Econ 301
Intermediate Microeconomics
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Problem set 9
(due Tuesday, April 19th, before class)

Problem 1 (Cost curves)

The GMC company is considering building a new car factory in China. The total (fixed) cost of the investment is $F = 4$ (in billions of dollars). When built, the factory will allow to produce y cars at the (variable) cost given by

$$c(y) = 4y^2$$

a) Does the technology used in the new factory exhibit increasing, decreasing or constant returns to scale (ignore the fixed costs in this point)?

b) Find a total costs (TC) of producing 1, 2 and 4 cars. In the graph $(y, COST)$ plot a TC curve, and decompose it into a fixed cost curve and a variable cost curve by adding the two curves to your graph.

c) Find the values of the average fixed cost (AFC) for three levels of production $y = 1$, 2 and 4. Plot a AFC curve in a separate graph. What happens to the AFC when production becomes very large (close to infinity) and when it is very small (close to zero). Explain.

d) Find the values of the average variable cost AVC for $y = 1$, 2 and 4, and mark them in the graph from point c). Connect the three points to obtain the AVC curve.

e) Find the values of the average total cost ATC for $y = 1$, 2 and 4 and mark them in your graph from c). Connect the three points to obtain the ATC curve. What are the values of ATC when the production is very small and very large? Explain which of the two components of ATC - AFC or AVC - dominates in each of the two extremes. Why?

f) Find analytically the minimal efficient scale (MES), y^{MES}, ATC^{MES} for the considered car technology.

g) Find analytically marginal cost MC curve. In a new graph plot the MC curve, together with the ATC, marking the MES.

e) Explain intuitively why or why not the MC curve cuts or does not cut the ATC curve at the MES.

f) Harder: find analytically a minimal efficient scale y^{MES}, and ATC^{MES} as a function of F (parameter). How do the two values depend on the level of F?

Problem 2 (Supply curve of GMC)

Suppose GMC from Problem 1 is maximizing a profit given by

$$\pi = py - TC(y)$$

.a) Find analytically the optimal level of production for each of the three price levels $p = 4$, $p = 8$, $p = 16$? (Hint: first derive the secret of happiness $MC = p$, then find the level of production y and finally check whether the maximal profit is non-negative. If it is non-negative, then y you have found is optimal, theorizes the optimal production is zero.

b) Find analytically a car supply function of GMC, $y(p)$. Hint: it should have the following form

$$y(p) = \left\{ \begin{array}{ll} 0 & \text{for } p < sth \\ sth & \text{for } p \geq sth \end{array} \right\}$$

where sth should be replaced with proper numbers or functions.

c) Plot your supply function in the graph, adding the ATC function.

d) Find supply as in b,c for $F = 1$ (instead of $F = 4$). How is your supply function affected by the change of F? Is it steeper? Hint: use the value you have calculated in f, Problem 1.

Problem 3 (Equilibrium with N firms)

Suppose the car industry in China is regulated (companies must have licences to sell on the Chinese market). Assume that before GMC entry, there are already two firms operating in China. They are all identical to the GMC from Problem 1.

a) find aggregate supply of the car industry, assuming that GMC has a licence and builds the third factory.

b) Suppose the aggregate demand for cars in China is $D(p) = 8 - \frac{1}{8}p$. Find the equilibrium price, individual and aggregate level of production and the level of individual profit.

c) How much (maximally) GMC is willing to pay for the licence to enter the market? (Hint: it will pay at most the value of the profit it makes after paying fixed cost.)

Problem 4 (Free entry and market structure)

Suppose now Chinese government liberalizes the car industry, so that no license is required anymore (in such a case we have free entry.)

a) Predict the number of firms producing cars, the level of production and the level of profit by each firm, the price of a car and given demand for cars equal to $D(p) = 8 - \frac{1}{8}p$, and costs are as in Problem 1 ($F = 4$ and $C(y) = 4y^2$).

b) Find a number of firms given different levels of fixed cost F

F	64	16	4	$\frac{1}{4}$	$\frac{1}{16}$
N					

(Hint: instead of calculating N for each value of F, is it much faster to find a function $N(F)$ where F is a parameter and only then plug concrete values of F. Use the values for MES from point f) in Problem 1)

c) for which values of F should we observe monopoly, oligopoly or nearly perfectly competitive car industry?

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