Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Final Exam (Group A)

You have 2h to complete the exam. The final consists of 6 questions (15+10+15+25+20+15=100).

Problem 1. (Choice with Cobb-Douglas preferences)

Sara spends her income on books x_1 and food x_2. The prices of the two commodities are $p_1 = p_2 = 5$ and her income is $m = 100$. Sara's utility function is given by

$$U(x_1, x_2) = (x_1)^3 (x_2)^3 .$$

a) Find analytically Sara's MRS as a function of (x_1, x_2) (give a function) and determine its value for consumption bundle $(x_1, x_2) = (6, 2)$. Provide economic and geometric interpretation of MRS at this bundle (one sentence + graph).

b) Give two secrets of happiness that determine Sara's optimal choice (two equation). Explain why violation of any of them implies that the bundle cannot be optimal (one sentence for each condition).

c) Find Sara's optimal choice (two numbers) and mark the optimal bundle in the commodity space.

d) Using magic formulas for Cobb-Douglas preferences argue that both commodities are ordinary commodities. (formulas and one sentence)

Problem 2. (Intertemporal choice with perfect substitutes)

Josh chooses a consumption plans for two periods. His income in the two periods is $(\omega_1, \omega_2) = (\$30, \$60)$ and the utility function is

$$U(x_1, x_2) = x_1 + \frac{1}{8} x_2.$$

a) Propose some other utility function that gives a higher level of utility for any bundle (x_1, x_2), which represents the same preferences. (utility function)

b) Plot intertemporal budget set of Josh for interest rate $r = 100\%$. Find PV and FV of the endowment cash flow and depict the two values in the graph. On the budget line mark all consumption plans that involve borrowing.

c) Find optimal consumption plan $(x_1, x_2)$and show it in the graph (give two numbers). Is your solution interior?

Problem 3. (Equilibrium)

Consider an economy with two goods: clothing x_1 and food x_2. Onur's initial endowement is $\omega^O = (80, 20)$ and Janet initially has $\omega^J = (20, 30)$. Utility functions of Onur and Janet are given by

$$U^i(x_1, x_2) = \frac{1}{4} \ln(x_1) + \frac{1}{4} \ln(x_2).$$

.

a) Plot an Edgeworth box and mark the point corresponding to the initial endowments.

b) Give the definition of a Pareto efficient allocation (one sentence) and provide its equivalent characterization in terms of MRS (equation). Verify whether the endowment allocation is Pareto efficient (compare two numbers).

c) Find the prices and the allocation in the competitive equilibrium (six numbers)

d) Using MRS condition demonstrate that the competitive allocation is Pareto efficient.

Problem 4.(Short questions)

a) You are renting a home that gives you \$1000 each month in form of rent (forever). Find PV of the cashflow if the monthly interest rate is $r = 1\%$.

b) Demonstrate that production function $f(K, L) = K^{0.3}L^{0.3}$ exhibits decreasing returns to scale. (use "lambda" argument). Without any calculations sketch the cost curve associated with this production function.

c) Suppose fixed cost is $F = 2$ and variable cost is $c(y) = 2y^2$. Find ATC^{MES} and y^{MES}. Give formula for a supply function of individual firm and plot it in a graph. Find equilibrium price and aggregate output in an industry with 4 firms, assuming demand $y = 10 - p$.

d) Give a von Neumann-Morgenstern utility function over lotteries for a Bernoulli utility function is $u(c) = \ln c$ and the probability of each state is 0.5 (formula). Is a consumer with this utility function risk loving, risk averse or risk neutral? (choose one)

e) In a market for second-hand vehicles there are two types of cars: lemons (bad quality cars) and plums (good quality ones). The value of a car depends on its type and is given by

	Lemon	Plum
Seller	0	20
Buyer	10	26

Are plums going to be traded if the probability of a lemon is $\frac{1}{2}$? (compare relevant numbers).

Problem 5.(Market Power)

Consider an industry with inverse demand $p(y) = 200 - y$ and total cost $TC = 40y$.

a) What are the total gains to trade in this industry? (number). Find the HHI index of this industry with one firm, a monopoly. (one number)

b) Find the optimal level of output and the price of a monopoly assuming uniform pricing (give two numbers). Illustrate its choice in a graph. Mark a DWL.

c) Find profit and a DWL if monopoly uses the first-degree price discrimination.

d) Find the aggregate output, the price and the markup in a Cournot-Nash equilibrium with N firms (all functions of N). What is the limit of the markup function as N goes to infinity? Why?

Problem 6.(Public good)

Alfonsia and Betonia are two countries that are members of the same military alliance. Their security depends positively on joint military spending $x^A + x^B$ of the two countries. Thus, Alfonsia's "utility" net of cost of military spending is given by

$$U^A = 2\ln(x^A + x^B) - x^A$$

and the analogous function for Bretonia is

$$U^B = 4\ln(x^A + x^B) - x^B.$$

a) Find the best response functions for Alfonsia and Bretonia (two formulas) and plot them in the coordinate system (x^A, x^B).

b) Find the Nash Equilibrium (give two numbers). Is one of the two countries free riding? If yes, which one?

c) Find the efficient level of military spending of the alliance (one number). Is the efficient spending smaller or bigger than the one observed in the Nash equilibrium? Why? (one sentence)

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Final Exam (Group B)

You have 2h to complete the exam. The final consists of 6 questions (15+10+15+25+20+15=100).

Problem 1. (Choice with Cobb-Douglas preferences)

Sara spends her income on books x_1 and food x_2. The prices of the two commodities are $p_1 = p_2 = 10$ and her income is $m = 200$. Sara's utility function is given by

$$U(x_1, x_2) = (x_1)^6 (x_2)^6 .$$

a) Find analytically Sara's MRS as a function of (x_1, x_2) (give a function) and determine its value for consumption bundle $(x_1, x_2) = (2, 6)$. Provide economic and geometric interpretation of MRS at this bundle (one sentence + graph).

b) Give two secrets of happiness that determine Sara's optimal choice (two equation). Explain why violation of any of them implies that the bundle cannot be optimal (one sentence for each condition).

c) Find Sara's optimal choice (two numbers) and mark the optimal bundle in the commodity space.

d) Using magic formulas for Cobb-Douglas preferences argue that both commodities are ordinary commodities. (formulas and one sentence)

Problem 2. (Intertemporal choice with perfect substitutes)

Josh chooses a consumption plans for two periods. His income in the two periods is $(\omega_1, \omega_2) = (\$30, \$60)$ and the utility function is

$$U(x_1, x_2) = x_1 + \frac{1}{3}x_2.$$

a) Propose some other utility function that gives a higher level of utility for any bundle (x_1, x_2), which represents the same preferences. (utility function)

b) Plot intertemporal budget set of Josh for interest rate $r = 100\%$. Find PV and FV of the endowment cash flow and depict the two values in the graph. On the budget line mark all consumption plans that involve borrowing.

c) Find optimal consumption plan $(x_1, x_2)$and show it in the graph (give two numbers). Is your solution interior?

Problem 3. (Equilibrium)

Consider an economy with two goods: clothing x_1 and food x_2. Onur's initial endowement is $\omega^O = (40, 10)$ and Janet initially has $\omega^J = (10, 15)$. Utility functions of Onur and Janet are given by

$$U^i(x_1, x_2) = \frac{1}{4}\ln(x_1) + \frac{1}{4}\ln(x_2).$$

.

a) Plot an Edgeworth box and mark the point corresponding to the initial endowments.

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c) Suppose fixed cost is $F = 4$ and variable cost is $c(y) = 4y^2$. Find ATC^{MES} and y^{MES}. Give formula for a supply function of individual firm and plot it in a graph. Find equilibrium price and aggregate output in an industry with 8 firms, assuming demand $y = 20 - p$.

d) Give a von Neumann-Morgenstern utility function over lotteries for a Bernoulli utility function is $u(c) = \ln c$ and the probability of each state is 0.5 (formula). Is a consumer with this utility function risk loving, risk averse or risk neutral? (choose one)

e) In a market for second-hand vehicles there are two types of cars: lemons (bad quality cars) and plums (good quality ones). The value of a car depends on its type and is given by

	Lemon	Plum
Seller	0	20
Buyer	10	40

Are plums going to be traded if the probability of a lemon is $\frac{1}{2}$? (compare relevant numbers).

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Consider an industry with inverse demand $p(y) = 200 - y$ and total cost $TC = 40y$.

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c) Find the efficient level of military spending of the alliance (one number). Is the efficient spending smaller or bigger than the one observed in the Nash equilibrium? Why? (one sentence)

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301 Final, Spring 2015 - Solutions
(Version A)

Problem 1.

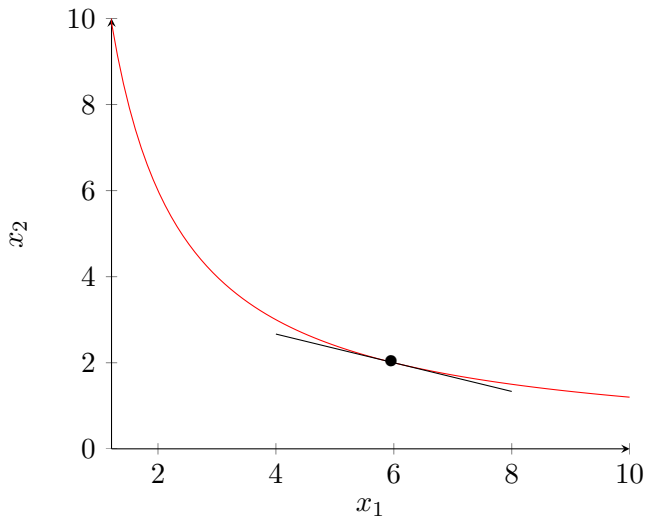
a)

$$MRS(x_1, x_2) = -\frac{(\frac{\partial U}{\partial x_1})}{(\frac{\partial U}{\partial x_2})} = -\frac{3x_1^2 x_2^3}{3x_1^3 x_2^2} = -\frac{x_2}{x_1} \tag{1}$$

$$MRS(6, 2) = -\frac{2}{6} = -\frac{1}{3} \tag{2}$$

Economic interpretation: The MRS is the rate at which Sara is willing to give up x_1 for x_2.

Geometric Interpretation: $MRS(x_1, x_2)$ is the slope of Sara's indifference curve at the point (x_1, x_2).



b) Sara's two secrets of happiness are 1) Sara spends all of her income (and no more): $p_1x_1 + p_2x_2 = w$ and 2) The rate at which Sara is willing to give up x_1 for x_2 is equal to the price of x_1 relative to the price of x_2: $MRS(x_1, x_2) = -\frac{p_1}{p_2}$. These become

$$5x_1 + 5x_2 = 100 \tag{3}$$

$$\frac{x_1}{x_2} = 1 \tag{4}$$

c) Solving these two equations, we get $x_1 = x_2 = 10$

d) Magic formulas:

$$x_1 = \frac{a}{a+b}\frac{m}{p_1} = \frac{1}{2}\frac{m}{p_1} \tag{5}$$

$$x_2 = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{2}\frac{m}{p_2} \tag{6}$$

x_1 is decreasing in p_1 and x_2 is decreasing in p_2. So, both x_1 and x_2 are ordinary commodities.

Problem 2.

a) We can take the monotone transformation $\tilde{U}(x_1, x_2) = 8(U(x_1, x_2)) = 8x_1 + x_2$. $\tilde{U}$ and U represent the same preferences since the bundles have the same utility ordering, but $\tilde{U}(x_1, x_2) > U(x_1, x_2)$ for every bundle (x_1, x_2).

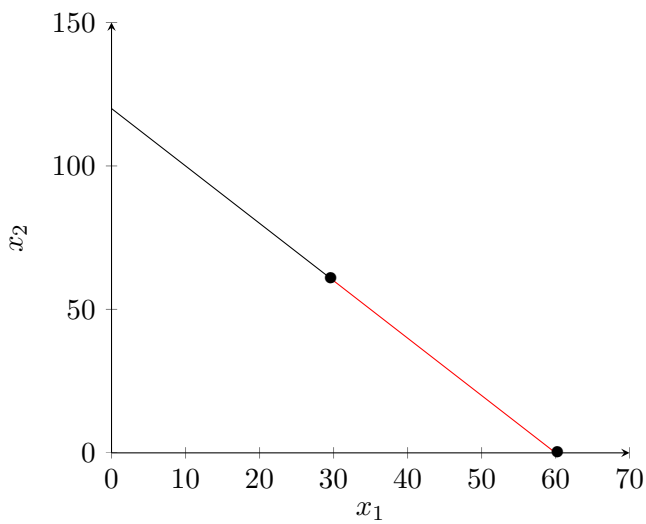
b) Since $r = 100\%$, the slope of the intertemporal budget line is $-(1 + r) = -2$.

The present and future values of the endowment are

$$PV = \omega_1 + \frac{\omega_2}{1+r} = 30 + \frac{60}{2} = 60 \tag{7}$$

$$FV = (1 + r)\omega_1 + \omega_2 = (1 + r)PV = 2(60) = 120 \tag{8}$$

The PV is where the intertemporal budget line crosses the x_1-axis, and the FV is where the intertemporal budget line crosses the x_2-axis. The red portion of the line, to the right of the endowment $(30, 60)$, are all the consumption plans that involve borrowing.



c) We can interpret the prices as $p_1 = 1 + r = 2$ and $p_2 = 1$, so $\frac{p_1}{p_2} = 2$.

Note that $|MRS(x_1, x_2)| = 8 > 2 = \frac{p_1}{p_2}$, so Josh will buy only x_1, and none of x_2. Therefore, $x_2 = 0$. The budget constraint then gives $2x_1 = 2\omega_1 + \omega_2 = 2(30) + 60$, or $x_1 = 60$.

The solution $(x_1, x_2) = (60, 0)$ is not interior. It is at the bottom right corner of the budget line.

Problem 3.

a) The total endowments are $\omega_1 = \omega_1^O + \omega_1^J = 80 + 20 = 100$ and $\omega_2 = \omega_2^O + \omega_2^J = 20 + 30 = 50$ so the Edgeworth box has dimensions 100 by 50. Put Onur on the bottom left origin and Janet on the upper right origin. The endowment is 80 units left and 20 units up from Onur's origin. This point is also 20 units left and 30 units down from Janet's origin.

b) Definition: A Pareto efficient allocation is where no other allocation can make one person off without hurting the other person.

Equivalent Characteristic: An allocation $(x_1^O, x_2^O, x_1^J, x_2^J)$ is Pareto efficient if $MRS(x_1^O, x_2^O) = MRS(x_1^J, x_2^J)$.

Since the utility is Cobb-Douglas with $a = b = \frac{1}{4}$, then $MRS(x_1, x_2) = \frac{a}{b}\frac{x_2}{x_1} = \frac{x_1}{x_2}$. We can check the Pareto efficiency of the endowment point:

$MRS(\omega_1^O, \omega_2^O) = \frac{\omega_2^O}{\omega_1^O} = 4 \neq \frac{2}{3} = \frac{\omega_2^J}{\omega_1^J} = MRS(\omega_1^J, \omega_2^J)$ so the endowment point is not Pareto optimal.

c) Cobb-Douglas magic formulas with parameters $a = b = \frac{1}{4}$ are

$$x_1^O = \frac{1}{2}\frac{m^O}{p_1} \tag{9}$$

$$x_2^O = \frac{1}{2}\frac{m^O}{p_2} \tag{10}$$

$$x_1^J = \frac{1}{2}\frac{m^J}{p_1} \tag{11}$$

$$x_2^J = \frac{1}{2}\frac{m^J}{p_2} \tag{12}$$

m^O and m^J are the total endowments of Onur and Janet, respectively, in terms of dollars.

Plugging these into the market clearing conditions $x_1^O + x_1^J = \omega_1$ and $x_2^O + x_2^J = \omega_2$ (where ω_1 and ω_2 are the aggregate endowments of goods 1 and 2, respectively), we get

$$\frac{1}{2}\frac{1}{p_1}(m^O + m^J) = \omega_1 = 80 + 20 = 100 \tag{13}$$

$$\frac{1}{2}\frac{1}{p_2}(m^O + m^J) = \omega_2 = 20 + 30 = 50 \tag{14}$$

Let us set $p_2 = 1$ and find the relative price p_1. The second equation gives $(m^O + m^J) = 100$. We can substitute this into the first equation to get $\frac{1}{2}\frac{1}{p_1}(100) = 100$, or $p_1 = \frac{1}{2}$.

Now, the dollar endowments are $m^O = \frac{1}{2}(80) + 20 = 60$ and $m^J = \frac{1}{2}(20) + 30 = 40$

Plugging all of these back into the magic formulas gives the equilibrium allocation:

$x_1^O = 60, x_2^O = 30, x_1^J = 40, x_2^J = 20.$

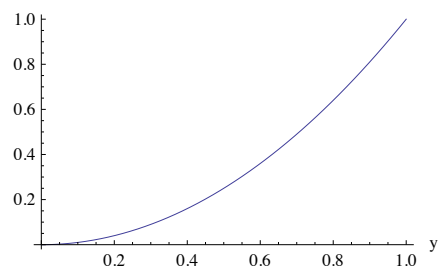
d) $MRS(x_1^O, x_2^O) = \frac{60}{30} = \frac{1}{2} = \frac{20}{40} = MRS(x_1^J, x_2^J)$, so the equilibrium allocation is indeed Pareto efficient.

Problem 4

a) $PV = \$1000/r = \$1000/0.01 = 100,000.$

b) $f(\lambda K, \lambda L) = \lambda^{0.6} K^{0.3} L^{0.3} < \lambda f(K, L)$, DRS.

Figure 1:



c) $MC = ATC \Rightarrow 4y = 2/y + 2y \Rightarrow y^{\text{MES}} = 1,\ ATC^{\text{MES}} = 4.$
$y^{\text{each firm}} = p/4,\ y^{\text{aggregate}} = p.$ Demand=Supply$\Rightarrow 10 - p = p \Rightarrow$
$p = 5,\ y^{\text{aggregate}} = 5$

d) $EU = \ln c_1/2 + \ln c_2/2$, risk averse as the Bernoulli utility is concave.

e) Expected value for the buyer if both types of sellers are in the market is $(10+26)/2 = 18$, less than the plum seller's valuation 20, thus plum won't be traded.

Problem 5

a) Total Gain is $160^2/2 = 12800$;
HHI $= 100^2 = 10,000.$

b) MR=MC $\Rightarrow 200 - 2y = 40 \Rightarrow y = 80 \Rightarrow p = 120.$

c) Profit is equal to total gain from trade found in a), 12800;
There's no DWL.

d) For a firm i, $\pi = (200 - y_i - \sum_{j \neq i} y_j)y_1 - 40y_1$, $MR = 160 - 2y_1 - \sum_{j \neq i} y_j$, $MR = 0 \Rightarrow y_i = \frac{160 - \sum_{j \neq i}^{N} y_j}{2}$, all y are equal$\Rightarrow y_i = \frac{160-(N-1)y_i}{2} \Rightarrow y_i = \frac{160}{N+1}$, $p = 200 - \frac{160N}{N+1} = \frac{40N}{N+1} + \frac{200}{N+1} \overset{N \to \infty}{=} 40$. Markup approaches 1 when N shoots to infinity.

Problem 6

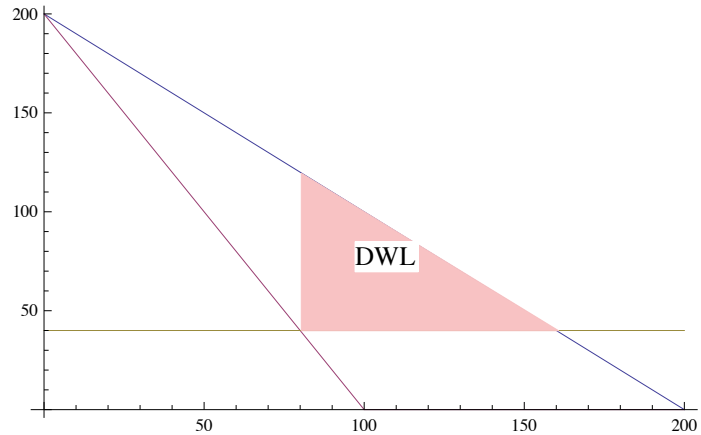


Figure 2:

a) $MU^A = 0 \Rightarrow \frac{2}{x^A + x^B} = 1 \Rightarrow x^A = 2 - x^B$ if $x^B < 2$ and $x^A = 0$ if $x^B \geq 2$. Similarly, $x^B = 4 - x^A$ if $x^A < 4$ and $x^B = 0$ if $x^A \geq 4$.

b) $x^A = 0$, $x^B = 4$. A is free riding as he values the military spending less than B.

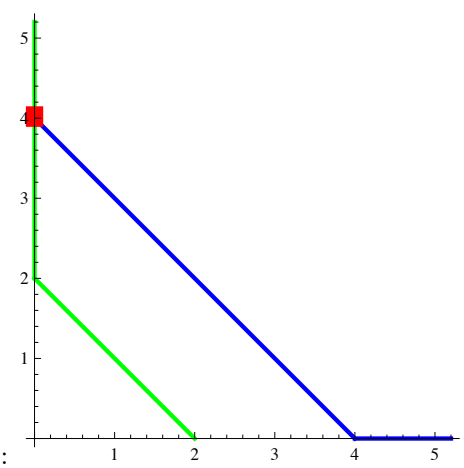


Figure 3:

c) Let $x = x^A + x^B$, $U^{\text{joint}} = 6\ln(x) - x$, $MU^{\text{joint}} = 0 \Rightarrow x = 6$, it's greater than the sum of Nash equilibrium level since the maximizing joint utility internalizes the positive externality.

301 Final, Spring 2015 - Solutions
(Version B)

Problem 1.

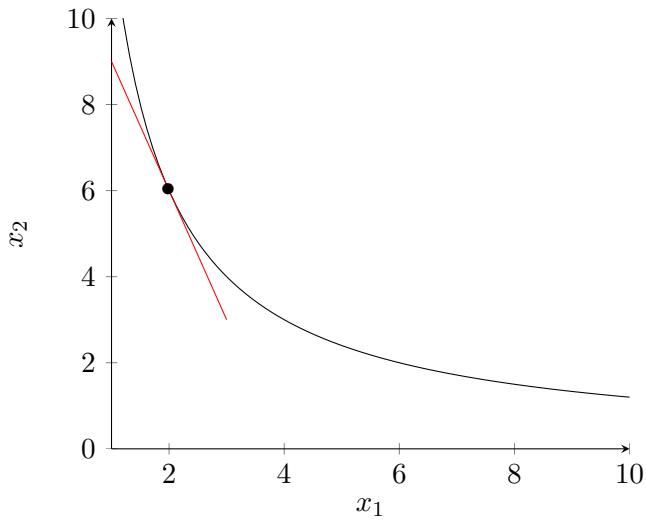
a)

$$MRS(x_1, x_2) = -\frac{(\frac{\partial U}{\partial x_1})}{(\frac{\partial U}{\partial x_2})} = -\frac{6x_1^5 x_2^6}{6x_1^6 x_2^5} = -\frac{x_2}{x_1} \tag{1}$$

$$MRS(6, 2) = -\frac{6}{2} = -3 \tag{2}$$

Economic interpretation: The MRS is the rate at which Sara is willing to give up x_1 for x_2.

Geometric Interpretation: $MRS(x_1, x_2)$ is the slope of Sara's indifference curve at the point (x_1, x_2).



b) Sara's two secrets of happiness are 1) Sara spends all of her income (and no more): $p_1x_1 + p_2x_2 = w$ and 2) The rate at which Sara is willing to give up x_1 for x_2 is equal to the price of x_1 relative to the price of x_2: $MRS(x_1, x_2) = -\frac{p_1}{p_2}$. These become

$$10x_1 + 10x_2 = 200 \tag{3}$$

$$\frac{x_1}{x_2} = 1 \tag{4}$$

c) Solving these two equations, we get $x_1 = x_2 = 10$

d) Magic formulas:

$$x_1 = \frac{a}{a+b}\frac{m}{p_1} = \frac{1}{2}\frac{m}{p_1} \tag{5}$$

$$x_2 = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{2}\frac{m}{p_2} \tag{6}$$

x_1 is decreasing in p_1 and x_2 is decreasing in p_2. So, both x_1 and x_2 are ordinary commodities.

Problem 2.

a) For example, take the monotone transformation $\tilde{U}(x_1, x_2) = 3(U(x_1, x_2)) = 3x_1 + x_2$. $\tilde{U}$ and U represent the same preferences since the bundles have the same utility ordering, but $\tilde{U}(x_1, x_2) > U(x_1, x_2)$ for every bundle (x_1, x_2).

b) Since $r = 100\%$, the slope of the intertemporal budget line is $-(1 + r) = -2$.

The present and future values of the endowment are

$$PV = \omega_1 + \frac{\omega_2}{1+r} = 30 + \frac{60}{2} = 60 \tag{7}$$

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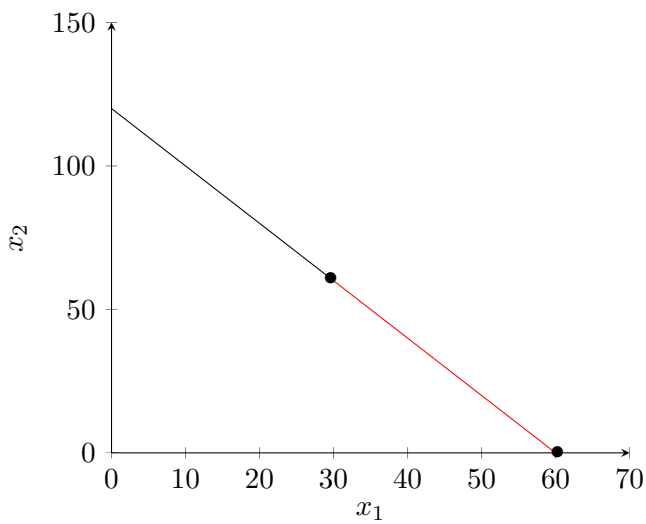

c) We can interpret the prices as $p_1 = 1 + r = 2$ and $p_2 = 1$, so $\frac{p_1}{p_2} = 2$.

Note that $|MRS(x_1, x_2)| = 3 > 2 = \frac{p_1}{p_2}$, so Josh will buy only x_1, and none of x_2. Therefore, $x_2 = 0$. The budget constraint then gives $2x_1 = 2\omega_1 + \omega_2 = 2(30) + 60$, or $x_1 = 60$.

The solution $(x_1, x_2) = (60, 0)$ is not interior. It is at the bottom right corner of the budget line.

Problem 3.

a) The total endowments are $\omega_1 = \omega_1^O + \omega_1^J = 40 + 10 = 50$ and $\omega_2 = \omega_2^O + \omega_2^J = 10 + 15 = 25$ so the Edgeworth box has dimensions 50 by 25. Put Onur on the bottom left origin and Janet on the upper right origin. The endowment is 40 units left and 10 units up from Onur's origin. This point is also 10 units left and 15 units down from Janet's origin.

b) Definition: A Pareto efficient allocation is where no other allocation can make one person off without hurting the other person.

Equivalent Characteristic: An allocation $(x_1^O, x_2^O, x_1^J, x_2^J)$ is Pareto efficient if $MRS(x_1^O, x_2^O) = MRS(x_1^J, x_2^J)$.

Since the utility is Cobb-Douglas with $a = b = \frac{1}{4}$, then $MRS(x_1, x_2) = \frac{a}{b}\frac{x_2}{x_1} = \frac{x_1}{x_2}$. We can check the Pareto efficiency of the endowment point:

$MRS(\omega_1^O, \omega_2^O) = \frac{\omega_2^O}{\omega_1^O} = \frac{1}{4} \neq \frac{3}{2} = \frac{\omega_2^J}{\omega_1^J} = MRS(\omega_1^J, \omega_2^J)$ so the endowment point is not Pareto optimal.

c) Cobb-Douglas magic formulas with parameters $a = b = \frac{1}{4}$ are

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$$x_1^J = \frac{1}{2}\frac{m^J}{p_1} \tag{11}$$

$$x_2^J = \frac{1}{2}\frac{m^J}{p_2} \tag{12}$$

m^O and m^J are the total endowments of Onur and Janet, respectively, in terms of dollars.

Plugging these into the market clearing conditions $x_1^O + x_1^J = \omega_1$ and $x_2^O + x_2^J = \omega_2$ (where ω_1 and ω_2 are the aggregate endowments of goods 1 and 2, respectively), we get

$$\frac{1}{2}\frac{1}{p_1}(m^O + m^J) = \omega_1 = 40 + 10 = 50 \tag{13}$$

$$\frac{1}{2}\frac{1}{p_2}(m^O + m^J) = \omega_2 = 10 + 15 = 25 \tag{14}$$

Let us set $p_2 = 1$ and find the relative price p_1. The second equation gives $(m^O + m^J) = 50$. We can substitute this into the first equation to get $\frac{1}{2}\frac{1}{p_1}(50) = 50$, or $p_1 = \frac{1}{2}$.

Now, the dollar endowments are $m^O = \frac{1}{2}(40) + 10 = 30$ and $m^J = \frac{1}{2}(10) + 15 = 20$

Plugging all of these back into the magic formulas gives the equilibrium allocation:

$x_1^O = 30, x_2^O = 15, x_1^J = 20, x_2^J = 10.$

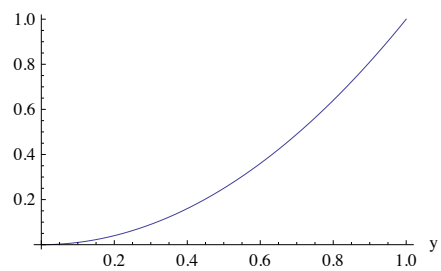
d) $MRS(x_1^O, x_2^O) = \frac{30}{15} = \frac{1}{2} = \frac{10}{20} = MRS(x_1^J, x_2^J)$, so the equilibrium allocation is indeed Pareto efficient.

Problem 4

a) $PV = \$1000/r = \$1000/0.01 = 100,000.$

b) $f(\lambda K, \lambda L) = \lambda^{0.6} K^{0.3} L^{0.3} < \lambda f(K, L)$, DRS.

Figure 1:



c) $MC = ATC \Rightarrow 8y = 4/y + 4y \Rightarrow y^{\text{MES}} = 1,\ ATC^{\text{MES}} = 8.$
$y^{\text{each firm}} = p/8,\ y^{\text{aggregate}} = p.$ Demand=Supply$\Rightarrow 20 - p = p \Rightarrow$
$p = 10,\ y^{\text{aggregate}} = 10$

d) $EU = \ln c_1/2 + \ln c_2/2$, risk averse as the Bernoulli utility is concave.

e) Expected value for the buyer if both types of sellers are in the market is $(10 + 40)/2 = 25$, greater than the plum seller's valuation 20, thus plum will be traded.

Problem 5

a) Total Gain is $160^2/2 = 12800$;
HHI $= 100^2 = 10,000.$

b) MR=MC $\Rightarrow 200 - 2y = 40 \Rightarrow y = 80 \Rightarrow p = 120.$

c) Profit is equal to total gain from trade found in a), 12800;
There's no DWL.

d) For a firm i, $\pi = (200 - y_i - \sum_{j \neq i} y_j)y_1 - 40y_1$, $MR = 160 - 2y_1 - \sum_{j \neq i} y_j$, $MR = 0 \Rightarrow y_i = \frac{160 - \sum_{j \neq i}^{N} y_j}{2}$, all y are equal$\Rightarrow y_i = \frac{160-(N-1)y_i}{2} \Rightarrow y_i = \frac{160}{N+1}$, $p = 200 - \frac{160N}{N+1} = \frac{40N}{N+1} + \frac{200}{N+1} \overset{N \to \infty}{=} 40$. Markup approaches 1 when N shoots to infinity.

Problem 6

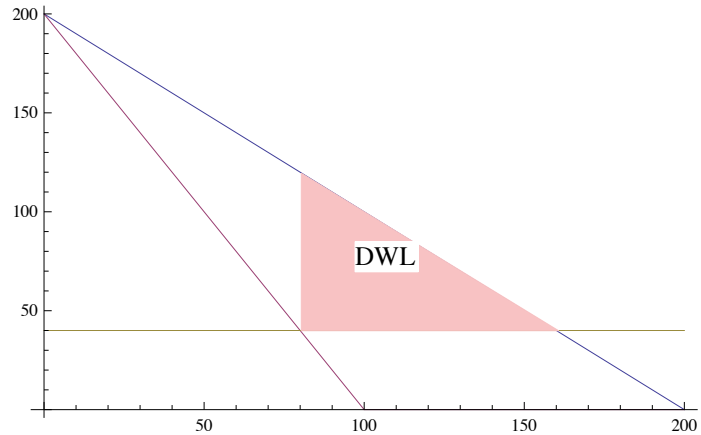


Figure 2:

a) $MU^A = 0 \Rightarrow \frac{3}{x^A + x^B} = 1 \Rightarrow x^A = 3 - x^B$ if $x^B < 3$ and $x^A = 0$ if $x^B \geq 3$. Similarly, $x^B = 4 - x^A$ if $x^A < 4$ and $x^B = 0$ if $x^A \geq 4$.

b) $x^A = 0$, $x^B = 4$. A is free riding as he values the military spending less than B.

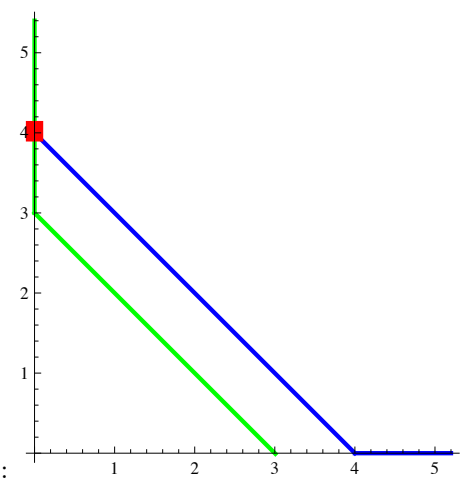


Figure 3:

c) Let $x = x^A + x^B$, $U^{\text{joint}} = 7\ln(x) - x$, $MU^{\text{joint}} = 0 \Rightarrow x = 7$, it's greater than the sum of Nash equilibrium level since the maximizing joint utility internalizes the positive externality.