

Econ 301 Intermediate Microeconomics

Problem Set 5

Problem 1 (Intertemporal Choice)

Gerald is a CEO in Brainies Consulting Inc. His income in the first year is $m_1 = \$200$ and in the second $m_2 = \$200$. Assume that the interest rate is $r = 100\%$: (His time horizon is limited to these two years.)

- Find PV and FV of Gerald's income.
- Show on the graph $(C_1; C_2)$ Gerald's budget set. Mark PV , FV and the slope of his budget line
- Explain what borrowing/lending strategy gives Gerald each of the two "extreme" consumption points. (how much does he borrow/lend in the first period, how much does he pay back/ receive in the second period?)
- Suppose his utility function is

$$U(C_1; C_2) = \ln C_1 + \ln C_2$$

Find analytically Gerald's optimal choice and show it on the graph. Does the optimal consumption involve saving or borrowing?

Problem 2 (Intertemporal Choice)

The preferences of a manager and a sportsman are given by

$$U(C_1; C_2) = \ln C_1 + \frac{1}{1 + \delta} \ln C_2$$

Suppose $r = \delta = 100\%$.

- Give economic interpretation to coefficient δ .
- Derive the optimal consumption plan of a manager (C_1, C_1) if his income in two periods is given by $m = (0, 3000)$. Find optimal level of savings S in the first period.
- Derive optimal consumption plan of a sportsman, (C_1, C_1) if his income in two periods is given by $m = (1500, 0)$. Find optimal level of savings in the first period.
- Are the manager and the sportsman smoothing their consumption profiles?
- Show, using one of the secrets of happiness, that the optimal consumption of the manager is decreasing over time when $r < \delta$.

Problem 3 (Annuity and Perpetuity)

- Derive PV formula for perpetuity with payment x and interest equal to r ,
- Derive PV formula for annuity that pays x , maturity is T and interest rate is r .

Problem 4 (Present Value, use a calculator)

a) Buying or renting: You are moving in to Madison (you are going to stay here forever!) and you would like to find an apartment. You can either buy it or rent it. The monthly rent is \$500 and the monthly interest rate is 0.1%. Alternatively, you can purchase the apartment, paying \$600,000: How are you going to finance your new accommodation? Argue using the PV formula.

b) You take a loan to buy a car that costs \$4000. What is your monthly payment if you want to pay back the loan after 3 years, (36 payments) and the monthly interest rate is 0.5%: (Hint: Loan with constant payments is like annuity!)

c) You are hired by Merrill Lynch – a financial management and advisory firm – to help assess the value of a T-bond (T-bond is a bond issued by the Department of the Treasury) with the face value $F = \$1000$; coupon $c = \$100$ (paid annually till $T - 1$ (included) and face value F paid in T) and time to maturity equal to $T = 10$ years. The interest rate is equal to $r = 10\%$. Find the PV of such bond. Is it a good or bad deal to buy such bond for \$900?: Explain

d) You want to receive \$40,000 per year when retired (61 and 80). How much do you have to save between 21-60 years. (Interest rate is 5%)

e) You save \$20,000 per year when working (21 and 60). How much will you consume per year when you retire (age 61–80) (Interest rate is 5%)

Problem 5 (Life cycle problems, use Excel program)

a) You are going to earn $m_t = \$200,000$ when working (age 21 and 60), and then you are going to live for the next 20 years with $m_t = \$0$. Find the constant level of consumption C during years (21-80) that can be financed by your income (Interest rate is 5%). Find the level of savings $S_t = m_t - C$ for periods $t < 60$ and for $t > 60$

b) Consider the problem in a). Assume that you inherit \$1000,000 (one million \$) apart from your regular income, at the age of 20. Find C and S_t ?

c) Consider the problem in b). Assume that you also leave your offspring a bequest of \$1000,000 (one million \$). Find C and S_t

d) Optional: Accumulated wealth in period t is given by $W_t = W_{t-1}(1 + r) + s_t$ that is it coincides with wealth from the previous period plus interest and savings in t . The agent's initial wealth is $W_{20} = 0$. Using Excel sheet, plot the behavior of wealth W_t over your life time for points a-c. When the agent's wealth attains its maximum? What can you say about the wealth level at the end of life: W_{80} ?