Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to problem set 9

(due Thursday, April 19th, before class)

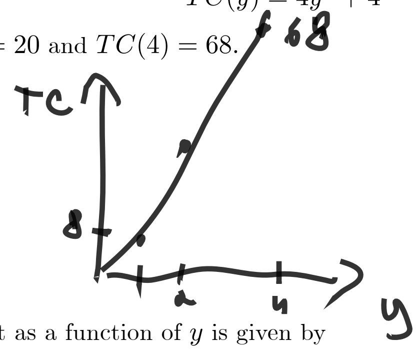
Problem 1 (Cost curves)

a) Decreasing - the exponent in the cost function is greater than one.

b) Total cost function is

$$TC(y) = 4y^2 + 4$$

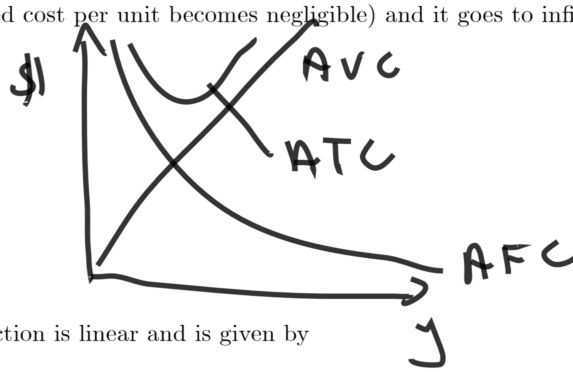
hence $TC(1) = 8$, $TC(2) = 20$ and $TC(4) = 68$.



c) The average fixed cost as a function of y is given by

$$AFC\,(y) = \frac{4}{y}$$

and hence $AFC\,(1) = 4$, $AFC\,(2) = 2$ and $AFC\,(4) = 1$. It becomes zero as y gets larger (with large production the constant fixed cost per unit becomes negligible) and it goes to infinity as y approaches zero.



d) The average cost function is linear and is given by

$$AVC\,(y) = 4y$$

and hence $AVC\,(1) = 4$, $AVC\,(2) = 8$ and $AVC\,(4) = 16$

e) Average total cost is given by

$$ATC = AVC + AFC = 4y + \frac{4}{y}$$

and hence $ATC\,(1) = 8$, $ATC\,(2) = 10$ and $AVC\,(4) = 17$. On the graph is a vertical sum of the two curves. For $y = 1$, 2 and 4 For small production, the average variable cost is negligible and hence ATC is dominated by AFC. When production is large, then it is AVC that dominates.

f) Function ATC

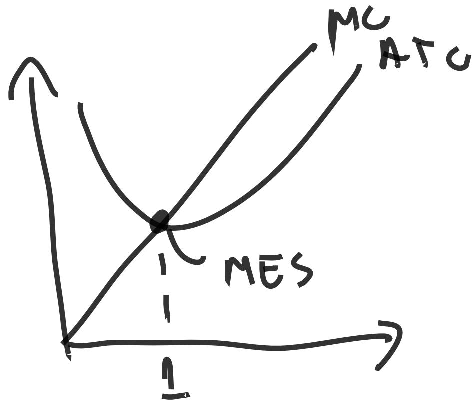
$$ATC = \frac{4}{y} + 4y$$

attains minimum for

$$y^{MES} = 1 \text{ and } ATC^{MES} = 8$$

g) Marginal cost curve is.

$$MC = 8y$$



e) Let y^* be a production level for which ATC and MC cross. We will argue that $y^* = y^{MES}$ and hence the two lines cross where ATC attains minimum. For all $y < y^*$ marginal cost is below the average one. Therefore by increasing production we add a unit that is cheaper to produce than the average cost - hence the average goes down. This implies that for all such $y < y^*$ ATC is downwar slopping. By similar argument for all $y > y^*$ ATC function is increasing. Consequently at y^* ATC attains minimum and hence $y^* = y^{MES}$.

f) The functions are

$$y^{MES} = \frac{1}{2}\sqrt{F}, \; ATC^{MES} = 4\sqrt{F}$$

(I took a derivative of ATC with respect to y and equalized it to zero, and solved for y but I kept F as a parameter)

Problem 2 (Supply curve of GMC)

a) Condition $p = MC$ gives

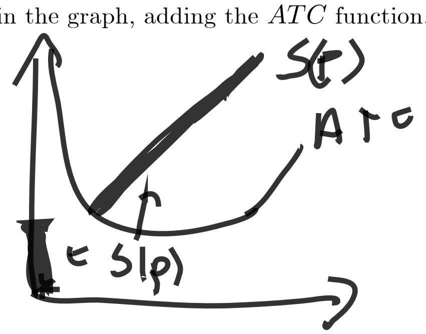
$$y = \frac{1}{8}p$$

For $y(4) = \frac{1}{2}$ we have negative profit hence $y = 0$, for $p = 8$, production is $y = 1$ and profit is zero and finally for $p = 16$ production is $y = 2$ and is associated with strictly positive profit.

b) Supply function is given by

$$y(p) = \left\{ \begin{array}{ll} 0 & \text{for } p < 8 \\ \frac{1}{8}p & \text{for } \; p \geq 8 \end{array} \right\}$$

c) Plot your supply function in the graph, adding the ATC function.



d) Only the "reservation price" at which firms shuts down the production changes $ATC^{MES} = 4\sqrt{F} = 4$

$$y(p) = \left\{ \begin{array}{ll} 0 & \text{for } p < 4 \\ \frac{1}{8}p & \text{for } \; p \geq 4 \end{array} \right\}$$

hence the new supply function has the same slope.

Problem 3 (Equilibrium with N firms)

a) The aggregate supply with three symmetric firms is

$$y\left(p\right)=\left\{\begin{array}{cc} 0 & \text{for } p<4 \\ 3\frac{1}{8}p & \text{for } \ p\geq 4 \end{array}\right\}$$

b) Price is determined from the market clearing condition

$$S\left(p\right)=D\left(p\right)$$

which gives

$$3\times\frac{1}{8}p=S\left(p\right)=D\left(p\right)=8-\frac{1}{8}p$$

therefore equilibrium price is

$$p=16$$

the level of production is

$$y=2 \text{ and } S\left(16\right)=8$$

and the individual profit is

$$\pi=16\times 2-16-4=12>0$$

c) Maximally it will pay \$12

Problem 4 (Free entry and market structure)

a) The price in equilibrium with entry is equal to $ATC^{MES}=4\sqrt{F}$ and the level of individual production is $y^{MES}=\frac{1}{2}\sqrt{F}$. The number of firms can be determined from the market clearing conditon

$$S(p)=D\left(p\right)$$

$$N\times\frac{1}{2}\sqrt{F}=8-\frac{1}{8}4\sqrt{F}$$

hence

$$N=\frac{16}{\sqrt{F}}-1$$

therefore

$$N=\frac{16}{\sqrt{4}}-1\ \ =7$$

b) The numbers of firms are

F	64	16	4	$\frac{1}{4}$	$\frac{1}{16}$
N	1	3	7	31	63

c) Market structuers are
Monopoly $F=64$
Oligopoly $F=16$, $F=4$
Perfect competition $F=\frac{1}{4}$, $F=\frac{1}{16}$