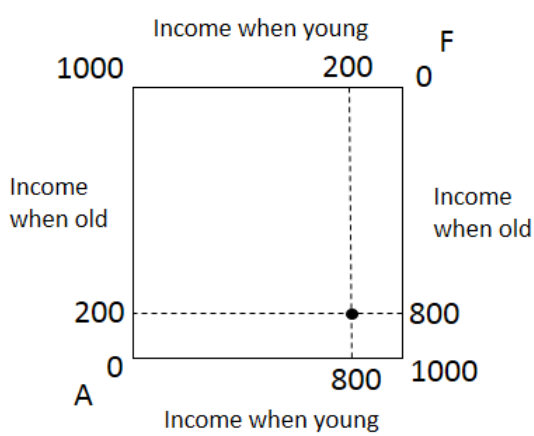
e) Find Lionel's optimal choice given prices $p_1 = p_2 = 5$ and $m = 45$ (two numbers). Is the optimal choice unique? (yes/no answer) **(5pt)**

$|MRS| = 1 = \frac{p_1}{p_2}$. Then, any bundle (x_1, x_2) that satisfies the budget constraint, i.e., $5x_1 + 5x_2 = 45$ is an optimal bundle and so the optimal bundle is not unique.

Problem 2. (Equilibrium with Intertemporal Choice)

a) Plot the Edgeworth box and mark the point corresponding to endowments of Ambrosia and Fergus (graph). **(2pt)**

The graph looks like this, where the black dot is the endowment allocation:



b) Give a general economic definition of Pareto efficiency (one sentence). Write the equivalent condition for Pareto efficiency in terms of MRS (equation). Use your equation to verify whether the endowment is Pareto efficient. (argue formally using your condition) **(4pt)**

If an allocation is Pareto efficient, neither of the two agents can be made better off with a different allocation, without making the other agent worse off.

Formally, we need $MRS^A = MRS^F$

From the utility function, we can find the MRS: $MRS = -\frac{x_2}{2x_1}$

At the endowment, $MRS^A = MRS(800, 200) = -\frac{200}{2*800} = -\frac{1}{8}$ and $MRS^F = MRS(200, 800) = -\frac{800}{2*200} = -2$, so the marginal rates of substitution are not equal for the two agents and thus the endowment allocation is not Pareto efficient.

c) Find all the allocations that are Pareto efficient (derive formula for the contract curve). Plot the contract curve in the Edgeworth box. **(4pt)**

The contract curve contains all allocations for which the MRS of the two agents are equal and which exactly exhaust the total endowment of the two goods. Formally:

$$\frac{x_2^A}{x_1^A} = \frac{x_2^F}{x_1^F} \tag{7}$$

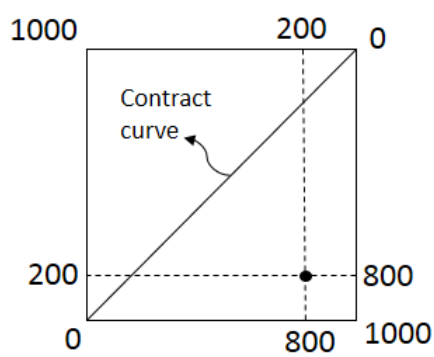
$$x_1^A + x_1^F = 1000 \implies x_1^F = 1000 - x_1^A \tag{8}$$

$$x_2^A + x_2^F = 1000 \implies x_2^F = 1000 - x_2^A \tag{9}$$

Plugging in (2) and (3) into (1), we get:

$$\frac{x_2^A}{x_1^A} = \frac{1000 - x_2^A}{1000 - x_1^A}$$

Cross-multiplying and solving gives: $x_1^A = x_2^A$ and $x_1^F = x_2^F$. The contract curve is the diagonal of the Edgeworth box:



d) Find the competitive equilibrium. Calculate borrowing and savings for both agents in your equilibrium. (Hint: use that for the intertemporal choice $1 + r = \frac{p_1}{p_2}$) **5pt**

First, calculate the "incomes" of the two agents based on their endowments, normalizing $p_2 = 1$:

$$m^A = 800p_1 + 200p_2 = 800p_1 + 200$$
$$m^F = 200p_1 + 800p_2 = 200p_1 + 800$$

Using "magic formulas", express the optimal consumption of x_1 for both agents:

$$x_1^A = \frac{a}{a+b} \cdot \frac{m^A}{p_1} = \frac{1}{3} \cdot \frac{800p_1 + 200}{p_1} \tag{10}$$

$$x_1^F = \frac{a}{a+b} \cdot \frac{m^F}{p_1} = \frac{1}{3} \cdot \frac{200p_1 + 800}{p_1} \tag{11}$$

The two agents together must exhaust the total endowment of x_1, so:

$$\frac{1}{3}.\frac{800p_1 + 200}{p_1} + \frac{1}{3}.\frac{200p_1 + 800}{p_1} = 1000 \implies p_1 = \frac{1}{2}$$

Substituting $p_1 = \frac{1}{2}$ into (4) and (5) and similarly for the optimal consumption of good 2, we get $x_1^A = x_2^A = 400$, $x_1^F = x_2^F = 600$.

Since Ambrosia is endowed with 800 when young and she only consumes $x_1^A = 400$, Ambrosia saves 400 dollars when young. Since Fergus is endowed with 200 when young and he consumes $x_1^F = 600$, Fergus borrows 400 dollars when young.

Problem 3. (Technology)

a) What can you say about the returns to scale (chose: IRS, CRS or DRS) and MPK (chose: increasing, constant, or decreasing). **(2pt)**

The technology exhibits decreasing returns to scale (DRS) and the MPK is decreasing.

b) Find the (variable) cost function $c(y)$ given that the prices of inputs are $w_K = w_L = 4.5$ (give a function). **(5pt)**

First, to find the optimal labor-to-capital ratio, set $\frac{MPK}{MPL} = \frac{w_K}{w_L}$

$$\frac{MPK}{MPL} = \frac{L^{\frac{3}{4}}L^{\frac{1}{4}}}{K^{\frac{3}{4}}K^{\frac{1}{4}}} = \frac{L}{K}$$
$$\frac{w_K}{w_L} = \frac{4.5}{4.5} = 1$$
$$\frac{L}{K} = 1 \implies L = K$$

Then, express the total cost as a function of one of the inputs by using $L = K$

$$C = w_K K + w_L L = 4.5K + 4.5L = 4.5K + 4.5K = 9K \tag{12}$$

Now, express output y as a function of that same input and use equation (6) to derive the final answer:

$$y = 3K^{\frac{1}{4}}L^{\frac{1}{4}} = 3K^{\frac{1}{4}}K^{\frac{1}{4}} = 3K^{\frac{1}{2}} \implies K = \frac{y^2}{9} \implies C(y) = 9K = y^2$$

c) Find the supply function of the individual competitive firm and plot it in the graph (give the formula, in the graph mark the prices for which the market will not open). **(4pt)**

To find its optimal output, the firm maximizes its profit function with respect to y:

$$\pi(y) = TR - TC = py - 16 - y^2$$
$$\frac{\partial \pi}{\partial y} = p - 2y = 0 \implies y = \frac{p}{2}$$

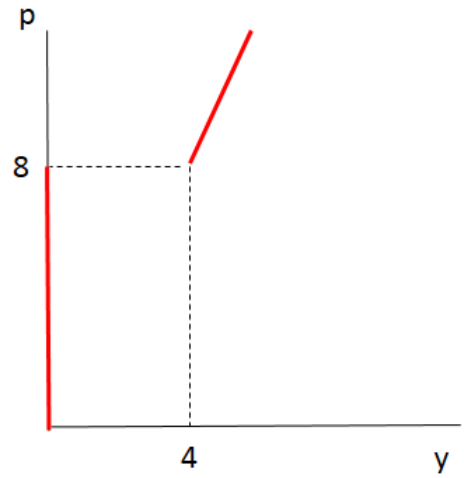
To find the prices for which the firm would not produce because it makes negative profits, we need to find y^{MES}:

$$MC = ATC$$
$$2y = \frac{16}{y} + y$$
$$\implies y^{MES} = 4 \implies ATC^{MES} = 8$$

Then, the supply function is given by:

$$y(p) = \begin{cases} \frac{p}{2} & \text{if } p \geq 8 \\ 0 & \text{if } p < 8 \end{cases}$$

The supply function is given in red:



d) Assume that the producers are competitive and there is free entry. Determine the number of firms operating in the industry in the long run if the demand is $D(p) = 20 - p$ (one number). **(4pt)**

In perfect competition, all firms make 0 profits in the long run. Thus, we know that firms operate at their minimal efficient scale, i.e., $y = y^{MES} = 4$ and $p = ATC^{MES} = 8$. Then, from the demand function, at a price of 8, the total quantity sold on the market is $Y = D(8) = 20 - 8 = 12$. The number of firms operating in the market is given by the total output Y divided by the individual firm output y and so $N = \frac{Y}{y} = \frac{12}{4} = 3$ firms.

Problem 4. (Short Questions)

a) A Bernoulli utility function is $u(x) = x^2$ and there are two states of the world which are equally likely. Find the certainty equivalent and the expected value of lottery $(0, 2)$ (two numbers). Which of the two is bigger (choose one)? Explain why (one sentence). **(4pt)**

The expected (von Neumann-Morgenstern) utility function is $U(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$.

The certainty equivalent (CE) of the lottery (0,2) solves the following equation:

$$\begin{aligned} U(CE, CE) &= U(0, 2) \\ \frac{1}{2}CE^2 + \frac{1}{2}CE^2 &= \frac{1}{2}0^2 + \frac{1}{2}2^2 \\ CE^2 &= 2 \\ CE &= \sqrt{2} \end{aligned}$$

The expected value of the lottery (0,2) is: $EV = \frac{1}{2}(0) + \frac{1}{2}(2) = 1$.

The expected value is smaller than the certainty equivalent, because the agent is risk-loving. In order to make him indifferent between playing the lottery or taking the certainty equivalent, we need to compensate him not only for the expected winnings of the lottery, but also for the utility he derives from taking the risk.

b) Consider an industry in which the market share of the dominant firm is 30%, while the market shares of the five other firms is 10% each. Find HHI index for this industry. Is the industry competitive, moderately concentrated, or concentrated? **(4pt)**

$HHI = (20)^2 + 8 \times (10)^2 = 1200 \in (1000, 1800) \implies$ the industry is moderately concentrated

c) Suppose there are two types of managers: talented with productivity 5 and not talented with productivity 3. The types are unobservable to employers and the competitive wage is given by the expected productivity of the manager. Is an MBA diploma from a program that takes three years to complete $e = 3$ a credible signal if the cost of effort for the not talented agent is

$$c(e) = e$$

(yes/no + one sentence explaining why). Find minimal e for which the MBA diploma is a credible signal. **(5pt)**

A not-talented manager who succeeds in convincing his employer that he is talented gains $5 - 3 = 2$ dollars. The cost to a not-talented manager of a 3-year MBA is $c(3) = 3$ dollars. Since the cost is more than the gain, this is a credible signal to the employer. The minimal e for which the MBA diploma is a credible signal solves: $c(e) = 2 \implies e = 2$ years.

d) Externality: Give two methods through which a government can achieve market efficiency in the presence of a negative externality. (two sentences for each method) **(2pt)**

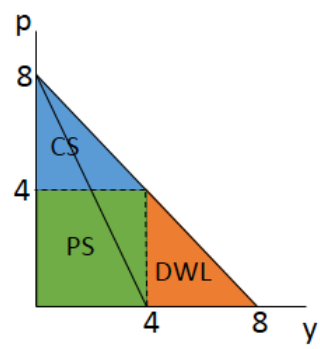
1) Taxes: The government can implement a tax on the negative externality that would make the producer of the externality feel the negative impact that his activity has on society

2) Creation of a missing market: For example, the government can create a market for pollution permits, where firms which pollute can trade permits and pollute only up to the quantity allowed by their permit.

Problem 5. (Market Power)

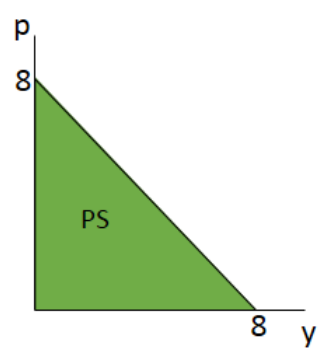
a) Find the level of production and the price chosen by a monopoly who is **not** allowed to price discriminate (give two numbers). Illustrate the choice using a graph. Find the consumer surplus (CS), the producer surplus (PS) and the deadweight loss (DWL) (give three numbers and mark them on the graph). **(3pt)**

The monopolist sets $MR = MC \implies 8 - 2y = 0 \implies y = 4$ and plugging into the demand function, the equilibrium price is $p = 8 - 4 = 4$. Welfare is given by $CS = 8$, $PS = 16$, $DWL = 8$.



b) Which pricing strategy of a monopoly gives rise to the Pareto efficient outcome (one sentence)? Find the consumer surplus (CS), the producer surplus (PS) and the deadweight loss (DWL) under your proposed strategy (give three numbers and mark them on the graph) **(4pt)**

A Pareto efficient monopoly pricing strategy is First Degree Price discrimination. Welfare under this scheme is given by $CS = DWL = 0$, $PS = 32$.



c) Find the individual level of production, the price and the profit of each firm in the Cournot-Nash equilibrium if there are two identical firms in the industry with cost fucntions $TC = 0$ (give three numbers) **(4pt)**

Let's find the best response function of firm 1 - this gives us the optimal output of firm 1 as a response to the output of firm 2.

$$\pi_1 = p(y)y_1 - TC(y_1) = (8 - y)y_1 - 0 = (8 - y_1 - y_2)y_1$$
$$\frac{\partial \pi_1}{\partial y_1} = 8 - y_2 - 2y_1 = 0$$
$$y_1 = 4 - \frac{y_2}{2}$$

Since the two firms are identical, they produce the same amount of output in equilibrium, i.e., $y_1 = y_2$. Substituting this into the best response function gives:

$$y_1 = 4 - \frac{y_1}{2} \implies y_1 = y_2 = \frac{8}{3}$$

The total output is then $y = y_1 + y_2 = \frac{16}{3}$ and so the equilibrium price is $p = 8 - y = \frac{8}{3}$. Each firm earns a profit of $\pi_1 = \pi_2 = \frac{8}{3} \times \frac{8}{3} - 0 = \frac{64}{9}$.

d) Find the joint profit of the two firms from part c) if they form a cartel. Explain the mechanism that prevents the formation of a (short-run) cartel in a Cournot-Nash equilibrium. **(4pt)**

If the two firms form a cartel, they would operate as a monopoly, so the equilibrium price and total output would be the same as in part a), $p = 4$, $y = 4$. The joint profit is then $\pi = py - TC = 16$. Since the total output is $y = 4$, then each firm produces $y_1 = y_2 = 2$. Suppose firm 1 decides to check what its best response is to firm 2 producing $y_2 = 2$. Using the best response function derived in part c), we get $y_1 = 4 - \frac{2}{2} = 3 > 2$. This means that firm 1 has an incentive to cheat and produce more than the agreed output of 2 units. This prevents the formation of short-lived cartels in a Cournot-Nash equilibrium.

Problem 6. (Provision of a Public Good)

a) Find analytically the best response of the US to any level of spending x^R (derive a function) and plot it in the coordinate system x^{US}, x^R. (Make sure you show optimal choice x^{US} for $x^R > 7$) **(5pt)**

$$\frac{\partial \pi^{US}}{\partial x_{US}} = \frac{7}{x^{US} + x^R} - 1 = 0$$
$$x^{US} = 7 - x^R$$

The best response function of the US is:

$$x^{US} = \begin{cases} 7 - x^R & \text{if } x^R \leq 7 \\ 0 & \text{if } x^R > 7 \end{cases}$$

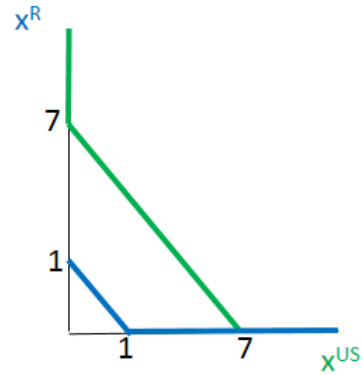
The plot is given below.

b) Find analytically the best response function of country R and add it to the graph in part a). **(5pt)**

$$\frac{\partial \pi^R}{\partial x_R} = \frac{1}{x^{US} + x^R} - 1 = 0$$
$$x^R = 1 - x^{US}$$

The best response function of the US is:

$$x^R = \begin{cases} 1 - x^R & \text{if } x^{US} \leq 1 \\ 0 & \text{if } x^{US} > 1 \end{cases}$$



c) Find the Nash equilibrium. What is the world's spending on science, x? Is the predicted outcome associated with free riding? If so by which country? **(4pt)**

The Nash equilibrium is $x^{US} = 7$, $x^R = 0$ and so the total spending on science is $x = x^{US} + x^R = 7$. Country R is free riding.

d) Find the Pareto efficient level of spending on research? Is it greater, smaller or equal to the one observed in markets (part c)? Explain intuitively why is it so? **(6pt)**

The Pareto efficient level of spending maximizes the world's total profit. Below, we use $x = x^{US} + x^R$

$$\pi^{TOTAL} = \pi^{US} + \pi^R = 8ln(x) - x$$
$$\frac{\partial \pi^{TOTAL}}{\partial x} = \frac{8}{x} - 1 = 0$$
$$x = 8 > 7$$

The Pareto efficient outcome is greater than the market outcome because the US values research more than the rest of the world, so country R is able to free ride.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final

You have 2h to complete the exam and the final consists of 6 questions (10+10+15+25+25+15=100).

Problem 1.

Ace consumes bananas x_1 and kiwis x_2. The prices of both goods are $p_1 = p_2 = 10$ and Ace's income is $m = 300$. His utility function is

$$U(x_1, x_2) = (x_1)^{20} (x_2)^{20}$$

a) Find analytically Ace's MRS as a function of (x_1, x_2) (give a function) and find its value for the consumption bundle $(x_1, x_2) = (80, 20)$. Give its economic and geometric interpretation (one sentence and find MRS on the graph)

b) Give two secrets of happiness that determine Ace's optimal choice of fruits (give two equation). Explain why violation of any of them implies that the bundle is not optimal (one sentence for each condition).

c) Show geometrically the optimum bundle of Ace – do not calculate it.

Problem 2.

Adria collects two types of rare coins: Jefferson Nickels $x_1$and Seated Half Dimes x_2. Her utility from a collection (x_1, x_2) is

$$U(x_1, x_2) = \min(x_1, x_2)$$

a) Propose a utility function that gives a higher level of utility for any (x_1, x_2), but represents the same preferences (give utility function).

b) Suppose the prices of the two types of coins are $p_1 = 4$ and $p_2 = 2$ for x_1, x_2 respectively and the Adria's income is $m = \$20$. Plot her budget set and find the optimal collection (x_1, x_2) and mark it in your graph (give two numbers)

c) Are the coins Giffen goods (yes or no and one sentence explaining why)?

d) Harder: Suppose Adria's provider of coins currently has only six Seated Half Dimes x_2 in stock (hence $x_2 \leq 6$). Plot a budget set with the extra constraint and find (geometrically) an optimal collection given the constraint.

Problem 3. (Equilibrium)

There are two commodities traded on the market: umbrellas x_1 and swimming suits x_2. Abigail has ten umbrellas and twenty swimming suits ($\omega^A = (10, 20)$). Gabriel has forty umbrellas and twenty swimming suits ($\omega^G = (40, 20)$). Abigail and Gabriel have identical utility functions given by

$$U^i(x_1, x_2) = \frac{1}{2}\ln(x_1) + \frac{1}{2}\ln(x_2)$$

a) Plot an Edgeworth box and mark the point corresponding to endowments of Abigail and Gabriel.

b) Give a definition of a Pareto efficient allocation (one sentence) and the equivalent condition in terms of MRS (equation). Verify whether endowment is Pareto efficient (two numbers+one sentence).

c) Find prices and an allocation of umbrellas and swimming suits in a competitive equilibrium and mark it in your graph.

d) Harder: Plot a contract curve in the Edgeworth box assuming utilities for two agents $U^A(x_1, x_2) = x_1 + x_2$ and $U^G(x_1, x_2) = x_1 + 2x_2$.

Problem 4.(Short questions)

a) You are going to pay taxes of \$20 every year, forever. Find the Present Value of your taxes if the yearly interest rate is $r = 10\%$.

b) Consider a lottery that pays 0 with probability $\frac{1}{2}$ and 4 with probability $\frac{1}{2}$ and a Bernoulli utility function is $u(x) = x^2$. Give a corresponding von Neuman-Morgenstern utility function. Find the certainty

equivalent of the lottery. Is it bigger or smaller than the expected value of the lottery? Why? (give a utility function, two numbers and one sentence.)

c) Give an example of a Cobb-Douglass production function that is associated with increasing returns to scale, increasing MPK and decreasing MPL (give a function). Without any calculations, sketch the average total cost function (ATC) associated with your production function.

d) Suppose the cost function is such that $ATC^{MES} = 2$ and $y^{MES} = 1$ and the demand is $D(p) = 4 - p$. Determine a number of firms in the industry given the free entry (and price taking). Is the industry monopolistic, duopolistic, oligopolistic or perfectly competitive? Find Herfindahl–Hirschman Index (HHI) of this industry (one number).

e) In a market for second-hand vehicles two types of cars can be traded: lemons (bad quality cars) and plums (good quality ones). The value of a car depends on its type and is given by

	Lemon	Plum
Seller	0	20
Buyer	10	26

Will we observe plums traded on the market if the probability of a lemon is equal to $\frac{1}{2}$? (compare two relevant numbers). Is the equilibrium outcome Pareto efficient (yes-no answer+ one sentence)? Give a threshold probability for which we might observe pooling equilibrium (number).

Problem 5.(Market Power)

Consider an industry with the inverse demand equal to $p(y) = 6 - y$, and suppose that the total cost function is $TC = 0$.

a) What are the total gains to trade in this industry? (give one number)

b) Find the level of production and the price if there is only one firm in the industry (i.e., we have a monopoly) charging a uniform price (give two numbers). Find demand elasticity at optimum. (give on number) Illustrate the choice using a graph. Mark a DWL.

c) Find the profit of the monopoly and a DWL given that monopoly uses the first degree price discrimination.

d) Find the individual and aggregate production and the price in a Cournot-Nash equilibrium given that there are two firms (give three numbers). Show DWL in the graph.

e) In which of the three cases, (b,c or d) the outcome is Pareto efficient? (chose one+ one sentence)

Problem 6.(Externality)

A bee keeper chooses the number of hives h. Each hive produces ten pounds of honey which sells at the price of \$2 per pound. The cost of holding h hives is $TC(h) = \frac{1}{2}h^2$. Consequently the profit of bee keeper is equal to

$$\pi_h(h) = 2h - \frac{1}{2}h^2$$

The hives are located next to an apple tree orchard. The bees pollinate the trees and hence the total production of apples $y = h + t$ is increasing in number of trees and bees. Apples sell for \$5 and the cost of t trees is $TC_t(t) = \frac{1}{2}t^2$. Therefore the profit of an orchard grower is

$$\pi_t(t) = 5(t + h) - \frac{1}{2}t^2$$

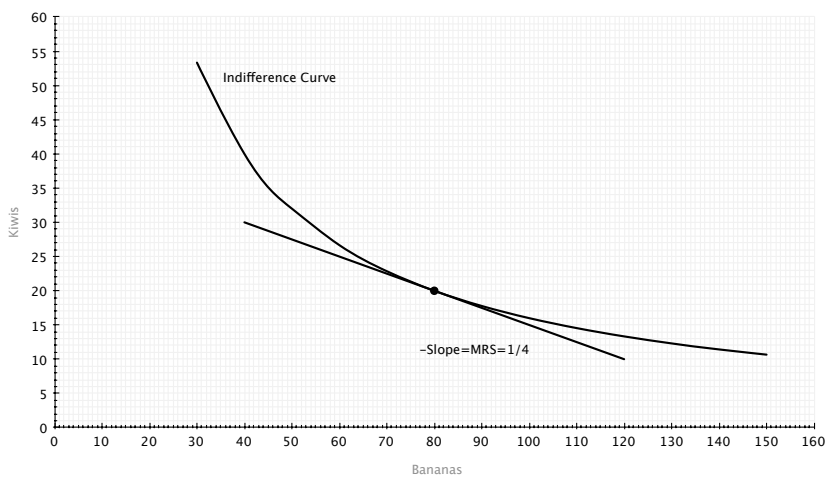
a) Market outcome: Find the level of hives h that maximizes the profit of a beekeeper and the number of trees that maximizes the profit of an orchard owner (assuming h optimal for a bee keeper) (two numbers)

b) Find the Pareto efficient level of h and t. Are the two values higher or smaller then the ones in a)? Why? (two numbers + one sentence)

Final Solutions
ECON 301
May 13, 2012

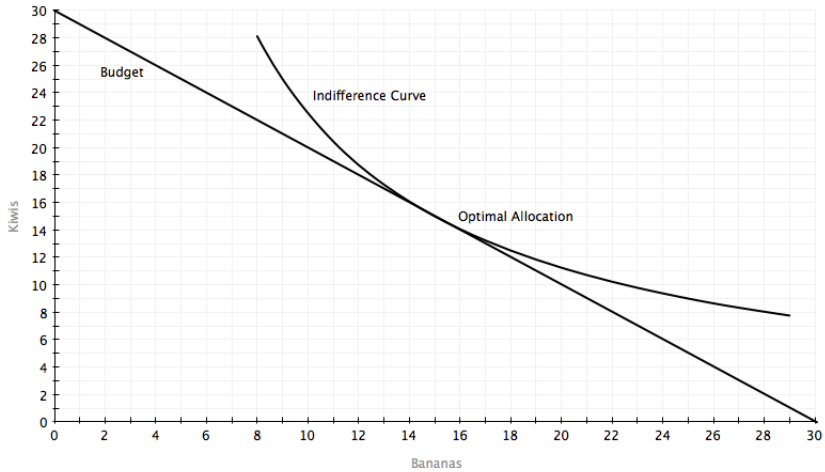
Problem 1

a) Because it is easier and more familiar, we will work with the monotonic transformation (and thus equivalent) utility function: $U(x_1, x_2) = \log x_1 + \log x_2$. $MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{1}{x_1}}{\frac{1}{x_2}} = \frac{x_2}{x_1}$. At $(x_1, x_2) = (80, 20)$, $MRS = \frac{20}{80} = \frac{1}{4}$. The MRS measures the rate a which you are willing to trade one good for the other. At a particular point in a graph, the MRS will be the negative of the slope of the indifference curve running through that point.



b)
- Budget: $10x_1 + 10x_2 = 300$. With a monotonic utility function like this one, the budget holds with equality because you can always make yourself better off by consuming more. Thus, it makes no sense to leave money unspent.
- $MRS = \frac{p_1}{p_2}$: The price at which you are willing to trade goods for one another (MRS) is the same as the rate at which you can trade the goods for one another (price ratio). Alternatively, you can think of this as the marginal utility per dollar spent on each good is the same: $\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$. If this does not hold you would be able to buy less of one good, spend that money one the other good, and gain more utility than you have lost.

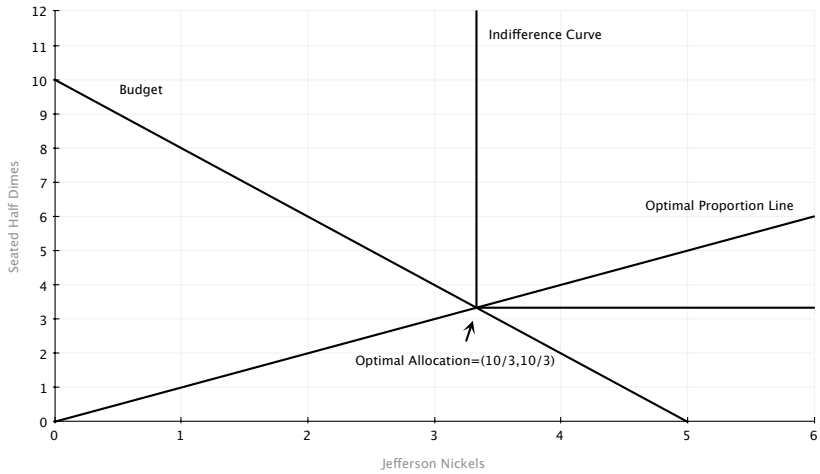
c) The optimal allocation is shown in the graph below



Problem 2

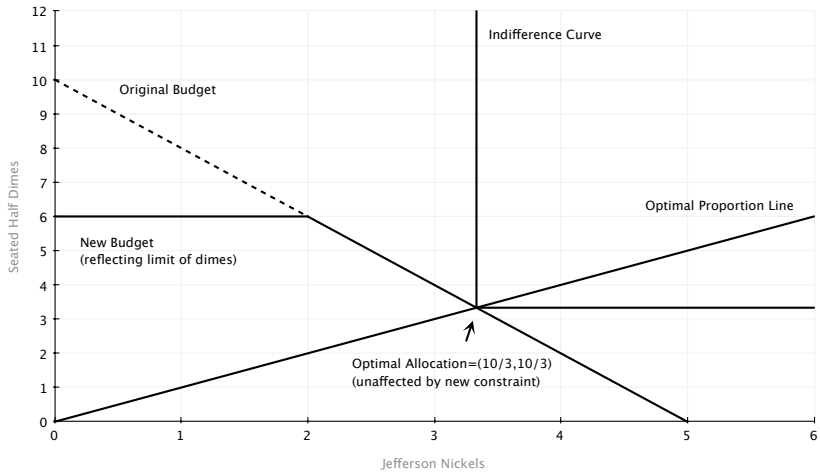
a) Lots of them exist. The most straightforward are $U(x_1, x_2) = A * min(x_1, x_2) + B$, with $A \geq 1$, $B \geq 0$, and $A + B > 1$. These represent the same preferences because they are monotonic transformations.

b) The optimal bundle occurs where the optimal proportion line, $x_1 = x_2$, crosses the budget line, $4x_1 + 2x_2 = 20$. This happens when $(x_1, x_2) = (\frac{10}{3}, \frac{10}{3})$.



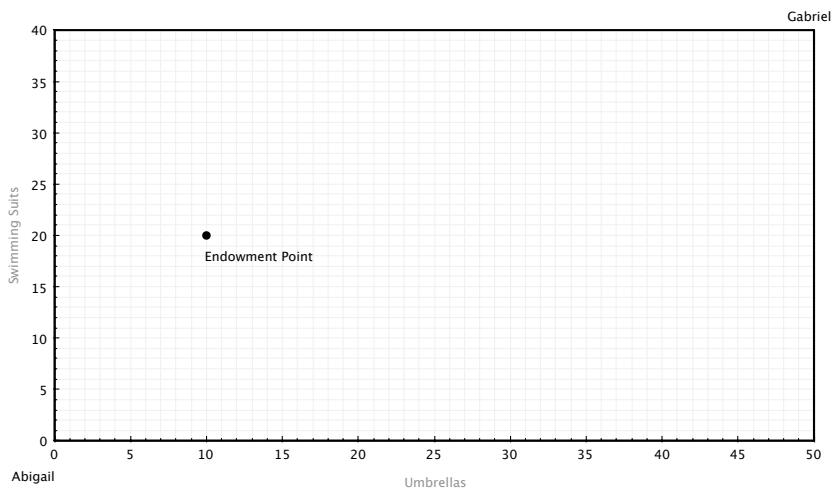
c) Giffen goods are goods that you consume more when their own price increases. Here we have $x_1 = x_2 = \frac{m}{p_1+p_2}$, so x_1 and x_2 are decreaseing in their own price: not Giffen goods.

d) The additional constraint is shown in the graph below, but it is not binding.



Problem 3

a) The Edgeworth box is shown below



b) An allocation is pareto efficient if there are no trades that can make at least one person better off without hurthing the other person. This happens when $MRS_A = MRS_G$. The MRS for both Abigail and Gabriel is $\frac{x_2}{x_1}$. At the endowment point we have $MRS_A = \frac{20}{10}$, and $MRS_B = \frac{20}{40}$. These are not equal so we were not endowed with a pareto efficient allocation.

c) First, the equlibrium only determines relative prices so we are free to normalize one price. Let's say $p_2 = 1$. Abigail and Gabriel have identical Cobb-Douglas preferences so we can use our magic formulas. For x_1:

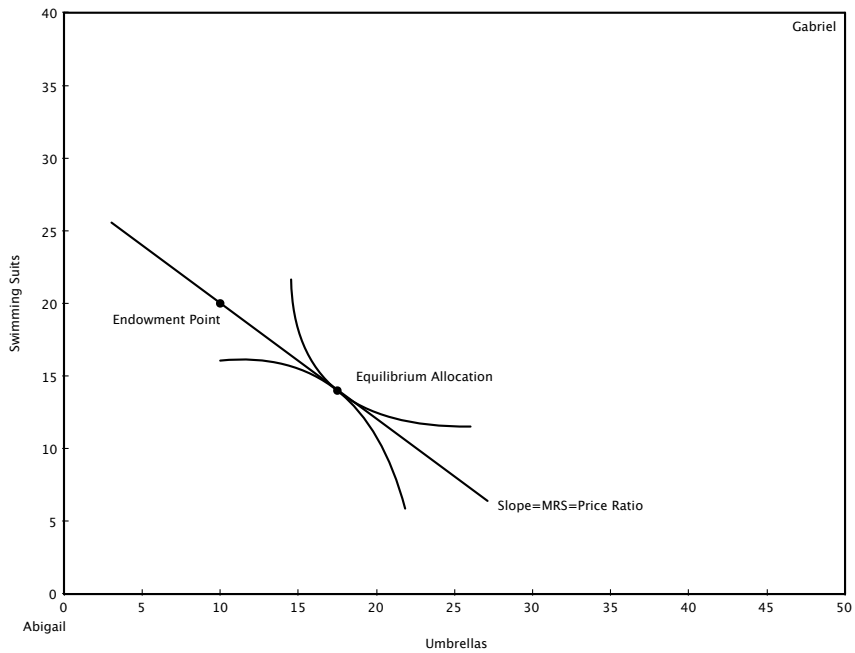
$$x_1^A = \frac{a}{a+b}\frac{m_A}{p_1} = \frac{1}{2}\frac{10p_1+20}{p_1} = 5 + \frac{10}{p_1}$$
$$x_1^G = 20 + \frac{10}{p_1}$$

We can use these two relationships along with the market clearing condition, $x_1^A + x_1^G = 50$, to solve for p_1.

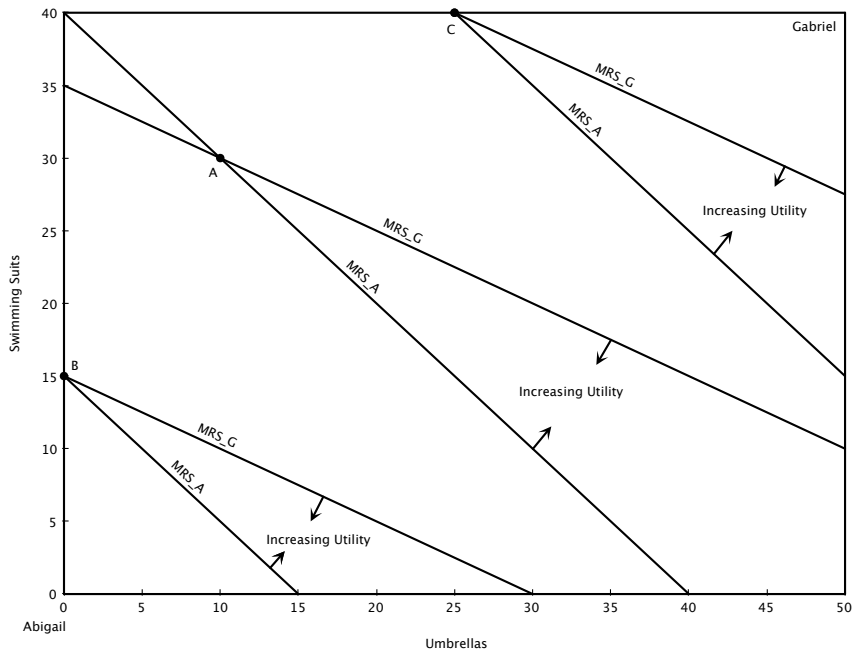
$$50 - x_1^A = 20 + \frac{10}{p_1}$$
$$50 - 5 - \frac{10}{p_1} = 20 + \frac{10}{p_1}$$
$$\Rightarrow p_1 = \frac{4}{5}$$

At this price we have $x_1^A = 5 + \frac{10}{\frac{4}{5}} = 17.5$, $x_1^G = 20 + \frac{10}{p_1} = 32.5$. Using the magic formulas for x_2 we have $x_2^A = 5p_1 + 10 = 14$, $x_2^G = 20p_1 + 10 = 26$. To summarize:

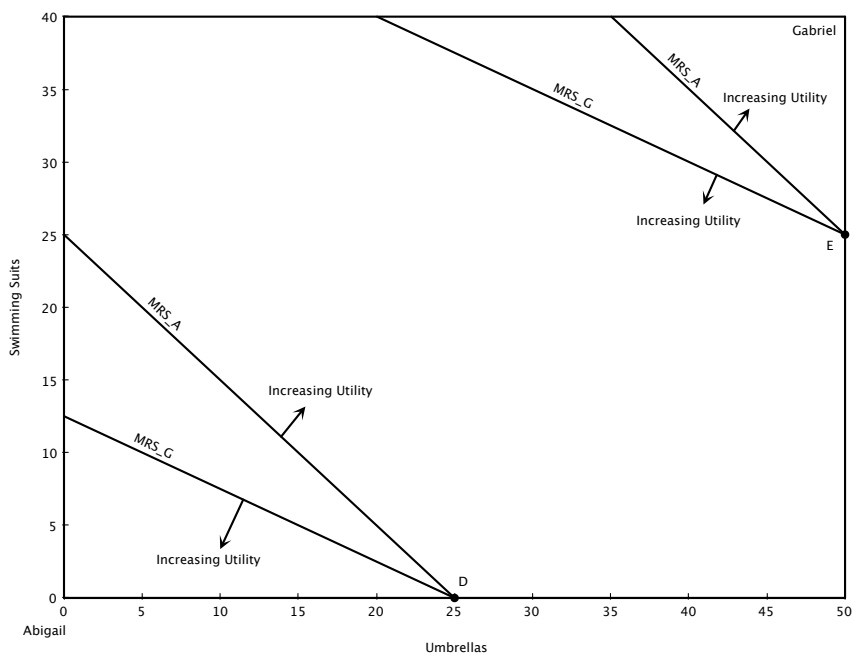
$$(p_1, p_2) = (\tfrac{4}{5}, 1)$$
$$(x_1^A, x_2^A) = (17.5, 14)$$
$$(x_1^G, x_2^G) = (32.5, 26)$$



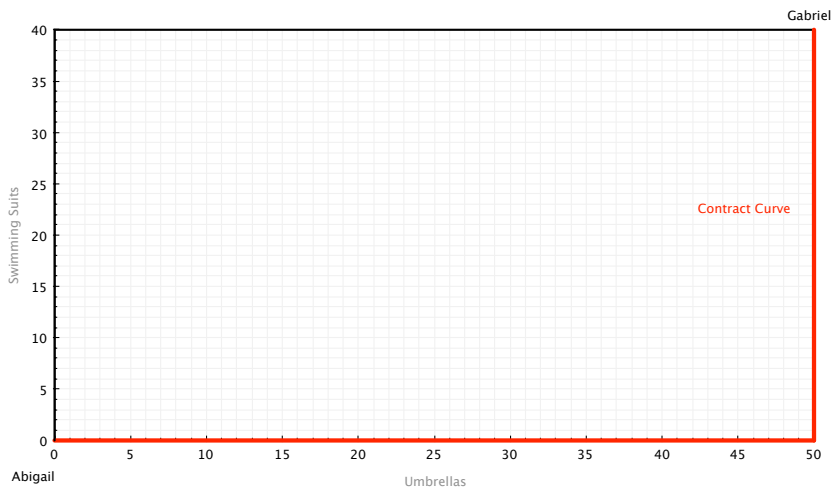
d) $MRS_A = 1$, and $MRS_G = 2$, so our condition for pareto optimality at an interior solution can never be satisfied. However, this doesn't mean there are not pareto efficient allocations. Instead, let's think about several types of allocations in the Edgeworth box and see if they are pareto optimal. First, consider an interior point (A in the figure below), a point on the left border (B), and a point on the top border (C). In each case, both Abigail and Gabriel agree upon which way to move in order to increase their utility, meaning there are pareto improvements.



In contrast, if we look at a point on the bottom border (D), or one on the right border (E), we see that Abigail and Gabriel want to move in different directions to improve utility. This means the points are pareto optimal.



To summarize, the contract curve of pareto optimal allocations consists of the bottom and right borders of the Edgeworth box.



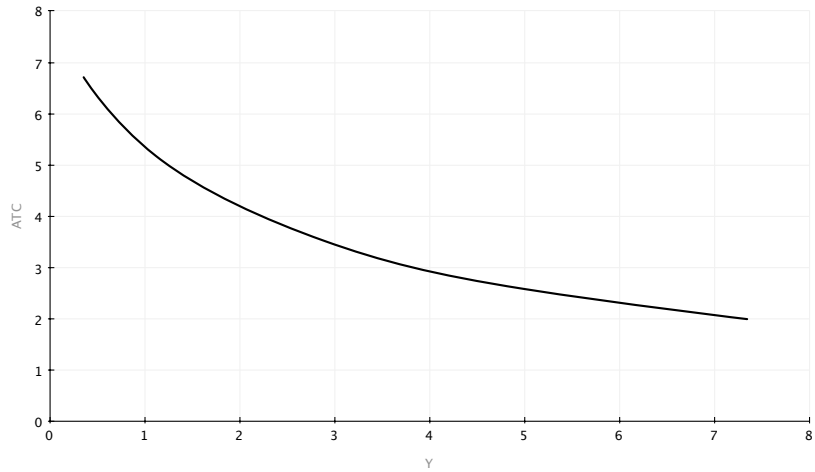
Alternative Argument: Let's normalize $p_2 = 1$ as usual, and then think about restrictions on p_1 that will allow the market to clear. If $p_1 < \frac{1}{2}$ then both Abigail and Gabriel only want to consume x_1, which is infeasible. If $p_1 > 1$, then both Abigail and Gabriel only want to consume x_2, which is also infeasible. If $\frac{1}{2} < p_1 < 1$ then Abigail only wants x_1, while Gabriel only wants x_2, so this corner solution will be feasible. If $p_1 = \frac{1}{2}$ Abigail only wants x_1, while Gabriel is indifferent between x_1 and x_2. Thus, the bottom border of the Edgeworth box (where Abigail has no x_2) is feasible. If $p_1 = 1$ Gabriel only wants x_2, while Abigail is indifferent between x_1 and x_2. Thus, the right border of the Edgeworth box (where Gabriel has no x_1) is feasible.

Problem 4

a) We use the formula for the present value of a perpetuity: $PV = \frac{20}{0.1} = 200$.

b) If we call x_w wealth if you win the lottery, and x_l wealth if you lose, then the von Neuman-Morgenstern expected utility function is $U(x_w, x_l) = \frac{1}{2}x_w^2 + \frac{1}{2}x_l^2$. The certainty equivalent is defined by $ce^2 = \frac{1}{2}4^2 + \frac{1}{2}0^2 \Rightarrow ce = 2.83$. The expected value of the lottery is $\frac{1}{2}4 + \frac{1}{2}0 = 2$. The certainty equivalent is larger than the expected value because the bernouli utility function is convex, which is also the same thing as saying this person is risk loving.

c) $F(K, L) = K^a L^b$, with $1 < a$, $0 < b < 1$, $a + b > 1$. We just know that ATC is decreasing due to the increasing returns to scale.



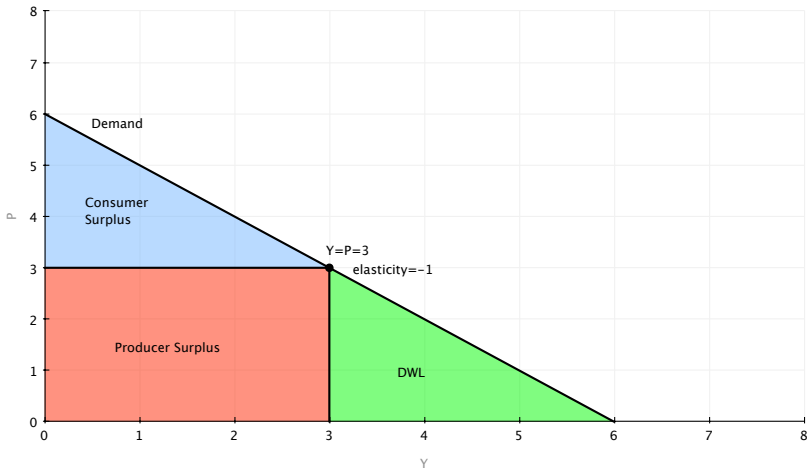
d) With free entry every firm will produce at minimum efficient scale (and make zero profits). If not, a firm could enter, produce at MES, and make positive profits. This would leave the firms originally producing at a level other than MES with negative profits. At $p = ATC^{MES} = 2$, $D(p) = 2$. Thus, it will take two firms producing at MES to satisfy this demand. We have a duopoly. $HHI = (\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{2}$.

e) We know the buyer won't pay more than his expected value for a car. Thus, we need this expected value to be greater than 20 to induce sellers of plums to participate. $\frac{1}{2} * 10 + \frac{1}{2} * 26 = 18 < 20$, so plums will not be sold. This outcome is not pareto efficient because what would be beneficial trades of plums will not occur. To get a pooling equilibrium (where both types of sellers sell) we need $10\pi + 26(1 - \pi) \geq 20 \Rightarrow \pi \leq \frac{3}{8}$.

Problem 5

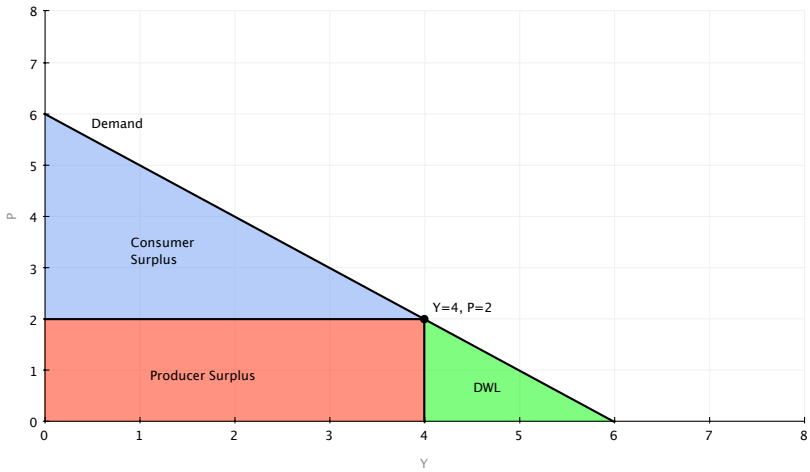
a) The competetive market is pareto efficient so it will provide the benchmark for total gains from trade. Firms in this competitive market produce at $p = MC = 0$, and make no profit. At $p = 0$ consumers purchase 6 units. This leaves consumer surplus (which is the same as total surplus) of $\frac{1}{2} * 6 * 6 = 18$.

b) A monopolist chooses y to $\max(6 - y)y - 0$. The FOC of this problem is $6 - 2y = 0 \Rightarrow y = 3$. They charge price $p = 3$. Demand elasticity is defined by $\epsilon = \frac{dy}{dp}\frac{p}{y}$. At the market equilibrium we have $\epsilon = -1 * \frac{3}{3} = -1$.



c) First degree price discrimination means that the monopolist can charge each customer the maximum price that individual is willing to pay. This outcome is efficient (DWL=0) because all possible beneficial trades occur, but now the monopolist has captured the entire gains from trade of 18.

d) Both firms participate in a symetric Cournot-Nash game where they choose their own quantity in response to the other firm's quantity. That is, firm 1 chooses y_1 to $\max(6-y_1-y_2)y_1$. The FOC of this problem is $6-2y_1-y_2=0$. Thus, the best response function for firm 1 is $y_1 = 3-\frac{1}{2}y_2$. Because the game is symetric (firm 2 faces the same type of decision) we can write down firm 2's best response function $y_2 = 3-\frac{1}{2}y_1$. We solve these best response functions together to locate the Nash equilibrium. This gives $y_1 = y_2 = 2$. Total production is 4, leaving $p = 2$.



e) Both b) and d) have DWL's, but as argued in c), first degree price discrimination is pareto efficient.

Problem 6

a) We will first determine the optimal number of hives for the bee keeper, and then see how the orchard owner will respond to this choice. The bee keeper chooses h to max $2h - \frac{1}{2}h^2$. The FOC for this problem is $h = 2$. Given this choice of h, the orchard owner chooses t to max $5(t+2) - \frac{1}{2}t^2$. The FOC for this problem is $t = 5$.

b) To find the pareto optimal outcome the bee keeper and orchard owner team up to choose both h and t to maximize the joint profit: max $5t + 7h - \frac{1}{2}t^2 - \frac{1}{2}h^2$. The FOC of this problem for h is $h = 7$, and the FOC for t is $t = 5$. The number of trees is the same because h does not affect this choice (h isn't in the FOC for t), but h is higher when maximizing the joint profit because on his own, the bee keeper doesn't care how his supply of bees helps the orchard owner.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final

You have 2h to complete the exam. The final consists of 6 questions (10+10+15+25+25+15=100).

Problem 1.

Ace consumes bananas x_1 and kiwis x_2. The prices of both goods are $p_1 = 4, p_2 = 10$ and Ace's income is $m = 120$. His utility function is

$$U(x_1, x_2) = (x_1)^{20} (x_2)^{40}$$

a) Find analytically Ace's MRS as a function of (x_1, x_2) (give a function) and find its value for the consumption bundle $(x_1, x_2) = (20, 20)$. Give its economic and geometric interpretation (one sentence and find MRS on the graph)

b) Give two secrets of happiness that determine Ace's optimal choice of fruits (give two equation). Explain why violation of any of them implies that the bundle is not optimal (one sentence for each condition).

c) Using magic forumula find the optimal bundle of Ace (two numbers), and show geometrically the .

Problem 2.

Adria collects two types of rare coins: Jefferson Nickels $x_1$and Seated Half Dimes x_2. Her utility from a collection (x_1, x_2) is

$$U(x_1, x_2) = x_1 + x_2$$

a) Propose a utility function that gives a higher level of utility for any (x_1, x_2), but represents the same preferences (give utility function).

b) Suppose the prices of the two types of coins are $p_1 = 4$ and $p_2 = 2$ for x_1, x_2 respectively and the Adria's income is $m = \$20$. Plot her budget set and find the optimal collection (x_1, x_2) and mark it in your graph (give two numbers)

c) Are the coins Giffen goods (yes or no and one sentence explaining why)?

d) Harder: Suppose Adria's provider of coins currently has only six Seated Half Dimes x_2 in stock (hence $x_2 \leq 6$). Plot a budget set with the extra constraint and find (geometrically) an optimal collection given the constraint.

Problem 3. (Equilibrium)

There are two commodities traded on the market: umbrellas x_1 and swimming suits x_2. Abigail has ten umbrellas and twenty swimming suits ($\omega^A = (10, 20)$). Gabriel has forty umbrellas and twenty swimming suits ($\omega^G = (40, 20)$). Abigail and Gabriel have identical utility functions given by

$$U^i(x_1, x_2) = \frac{1}{2} \ln(x_1) + \frac{1}{2} \ln(x_2)$$

a) Plot an Edgeworth box and mark the point corresponding to endowments of Abigail and Gabriel.

b) Give a definition of a Pareto efficient allocation (one sentence) and the equivalent condition in terms of MRS (equation). Verify whether endowment is Pareto efficient (two numbers+one sentence).

c) Find prices and an allocation of umbrellas and swimming suits in a competitive equilibrium and mark it in your graph.

d) Harder: Plot a contract curve in the Edgeworth box assuming utilities for two agents $U^i(x_1, x_2) = \min(x_1, x_2)$.

Problem 4.(Short questions)

a) You are going to pay taxes of \$200 every year, forever. Find the Present Value of your taxes if the yearly interest rate is $r = 10\%$.

b) Consider a lottery that pays 0 with probability $\frac{1}{2}$ and 16 with probability $\frac{1}{2}$ and a Bernoulli utility function is $u(x) = \sqrt{x}$. Give a corresponding von Neuman-Morgenstern utility function. Find the certainty

equivalent of the lottery. Is it bigger or smaller than the expected value of the lottery? Why? (give a utility function, two numbers and one sentence.)

c) Give an example of a Cobb-Douglass production function that is associated with increasing returns to scale, decreasing MPK and decreasing MPL (give a function). Without any calculations, sketch the average total cost function (ATC) associated with your production function.

d) Let the variable cost be $c(y) = y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). Given demand $D(p) = 8 - p$ determine a number of firms in the industry assuming free entry (and price taking). Is the industry monopolistic, duopolistic, oligopolistic or perfectly competitive? Find Herfindahl–Hirschman Index (HHI) of this industry (one number).

e) In a market for second-hand vehicles two types of cars can be traded: lemons (bad quality cars) and plums (good quality ones). The value of a car depends on its type and is given by

	Lemon	Plum
Seller	0	20
Buyer	10	26

Will we observe plums traded on the market if the probability of a lemon is equal to $\frac{1}{2}$? (compare two relevant numbers). Is the equilibrium outcome Pareto efficient (yes-no answer+ one sentence)? Give a threshold probability for which we might observe pooling equilibrium (number).

Problem 5.(Market Power)

Consider an industry with the inverse demand equal to $p(y) = 6 - y$, and suppose that the total cost function is $TC = 2y$.

a) What are the total gains to trade in this industry? (give one number)

b) Find the level of production and the price if there is only one firm in the industry (i.e., we have a monopoly) charging a uniform price (give two numbers). Find demand elasticity at optimum. (give on number) Illustrate the choice using a graph. Mark a DWL.

c) Find the profit of the monopoly and a DWL given that monopoly uses the first degree price discrimination.

d) Find the individual and aggregate production and the price in a Cournot-Nash equilibrium given that there are two firms (give three numbers). Show DWL in the graph.

e) In which of the three cases, (b,c or d) the outcome is Pareto efficient? (chose one+ one sentence)

Problem 6.(Externality)

A bee keeper chooses the number of hives h. Each hive produces one pound of honey which sells at the price of \$10 per pound. The cost of holding h hives is $TC(h) = \frac{1}{2}h^2$. Consequently the profit of bee keeper is equal to

$$\pi_h(h) = 10h - \frac{1}{2}h^2$$

The hives are located next to an apple tree orchard. The bees pollinate the trees and hence the total production of apples $y = h + t$ is increasing in number of trees and bees. Apples sell for \$3 and the cost of t trees is $TC_t(t) = \frac{1}{2}t^2$. Therefore the profit of an orchard grower is

$$\pi_t(t) = 3(t + h) - \frac{1}{2}t^2$$

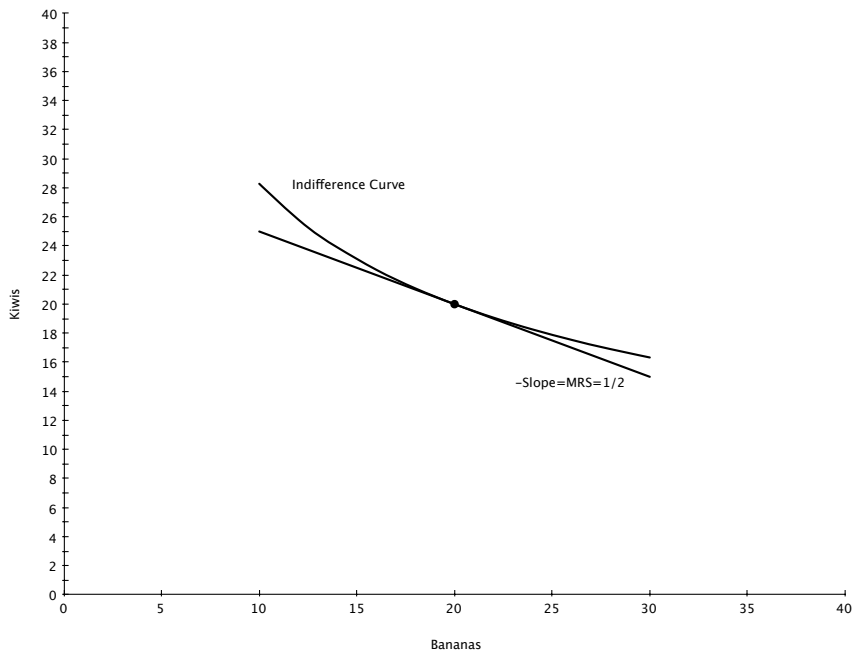
a) Market outcome: Find the level of hives h that maximizes the profit of a beekeeper and the number of trees that maximizes the profit of an orchard owner (assuming h optimal for a bee keeper) (two numbers)

b) Find the Pareto efficient level of h and t. Are the two values higher or smaller then the ones in a)? Why? (two numbers + one sentence)

Makeup Final Solutions
ECON 301
May 15, 2012

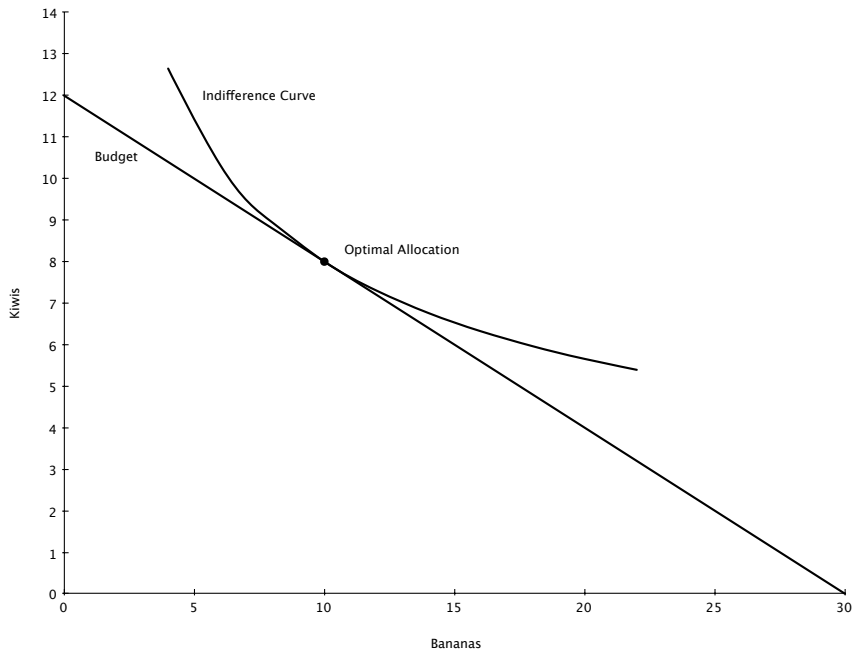
Problem 1

a) Because it is easier and more familiar, we will work with the monotonic transformation (and thus equivalent) utility function: $U(x_1, x_2) = \log x_1 + 2 \log x_2$. $MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{1}{x_1}}{\frac{2}{x_2}} = \frac{x_2}{2x_1}$. At $(x_1, x_2) = (20, 20)$, $MRS = \frac{20}{40} = \frac{1}{2}$. The MRS measures the rate a which you are willing to trade one good for the other. At a particular point in a graph, the MRS will be the negative of the slope of the indifference curve running through that point.



b)
- Budget: $4x_1 + 10x_2 = 120$. With a monotonic utility function like this one, the budget holds with equality because you can always make yourself better off by consuming more. Thus, it makes no sense to leave money unspent.
- $MRS = \frac{p_1}{p_2}$: The price at which you are willing to trade goods for one another (MRS) is the same as the rate at which you can trade the goods for one another (price ratio). Alternatively, you can think of this as the marginal utility per dollar spent on each good is the same: $\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$. If this does not hold you would be able to buy less of one good, spend that money one the other good, and gain more utility than you have lost.

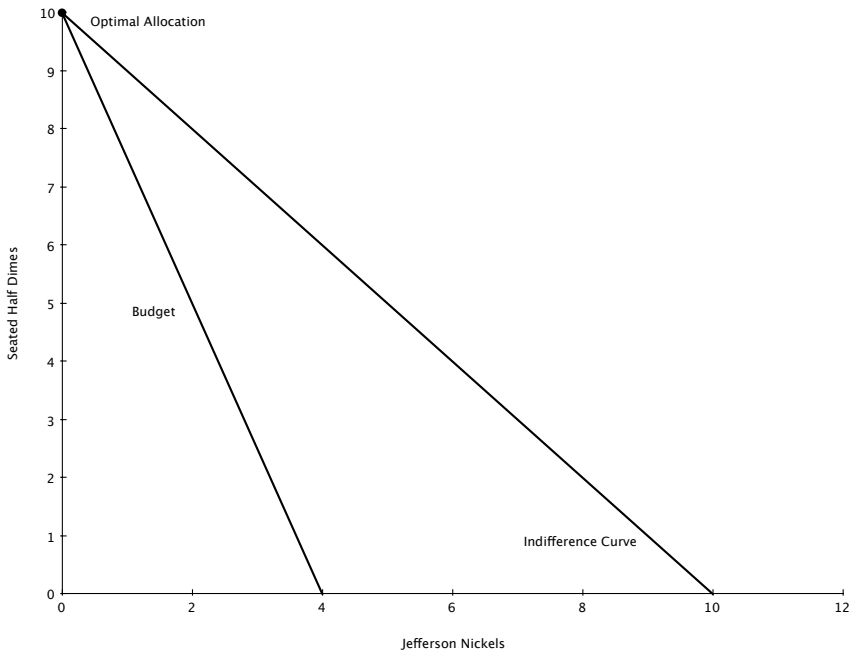
c) The optimal allocation is shown in the graph below



Problem 2

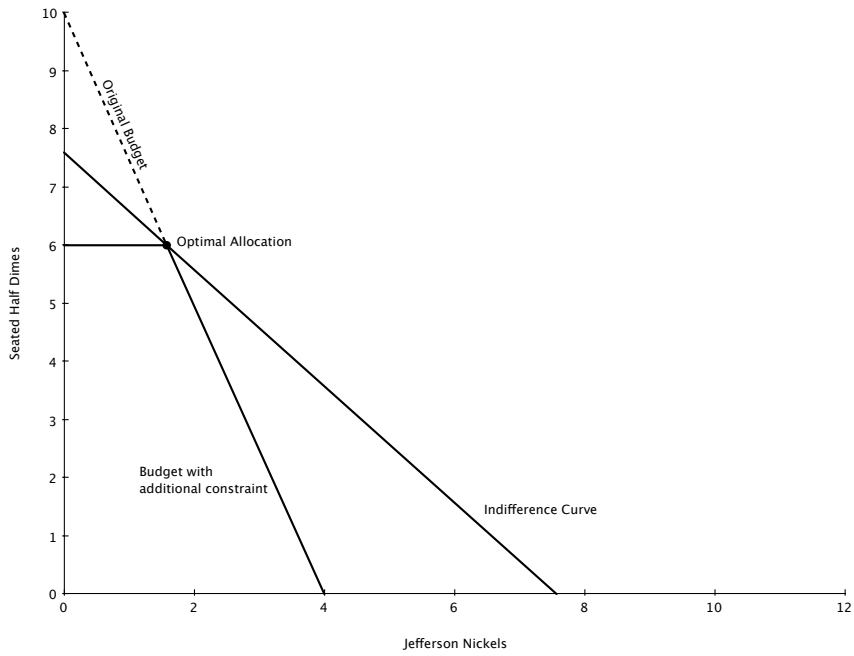
a) Lots of them exist. The most straightforward are $U(x_1, x_2) = A * (x_1 + x_2) + B$, with $A \geq 1$, $B \geq 0$, and $A + B > 1$. These represent the same preferences because they are monotonic transformations.

b) Since we are dealing with perfect substitues we know we will have a corner solution. We will choose only the good that delivers utility in the least expensive manner. Because each unit of x_1 and x_2 give the same amount of utility, this will be the cheaper good, x_2. At $p_2 = 2$ and $m = 20$ we can afford $x_2 = 10$.



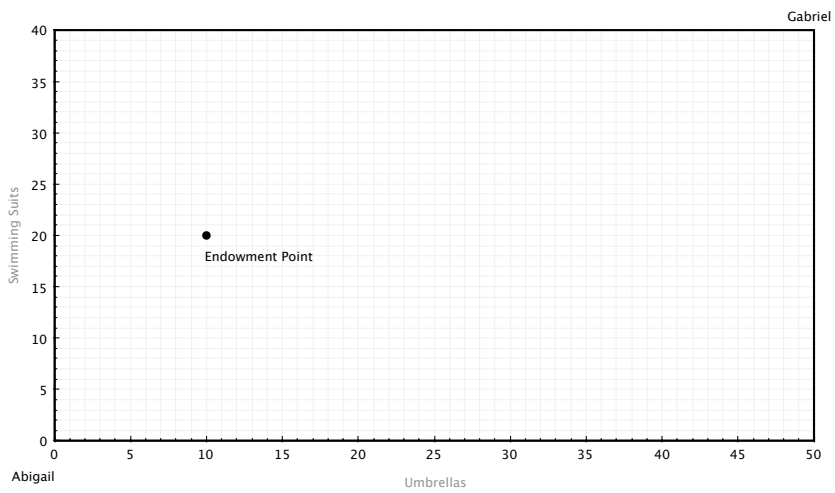
c) Giffen goods are goods that you consume more when their own price increases. Here you spend all your money on the cheaper good. As the price of that good increases you can buy less of it, until it becomes the more expensive good at which point you switch entirely to the other good: not Giffen goods.

d) As shown in the graph below, the additional constraint forces you to start buying Jefferson Nickels after all 6 Seated Half Dimes have been purchased.



Problem 3

a) The Edgeworth box is shown below



b) An allocation is pareto efficient if there are no trades that can make at least one person better off without hurthing the other person. This happens when $MRS_A = MRS_G$. The MRS for both Abigail and Gabriel is $\frac{x_2}{x_1}$. At the endowment point we have $MRS_A = \frac{20}{10}$, and $MRS_B = \frac{20}{40}$. These are not equal so we were not endowed with a pareto efficient allocation.

c) First, the equlibrium only determines relative prices so we are free to normalize one price. Let's say $p_2 = 1$. Abigail and Gabriel have identical Cobb-Douglas preferences so we can use our magic formulas. For x_1:

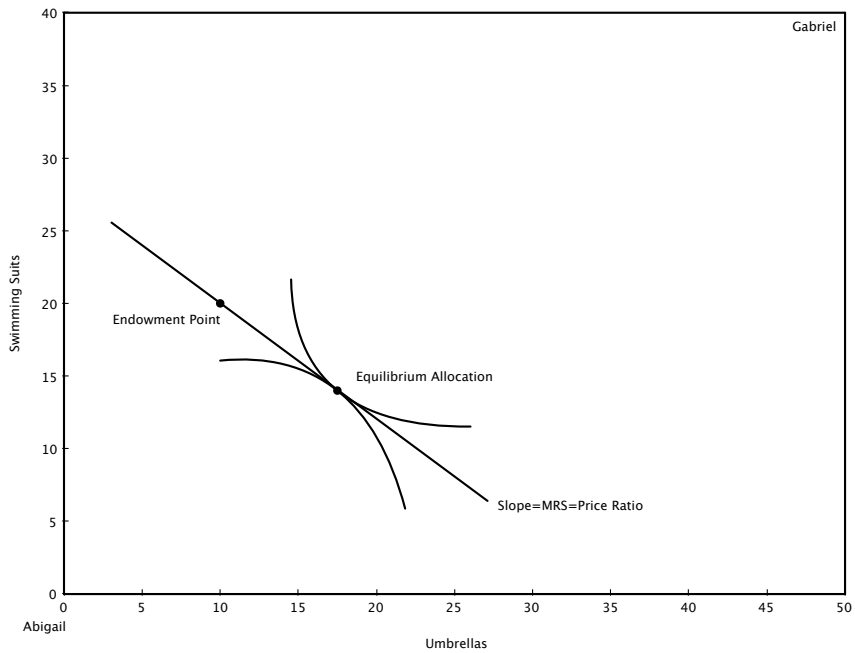
$$x_1^A = \frac{a}{a+b}\frac{m_A}{p_1} = \frac{1}{2}\frac{10p_1+20}{p_1} = 5 + \frac{10}{p_1}$$
$$x_1^G = 20 + \frac{10}{p_1}$$

We can use these two relationships along with the market clearing condition, $x_1^A + x_1^G = 50$, to solve for p_1.

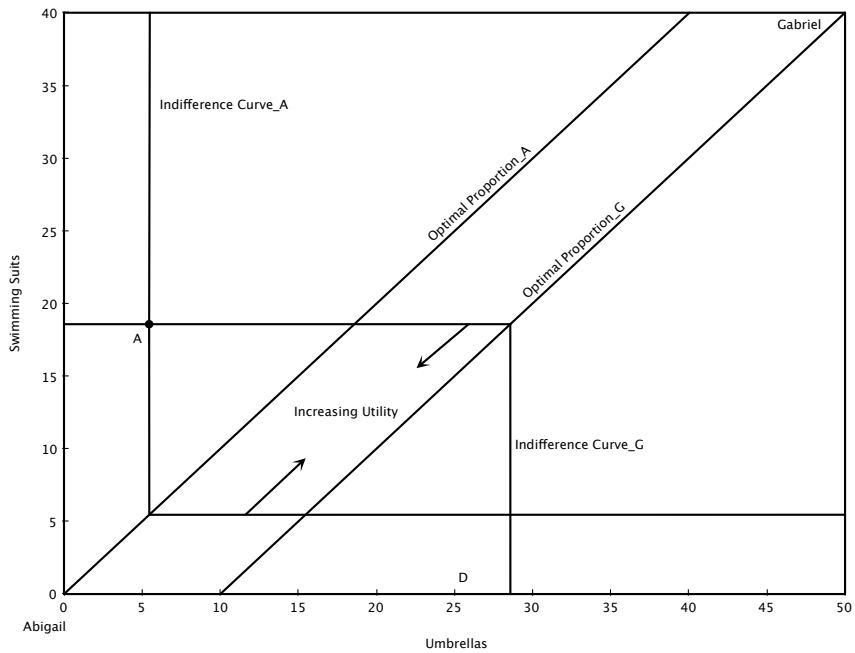
$$50 - x_1^A = 20 + \frac{10}{p_1}$$
$$50 - 5 - \frac{10}{p_1} = 20 + \frac{10}{p_1}$$
$$\Rightarrow p_1 = \frac{4}{5}$$

At this price we have $x_1^A = 5 + \frac{10}{\frac{4}{5}} = 17.5$, $x_1^G = 20 + \frac{10}{p_1} = 32.5$. Using the magic formulas for x_2 we have $x_2^A = 5p_1 + 10 = 14$, $x_2^G = 20p_1 + 10 = 26$. To summarize:

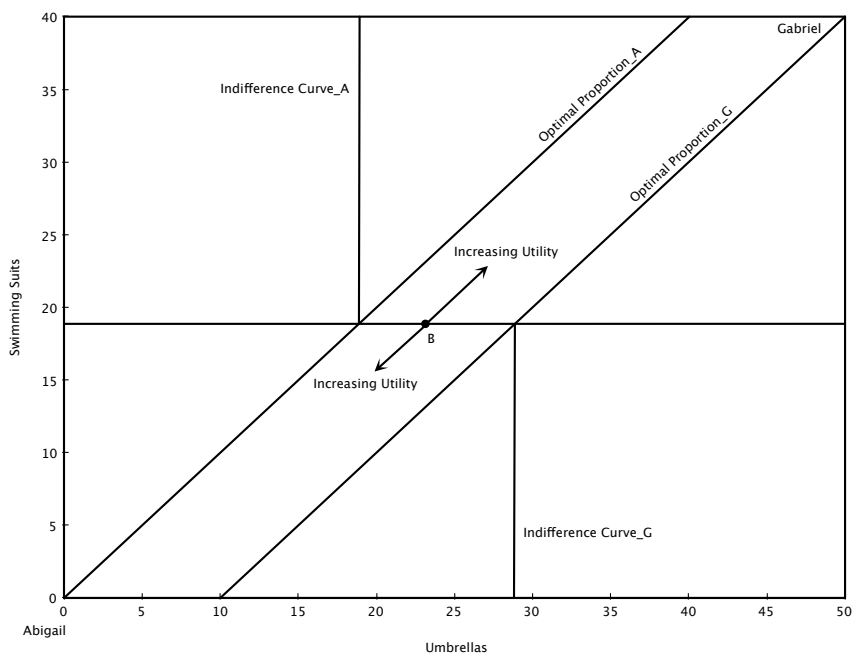
$$(p_1, p_2) = (\tfrac{4}{5}, 1)$$
$$(x_1^A, x_2^A) = (17.5, 14)$$
$$(x_1^G, x_2^G) = (32.5, 26)$$



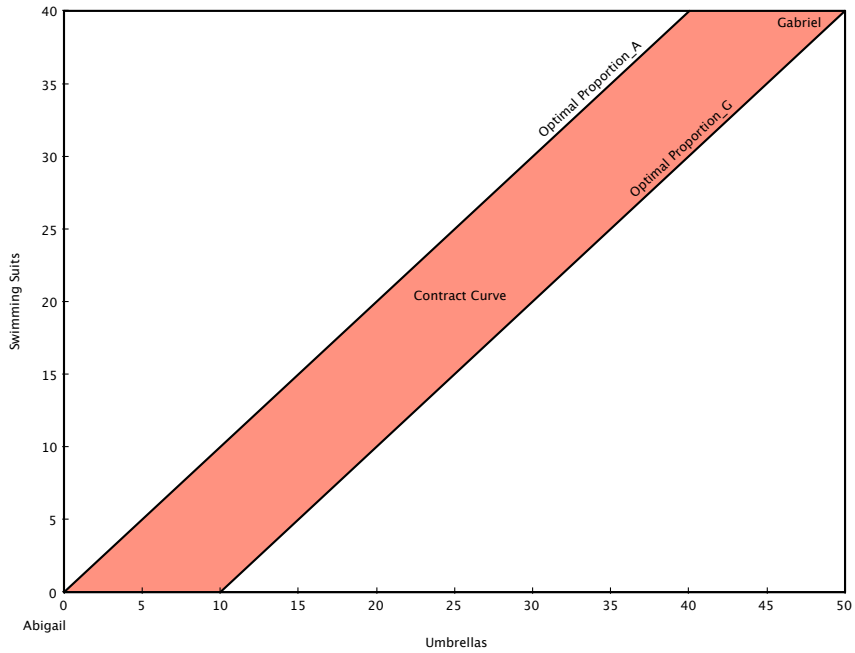
d) With perfect complements the MRS is not defined at the optimal point, so we can't equate them to find the contract curve. The optimal proportion line for both Abigail and Gabriel is where $x_1 = x_2$, but because the Edgeworth box is not square these lines do not coincide. However, this doesn't mean there are not pareto efficient allocations. Instead, let's think about several types of allocations in the Edgeworth box and see if they are pareto optimal. First, consider a point outside the two optimal proportion lines (A in the figure below). Both Abigail and Gabriel agree upon which way to move in order to increase their utility, meaning is a pareto improvement.



In contrast, if we look at a point in between the two optimal proportion lines (B), we see that Abigail and Gabriel want to move in different directions to improve utility. This means the point is pareto optimal.



To summarize, the contract curve of pareto optimal allocations is the space in between the two optimal proportion lines.

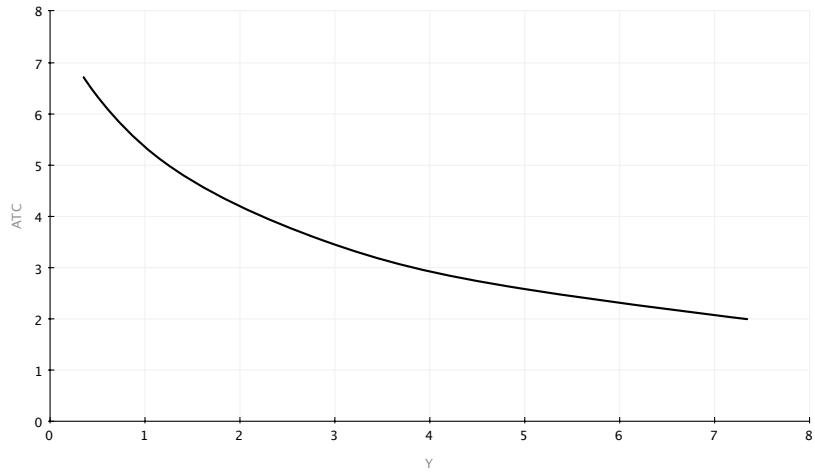


Problem 4

a) We use the formula for the present value of a perpetuity: $PV = \frac{20}{0.1} = 200$.

b) If we call x_w wealth if you win the lottery, and x_l wealth if you lose, then the von Neuman-Morgenstern expected utility function is $U(x_w, x_l) = \frac{1}{2}\sqrt{x_w} + \frac{1}{2}\sqrt{x_l}$. The certainty equivalent is defined by $\sqrt{ce} = \frac{1}{2}\sqrt{16} + \frac{1}{2}\sqrt{0} \Rightarrow ce = 4$. The expected value of the lottery is $\frac{1}{2}16 + \frac{1}{2}0 = 8$. The certainty equivalent is smaller than the expected value because the bernouli utility function is concave, which is also the same thing as saying this person is risk averse.

c) $F(K, L) = K^a L^b$, with $0 < a < 1$, $0 < b < 1$, $a + b > 1$. We just know that ATC is decreasing due to the increasing returns to scale.



d) Total cost is given by $y^2 + 4$, which makes $ATC = y + \frac{4}{y}$. We minimize this function to find ATC^{MES} and y^{MES}. Since it is a convex function the FOC will find the minimum. The FOC is $1 - \frac{4}{y^2} = 0 \Rightarrow y^{MES} = 2$. Then, $ATC^{MES} = 2 + \frac{4}{2} = 4$. With free entry every firm will produce at minimum efficient scale (and make zero profits). If not, a firm could enter, produce at MES, and make positive profits. This would leave the firms originally producing at a level other than MES with negative profits. At $p = ATC^{MES} = 4$, $D(p) = 4$. Thus, it will take two firms producing at MES to satisfy this demand. We have a duopoly. $HHI = (\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{2}$.

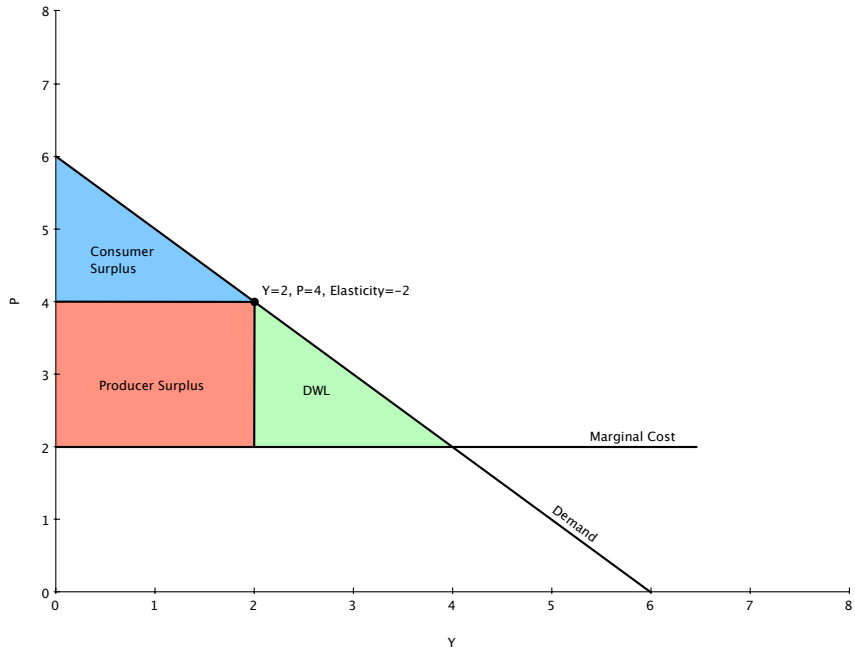
e) We know the buyer won't pay more than his expected value for a car. Thus, we need this expected value to be greater than 20 to induce sellers of plums to participate. $\frac{1}{2} * 10 + \frac{1}{2} * 26 = 18 < 20$, so plums will not be sold. This outcome is not pareto efficient because what would be beneficial trades of plums will not occur. To get a pooling equilibrium (where both types of sellers sell) we need $10\pi + 26(1 - \pi) \geq 20 \Rightarrow \pi \leq \frac{3}{8}$.

Problem 5

a) The competetive market is pareto efficient so it will provide the benchmark for total gains from trade. Firms in this competitive market produce at $p = MC = 2$, and make no profit. At $p = 2$ consumers purchase 4 units. This leaves consumer surplus (which is the same as total surplus) of $\frac{1}{2} * (6 - 2) * 4 = 8$.

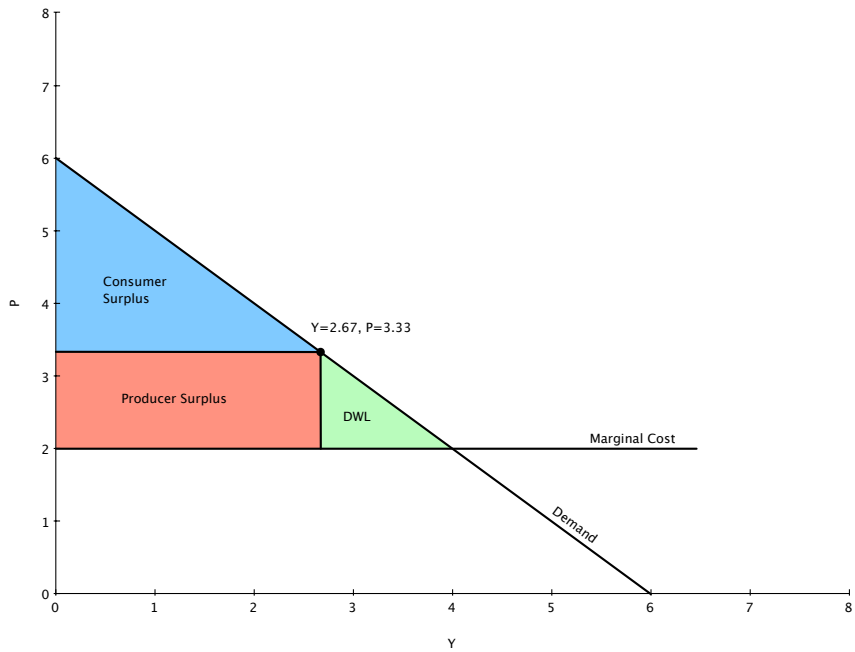
b) A monopolist chooses y to $\max(6 - y)y - 2y$. The FOC of this problem is $6 - 2y = 2 \Rightarrow y = 2$. They charge price $p = 4$. Demand elasticity is defined by $\epsilon = \frac{dy}{dp}\frac{p}{y}$. At the market equilibrium

we have $\epsilon = -1 * \frac{4}{2} = -2$.



c) First degree price discrimination means that the monopolist can charge each customer the maximum price that individual is willing to pay, and will do so as long as that price is larger than the marginal cost of 2. This outcome is efficient (DWL=0) because all possible beneficial trades occur, but now the monopolist has captured the entire gains from trade of 8.

d) Both firms participate in a symetric Cournot-Nash game where they choose their own quantity in response to the other firm's quantity. That is, firm 1 chooses y_1 to $\max(6-y_1-y_2)y_1 - 2y_1$. The FOC of this problem is $4 - 2y_1 - y_2 = 0$. Thus, the best response function for firm 1 is $y_1 = 2 - \frac{1}{2}y_2$. Because the game is symetric (firm 2 faces the same type of decision) we can write down firm 2's best response function $y_2 = 2 - \frac{1}{2}y_1$. We solve these best response functions together to locate the Nash equilibrium. This gives $y_1 = y_2 = \frac{4}{3}$. Total production is $2\frac{2}{3}$, leaving $p = 3\frac{1}{3}$.



e) Both b) and d) have DWL's, but as argued in c), first degree price discrimination is pareto efficient.

Problem 6

a) We will first determine the optimal number of hives for the bee keeper, and then see how the orchard owner will respond to this choice. The bee keeper chooses h to max $10h - \frac{1}{2}h^2$. The FOC for this problem is $h = 10$. Given this choice of h, the orchard owner chooses t to max $3(t + 10) - \frac{1}{2}t^2$. The FOC for this problem is $t = 3$.

b) To find the pareto optimal outcome the bee keeper and orchard owner team up to choose both h and t to maximize the joint profit: max $3t + 13h - \frac{1}{2}t^2 - \frac{1}{2}h^2$. The FOC of this problem for h is $h = 13$, and the FOC for t is $t = 3$. The number of trees is the same because h does not affect this choice (h isn't in the FOC for t), but h is higher when maximizing the joint profit because on his own, the bee keeper doesn't care how his supply of bees helps the orchard owner.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (A)

You have 2h to complete the exam and the final consists of 6 questions (15+10+25+15+20+15=100).

Problem 1. (Consumer Choice)

Jeremy's favorite flowers are tulips x_1 and daffodils x_2. Suppose $p_1 = 2$, $p_2 = 4$ and $m = 40$.

a) Write down Jeremy's budget constraint (a formula) and plot all Jeremy's affordable bundles in the graph (his budget set). Find the slope of a budget line (number). Give an economic interpretation for the slope of the budget line (one sentence).

b) Jeremy's utility function is given by

$$U(x_1, x_2) = \sqrt{(\ln x_1 + \ln x_2)^2 + 7}$$

Propose a simpler utility function that represents the same preferences (give a formula). Explain why your utility represents the same preferences (one sentence).

c)[1] Plot Jeremy's indifference curve map (graph), find MRS analytically (give a formula) and find its value at bundle $(2, 4)$ (one number). Give economic interpretation of this number (one sentence). Mark its value in the graph.

d) Write down two secrets of happiness (two equalities) that allow determining the optimal bundle. Provide their geometric interpretation (one sentence for each). Find the optimal bundle (x_1, x_2) (two numbers). Is your solution interior? (a yes -no answer)

e) Hard: Find the optimal bundle given $p_1 = 2$, $p_2 = 4$ and $m = 40$ assuming $U(x_1, x_2) = 2x_1 + 3x_2$ (two numbers). Is your solution interior? (a yes -no answer)

Problem 2. (Producers)

Consider production function given by $F(K, L) = 3K^{\frac{1}{4}}L^{\frac{1}{4}}$.

a) Using the λ argument demonstrate that production function exhibits decreasing returns to scale.

b) Derive the cost function given $w_K = w_L = 9$.

c) Derive a supply function of a competitive firm, assuming the cost function from b) and fixed cost $F = 2$ (give a formula for $y(p)$). Plot the supply function in a graph, marking the threshold price below which a firm chooses inaction.

Problem 3. (Competitive Equilibrium)

Consider an economy with apples and oranges. Dustin's endowment of two commodities is given by $\omega^D = (8, 2)$ and Kate's endowment is $\omega^K = (2, 8)$. The utility functions of Dustin and Kate are the same and given by

$$U^i\left(x_1^i, x_2^i\right) = 5\ln x_1^i + 5\ln x_2^i$$

where $i = D, K$.

a) Plot the Edgeworth box and mark the point corresponding to the initial endowments.

b) Give a general definition of Pareto efficient allocation x (one sentence) and state its equivalent condition in terms of MRS (one sentence, you do not need to prove the equivalence).

c) Using the "MRS" condition verify that the initial endowments are not Pareto efficient.

d) Find a competitive equilibrium (six numbers). Provide an example of a competitive equilibrium with some other prices (six numbers).

e) Using MRS condition verify that the competitive allocation is Pareto efficient.

f) Hard: Find prices p_1, p_2 in a competitive equilibrium for identical preferences of two agents $U(x_1, x_2) = 2x_1 + 3x_2$ (two numbers, no calculations). Explain why any two prices that give rise to a relative price higher than p_1/p_2 cannot be equilibrium prices (which condition of equilibrium fails?)

[1] If you do not know the answer to b), to get partial credit in points c)-e) your can assume $U(x_1, x_2) = x_1 x_2$.

Problem 4. (Short Questions)

a) Uncertainty: Find the certainty equivalent of a lottery which, in two equally likely states, pays $(0, 9)$. Bernoulli utility function is $u(c) = \sqrt{c}$ (one number). Is the certainty equivalent smaller or bigger than the expected value of a lottery 4.5. Why? (one sentence)

b) Market for lemons: In a market for racing horses one can find two types of animals: champions (Plums) and ordinary recreational horses (Lemons). Buyers can distinguish between the two types only long after they buy a horse. The values of the two types of horses for buyers and sellers are summarized in the table

	Lemon	Plum
Seller	1	4
Buyer	2	5

Are champions (Plums) going to be traded if probability of Lemons is $\frac{1}{2}$. (yes-no). Why? (a one sentence argument that involves the expected value of a horse to a buyer)

c) Signaling: The productivity of high ability workers (and hence the competitive wage rate) is 1000 while productivity of low ability workers is only 400. To determine the type, employer can, first offer an internship program with the length of x months, during which a worker has to demonstrate her high productivity. A low ability worker by putting extra effort can mimic high ability performance, which costs him $c(x) = 200x$. Find the minimal length x for which the internship becomes a credible signal of high ability. (one number)

d) PV of Perpetuity: You can rent an apartment paying 1000 per month (starting next month, till the "end of the world") or you can buy the apartment for 100.000. Which option are you going to chose if monthly interest rate is $r = 2\%$? (find the PV of rent and compare two numbers)

Problem 5. (Market Power)

Consider a monopoly facing the inverse demand $p(y) = 25 - y$, and with total cost $TC(y) = 5y$.

a) Find the marginal revenue of a monopoly, $MR(y)$ and depict it in a graph together with the demand (formula +graph). Which is bigger: price or marginal revenue? Why? (one sentence)

b) Find the optimal level of production and price (two numbers). Illustrate the optimal choice in a graph, depicting Consumer and Producer Surplus, and DWL (three numbers +graph).

c) Find equilibrium markup (one number).

d) First Degree Price Discrimination: Find Total Surplus, Consumer, Producer Surplus and DWL if monopoly can perfectly discriminate among buyers and quantities. (four numbers +graph)

e) Hard: find the individual level of production and price in a Cournot-Nash equilibrium with N identical firms with cost $TC(y) = 5y$, both as a function of N (two formulas). Argue that the equilibrium price converges to the marginal cost as N goes to infinity.

Problem 6. (Public good: Music downloads)

Freddy and Miriam share the same collection of songs downloaded from i-tunes (they have one PC). Each song costs 1. If Freddy downloads x^F and Miriam x^M, their collection contains $x^F + x^M$ and utility of Freddy (net of the cost) is given by

$$u^F\left(x^F\right) = 200\ln(x^F + x^M) - x^F,$$

while Miriam's utility (net of the cost) is

$$u^M\left(x^M\right) = 100\ln(x^F + x^M) - x^M,$$

(Observe that Freddy is more into music than Miriam.)

a) Find optimal number of downloads by Freddy x^F (his best response) for any choice of Miriam x^M (formula $x^F = R^F(x^M)$). Plot the best response in the coordinate system (x^F, x^M).

(Hint: You do not need prices. Utility functions are net cost and hence you just have to take the derivative with respect to x^F and equalize it to zero).

b) Find the optimal number of downloads by Miriam x^M, (her best response) for any choice of Freddy x^F and plot it in the coordinate system from point a).

c) Find the number of downloads in the Nash equilibrium (two numbers). Do we observe the free riding problem? (yes-no + one sentence)

d) Hard: Find Pareto efficient number of downloads $x = x^M + x^F$ (one number). Compare the Pareto efficient level of x with the equilibrium one. Which is bigger and why?

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Final Exam (B)

You have 2h to complete the exam and the final consists of 6 questions (15+10+25+15+20+15=100).

Problem 1. (Consumer Choice)

Jeremy's favorite flowers are tulips x_1 and daffodils x_2. Suppose $p_1 = 5$, $p_2 = 10$ and $m = 100$.

a) Write down Jeremy's budget constraint (a formula) and plot all Jeremy's affordable bundles in the graph (his budget set). Find the slope of a budget line (number). Give an economic interpretation for the slope of the budget line (one sentence).

b) Jeremy's utility function is given by

$$U(x_1, x_2) = \sqrt{(3\ln x_1 + 3\ln x_2)^4 + 8}$$

Propose a simpler utility function that represents the same preferences (give a formula). Explain why your utility represents the same preferences (one sentence).

c)[2] Plot Jeremy's indifference curve map (graph), find MRS analytically (give a formula) and find its value at bundle $(2, 4)$ (one number). Give economic interpretation of this number (one sentence). Mark its value in the graph.

d) Write down two secrets of happiness (two equalities) that allow determining the optimal bundle. Provide their geometric interpretation (one sentence for each). Find the optimal bundle (x_1, x_2) (two numbers). Is your solution interior? (a yes -no answer)

e) Hard: Find the optimal bundle given $p_1 = 5$, $p_2 = 10$ and $m = 100$ assuming $U(x_1, x_2) = 2x_1 + 3x_2$ (two numbers). Is your solution interior? (a yes -no answer)

Problem 2. (Producers)

Consider production function given by $F(K, L) = 5K^{\frac{1}{4}}L^{\frac{1}{4}}$.

a) Using the λ argument demonstrate that production function exhibits decreasing returns to scale.

b) Derive the cost function given $w_K = w_L = 25$.

c) Derive a supply function of a competitive firm, assuming the cost function from b) and fixed cost $F = 2$ (give a formula for $y(p)$). Plot the supply function in a graph, marking the threshold price below which a firm chooses inaction.

Problem 3. (Competitive Equilibrium)

Consider an economy with apples and oranges. Dustin's endowment of two commodities is given by $\omega^D = (20, 10)$ and Kate's endowment is $\omega^K = (10, 20)$. The utility functions of Dustin and Kate are the same and given by

$$U^i\left(x_1^i, x_2^i\right) = 4\ln x_1^i + 4\ln x_2^i$$

where $i = D, K$.

a) Plot the Edgeworth box and mark the point corresponding to the initial endowments.

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c) Using the "MRS" condition verify that the initial endowments are not Pareto efficient.

d) Find a competitive equilibrium (six numbers). Provide an example of a competitive equilibrium with some other prices (six numbers).

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f) Hard: Find prices p_1, p_2 in a competitive equilibrium for identical preferences of two agents $U(x_1, x_2) = 2x_1 + 3x_2$ (two numbers, no calculations). Explain why any two prices that give rise to a relative price higher than p_1/p_2 cannot be equilibrium prices (which condition of equilibrium fails?)

[2] If you do not know the answer to b), to get partial credit in points c)-e) your can assume $U(x_1, x_2) = x_1 x_2$.

Problem 4. (Short Questions)

a) Uncertainty: Find the certainty equivalent of a lottery which, in two equally likely states, pays $(16, 0)$. Bernoulli utility function is $u(c) = \sqrt{c}$ (one number). Is the certainty equivalent smaller or bigger than the expected value of a lottery 8. Why? (one sentence)

b) Market for lemons: In a market for racing horses one can find two types of animals: champions (Plums) and ordinary recreational horses (Lemons). Buyers can distinguish between the two types only long after they buy a horse. The values of the two types of horses for buyers and sellers are summarized in the table

	Lemon	Plum
Seller	1	6
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Are champions (Plums) going to be traded if probability of Lemons is $\frac{1}{2}$. (yes-no). Why? (a one sentence argument that involves the expected value of a horse to a buyer)

c) Signaling: The productivity of high ability workers (and hence the competitive wage rate) is 1000 while productivity of low ability workers is only 400. To determine the type, employer can, first offer an internship program with the length of x months, during which a worker has to demonstrate her high productivity. A low ability worker by putting extra effort can mimic high ability performance, which costs him $c(x) = 200x$. Find the minimal length x for which the internship becomes a credible signal of high ability. (one number)

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Problem 5. (Market Power)

Consider a monopoly facing the inverse demand $p(y) = 40 - y$, and with total cost $TC(y) = 20y$.

a) Find the marginal revenue of a monopoly, $MR(y)$ and depict it in a graph together with the demand (formula +graph). Which is bigger: price or marginal revenue? Why? (one sentence)

b) Find the optimal level of production and price (two numbers). Illustrate the optimal choice in a graph, depicting Consumer and Producer Surplus, and DWL (three numbers +graph).

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$$u^F\left(x^F\right) = 200\ln(x^F + x^M) - x^F,$$

while Miriam's utility (net of the cost) is

$$u^M\left(x^M\right) = 100\ln(x^F + x^M) - x^M,$$

(Observe that Freddy is more into music than Miriam.)

a) Find optimal number of downloads by Freddy x^F (his best response) for any choice of Miriam x^M (formula $x^F = R^F(x^M)$). Plot the best response in the coordinate system (x^F, x^M).

(Hint: You do not need prices. Utility functions are net cost and hence you just have to take the derivative with respect to x^F and equalize it to zero).

b) Find the optimal number of downloads by Miriam x^M, (her best response) for any choice of Freddy x^F and plot it in the coordinate system from point a).

c) Find the number of downloads in the Nash equilibrium (two numbers). Do we observe the free riding problem? (yes-no + one sentence)

d) Hard: Find Pareto efficient number of downloads $x = x^M + x^F$ (one number). Compare the Pareto efficient level of x with the equilibrium one. Which is bigger and why?

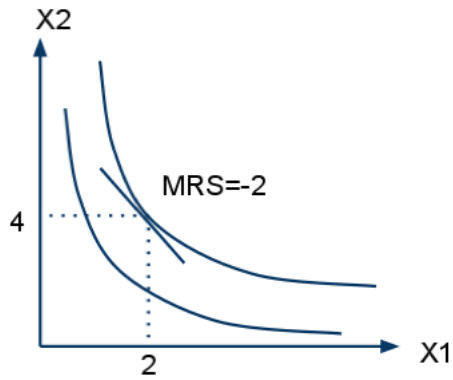
Econ 301 Final Exam Solutions - Spring 2011
Group A

Problem 1 - Consumer Choice

a) The budget constraint is $2x_1 + 4x_2 = 40$. The slope of the budget line is $-\frac{p_1}{p_2} = -\frac{2}{4} = -0.5$. Interpretation of the slope: the relative price of one tulip in the market is $\frac{1}{2}$ daffodil.

b) $U = \ln x_1 + \ln x_2$ or $U = x_1 x_2$ will represent the same preferences, since the operations of taking square root, adding a constant and taking the square of a function are all monotone transformations.

c)$MRS = -\frac{MU_{x_1}}{MU_{x_2}} = -\frac{x_2}{x_1}$. At $(2, 4)$ $MRS = -2$. Interpretation: given 2 tulips and 4 daffodils, Jeremy is willing to trade 1 tulip for 2 daffodils.



d) The two secrets of happiness are

$$2x_1 + 4x_2 = 40$$
$$-\frac{x_2}{x_1} = -0.5$$

The first condition implies the optimal bundle lies on the budget line. The second condition guarantees the indifference curve is tangent to the budget line at optimum. (MRS equals the slope of the budget line)
The optimal bundle is $(x_1, x_2) = (10, 5)$. The solution is interior since $x_1 \neq 0$ and $x_2 \neq 0$.

e) The function $U = 2x_1 + 3x_2$ represents preferences over perfect substitutes. $\frac{MU_1}{p_1} = \frac{2}{2} = 1 > \frac{MU_2}{p_2} = \frac{3}{4}$. Hence good 1 only will be consumed, and optimal $(x_1, x_2) = (20, 0)$. The solution is not interior.

Problem 2 - Producers

a) Take $\lambda > 1$. $F(\lambda K, \lambda L) = 3(\lambda K)^{\frac{1}{4}}(\lambda L)^{\frac{1}{4}} = \lambda^{\frac{1}{2}} \cdot 3K^{\frac{1}{4}}L^{\frac{1}{4}}$
$\lambda F(K, L) = \lambda \cdot 3K^{\frac{1}{4}}L^{\frac{1}{4}}$. Then $F(\lambda K, \lambda L) < \lambda F(K, L)$ and the production function exhibits decreasing returns to scale.

b) First apply the cost-minimization condition:

$$\frac{MP_K}{MP_L} = \frac{w_K}{w_L}$$

Given $w_K = w_L = 9$, $\frac{w_K}{w_L} = 1$. Plug in $MP_K = \frac{3}{4}K^{-\frac{3}{4}}L^{\frac{1}{4}}$ and $MP_L = \frac{3}{4}K^{\frac{1}{4}}L^{-\frac{3}{4}}$:

$$\frac{\frac{3}{4}K^{-\frac{3}{4}}L^{\frac{1}{4}}}{\frac{3}{4}K^{\frac{1}{4}}L^{-\frac{3}{4}}} = 1$$

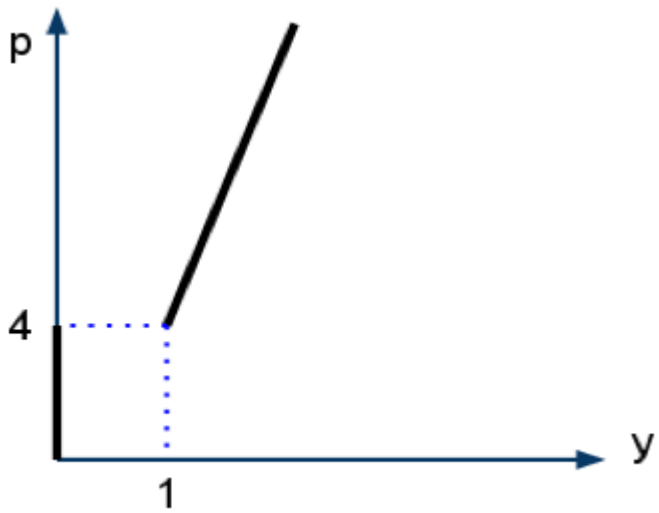
Therefore in optimum $K = L$, which implies $y = 3\sqrt{K}$ and $y = 3\sqrt{L}$, so $K = L = \frac{y^2}{9}$.
Plug the result into the cost function: $c(y) = w_K K + w_L L = 9\frac{y^2}{9} + 9\frac{y^2}{9} = 2y^2$.

c) A competitive firm facing price p, variable costs $2y^2$ and fixed costs 2 maximizes

$$\pi(y) = py - 2y^2 - 2$$

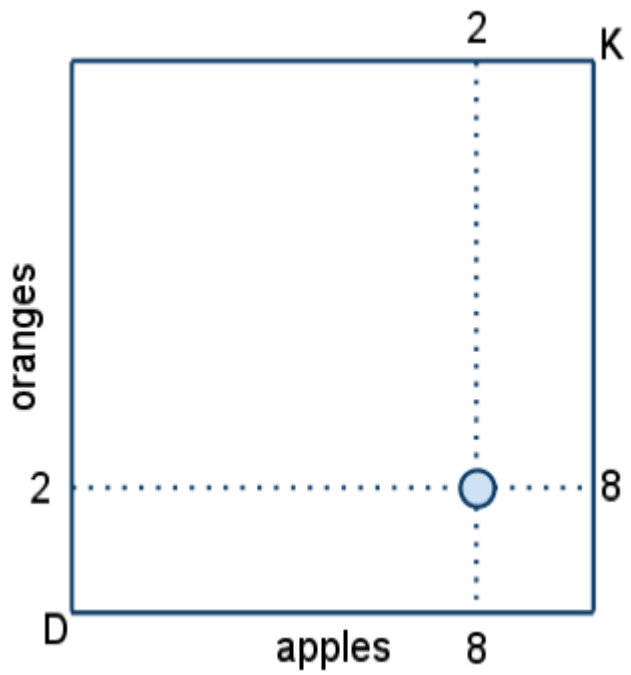
The profit-maximizing output solves $\pi^{'} = 0 : p - 4y = 0$. So the supply function is $y(p) = \frac{p}{4}$ provided $\pi \geq 0$. The profit is non-negative as long as $MC \geq ATC$, or $4y \geq \frac{2y^2+2}{y}$, so $y \geq 1$ and thus the threshold price is 4. The answer is

$$y(p) = \begin{cases} p/4 & if \quad p \geq 4 \\ 0 & if \quad p \leq 4 \end{cases}$$



Problem 3 - Competitive Equilibrium

a) The total endowment in the economy is $w = (10, 10)$.



b) An allocation is Pareto efficient if there is no way to make one agent better off without hurting the other one. The condition for Pareto efficiency is $MRS^D = MRS^K$.

c) $MRS^i = -\frac{MU_1^i}{MU_2^i} = -\frac{x_2^i}{x_1^i}$. Given the initial endowments are $w^D = (8, 2)$, $w^K = (2, 8)$

$$MRS^D = -\frac{x_2^D}{x_1^D} = -\frac{2}{8} = -0.25$$

$$MRS^K = -\frac{x_2^K}{x_1^K} = -\frac{8}{2} = -4$$

$MRS^D \neq MRS^K$, hence the initial allocation is not Pareto efficient.

d) A competitive equilibrium is an allocation $(x_1^D, x_2^D, x_1^K, x_2^K)$ and a vector of prices (p_1, p_2) such that
- consumption bundles are optimal given the prices
- markets clear

If we normalize $p_2 = 1$, the incomes (the cost of the initial endowments) are:

$$m^D = 8p_1 + 2$$

$$m^K = 2p_1 + 8$$

Using the formula for Cobb-Douglas utility function, the optimal choices for good 1 are:

$$x_1^D = \frac{1}{2}\frac{8p_1 + 2}{p_1}$$

$$x_1^K = \frac{1}{2}\frac{2p_1 + 8}{p_1}$$

Since the markets must clear, it must be that $x_1^D + x_1^K = 10$, so

$$\frac{1}{2}\frac{8p_1 + 2}{p_1} + \frac{1}{2}\frac{2p_1 + 8}{p_1} = 10$$

therefore $p_1 = 1$ and hence the equilibrium allocation is $(x_1^D, x_2^D, x_1^K, x_2^K) = (5, 5, 5, 5)$.
Since what matters is the price ratio, not the prices, the same allocation with different prices such that $\frac{p_1}{p_2} = 1$ will constitute a different competitive equilibrium.

e)

$$MRS^D = -\frac{x_2^D}{x_1^D} = -\frac{5}{5} = -1 = MRS^K$$

f) In order for consumption bundles to be optimal given the prices, it must be that

$$MRS^D = MRS^K = -\frac{p_1}{p_2}$$

Hence $\frac{p_1}{p_2} = \frac{2}{3}$ in equilibrium. If $\frac{p_1}{p_2} > \frac{2}{3}$, both agents will consume good 2 only, and the market clearing condition fails.

Problem 4 - Short Questions

a) The expected utility of the lottery is $EU = \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{9} = 1.5$. The certainty equivalent is a number that gives the same utility as the lottery in expectation, so it solves $\sqrt{CE} = 1.5$. Hence $CE = 2.25$. The certainty equivalent is less than the expected payoff of the lottery since the agent is risk-averse.

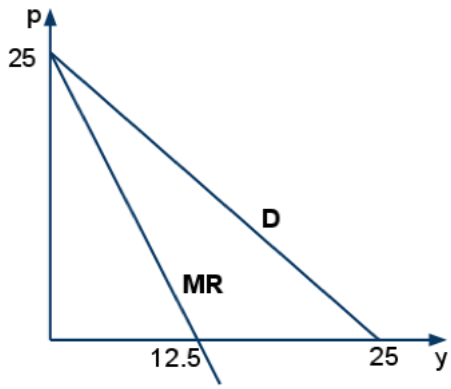
b) The expected value of a horse to a buyer equals $\frac{1}{2}2 + \frac{1}{2}5 = 3.5$, which is less than 4 - the value of a Plum horse to a seller. Hence Plums won't be traded in the market.

c) The internship becomes a credible signal of high ability if the low ability workers choose not to accept the internship offer: $1000 - 200x \leq 400$. So minimal length should be 3.

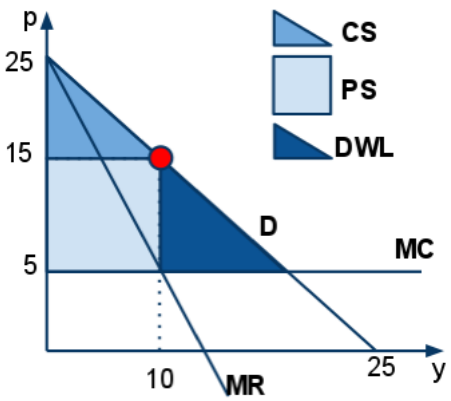
d) The present value of renting is $PV = \frac{1000}{0.02} = 50000$. It's less than the price of the apartment (100 000), so renting is cheaper.

Problem 5 - Market Power

a) Marginal revenue is the derivative of the total revenue. $TR(y) = p(y)y = (25 - y)y$. So $MR(y) = 25 - 2y$. Marginal revenue is smaller than the price because in order to sell an additional unit of output, the monopolist has to decrease price for all the units he is willing to sell.



b) In optimum $MR = MC$, so $25 - 2y^M = 5$, and $y^M = 10$. $p^M = 25 - y^M = 15$.
$CS = \frac{1}{2} \cdot (25 - 15) \cdot 10 = 50$, $PS = (15 - 5) \cdot 10 = 100$, $DWL = \frac{1}{2} \cdot (15 - 5) \cdot (20 - 10) = 50$.



c) Markup is determined by the formula $\frac{P}{MC} = \frac{15}{5} = 3$. Another way to calculate it is via elasticity: $\frac{1}{1+\frac{1}{e}} = \frac{1}{1-\frac{1}{3/2}} = 3$.

d) Under perfect price discrimination the monopolist sells as long as $P \geq MC$ and extracts full surplus. Hence $TS = PS = \frac{1}{2} \cdot (25 - 5) \cdot 20 = 200$. $CS = DWL = 0$.

e) Let Y denote total output in the industry and y be output of an individual firm. An individual firm chooses y to maximize

$$\pi = (25 - Y)y - 5y$$

$\pi' = 0$ gives $25 - Y - y - 5 = 0$. In equilibrium every firm anticipates the same behavior from every other firm, so $Y = ny$. Thus $(25 - 5) - (n + 1)y = 0$ and $y = \frac{20}{n+1}$.
$p = 25 - Y = 25 - ny = 25 - \frac{20n}{n+1}$. As $n \to \infty$, $\frac{20n}{n+1} \to 20$ and hence $p \to 5$.

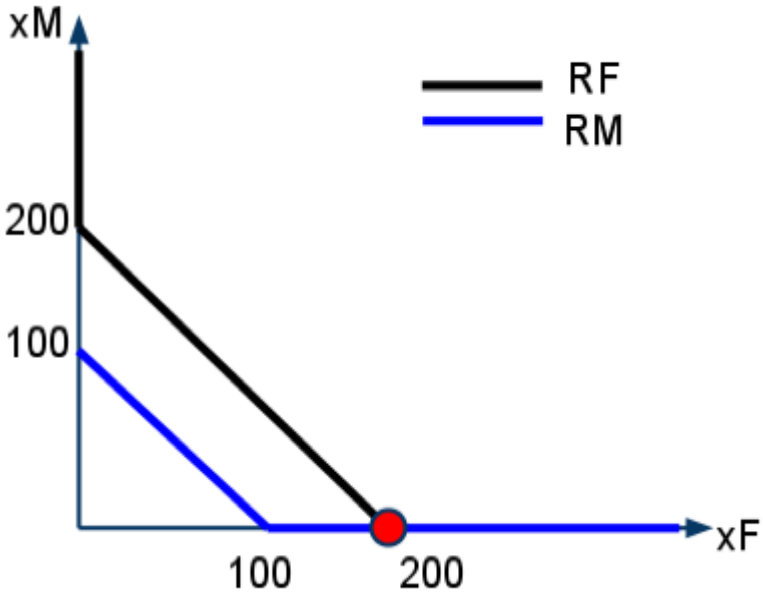
Problem 6 - Public Goods

a) Freddy's best response solves $\frac{du^F}{dx^F} = 0$:
$\frac{200}{x^F+x^M} - 1 = 0$, so Freddy's best response is

$$R^F(x^M) = \begin{cases} 200 - x^M & if \quad x^M \leq 200 \\ 0 & if \quad x^M > 200 \end{cases}$$

b) In the same way Miriam's best response is derived from $\frac{du^M}{dx^M} = 0$:
$\frac{100}{x^F+x^M} - 1 = 0$, hence

$$R^M(x^F) = \begin{cases} 100 - x^F & if \quad x^F \leq 100 \\ 0 & if \quad x^F > 100 \end{cases}$$



c) The equilibrium is the intersection of best responses: $(x^F, x^M) = (200, 0)$. Miriam free rides because she values the collection less than Freddy does.

d) In Pareto efficient case the sum of utilities is maximized with respect to $x^F + x^M$:

$$u^F + u^M = 300 \ln(x^F + x^M) - (x^F + x^M)$$

The efficient number of downloads is $x^F + x^M = 300$. It's greater than the equilibrium one because one agent's downloads create a positive externality for the other agents.

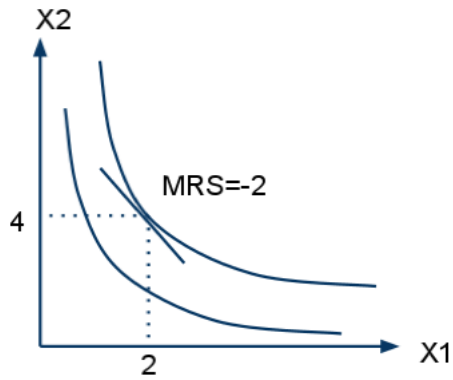
Econ 301 Final Exam Solutions - Spring 2011
Group B

Problem 1 - Consumer Choice

a) The budget constraint is $5x_1+10x_2 = 100$. The slope of the budget line is $-\frac{p_1}{p_2} = -\frac{5}{10} = -0.5$. Interpretation of the slope: the relative price of one tulip in the market is $\frac{1}{2}$ daffodil.

b) $U = \ln x_1 + \ln x_2$ or $U = x_1x_2$ will represent the same preferences, since the operations of taking square root, adding a constant and taking the square of a function are all monotone transformations.

c)$MRS = -\frac{MU_{x_1}}{MU_{x_2}} = -\frac{x_2}{x_1}$. At $(2, 4)$ $MRS = -2$. Interpretation: given 2 tulips and 4 daffodils, Jeremy is willing to trade 1 tulip for 2 daffodils.



d) The two secrets of happiness are

$$5x_1 + 10x_2 = 100$$
$$-\frac{x_2}{x_1} = -0.5$$

The first condition implies the optimal bundle lies on the budget line. The second condition guarantees the indifference curve is tangent to the budget line at optimum. (MRS equals the slope of the budget line)
The optimal bundle is $(x_1, x_2) = (10, 5)$. The solution is interior since $x_1 \neq 0$ and $x_2 \neq 0$.

e) The function $U = 2x_1 + 3x_2$ represents preferences over perfect substitutes. $\frac{MU_1}{p_1} = \frac{2}{5} > \frac{MU_2}{p_2} = \frac{3}{10}$. Hence good 1 only will be consumed, and optimal $(x_1, x_2) = (20, 0)$. The solution is not interior.

Problem 2 - Producers

a) Take $\lambda > 1$. $F(\lambda K, \lambda L) = 5(\lambda K)^{\frac{1}{4}}(\lambda L)^{\frac{1}{4}} = \lambda^{\frac{1}{2}} \cdot 5K^{\frac{1}{4}}L^{\frac{1}{4}}$
$\lambda F(K, L) = \lambda \cdot 5K^{\frac{1}{4}}L^{\frac{1}{4}}$. Then $F(\lambda K, \lambda L) < \lambda F(K, L)$ and the production function exhibits decreasing returns to scale.

b) First apply the cost-minimization condition:

$$\frac{MP_K}{MP_L} = \frac{w_K}{w_L}$$

Given $w_K = w_L = 25$, $\frac{w_K}{w_L} = 1$. Plug in $MP_K = \frac{5}{4}K^{-\frac{3}{4}}L^{\frac{1}{4}}$ and $MP_L = \frac{5}{4}K^{\frac{1}{4}}L^{-\frac{3}{4}}$:

$$\frac{\frac{5}{4}K^{-\frac{3}{4}}L^{\frac{1}{4}}}{\frac{5}{4}K^{\frac{1}{4}}L^{-\frac{3}{4}}} = 1$$

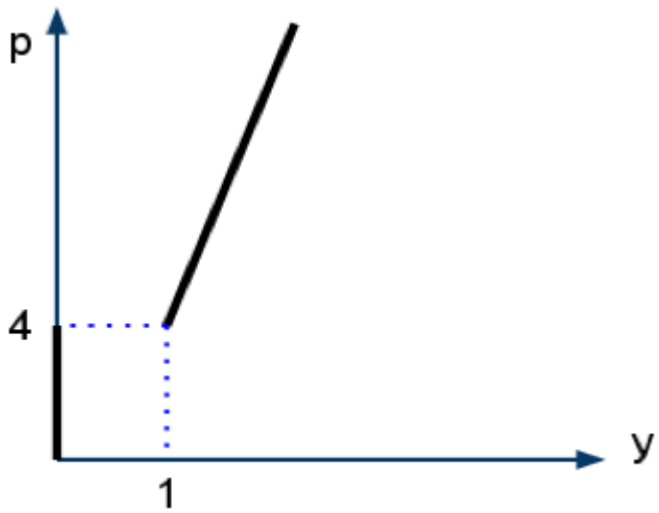
Therefore in optimum $K = L$, which implies $y = 5\sqrt{K}$ and $y = 5\sqrt{L}$, so $K = L = \frac{y^2}{25}$.
Plug the result into the cost function: $c(y) = w_K K + w_L L = 25\frac{y^2}{25} + 25\frac{y^2}{25} = 2y^2$.

c) A competitive firm facing price p, variable costs $2y^2$ and fixed costs 2 maximizes

$$\pi(y) = py - 2y^2 - 2$$

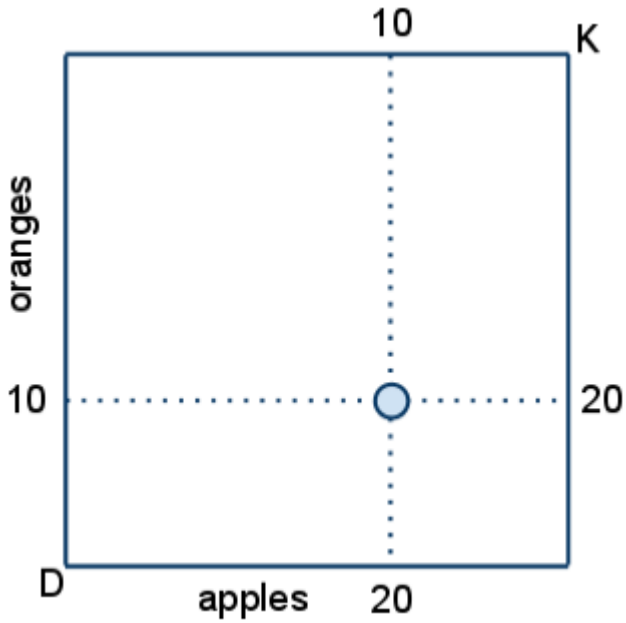
The profit-maximizing output solves $\pi^{'} = 0 : p - 4y = 0$. So the supply function is $y(p) = \frac{p}{4}$ provided $\pi \geq 0$. The profit is non-negative as long as $MC \geq ATC$, or $4y \geq \frac{2y^2+2}{y}$, so $y \geq 1$ and thus the threshold price is 4. The answer is

$$y(p) = \begin{cases} p/4 & if \quad p \geq 4 \\ 0 & if \quad p \leq 4 \end{cases}$$



Problem 3 - Competitive Equilibrium

a) The total endowment in the economy is $w = (30, 30)$.



b) An allocation is Pareto efficient if there is no way to make one agent better off without hurting the other one. The condition for Pareto efficiency is $MRS^D = MRS^K$.

c) $MRS^i = -\frac{MU_1^i}{MU_2^i} = -\frac{x_2^i}{x_1^i}$. Given the initial endowments are $w^D = (20, 10)$, $w^K = (10, 20)$

$$MRS^D = -\frac{x_2^D}{x_1^D} = -\frac{10}{20} = -0.5$$

$$MRS^K = -\frac{x_2^K}{x_1^K} = -\frac{20}{10} = -2$$

$MRS^D \neq MRS^K$, hence the initial allocation is not Pareto efficient.

d) A competitive equilibrium is an allocation $(x_1^D, x_2^D, x_1^K, x_2^K)$ and a vector of prices (p_1, p_2) such that
- consumption bundles are optimal given the prices
- markets clear

If we normalize $p_2 = 1$, the incomes (the cost of the initial endowments) are:

$$m^D = 20p_1 + 10$$

$$m^K = 10p_1 + 20$$

Using the formula for Cobb-Douglas utility function, the optimal choices for good 1 are:

$$x_1^D = \frac{1}{2}\frac{20p_1 + 10}{p_1}$$

$$x_1^K = \frac{1}{2}\frac{10p_1 + 20}{p_1}$$

Since the markets must clear, it must be that $x_1^D + x_1^K = 30$, so

$$\frac{1}{2}\frac{20p_1 + 10}{p_1} + \frac{1}{2}\frac{10p_1 + 20}{p_1} = 30$$

therefore $p_1 = 1$ and hence the equilibrium allocation is $(x_1^D, x_2^D, x_1^K, x_2^K) = (15, 15, 15, 15)$. Since what matters is the price ratio, not the prices, the same allocation with different prices such that $\frac{p_1}{p_2} = 1$ will constitute a different competitive equilibrium.

e)

$$MRS^D = -\frac{x_2^D}{x_1^D} = -\frac{15}{15} = -1 = MRS^K$$

f) In order for consumption bundles to be optimal given the prices, it must be that

$$MRS^D = MRS^K = -\frac{p_1}{p_2}$$

Hence $\frac{p_1}{p_2} = \frac{2}{3}$ in equilibrium. If $\frac{p_1}{p_2} > \frac{2}{3}$, both agents will consume good 2 only, and the market clearing condition fails.

Problem 4 - Short Questions

a) The expected utility of the lottery is $EU = \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{16} = 2$. The certainty equivalent is a number that gives the same utility as the lottery in expectation, so it solves $\sqrt{CE} = 2$. Hence $CE = 4$. The certainty equivalent is less than the expected payoff of the lottery since the agent is risk-averse.

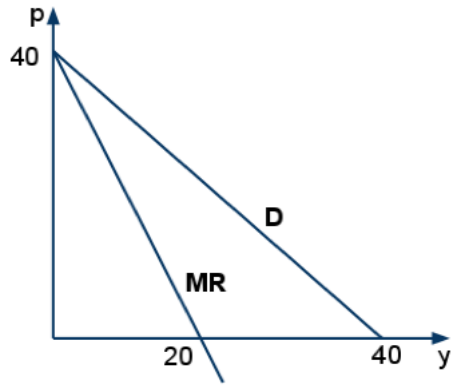
b) The expected value of a horse to a buyer equals $\frac{1}{2}2 + \frac{1}{2}8 = 5$, which is less than 6 - the value of a Plum horse to a seller. Hence Plums won't be traded in the market.

c) The internship becomes a credible signal of high ability if the low ability workers choose not to accept the internship offer: $1000 - 200x \leq 400$. So minimal length should be 3.

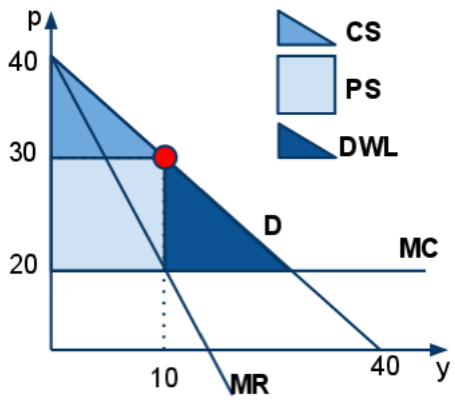
d) The present value of renting is $PV = \frac{1000}{0.02} = 50000$. It's more than the price of the apartment (30 000), so purchasing the apartment is cheaper.

Problem 5 - Market Power

a) Marginal revenue is the derivative of the total revenue. $TR(y) = p(y)y = (40 - y)y$. So $MR(y) = 40 - 2y$. Marginal revenue is smaller than the price because in order to sell an additional unit of output, the monopolist has to decrease price for all the units he is willing to sell.



b) In optimum $MR = MC$, so $40 - 2y^M = 20$, and $y^M = 10$. $p^M = 40 - y^M = 30$.
$CS = \frac{1}{2} \cdot (40 - 30) \cdot 10 = 50$, $PS = (30 - 20) \cdot 10 = 100$, $DWL = \frac{1}{2} \cdot (30 - 20) \cdot (20 - 10) = 50$.



c) Markup is determined by the formula $\frac{P}{MC} = \frac{30}{20} = 1.5$. Another way to calculate it is via elasticity: $\frac{1}{1+\frac{1}{e}} = \frac{1}{1-\frac{1}{3}} = 1.5$.

d) Under perfect price discrimination the monopolist sells as long as $P \geq MC$ and extracts full surplus. Hence $TS = PS = \frac{1}{2} \cdot (40 - 20) \cdot 20 = 200$. $CS = DWL = 0$.

e) Let Y denote total output in the industry and y be output of an individual firm. An individual firm chooses y to maximize

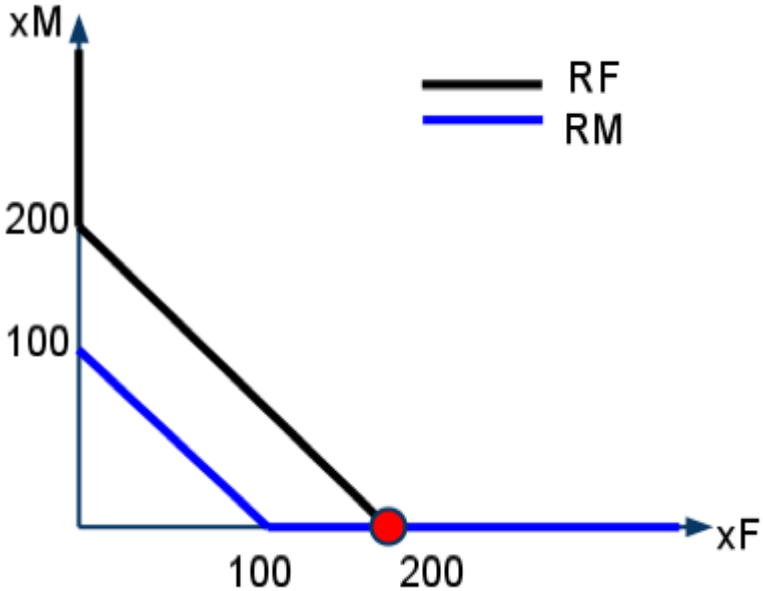
$$\pi = (40 - Y)y - 20y$$

$\pi' = 0$ gives $40 - Y - y - 20 = 0$. In equilibrium every firm anticipates the same behavior from every other firm, so $Y = ny$. Thus $(40 - 20) - (n + 1)y = 0$ and $y = \frac{20}{n+1}$.
$p = 40 - Y = 40 - ny = 40 - \frac{20n}{n+1}$. As $n \to \infty$, $\frac{20n}{n+1} \to 20$ and hence $p \to 20$.

Problem 6 - Public Goods

a) Freddy's best response solves $\frac{du^F}{dx^F} = 0$:
$\frac{200}{x^F+x^M} - 1 = 0$, so Freddy's best response is

$$R^F(x^M) = \begin{cases} 200 - x^M & if \quad x^M \leq 200 \\ 0 & if \quad x^M > 200 \end{cases}$$



b) In the same way Miriam's best response is derived from $\frac{du^M}{dx^M} = 0$:
$\frac{100}{x^F+x^M} - 1 = 0$, hence

$$R^M(x^F) = \begin{cases} 100 - x^F & if \quad x^F \leq 100 \\ 0 & if \quad x^F > 100 \end{cases}$$

c) The equilibrium is the intersection of best responses: $(x^F, x^M) = (200, 0)$. Miriam free rides because she values the collection less than Freddy does.

d) In Pareto efficient case the sum of utilities is maximized with respect to $x^F + x^M$:

$$u^F + u^M = 300 \ln(x^F + x^M) - (x^F + x^M)$$

The efficient number of downloads is $x^F + x^M = 300$. It's greater than the equilibrium one because one agent's downloads create a positive externality for the other agents.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final

You have 2h to complete the exam. The final consists of 6 questions (10+10+15+25+25+15=100).

Problem 1.

Ace consumes bananas x_1 and kiwis x_2. The prices of both goods are $p_1 = 4, p_2 = 10$ and Ace's income is $m = 120$. His utility function is

$$U(x_1, x_2) = (x_1)^{20} (x_2)^{40}$$

a) Find analytically Ace's MRS as a function of (x_1, x_2) (give a function) and find its value for the consumption bundle $(x_1, x_2) = (20, 20)$. Give its economic and geometric interpretation (one sentence and find MRS on the graph)

b) Give two secrets of happiness that determine Ace's optimal choice of fruits (give two equation). Explain why violation of any of them implies that the bundle is not optimal (one sentence for each condition).

c) Using magic forumula find the optimal bundle of Ace (two numbers), and show geometrically the .

Problem 2.

Adria collects two types of rare coins: Jefferson Nickels $x_1$and Seated Half Dimes x_2. Her utility from a collection (x_1, x_2) is

$$U(x_1, x_2) = x_1 + x_2$$

a) Propose a utility function that gives a higher level of utility for any (x_1, x_2), but represents the same preferences (give utility function).

b) Suppose the prices of the two types of coins are $p_1 = 4$ and $p_2 = 2$ for x_1, x_2 respectively and the Adria's income is $m = \$20$. Plot her budget set and find the optimal collection (x_1, x_2) and mark it in your graph (give two numbers)

c) Are the coins Giffen goods (yes or no and one sentence explaining why)?

d) Harder: Suppose Adria's provider of coins currently has only six Seated Half Dimes x_2 in stock (hence $x_2 \leq 6$). Plot a budget set with the extra constraint and find (geometrically) an optimal collection given the constraint.

Problem 3. (Equilibrium)

There are two commodities traded on the market: umbrellas x_1 and swimming suits x_2. Abigail has ten umbrellas and twenty swimming suits ($\omega^A = (10, 20)$). Gabriel has forty umbrellas and twenty swimming suits ($\omega^G = (40, 20)$). Abigail and Gabriel have identical utility functions given by

$$U^i(x_1, x_2) = \frac{1}{2} \ln(x_1) + \frac{1}{2} \ln(x_2)$$

a) Plot an Edgeworth box and mark the point corresponding to endowments of Abigail and Gabriel.

b) Give a definition of a Pareto efficient allocation (one sentence) and the equivalent condition in terms of MRS (equation). Verify whether endowment is Pareto efficient (two numbers+one sentence).

c) Find prices and an allocation of umbrellas and swimming suits in a competitive equilibrium and mark it in your graph.

d) Harder: Plot a contract curve in the Edgeworth box assuming utilities for two agents $U^i(x_1, x_2) = \min(x_1, x_2)$.

Problem 4.(Short questions)

a) You are going to pay taxes of \$200 every year, forever. Find the Present Value of your taxes if the yearly interest rate is $r = 10\%$.

b) Consider a lottery that pays 0 with probability $\frac{1}{2}$ and 16 with probability $\frac{1}{2}$ and a Bernoulli utility function is $u(x) = \sqrt{x}$. Give a corresponding von Neuman-Morgenstern utility function. Find the certainty

equivalent of the lottery. Is it bigger or smaller than the expected value of the lottery? Why? (give a utility function, two numbers and one sentence.)

c) Give an example of a Cobb-Douglass production function that is associated with increasing returns to scale, decreasing MPK and decreasing MPL (give a function). Without any calculations, sketch the average total cost function (ATC) associated with your production function.

d) Let the variable cost be $c(y) = y^2$ and fixed cost $F = 4$. Find ATC^{MES} and y^{MES} (two numbers). Given demand $D(p) = 8 - p$ determine a number of firms in the industry assuming free entry (and price taking). Is the industry monopolistic, duopolistic, oligopolistic or perfectly competitive? Find Herfindahl–Hirschman Index (HHI) of this industry (one number).

e) In a market for second-hand vehicles two types of cars can be traded: lemons (bad quality cars) and plums (good quality ones). The value of a car depends on its type and is given by

	Lemon	Plum
Seller	0	20
Buyer	10	26

Will we observe plums traded on the market if the probability of a lemon is equal to $\frac{1}{2}$? (compare two relevant numbers). Is the equilibrium outcome Pareto efficient (yes-no answer+ one sentence)? Give a threshold probability for which we might observe pooling equilibrium (number).

Problem 5.(Market Power)

Consider an industry with the inverse demand equal to $p(y) = 6 - y$, and suppose that the total cost function is $TC = 2y$.

a) What are the total gains to trade in this industry? (give one number)

b) Find the level of production and the price if there is only one firm in the industry (i.e., we have a monopoly) charging a uniform price (give two numbers). Find demand elasticity at optimum. (give on number) Illustrate the choice using a graph. Mark a DWL.

c) Find the profit of the monopoly and a DWL given that monopoly uses the first degree price discrimination.

d) Find the individual and aggregate production and the price in a Cournot-Nash equilibrium given that there are two firms (give three numbers). Show DWL in the graph.

e) In which of the three cases, (b,c or d) the outcome is Pareto efficient? (chose one+ one sentence)

Problem 6.(Externality)

A bee keeper chooses the number of hives h. Each hive produces one pound of honey which sells at the price of \$10 per pound. The cost of holding h hives is $TC(h) = \frac{1}{2}h^2$. Consequently the profit of bee keeper is equal to

$$\pi_h(h) = 10h - \frac{1}{2}h^2$$

The hives are located next to an apple tree orchard. The bees pollinate the trees and hence the total production of apples $y = h + t$ is increasing in number of trees and bees. Apples sell for \$3 and the cost of t trees is $TC_t(t) = \frac{1}{2}t^2$. Therefore the profit of an orchard grower is

$$\pi_t(t) = 3(t + h) - \frac{1}{2}t^2$$

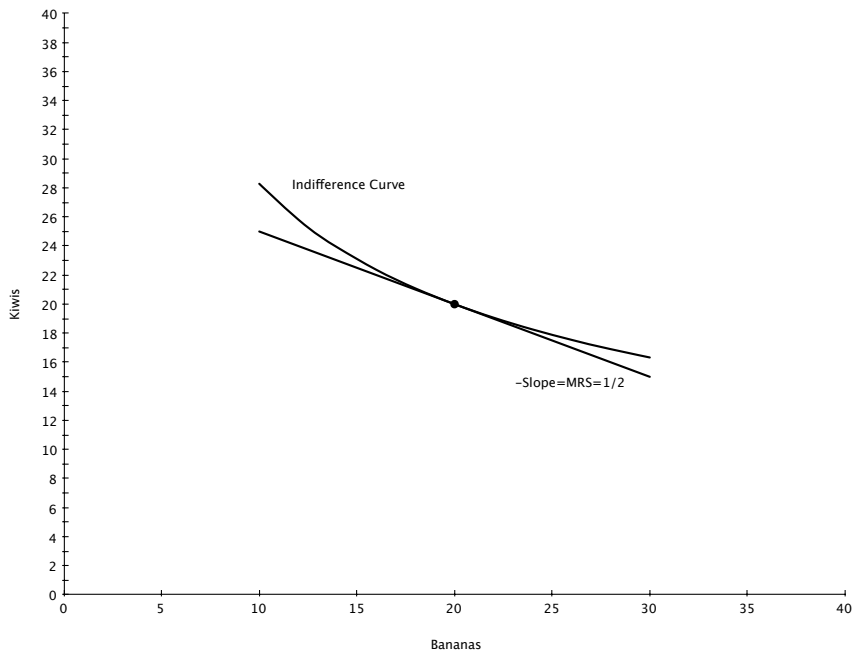
a) Market outcome: Find the level of hives h that maximizes the profit of a beekeeper and the number of trees that maximizes the profit of an orchard owner (assuming h optimal for a bee keeper) (two numbers)

b) Find the Pareto efficient level of h and t. Are the two values higher or smaller then the ones in a)? Why? (two numbers + one sentence)

Makeup Final Solutions
ECON 301
May 15, 2012

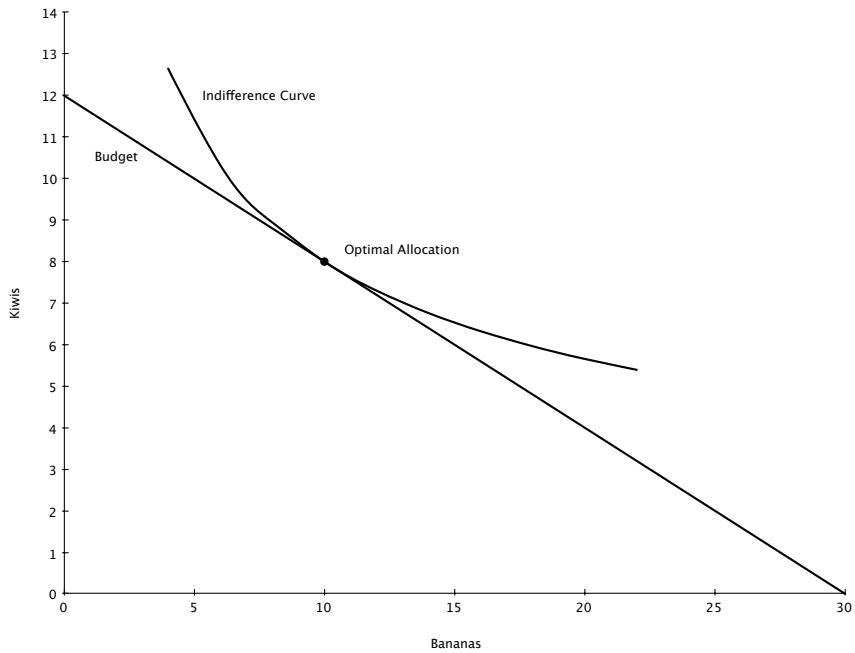
Problem 1

a) Because it is easier and more familiar, we will work with the monotonic transformation (and thus equivalent) utility function: $U(x_1, x_2) = \log x_1 + 2 \log x_2$. $MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{1}{x_1}}{\frac{2}{x_2}} = \frac{x_2}{2x_1}$. At $(x_1, x_2) = (20, 20)$, $MRS = \frac{20}{40} = \frac{1}{2}$. The MRS measures the rate a which you are willing to trade one good for the other. At a particular point in a graph, the MRS will be the negative of the slope of the indifference curve running through that point.



b)
- Budget: $4x_1 + 10x_2 = 120$. With a monotonic utility function like this one, the budget holds with equality because you can always make yourself better off by consuming more. Thus, it makes no sense to leave money unspent.
- $MRS = \frac{p_1}{p_2}$: The price at which you are willing to trade goods for one another (MRS) is the same as the rate at which you can trade the goods for one another (price ratio). Alternatively, you can think of this as the marginal utility per dollar spent on each good is the same: $\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$. If this does not hold you would be able to buy less of one good, spend that money one the other good, and gain more utility than you have lost.

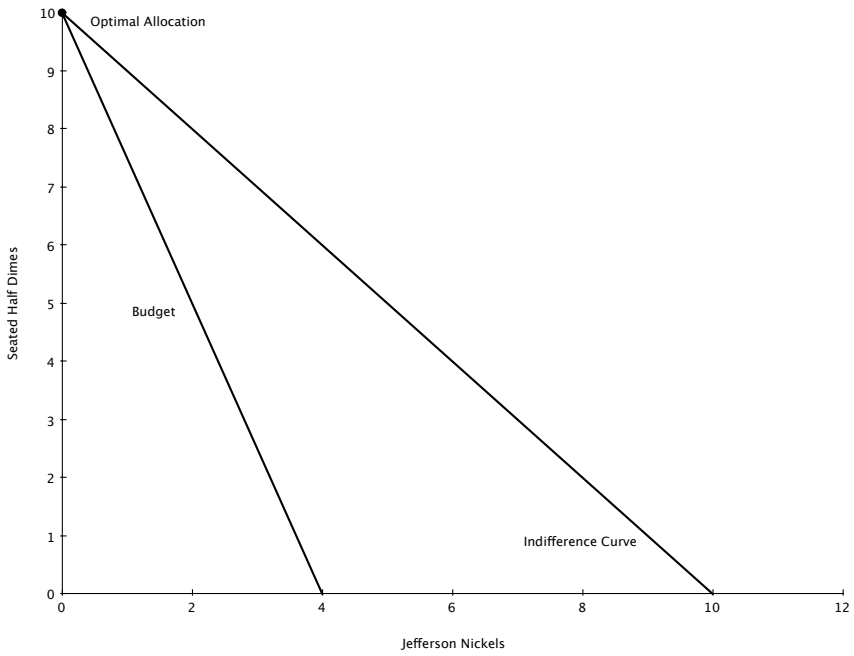
c) The optimal allocation is shown in the graph below



Problem 2

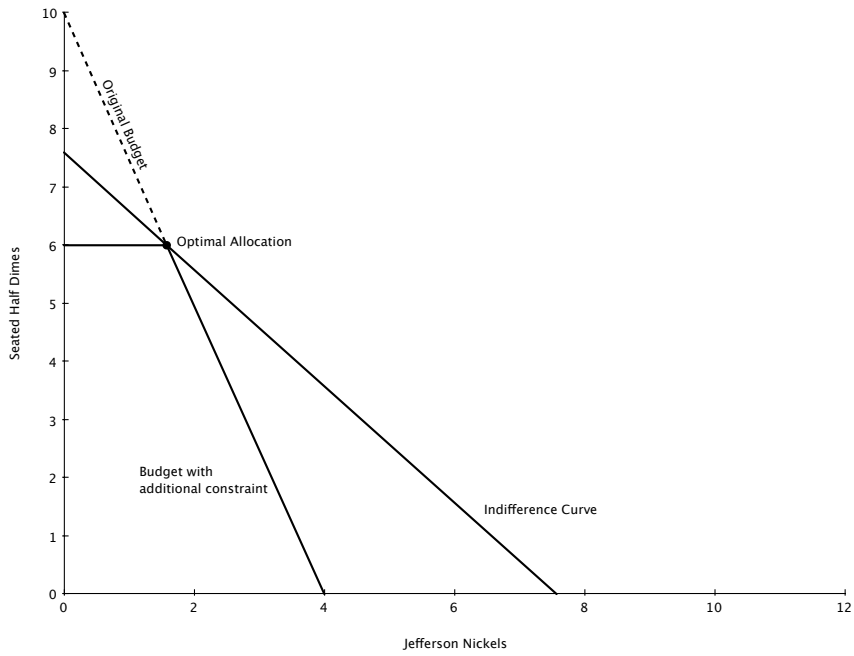
a) Lots of them exist. The most straightforward are $U(x_1, x_2) = A * (x_1 + x_2) + B$, with $A \geq 1$, $B \geq 0$, and $A + B > 1$. These represent the same preferences because they are monotonic transformations.

b) Since we are dealing with perfect substitues we know we will have a corner solution. We will choose only the good that delivers utility in the least expensive manner. Because each unit of x_1 and x_2 give the same amount of utility, this will be the cheaper good, x_2. At $p_2 = 2$ and $m = 20$ we can afford $x_2 = 10$.



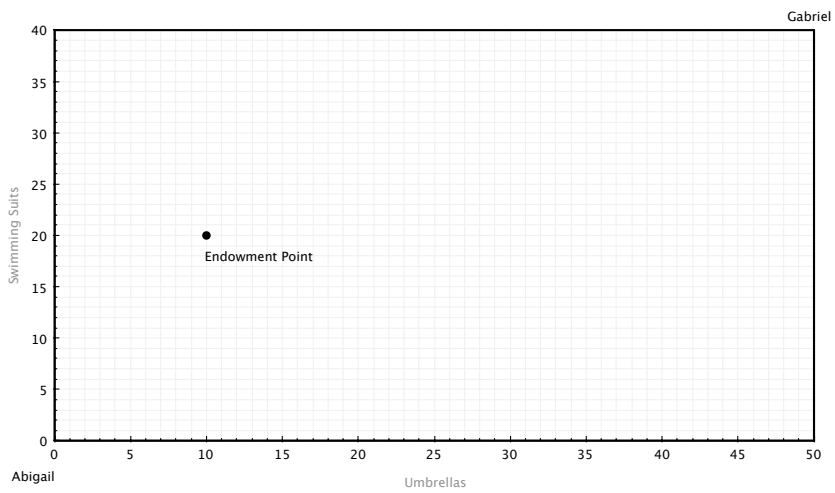
c) Giffen goods are goods that you consume more when their own price increases. Here you spend all your money on the cheaper good. As the price of that good increases you can buy less of it, until it becomes the more expensive good at which point you switch entirely to the other good: not Giffen goods.

d) As shown in the graph below, the additional constraint forces you to start buying Jefferson Nickels after all 6 Seated Half Dimes have been purchased.



Problem 3

a) The Edgeworth box is shown below



b) An allocation is pareto efficient if there are no trades that can make at least one person better off without hurthing the other person. This happens when $MRS_A = MRS_G$. The MRS for both Abigail and Gabriel is $\frac{x_2}{x_1}$. At the endowment point we have $MRS_A = \frac{20}{10}$, and $MRS_B = \frac{20}{40}$. These are not equal so we were not endowed with a pareto efficient allocation.

c) First, the equlibrium only determines relative prices so we are free to normalize one price. Let's say $p_2 = 1$. Abigail and Gabriel have identical Cobb-Douglas preferences so we can use our magic formulas. For x_1:

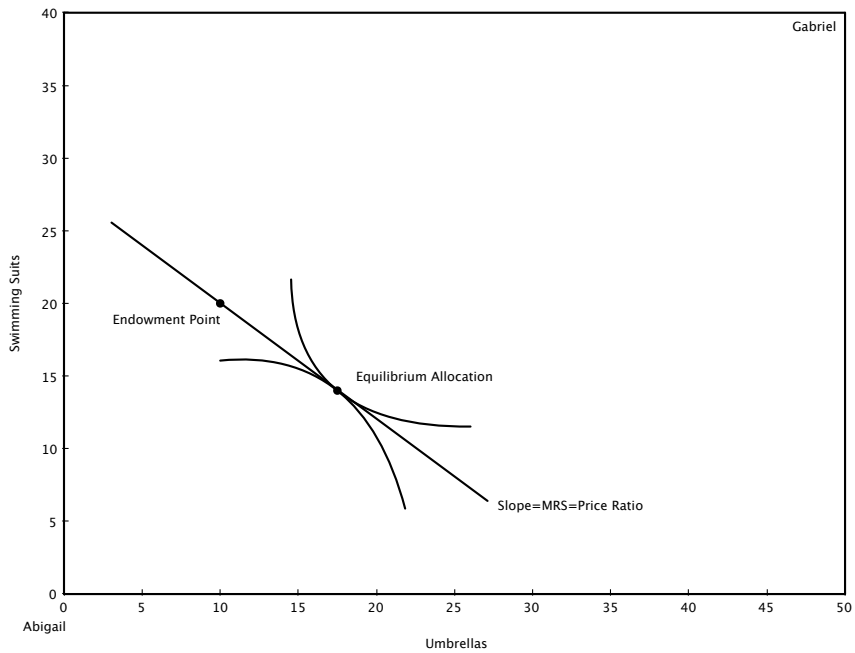
$$x_1^A = \frac{a}{a+b}\frac{m_A}{p_1} = \frac{1}{2}\frac{10p_1+20}{p_1} = 5 + \frac{10}{p_1}$$
$$x_1^G = 20 + \frac{10}{p_1}$$

We can use these two relationships along with the market clearing condition, $x_1^A + x_1^G = 50$, to solve for p_1.

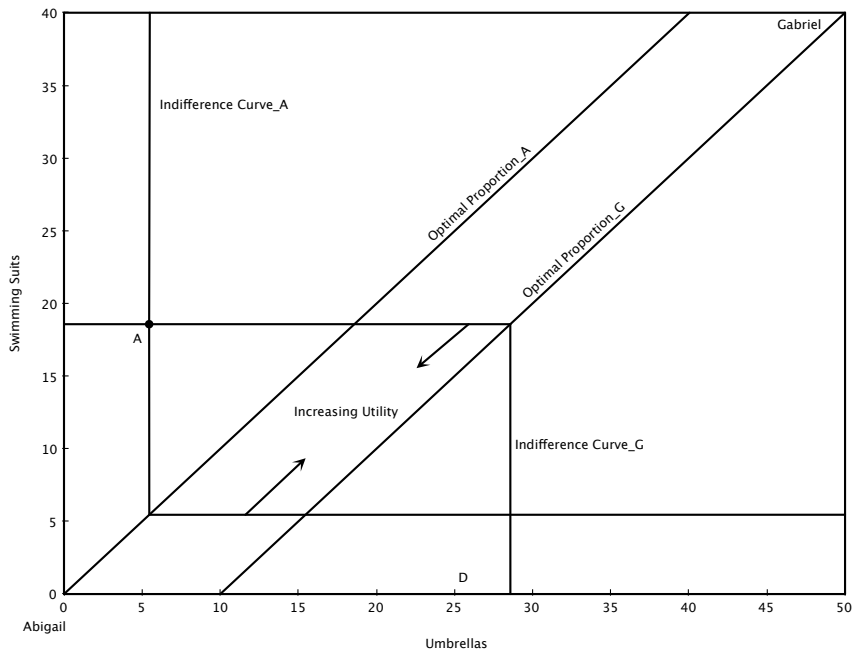
$$50 - x_1^A = 20 + \frac{10}{p_1}$$
$$50 - 5 - \frac{10}{p_1} = 20 + \frac{10}{p_1}$$
$$\Rightarrow p_1 = \frac{4}{5}$$

At this price we have $x_1^A = 5 + \frac{10}{\frac{4}{5}} = 17.5$, $x_1^G = 20 + \frac{10}{p_1} = 32.5$. Using the magic formulas for x_2 we have $x_2^A = 5p_1 + 10 = 14$, $x_2^G = 20p_1 + 10 = 26$. To summarize:

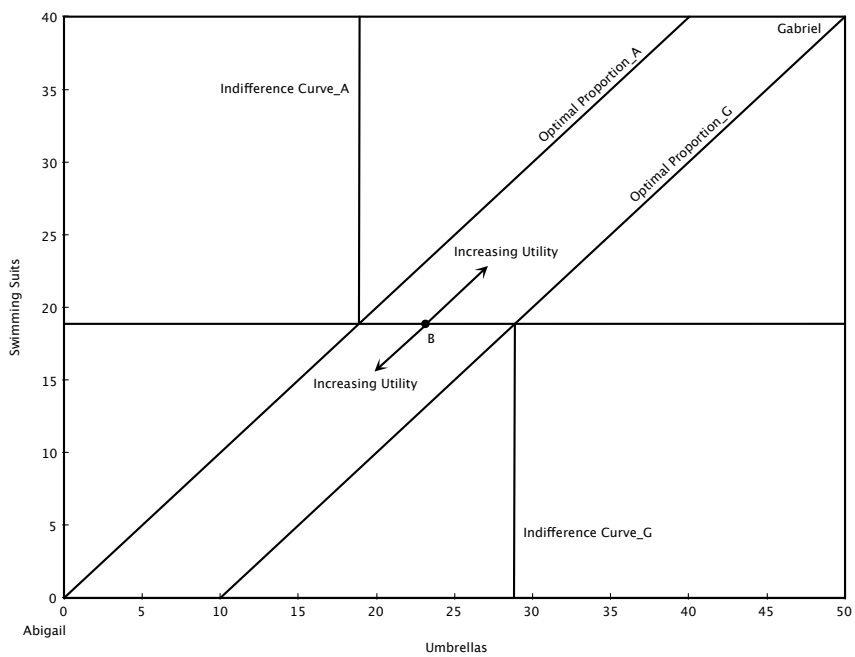
$$(p_1, p_2) = (\tfrac{4}{5}, 1)$$
$$(x_1^A, x_2^A) = (17.5, 14)$$
$$(x_1^G, x_2^G) = (32.5, 26)$$



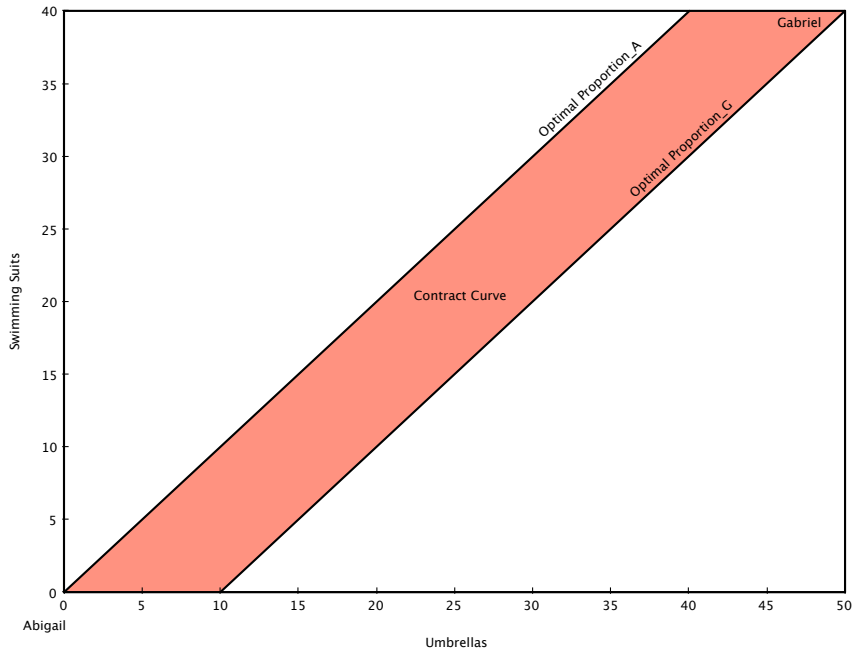
d) With perfect complements the MRS is not defined at the optimal point, so we can't equate them to find the contract curve. The optimal proportion line for both Abigail and Gabriel is where $x_1 = x_2$, but because the Edgeworth box is not square these lines do not coincide. However, this doesn't mean there are not pareto efficient allocations. Instead, let's think about several types of allocations in the Edgeworth box and see if they are pareto optimal. First, consider a point outside the two optimal proportion lines (A in the figure below). Both Abigail and Gabriel agree upon which way to move in order to increase their utility, meaning is a pareto improvement.



In contrast, if we look at a point in between the two optimal proportion lines (B), we see that Abigail and Gabriel want to move in different directions to improve utility. This means the point is pareto optimal.



To summarize, the contract curve of pareto optimal allocations is the space in between the two optimal proportion lines.

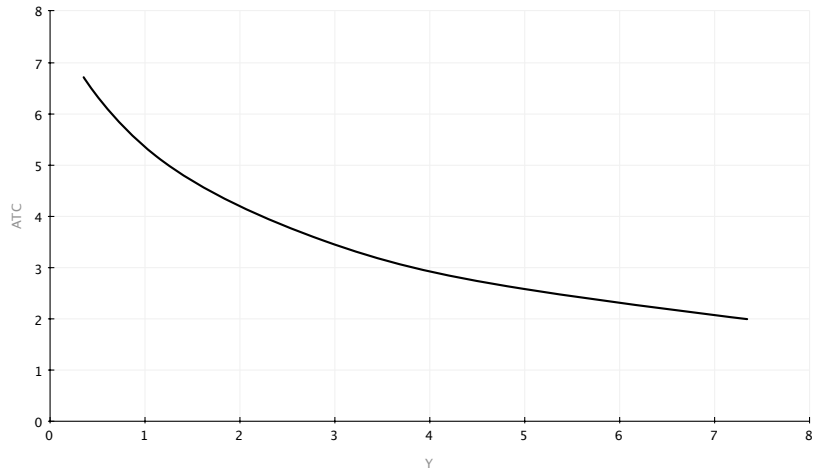


Problem 4

a) We use the formula for the present value of a perpetuity: $PV = \frac{20}{0.1} = 200$.

b) If we call x_w wealth if you win the lottery, and x_l wealth if you lose, then the von Neuman-Morgenstern expected utility function is $U(x_w, x_l) = \frac{1}{2}\sqrt{x_w} + \frac{1}{2}\sqrt{x_l}$. The certainty equivalent is defined by $\sqrt{ce} = \frac{1}{2}\sqrt{16} + \frac{1}{2}\sqrt{0} \Rightarrow ce = 4$. The expected value of the lottery is $\frac{1}{2}16 + \frac{1}{2}0 = 8$. The certainty equivalent is smaller than the expected value because the bernouli utility function is concave, which is also the same thing as saying this person is risk averse.

c) $F(K, L) = K^a L^b$, with $0 < a < 1$, $0 < b < 1$, $a + b > 1$. We just know that ATC is decreasing due to the increasing returns to scale.



d) Total cost is given by $y^2 + 4$, which makes $ATC = y + \frac{4}{y}$. We minimize this function to find ATC^{MES} and y^{MES}. Since it is a convex function the FOC will find the minimum. The FOC is $1 - \frac{4}{y^2} = 0 \Rightarrow y^{MES} = 2$. Then, $ATC^{MES} = 2 + \frac{4}{2} = 4$. With free entry every firm will produce at minimum efficient scale (and make zero profits). If not, a firm could enter, produce at MES, and make positive profits. This would leave the firms originally producing at a level other than MES with negative profits. At $p = ATC^{MES} = 4$, $D(p) = 4$. Thus, it will take two firms producing at MES to satisfy this demand. We have a duopoly. $HHI = (\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{2}$.

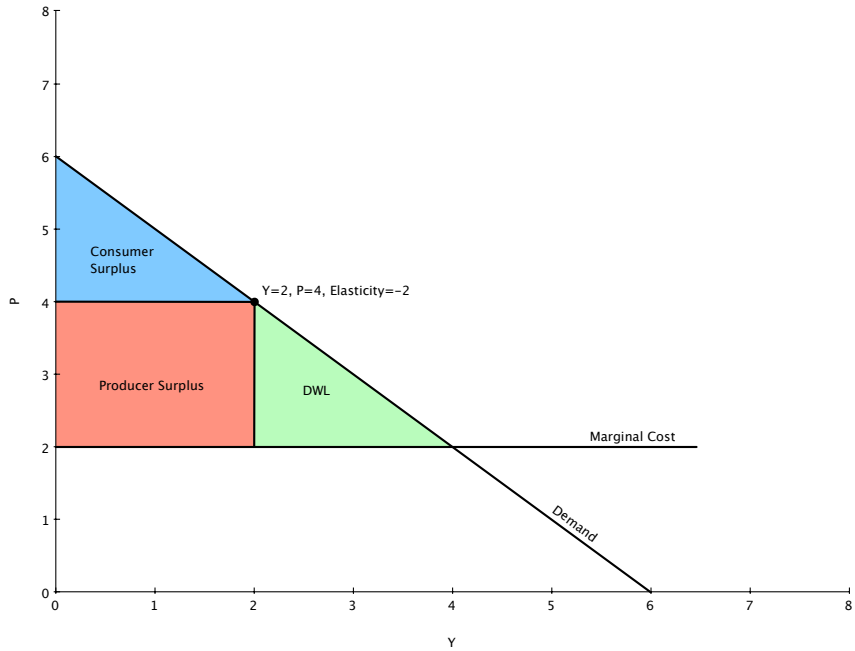
e) We know the buyer won't pay more than his expected value for a car. Thus, we need this expected value to be greater than 20 to induce sellers of plums to participate. $\frac{1}{2} * 10 + \frac{1}{2} * 26 = 18 < 20$, so plums will not be sold. This outcome is not pareto efficient because what would be beneficial trades of plums will not occur. To get a pooling equilibrium (where both types of sellers sell) we need $10\pi + 26(1 - \pi) \geq 20 \Rightarrow \pi \leq \frac{3}{8}$.

Problem 5

a) The competetive market is pareto efficient so it will provide the benchmark for total gains from trade. Firms in this competitive market produce at $p = MC = 2$, and make no profit. At $p = 2$ consumers purchase 4 units. This leaves consumer surplus (which is the same as total surplus) of $\frac{1}{2} * (6 - 2) * 4 = 8$.

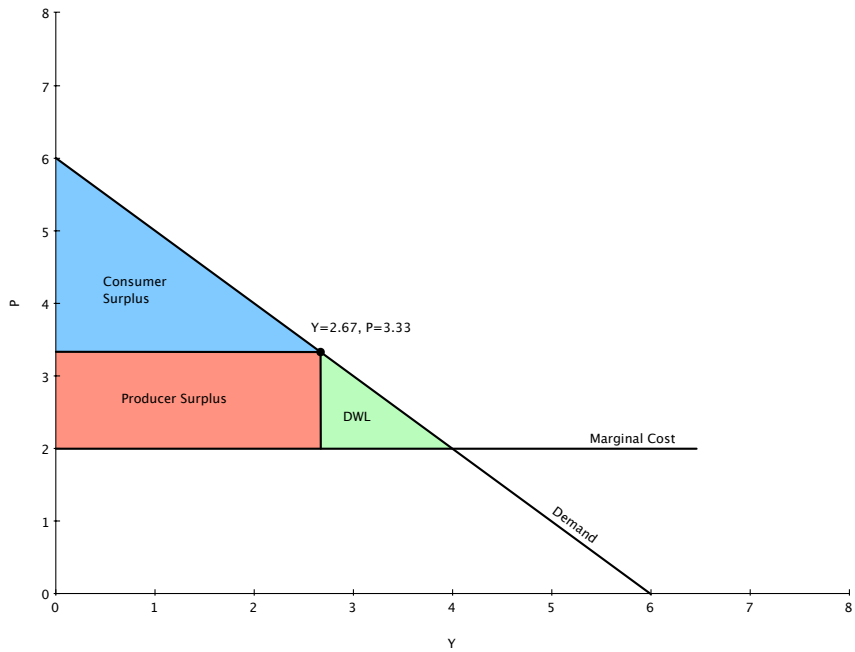
b) A monopolist chooses y to $\max(6 - y)y - 2y$. The FOC of this problem is $6 - 2y = 2 \Rightarrow y = 2$. They charge price $p = 4$. Demand elasticity is defined by $\epsilon = \frac{dy}{dp}\frac{p}{y}$. At the market equilibrium

we have $\epsilon = -1 * \frac{4}{2} = -2$.



c) First degree price discrimination means that the monopolist can charge each customer the maximum price that individual is willing to pay, and will do so as long as that price is larger than the marginal cost of 2. This outcome is efficient (DWL=0) because all possible beneficial trades occur, but now the monopolist has captured the entire gains from trade of 8.

d) Both firms participate in a symetric Cournot-Nash game where they choose their own quantity in response to the other firm's quantity. That is, firm 1 chooses y_1 to $\max(6-y_1-y_2)y_1 - 2y_1$. The FOC of this problem is $4 - 2y_1 - y_2 = 0$. Thus, the best response function for firm 1 is $y_1 = 2 - \frac{1}{2}y_2$. Because the game is symetric (firm 2 faces the same type of decision) we can write down firm 2's best response function $y_2 = 2 - \frac{1}{2}y_1$. We solve these best response functions together to locate the Nash equilibrium. This gives $y_1 = y_2 = \frac{4}{3}$. Total production is $2\frac{2}{3}$, leaving $p = 3\frac{1}{3}$.



e) Both b) and d) have DWL's, but as argued in c), first degree price discrimination is pareto efficient.

Problem 6

a) We will first determine the optimal number of hives for the bee keeper, and then see how the orchard owner will respond to this choice. The bee keeper chooses h to max $10h - \frac{1}{2}h^2$. The FOC for this problem is $h = 10$. Given this choice of h, the orchard owner chooses t to max $3(t + 10) - \frac{1}{2}t^2$. The FOC for this problem is $t = 3$.

b) To find the pareto optimal outcome the bee keeper and orchard owner team up to choose both h and t to maximize the joint profit: max $3t + 13h - \frac{1}{2}t^2 - \frac{1}{2}h^2$. The FOC of this problem for h is $h = 13$, and the FOC for t is $t = 3$. The number of trees is the same because h does not affect this choice (h isn't in the FOC for t), but h is higher when maximizing the joint profit because on his own, the bee keeper doesn't care how his supply of bees helps the orchard owner.