You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+25+25+10=100 points). Make sure that all your graphs have appropriate labels.

Problem 1 (40p) (Optimal choice)
Isabelle spends her income on two types of commodities, books $x_1$ and food $x_2$.

a) The price of a book is $p_1 = 2$ while the price of food is $p_2 = 4$. Isabelle’s income is $m = 16$. Depict Isabelle’s budget set. Find the relative price of a book in terms of food (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of the budget set? (one sentence)

b) How is Isabelle’s budget set affected by a gift of 5 books from her father (show the change in the graph)?

c) Isabelle’s utility function is $U(x_1, x_2) = x_1 x_2$.
Find the expression for an indifference curve that passes through bundle $(1, 2)$ (give formula). Plot the indifference curve in the graph.

d) Find Marginal Rate of Substitution (MRS) as a function of $x_1, x_2$ (give formula). Give the economic interpretation of MRS (one sentence). Find the value of the MRS for bundle $(4, 1)$ (one number). Which of the goods is more valuable at this bundle? What is the geometric interpretation of MRS? (one sentence).

e) Assume a budget set as in point a). Write down the two secrets of happiness that determine the optimal choice (two equations). Explain the economic intuition behind the two conditions (two sentences, one for each condition). Solve for the optimal choice.

f) Suppose the price of a book changes to $p_1 = 1$. Find new optimal choice (two numbers, use magic formula). Decompose the change of consumption $x_1$ into an income and substitution effect (two numbers).

Problem 2 (25p) (Perfect complements, Buying and Selling)
John’s initial endowment of apples and oranges is $(\omega_1, \omega_2) = (6, 8)$ and market prices of the two commodities are $p_1 = 2, p_2 = 1$, respectively.

a) Depict budget set in the graph. On the budget line, mark all the bundles that involve buying apples and selling oranges.

b) John’s utility function is $U(x_1, x_2) = \min(x_1, 2x_2)$.
What is the optimal proportion of the two commodities? (give the ratio of $x_1$ to $x_2$)? In the commodity space plot John’s indifference curves.

c) Write down the two secrets of happiness (two equations) and give the economic intuition behind each of them (one sentence for each).

d) Find optimal bundle $(x_1, x_2)$ (two numbers). Which of the two commodities is he buying and which is selling in optimum (one sentence)? Is your solution corner or interior?

Problem 3 (25p) (Perfect Substitutes)
Utility function if Liam is $U(x_1, x_2) = 2x_1 + x_2$.

a) Find Liam’s Marginal Rate of Substitution, (give formula). Which of the two commodities is more “precious” to her? (one sentence). Plot the indifference curve map in the commodity space.

b) Propose an alternative utility function that represents the same preferences (give formula).
c) For prices $p_1 = 3, p_2 = 1$ and income $m = 30$ plot the budget set. What is the relative price? (number)

d) Find the optimal consumption bundle (two numbers). In the graph with the indifference curves and the budget set mark the optimal choice.
e) Harder: Assume $p_2 = 1, m = 30$. Derive demand function for commodity $x_1$ (give formula) and plot it in the graph. Is this commodity ordinary or Giffen and why (one sentence)?

**Problem 4 (10p) (Choice and Engel Curve)**

Consider a Cobb-Douglass utility function

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$ 

a) Using the magic formulas from class, determine whether solution is interior, corner, or any of the two, depending on values of parameters (chose one, give two formulas and one sentence).

b) Assume $a = b = p_1 = p_2 = 2$. Find geometrically and determine analytically income offer curve and Engel curve (give two functions and depict them in the graphs). Give a definition of an inferior good (one sentence). Is $x_1$ a normal or inferior good and why? (one sentence).
Problem 1 (40p) (Optimal choice)

Isabelle spends her income on two types of commodities, books $x_1$ and food $x_2$.

a) The price of a book is $p_1 = 2$ while the price of food is $p_2 = 1$. Isabelle’s income is $m = 16$. Depict Isabelle's budget set. Find the relative price of a book in terms of food (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of the budget set? (one sentence)

b) How is Isabelle’s budget set affected by a gift of 5 books from her father (show the change in the graph)?

c) Isabelle’s utility function is $U(x_1, x_2) = x_1 x_2$. Find the expression for an indifference curve that passes through bundle (2, 1) (give formula). Plot the indifference curve in the graph.

d) Find Marginal Rate of Substitution (MRS) as a function of $x_1, x_2$ (give formula). Give the economic interpretation of MRS (one sentence). Find the value of the MRS for bundle (1, 4) (one number). Which of the goods is more valuable at this bundle? What is the geometric interpretation of MRS? (one sentence).

e) Assume a budget set as in point a). Write down the two secrets of happiness that determine the optimal choice (two equations). Explain the economic intuition behind the two conditions (two sentences, one for each condition). Solve for the optimal choice.

f) Suppose the price of a book changes to $p_1 = 1$. Find new optimal choice (two numbers, use magic formula). Decompose the change of consumption $x_1$ into an income and substitution effect (two numbers).

Problem 2 (25p) (Perfect complements, Buying and Selling)

John’s initial endowment of apples and oranges is $(ω_1, ω_2) = (4, 4)$ and market prices of the two commodities are $p_1 = 2, p_2 = 1$, respectively.

a) Depict budget set in the graph. On the budget line, mark all the bundles that involve buying apples and selling oranges.

b) John’s utility function is $U(x_1, x_2) = \min(2x_1, x_2)$. What is the optimal proportion of the two commodities? (give the ratio of $x_1$ to $x_2$)? In the commodity space plot John’s indifference curves.

c) Write down the two secrets of happiness (two equations) and give the economic intuition behind each of them (one sentence for each).

d) Find optimal bundle $(x_1, x_2)$ (two numbers). Which of the two commodities is he buying and which is selling in optimum (one sentence)? Is your solution corner or interior?

Problem 3 (25p) (Perfect Substitutes)

Utility function if Liam is $U(x_1, x_2) = 2x_1 + x_2$.

a) Find Liam’s Marginal Rate of Substitution, (give formula). Which of the two commodities is more “precious” to her? (one sentence). Plot the indifference curve map in the commodity space.

b) Propose an alternative utility function that represents the same preferences (give formula).
c) For prices $p_1 = 1, p_2 = 2$ and income $m = 16$ plot the budget set. What is the relative price? (number)

d) Find the optimal consumption bundle (two numbers). In the graph with the indifference curves and the budget set mark the optimal choice.

e) Harder: Assume $p_2 = 2, m = 16$. Derive demand function for commodity $x_1$ (give formula) and plot it in the graph. Is this commodity ordinary of Giffen and why (one sentence)?

**Problem 4 (10p) (Choice and Engel Curve)**

Consider a Cobb-Douglas utility function

$$U(x_1, x_2) = a \cdot \ln x_1 + b \ln x_2.$$ 

a) Using the magic formulas from class, determine whether solution is interior, corner, or any of the two, depending on values of parameters (chose one, give two formulas and one sentence).

b) Assume $a = b = p_1 = p_2 = 3$. Find geometrically and determine analytically income offer curve and Engel curve (give two functions and depict them in the graphs). Give a definition of an inferior good (one sentence). Is $x_1$ a normal or inferior good and why? (one sentence).
Econ 301  
Intermediate Microeconomics  
Prof. Marek Weretka  

Midterm 1 (C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+25+25+10=100 points). Make sure that all your graphs have appropriate labels.

Problem 1 (40p) (Optimal choice)
Isabelle spends her income on two types of commodities, books $x_1$ and food $x_2$.

a) The price of a book is $p_1 = 4$ while the price of food is $p_2 = 1$. Isabelle’s income is $m = 40$. Depict Isabelle’s budget set. Find the relative price of a book in terms of food (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of the budget set? (one sentence)

b) How is Isabelle’s budget set affected by a gift of 5 books from her father (show the change in the graph)?

c) Isabelle’s utility function is $U(x_1, x_2) = x_1 x_2$. Find the expression for an indifference curve that passes through bundle $(4, 1)$ (give formula). Plot the indifference curve in the graph.

d) Find Marginal Rate of Substitution (MRS) as a function of $x_1, x_2$ (give formula). Give the economic interpretation of MRS (one sentence). Which of the goods is more valuable at this bundle? What is the geometric interpretation of MRS? (one sentence).

e) Assume a budget set as in point a). Write down the two secrets of happiness that determine the optimal choice (two equations). Explain the economic intuition behind the two conditions (two sentences, one for each condition). Solve for the optimal choice.

f) Suppose the price of a book changes to $p_1 = 1$. Find new optimal choice (two numbers, use magic formula). Decompose the change of consumption $x_1$ into an income and substitution effect (two numbers).

Problem 2 (25p) (Perfect complements, Buying and Selling)
John’s initial endowment of apples and oranges is $(\omega_1, \omega_2) = (10, 2)$ and market prices of the two commodities are $p_1 = 1, p_2 = 2$, respectively.

a) Depict budget set in the graph. On the budget line, mark all the bundles that involve buying apples and selling oranges.

b) John’s utility function is $U(x_1, x_2) = \min(3x_1, x_2)$. What is the optimal proportion of the two commodities? (give the ratio of $x_1$ to $x_2$)? In the commodity space plot John’s indifference curves.

c) Write down the two secrets of happiness (two equations) and give the economic intuition behind each of them (one sentence for each).

d) Find optimal bundle $(x_1, x_2)$ (two numbers). Which of the two commodities is he buying and which is selling in optimum (one sentence)? Is your solution corner or interior?

Problem 3 (25p) (Perfect Substitutes)
Utility function if Liam is $U(x_1, x_2) = 3x_1 + x_2$.

a) Find Liam’s Marginal Rate of Substitution, (give formula). Which of the two commodities is more “precious” to her? (one sentence). Plot the indifference curve map in the commodity space.

b) Propose an alternative utility function that represents the same preferences (give formula).
c) For prices $p_1 = 2, p_2 = 1$ and income $m = 12$ plot the budget set. What is the relative price? (number)

   d) Find the optimal consumption bundle (two numbers). In the graph with the indifference curves and the budget set mark the optimal choice.

   e) Harder: Assume $p_2 = 1, m = 12$. Derive demand function for commodity $x_1$ (give formula) and plot it in the graph. Is this commodity ordinary of Giffen and why (one sentence)?

**Problem 4 (10p) (Choice and Engel Curve)**

Consider a Cobb-Douglass utility function

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$

a) Using the magic formulas from class, determine whether solution is interior, corner, or any of the two, depending on values of parameters (chose one, give two formulas and one sentence).

b) Assume $a = b = p_1 = p_2 = 2$. Find geometrically and determine analytically income offer curve and Engel curve (give two functions and depict them in the graphs). Give a definition of an inferior good (one sentence). Is $x_1$ a normal or inferior good and why? (one sentence).
Midterm 1 (D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (40+25+25+10=100 points). Make sure that all your graphs have appropriate labels.

Problem 1 (40p) (Optimal choice)
Isabelle spends her income on two types of commodities, books $x_1$ and food $x_2$.

a) The price of a book is $p_1 = 2$ while the price of food is $p_2 = 2$. Isabelle’s income is $m = 20$. Depict Isabelle’s budget set. Find the relative price of a book in terms of food (number). Give economic interpretation of the relative price (one sentence). Where can the relative price be seen in the graph of the budget set? (one sentence)

b) How is Isabelle’s budget set affected by a gift of 5 books from her father (show the change in the graph)?

c) Isabelle’s utility function is $U(x_1, x_2) = x_1 x_2$.
Find the expression for an indifference curve that passes though bundle $(1, 4)$ (give formula). Plot the indifference curve in the graph.

d) Find Marginal Rate of Substitution (MRS) as a function of $x_1, x_2$ (give formula). Give the economic interpretation of MRS (one sentence). Find the value of the MRS for bundle $(2, 2)$ (one number). Which of the goods is more valuable at this bundle? What is the geometric interpretation of MRS? (one sentence).

e) Assume a budget set as in point a). Write down the two secrets of happiness that determine the optimal choice (two equations). Explain the economic intuition behind the two conditions (two sentences, one for each condition). Solve for the optimal choice.

f) Suppose the price of a book changes to $p_1 = 1$. Find new optimal choice (two numbers, use magic formula). Decompose the change of consumption $x_1$ into an income and substitution effect (two numbers).

Problem 2 (25p) (Perfect complements, Buying and Selling)
John’s initial endowment of apples and oranges is $(ω_1, ω_2) = (8, 1)$ and market prices of the two commodities are $p_1 = 1, p_2 = 2$, respectively.

a) Depict budget set in the graph. On the budget line, mark all the bundles that involve buying apples and selling oranges.

b) John’s utility function is $U(x_1, x_2) = \min(x_1, 3x_2)$.
Find the optimal proportion of the two commodities? (give the ratio of $x_1$ to $x_2$)? In the commodity space plot John’s indifference curves.

c) Write down the two secrets of happiness (two equations) and give the economic intuition behind each of them (one sentence for each).

d) Find optimal bundle $(x_1, x_2)$ (two numbers). Which of the two commodities is he buying and which is selling in optimum (one sentence)? Is your solution corner or interior?

Problem 3 (25p) (Perfect Substitutes)
Utility function if Liam is $U(x_1, x_2) = 3x_1 + x_2$.

a) Find Liam’s Marginal Rate of Substitution, (give formula). Which of the two commodities is more “precious” to her? (one sentence). Plot the indifference curve map in the commodity space.

b) Propose an alternative utility function that represents the same preferences (give formula).
c) For prices $p_1 = 4, p_2 = 1$ and income $m = 12$ plot the budget set. What is the relative price? (number)

d) Find the optimal consumption bundle (two numbers). In the graph with the indifference curves and the budget set mark the optimal choice.

e) Harder: Assume $p_2 = 1, m = 12$. Derive demand function for commodity $x_1$ (give formula) and plot it in the graph. Is this commodity ordinary of Giffen and why (one sentence)?

**Problem 4 (10p) (Choice and Engel Curve)**

Consider a Cobb-Douglass utility function

$$U(x_1, x_2) = a \ln x_1 + b \ln x_2.$$ 

a) Using the magic formulas from class, determine whether solution is interior, corner, or any of the two, depending on values of parameters (chose one, give two formulas and one sentence).

b) Assume $a = b = p_1 = p_2 = 1$. Find geometrically and determine analytically income offer curve and Engel curve (give two functions and depict them in the graphs). Give a definition of an inferior good (one sentence). Is $x_1$ a normal or inferior good and why? (one sentence).
Question 1

(A)

$p_1 = 2$, $p_2 = 4$ and $m = 16$

- Relative price of books in terms of food: $\frac{p_1}{p_2} = \frac{1}{2}$
- The relative price of books in terms of food is the opportunity cost of a book.
- The relative price is the slope of the budget line (books in the x axis and food in the y axis).

(B) The gift creates a kink in her budget line. The bundles $(x_1 \leq 5, x_2 = 4)$ and $(13, 0)$ become affordable.
\( u(1, 2) = 2 \Rightarrow IC_{u=2} = \{(x_1, x_2) : x_2 = \frac{2}{x_1}\} \) or just \( x_2 = \frac{2}{x_1} \).

(D) \[ MRS(x_1, x_2) = -\frac{\partial u(x_1, x_2)}{\partial x_1} = -\frac{x_2}{x_1} \]

- The MRS measures how much \( x_2 \) the consumer is willing to give up to have one more unit of \( x_1 \).
- \( MRS(4, 1) = -\frac{1}{4} \)
- \( x_2 \) (food) is more valuable.
- \( MRS(x_1, x_2) \) is the slope of the IC at \((x_1, x_2)\).

(E) The two secrets of happiness are:
\[ 2x_1 + 4x_2 = 16 \]
\[ \frac{x_2}{x_1} = \frac{1}{2} \]

Since they are both goods you should consume all of your income. The second secret of happiness states that the Marginal utility per dollar of each good should be the same. Otherwise you could take the last dollar you spent on one good, instead spend it on the other good, and make yourself better off.

The optimal solution is:
\( x_1^* = 4, \ x_2^* = 2 \)

(F) Consider the new prices: \( p_1^{new} = 1, \ p_2^{new} = 4 \)

New optimal demand: From the magic formulas:
\[ x_1^{new} = \left(\frac{1}{2}\right) \frac{m}{p_1^{new}} = 8 \]
\[ x_2^{new} = \left(\frac{1}{2}\right) \frac{m}{p_2^{new}} = 2 \]

Substitution and Income Effects:
• Find the income $m'$ that makes the original bundle affordable at the new prices:

$$m' = p_1^{new} 4 + p_2^{new} 2 \Rightarrow m' = 12$$

• Use the magic formula to find $x_1(p_1^{new}, m')$:

$$x_1(p_1^{new}, m') = 6$$

• Substitution Effect:

$$SE = x_1(p_1^{new}, m') - x_1(p_1, m)$$

$$SE = 6 - 4$$

$$SE = 2$$

• Income Effect:

$$IE = x_1(p_1^{new}, m) - x_1(p_1^{new}, m')$$

$$IE = 8 - 6$$

$$IE = 2$$

• Total Effect $= IE + SE = 4$

**Question 2**

(A)

All points $(x_1, x_2)$ that are on the red line are such that the consumer is buying apples $(x_1)$ and selling oranges $(x_2)$.

(B) Optimal proportion is 2 units of $x_1$ for 1 unit of $x_2$. 
1. Since they are both goods you should consume all of your income.

2. Since they are perfect complements, you should consume in the correct proportion. It is important to point out that there is no MRS in this question. Actually, the utility function is not differentiable so MRS is no defined.

(D) Using the two secrets of happiness we can solve for $x_1$ and $x_2$. Specifically, plug the second secret into the first:

$$2x_1 + 0.5x_1 = 20 \Rightarrow 5x_1 = 40 \Rightarrow x_1 = 8$$

Using the optimal proportion and the optimal amount of $x_1$ we got in the previous step we have that $x_2 = 4$.

Therefore,

$$x_1 = 8 \text{ and } x_2 = 4$$

We can conclude that the solution is interior and John is buying apples (2 units) and selling oranges (4 units).

Question 3

(A)

- $MRS(x_1, x_2) = -\frac{MU_{x_1}}{MU_{x_2}} = -2$
- Good 1 is more valuable: (1) For every $(x_1, x_2)$, good 1 has a higher marginal utility; or (2) holding the level of utility fixed, he is willing to give up 2 units of good 2 to have one more of good 1 (MRS).
- The IC map is:
(B) Examples of monotonic transformations:

1. $\ln(u(x_1, x_2)) = \ln(2x_1 + x_2)$
2. $(u(x_1, x_2))^2 = (2x_1 + x_2)^2$
3. $2u(x_1, x_2) = 4x_1 + 2x_2$

(C) The relative price is $\frac{p_1}{p_2} = 3$

(D) Since we have perfect substitutes, the consumer will only buy the good with the highest marginal utility per dollar.

$$\frac{MU_1}{p_1} = \frac{2}{3}$$
$$\frac{MU_2}{p_2} = \frac{1}{1}$$

Therefore, $x_1 = 0$ and $x_2 = 30$
The demand for good 1 as a function of $p_1$ can be written as:

$$x_1(p_1) = \begin{cases} \frac{m}{p_1} & \text{if } 2 > p_1 \\ 0 & \text{if } 2 < p_1 \\ \left[0, \frac{m}{p_1}\right] & \text{if } 2 = p_1 \end{cases}$$

Plotting the graph, we obtain:

From the graph, we can see that $x_1$ is an ordinary good.

**Question 4**

(A) From the magic formula we have that:

$$x_1(p_1, m) = \left(\frac{a}{a + b}\right) \frac{m}{p_1}$$

$$x_2(p_2, m) = \left(\frac{b}{a + b}\right) \frac{m}{p_2}$$

Since $a > 0, b > 0, m > 0$ and $p_1, p_2 > 0$ it is impossible to have a corner solution.

(B) Assume $a = b = p_1 = p_2 = 2$. From the magic formulas we get that:

From the magic formulas we have that:
Rewriting it to have $m$ as a function of $x$, we get that the Engel Curves are:

\[ m = 4x_1 \]
\[ m = 4x_2 \]

Plotting the Engel Curves for both goods in the same graph we get:

Using the Engel Curves we get that the Income Offer Curve is:

\[ x_2 = x_1 \]

Plotting the Income Offer Curve in the commodity space we get:

An inferior good is such that as income increase you consume less of it. Since the Engel Curve for $x_1$ and $x_2$ are positive sloped we can conclude that they are normal goods.
Question 1

(A) \[ p_1 = 2, \ p_2 = 1 \text{ and } m = 16 \]

- Relative price of books in terms of food: \( \frac{p_1}{p_2} = \frac{2}{1} \)
- The relative price of books in terms of food is the opportunity cost of a book.
- The relative price is the slope of the budget line (books in the x axis and food in the y axis).

(B) The gift creates a kink in her budget line. The bundles \((x_1 \leq 5, x_2 = 16)\) and \((13, 0)\) become affordable.
(C) \( u(2,1) = 2 \Rightarrow IC_{u=2} = \{(x_1,x_2) : x_2 = \frac{2}{x_1} \} \) or just \( x_2 = \frac{2}{x_1} \).

(D)

\[
MRS(x_1,x_2) = \frac{\frac{\partial u(x_1,x_2)}{\partial x_1}}{\frac{\partial u(x_1,x_2)}{\partial x_2}} = -\frac{x_2}{x_1}
\]

- The MRS measures how much \( x_2 \) the consumer is willing to give up to have one more unit of \( x_1 \).
- \( MRS(1,4) = -4 \)
- \( x_1 \) (book) is more valuable.
- \( MRS(x_1,x_2) \) is the slope of the IC at \( (x_1,x_2) \).

(E) The two secrets of happiness are:

\[
2x_1 + x_2 = 16
\]

\[
\frac{x_2}{x_1} = 2
\]

Since they are both goods you should consume all of your income. The second secret of happiness states that the Marginal utility per dollar of each good should be the same. Otherwise you could take the last dollar you spent on one good, instead spend it on the other good, and make yourself better off.

The optimal solution is:

\[
x_1^* = 4, \ x_2^* = 8
\]

(F) Consider the new prices: \( p_1^{new} = 1, \ p_2^{new} = 1 \)

New optimal demand: From the magic formulas:

\[
x_1^{new} = \left( \frac{1}{2} \right) \frac{m}{p_1^{new}} = 8
\]

\[
x_2^{new} = \left( \frac{1}{2} \right) \frac{m}{p_2^{new}} = 8
\]

Substitution and Income Effects:
• Find the income $m'$ that makes the original bundle affordable at the new prices:

$$m' = p_1^{new} 4 + p_2^{new} 8 \Rightarrow m' = 12$$

• Use the magic formula to find $x_1(p_1^{new}, m')$:

$$x_1(p_1^{new}, m') = 6$$

• Substitution Effect:

$$SE = x_1(p_1^{new}, m') - x_1(p_1, m)$$
$$SE = 6 - 4$$
$$SE = 2$$

• Income Effect:

$$IE = x_1(p_1^{new}, m) - x_1(p_1^{new}, m')$$
$$IE = 8 - 6$$
$$IE = 2$$

• Total Effect $= IE + SE = 4$

**Question 2**

(A) All points $(x_1, x_2)$ that are on the red line are such that the consumer is buying apples $(x_1)$ and selling oranges $(x_2)$.

(B) Optimal proportion is 1 units of $x_1$ for 2 units of $x_2$.  

3
(C) Secrets of happiness:

1. Since they are both goods you should consume all of your income.

2. Since they are perfect complements, you should consume in the correct proportion. It is important to point out that there is no MRS in this question. Actually, the utility function is not differentiable so MRS is no defined.

(D) Using the two secrets of happiness we can solve for $x_1$ and $x_2$. Specifically, plug the second secret into the first:

$$2x_1 + 2x_1 = 12 \Rightarrow x_1 = 3$$

Using the optimal proportion and the optimal amount of $x_1$ we got in the previous step we have that $x_2 = 6$.

Therefore,

$$x_1 = 3 \text{ and } x_2 = 6$$

The solution is interior and John is selling apples (1 unit) and buying oranges (2 units).

Question 3

(A)

- $MRS(x_1, x_2) = -\frac{MU_{x_1}}{MU_{x_2}} = -2$

- Good 1 is more valuable: (1) For every $(x_1, x_2)$, good 1 has a higher marginal utility; or (2) holding the level of utility fixed, he is willing to give up 2 units of good 2 to have one more of good 1 (MRS).

- The IC map is:
(B) Examples of monotonic transformations:

1. \( \ln(u(x_1, x_2)) = \ln(2x_1 + x_2) \)
2. \( (u(x_1, x_2))^2 = (2x_1 + x_2)^2 \)
3. \( 2u(x_1, x_2) = 4x_1 + 2x_2 \)

(C) The relative price is \( \frac{p_1}{p_2} = 0.5 \)

(D)

Since we have perfect substitutes, the consumer will only buy the good with the highest marginal utility per dollar.

\[
\frac{MU_1}{p_1} = \frac{2}{1} \\
\frac{MU_2}{p_2} = \frac{1}{2}
\]

Therefore, \( x_1 = 16 \) and \( x_2 = 0 \)
The demand for good 1 as a function of $p_1$ can be written as:

$$x_1(p_1) = \begin{cases} \frac{m}{p_1} & \text{if } 4 > p_1 \\ 0 & \text{if } 4 < p_1 \\ \left[0, \frac{m}{p_1}\right] & \text{if } 4 = p_1 \end{cases}$$

Plotting the graph, we obtain:

From the graph, we can see that $x_1$ is an ordinary good.

**Question 4**

(A) From the magic formulas we have that:

$$x_1(p_1, m) = \left(\frac{a}{a + b}\right) \frac{m}{p_1}$$

$$x_2(p_2, m) = \left(\frac{b}{a + b}\right) \frac{m}{p_2}$$

Since $a > 0$, $b > 0$, $m > 0$ and $p_1, p_2 > 0$ it is impossible to have a corner solution.

(B) Assume $a = b = p_1 = p_2 = 3$. From the magic formulas we get that:

From the magic formulas we have that:
Rewriting it to have \( m \) as a function of \( x \), we get that the Engel Curves are:

\[
\begin{align*}
  m &= 6x_1 \\
  m &= 6x_2
\end{align*}
\]

Plotting the Engel Curves for both goods in the same graph we get:

Using the Engel Curves we get that the Income Offer Curve is:

\[ x_2 = x_1 \]

Plotting the Income Offer Curve in the commodity space we get:

An inferior good is such that as income increase you consume less of it. Since the Engel Curve for \( x_1 \) and \( x_2 \) are positive sloped we can conclude that they are normal goods.
Question 1

(A) 

\[ p_1 = 4, \; p_2 = 1 \text{ and } m = 40 \]

- Relative price of books in terms of food: \( \frac{p_1}{p_2} = 4 \)
- The relative price of books in terms of food is the opportunity cost of a book.
- The relative price is the slope of the budget line (books in the x axis and food in the y axis).

(B) The gift creates a kink in her budget line. The bundles \((x_1 \leq 5, x_2 = 40)\) and \((15, 0)\) become affordable.
(C) \( u(4, 1) = 4 \Rightarrow IC_{u=4} = \{(x_1, x_2) : x_2 = \frac{4}{x_1}\} \) or just \( x_2 = \frac{4}{x_1} \).

(D)

- \( MRS(x_1, x_2) = -\frac{\partial u(x_1, x_2)}{\partial x_1} \div \frac{\partial u(x_1, x_2)}{\partial x_2} = -\frac{x_2}{x_1} \)
- The MRS measures how much \( x_2 \) the consumer is willing to give up to have one more unit of \( x_1 \).
- \( MRS(2, 1) = -1/2 \)
- \( x_2 \) is more valuable.
- \( MRS(x_1, x_2) \) is the slope of the IC at \((x_1, x_2)\).

(E) The two secrets of happiness are:

\[
4x_1 + x_2 = 40
\]

\[
\frac{x_2}{x_1} = 4
\]

Since they are both goods you should consume all of your income. The second secret of happiness states that the Marginal utility per dollar of each good should be the same. Otherwise you could take the last dollar you spent on one good, instead spend it on the other good, and make yourself better off.

The optimal solution is:

\[
x_1^* = 5, \ x_2^* = 20
\]

(F) Consider the new prices: \( p_1^{\text{new}} = 1, \ p_2^{\text{new}} = 1 \)

**New optimal demand:** From the magic formulas:

\[
x_1^{\text{new}} = \left(\frac{1}{2}\right) \frac{m}{p_1^{\text{new}}} = 20
\]

\[
x_2^{\text{new}} = \left(\frac{1}{2}\right) \frac{m}{p_2^{\text{new}}} = 20
\]

**Substitution and Income Effects:**
• Find the income $m'$ that makes the original bundle affordable at the new prices:

$$m' = p_{1}^{new}5 + p_{2}^{new}20 \Rightarrow m' = 25$$

• Use the magic formula to find $x(P_{1}^{new}, m')$:

$$x(P_{1}^{new}, m') = 12.5$$

• Substitution Effect:

$$SE = x(P_{1}^{new}, m') - x(P_{1}, m)$$
$$SE = 12.5 - 5$$
$$SE = 7.5$$

• Income Effect:

$$IE = x(P_{1}^{new}, m) - x(P_{1}^{new}, m')$$
$$IE = 20 - 12.5$$
$$IE = 7.5$$

• Total Effect = $IE + SE = 15$

**Question 2**

(A) All points $(x_1, x_2)$ that are on the red line are such that the consumer is buying apples ($x_1$) and selling oranges ($x_2$).

(B) Optimal proportion is 1 unit of $x_1$ for 3 units of $x_2$. So, the ratio is $x_1 : x_2 = 1 : 3$. 

(C) Secrets of happiness:

\[ 14 = x_1 + 2x_2 \quad (1) \]
\[ 3x_1 = x_2 \quad (2) \]

1. Since they are both goods you should consume all of your income.

2. Since they are perfect complements, you should consume in the correct proportion. It is important to point out that there is no MRS in this question. Actually, the utility function is not differentiable so MRS is no defined.

(D) Using the two secrets of happiness we can solve for \( x_1 \) and \( x_2 \). Specifically, plug the second secret into the first:

\[ x_1 + 6x_1 = 14 \Rightarrow x_1 = 2 \]

Using the optimal proportion and the optimal amount of \( x_1 \) we got in the previous step we have that \( x_2 = 6 \).
Therefore,

\[ x_1 = 2 \text{ and } x_2 = 6 \]

The solution is interior and John is selling apples (8 units) and buying oranges (4 units).

**Question 3**

(A)

- \( MRS(x_1, x_2) = -\frac{MU_{x_1}}{MU_{x_2}} = -3 \)

- Good 1 is more valuable: (1) For every \((x_1, x_2)\), good 1 has a higher marginal utility; or (2) holding the level of utility fixed, he is willing to give up 3 units of good 2 to have one more of good 1 (MRS).

- The IC map is:
(B) Examples of monotonic transformations:
1. $\ln(u(x_1, x_2)) = \ln(3x_1 + x_2)$
2. $(u(x_1, x_2))^2 = (3x_1 + x_2)^2$
3. $2u(x_1, x_2) = 6x_1 + 2x_2$

(C) The relative price is $\frac{p_1}{p_2} = 2$

(D) Since we have perfect substitutes, the consumer will only buy the good with the highest marginal utility per dollar.

\[
\frac{MU_1}{p_1} = \frac{3}{2} \quad \frac{MU_2}{p_2} = \frac{1}{1}
\]

Therefore, $x_1 = 6$ and $x_2 = 0$
The demand for good 1 as a function of $p_1$ can be written as:

$$x_1(p_1) = \begin{cases} \frac{m}{p_1} & \text{if } 3 > p_1 \\ 0 & \text{if } 3 < p_1 \\ [0, \frac{m}{p_1}] & \text{if } 3 = p_1 \end{cases}$$

Plotting the graph, we obtain:

From the graph, we can see that $x_1$ is an ordinary good.

**Question 4**

(A) From the magic formulas we have that:

$$x_1(p_1, m) = \left( \frac{a}{a + b} \right) \frac{m}{p_1}$$

$$x_2(p_2, m) = \left( \frac{b}{a + b} \right) \frac{m}{p_2}$$

Since $a > 0$, $b > 0$, $m > 0$ and $p_1, p_2 > 0$ it is impossible to have a corner solution.

(B) Assume $a = b = p_1 = p_2 = 2$. From the magic formulas we get that:

From the magic formulas we have that:
Rewriting it to have $m$ as a function of $x$, we get that the Engel Curves are:

\[ m = 4x_1 \]
\[ m = 4x_2 \]

Plotting the Engel Curves for both goods in the same graph we get:

Using the Engel Curves we get that the Income Offer Curve is:

\[ x_2 = x_1 \]

Plotting the Income Offer Curve in the commodity space we get:

An inferior good is such that as income increase you consume less of it. Since the Engel Curve for $x_1$ and $x_2$ are positive sloped we can conclude that they are normal goods.
Question 1

(A) \[ p_1 = 2, \ p_2 = 2 \ and \ m = 20 \]

- Relative price of books in terms of food: \[ \frac{p_1}{p_2} = 1 \]
- The relative price of books in terms of food is the opportunity cost of a book.
- The relative price is the slope of the budget line (books in the x axis and food in the y axis).

(B) The gift creates a kink in her budget line. The bundles \( (x_1 \leq 5, x_2 = 10) \) and \( (15,0) \) become affordable.
(C) \( u(1, 4) = 4 \Rightarrow IC_{u=4} = \{(x_1, x_2): x_2 = \frac{4}{x_1}\} \) or just \( x_2 = \frac{4}{x_1} \).

(D)

- \( MRS(x_1, x_2) = -\frac{\partial u(x_1, x_2)}{\partial x_1} \frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{-x_2}{x_1} \)

- The MRS measures how much \( x_2 \) the consumer is willing to give up to have one more unit of \( x_1 \).
- \( MRS(2, 2) = -1 \)
- They have the same value to the consumer.
- \( MRS(x_1, x_2) \) is the slope of the IC at \((x_1, x_2)\).

(E) The two secrets of happiness are:

\[
2x_1 + 2x_2 = 20
\]

\[
\frac{x_2}{x_1} = 1
\]

Since they are both goods you should consume all of your income. The second secret of happiness states that the Marginal utility per dollar of each good should be the same. Otherwise you could take the last dollar you spent on one good, instead spend it on the other good, and make yourself better off.

The optimal solution is:

\[
x_1^* = 5, \ x_2^* = 5
\]

(F) Consider the new prices: \( p_1^{new} = 1, \ p_2^{new} = 2 \)

**New optimal demand:** From the magic formulas:

\[
x_1^{new} = \left(\frac{1}{2}\right) \frac{m}{p_1^{new}} = 10
\]

\[
x_2^{new} = \left(\frac{1}{2}\right) \frac{m}{p_2^{new}} = 5
\]

**Substitution and Income Effects:**
• Find the income $m'$ that makes the original bundle affordable at the new prices:

$$m' = p_1^{new} \times 5 + p_2^{new} \times 5 \Rightarrow m' = 15$$

• Use the magic formula to find $x_1(p_1^{new}, m')$:

$$x_1(p_1^{new}, m') = 7.5$$

• Substitution Effect:

$$SE = x_1(p_1^{new}, m') - x_1(p_1, m)$$
$$SE = 7.5 - 5$$
$$SE = 2.5$$

• Income Effect:

$$IE = x_1(p_1^{new}, m) - x_1(p_1^{new}, m')$$
$$IE = 10 - 7.5$$
$$IE = 2.5$$

• Total Effect = $IE + SE = 5$

**Question 2**

(A) All points $(x_1, x_2)$ that are on the red line are such that the consumer is buying apples ($x_1$) and selling oranges ($x_2$).

(B) Optimal proportion is 3 units of $x_1$ for 1 unit of $x_2$. 
(C) Secrets of happiness:

\[ 10 = x_1 + 2x_2 \]  
\[ x_1 = 3x_2 \]

1. Since they are both goods you should consume all of your income.

2. Since they are perfect complements, you should consume in the correct proportion. It is important to point out that there is no MRS in this question. Actually, the utility function is not differentiable so MRS is no defined.

(D) Using the two secrets of happiness we can solve for \( x_1 \) and \( x_2 \). Specifically, plug the second secret into the first:

\[ 3x_2 + 2x_2 = 10 \Rightarrow x_2 = 2 \]

Using the optimal proportion and the optimal amount of \( x_2 \) we got in the previous step we have that \( x_1 = 6 \).

Therefore,

\[ x_1 = 6 \text{ and } x_2 = 2 \]

The solution is interior and John is selling apples (2 units) and buying oranges (1 unit).

**Question 3**

(A)

- \( MRS(x_1, x_2) = -\frac{MU_{x_1}}{MU_{x_2}} = -3 \)

- Good 1 is more valuable: (1) For every \((x_1, x_2)\), good 1 has a higher marginal utility; or (2) holding the level of utility fixed, he is willing to give up 3 units of good 2 to have one more of good 1 (MRS).

- The IC map is:
(B) Examples of monotonic transformations:
1. $\ln(u(x_1, x_2)) = \ln(3x_1 + x_2)$
2. $(u(x_1, x_2))^2 = (3x_1 + x_2)^2$
3. $2u(x_1, x_2) = 6x_1 + 2x_2$

(C) The relative price is $\frac{p_1}{p_2} = 4$

(D)
Since we have perfect substitutes, the consumer will only buy the good with the highest marginal utility per dollar.

\[
\frac{MU_1}{p_1} = \frac{3}{4}
\]
\[
\frac{MU_2}{p_2} = \frac{1}{1}
\]

Therefore, $x_1 = 0$ and $x_2 = 12$
The demand for good 1 as a function of $p_1$ can be written as:

$$x_1(p_1) = \begin{cases} \frac{m}{p_1} & \text{if } 3 > p_1 \\ 0 & \text{if } 3 < p_1 \\ [0, \frac{m}{p_1}] & \text{if } 3 = p_1 \end{cases}$$

Plotting the graph, we obtain:

From the graph, we can see that $x_1$ is an ordinary good.

**Question 4**

(A) From the magic formulas we have that:

$$x_1(p_1, m) = \left( \frac{a}{a+b} \right) \frac{m}{p_1}$$

$$x_2(p_2, m) = \left( \frac{b}{a+b} \right) \frac{m}{p_2}$$

Since $a > 0$, $b > 0$, $m > 0$ and $p_1, p_2 > 0$ it is impossible to have a corner solution.

(B) Assume $a = b = p_1 = p_2 = 1$. From the magic formulas we get that:

From the magic formulas we have that:
Rewriting it to have $m$ as a function of $x$, we get that the Engel Curves are:

\[ m = 2x_1 \]
\[ m = 2x_2 \]

Plotting the Engel Curves for both goods in the same graph we get:

Using the Engel Curves we get that the Income Offer Curve is:

\[ x_2 = x_1 \]

Plotting the Income Offer Curve in the commodity space we get:

An inferior good is such that as income increase you consume less of it. Since the Engel Curve for $x_1$ and $x_2$ are positive sloped we can conclude that they are normal goods.