Problem Set 9: Solutions
ECON 301: Intermediate Microeconomics
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Problem 1 (Equilibrium with \( N \) Firms)

(a) First note that marginal cost is \( MC(y) = c'(y) = 8y \). Using the condition that \( p = MC(y) \), we have \( p = 8y \) \( \implies \) \( y = \frac{1}{8}p \). For prices equal to above \( ATC^{MES} \), the supply function is \( y = \frac{1}{8}p \), for prices below \( ATC^{MES} \), the firm would be operating at a loss if producing and so supply is \( y = 0 \).

We can find the minimum of \( ATC \), which is what \( ATC^{MES} \) is, either by (1) setting \( ATC' \) equal to 0 and solving for \( y \), or (2) equating \( ATC = MC \) and solving for \( y \) (since the minimum of \( ATC \) corresponds to the point at which \( ATC = MC \)). Both give us \( y^{MES} = 1 \) and \( ATC^{MES} = ATC(y^{MES}) = 8 \).

The individual supply is then:

\[
y(p) = \begin{cases} 
0 & \text{for } p < 8 \\
\frac{1}{8}p & \text{for } p \geq 8
\end{cases}
\]

(b) The aggregate supply with the three identical cost structures is \( y^{AGG}(p) = 3y(p) \) (the supply curves are added horizontally over the \( y \)-axis). Letting \( S(p) = y^{AGG}(p) \):

\[
S(p) = \begin{cases} 
0 & \text{for } p < 8 \\
\frac{3}{8}p & \text{for } p \geq 8
\end{cases}
\]

(c) We’ll first find the equilibrium price and aggregate level of production by equating \( S(p) = D(p) \):

\[
S(p) = D(p) \implies \frac{3}{8}p = 8 - \frac{1}{8}p \implies p = 16 \quad (\text{which is } \geq 8)
\]

and the aggregate level of production is \( S(16) = D(16) = 6 \).

The production of each firm is \( y(p) = 2 \) (since \( S(p) = 3y(p), \ 6 = 3y(p) \)), which gives profit \( \pi = TR - TC = 16 \cdot 2 - (4 \cdot 2^2 + 4) = 12 \) for each factory.

(d) The highest amount a firm would pay for the license is 12 (its profit when producing in this market, leaving it with 0 economic profit after paying the license fee).
Problem 2 (Free Entry and Market Structure)

(a) We’ll first solve this for a general fixed cost of $F$, as we saw in Problem Set 8. The price in equilibrium with free entry must be $p = ATC^{MES} = 4\sqrt{F}$ where $F$ is fixed cost with $y^{MES} = \frac{1}{2}\sqrt{F}$. At $p = ATC^{MES} = 4\sqrt{F}$, quantity demanded is

$$D(4\sqrt{F}) = 8 - \frac{1}{8}(4\sqrt{F})$$

and aggregate supply with $N$ firms is

$$S(4\sqrt{F}) = N\frac{1}{2}\sqrt{F}.$$  

Equating the two, we get

$$S(4\sqrt{F}) = D(4\sqrt{F}) \implies N\frac{1}{2}\sqrt{F} = 8 - \frac{1}{8}(4\sqrt{F}) \implies N = \frac{16}{\sqrt{F}} - 1.$$  

So for fixed cost $F = 4$, the number of firms in the market will be $N = 7$.

(b) The number of firms in the market for various fixed costs are shown below, using the formula we found above, $N = \frac{16}{\sqrt{F}} - 1$:

<table>
<thead>
<tr>
<th>Fixed Cost $F$:</th>
<th>64</th>
<th>16</th>
<th>4</th>
<th>$\frac{1}{4}$</th>
<th>$\frac{1}{16}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms $N$:</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>31</td>
<td>63</td>
</tr>
</tbody>
</table>

(c) The market structures are:

- Monopoly at $F = 64$
- Oligopoly at $F = 16$, $F = 4$
- Nearly perfect competition at $F = \frac{1}{4}$, $F = \frac{1}{16}$