Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to problem set 8

(due Tuesday, March 23th, before class)

Problem 1 (Cobb-Douglas)

a) what are the returns to scale for each function?

$$\begin{aligned}
F(\lambda K, \lambda L) &= \lambda^2 K^2 \lambda^2 L^2 = \lambda^4 K^2 L^2 \text{ (IRS)} \\
F(\lambda K, \lambda L) &= \lambda^{\frac{1}{3}} K^{\frac{1}{3}} \lambda^{\frac{2}{3}} L^{\frac{2}{3}} = \lambda K^{\frac{1}{3}} L^{\frac{2}{3}} \text{(CRS)} \\
F(\lambda K, \lambda L) &= \lambda^{\frac{1}{4}} K^{\frac{1}{4}} \lambda^{\frac{1}{4}} L^{\frac{1}{4}} = \lambda^{\frac{1}{2}} K^{\frac{1}{4}} L^{\frac{1}{4}} \text{ (DRS)}
\end{aligned}$$

b) Given the first production function $F(K, L) = K^2 L^2$ characterized by IRS, the first secret of happiness (for minimization of the cost) implies that

$$TRS = -\frac{w_K}{w_L} \Rightarrow -\frac{L}{K} = -1 \Rightarrow K = L$$

$$y = K^2 L^2 = K^2 K^2 = K^4 \Rightarrow K = y^{\frac{1}{4}} = L$$

and hence the cost function

$$C(y) = w_K K + w_L L = 1 \times K + 1 \times L = 2y^{\frac{1}{4}}$$

For production function $F(K, L) = K^{\frac{1}{3}} L^{\frac{2}{3}}$ characterized by CRS,

$$TRS = -\frac{w_K}{w_L} \Rightarrow -\frac{\frac{1}{3}}{\frac{2}{3}} \frac{L}{K} = -1 \Rightarrow L = 2K$$

$$\begin{aligned}
y &= K^{\frac{1}{3}} L^{\frac{2}{3}} = K^{\frac{1}{3}} (2K)^{\frac{2}{3}} = 2^{\frac{2}{3}} K \Rightarrow \\
K &= 2^{-\frac{2}{3}} y = 0.629\,96 y \\
L &= 2K = 2^{\frac{1}{3}} y = 1.259\,9 y
\end{aligned}$$

and hence the cost function

$$C(y) = w_K K + w_L L = 1 \times K + 1 \times L = 2^{-\frac{2}{3}} y + 2^{\frac{1}{3}} y = \left(2^{-\frac{2}{3}} + 2^{\frac{1}{3}}\right) y = 1.9y$$

Finally for production function $F(K, L) = K^{\frac{1}{4}} L^{\frac{1}{4}}$ characterized by DRS again

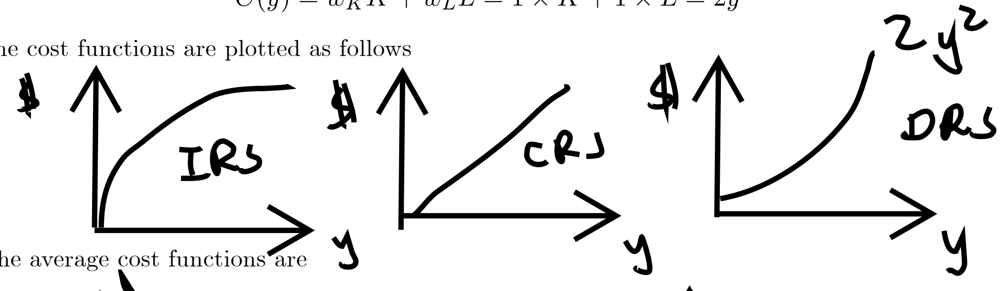
$$TRS = -\frac{w_K}{w_L} \Rightarrow -\frac{L}{K} = -1$$

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}} = K^{\frac{1}{4}} K^{\frac{1}{4}} = K^{\frac{1}{2}} \Rightarrow K = y^2 = L$$

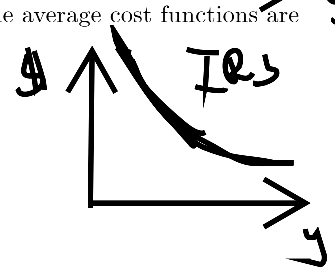
and hence the cost function

$$C(y) = w_K K + w_L L = 1 \times K + 1 \times L = 2y^2$$

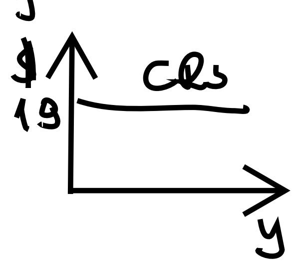
c) The cost functions are plotted as follows

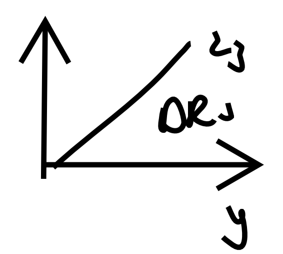


d) The average cost functions are



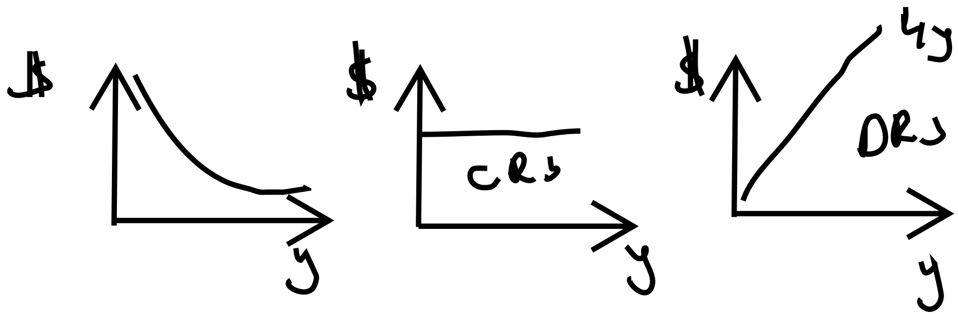
$$\begin{aligned}
AC(y) &= 2y^{-\frac{3}{4}} \\
AC(y) &= 1.9 \\
AC(y) &= 2y
\end{aligned}$$





and marginal costs are

$$
\begin{aligned}
MC(y) &= \frac{1}{2} y^{-\frac{3}{4}} \\
MC(y) &= 1.9 \\
MC(y) &= 4y
\end{aligned}
$$



Problem 2 (Perfect Complements)
a) what are the returns to scale for each function?

$$
\begin{aligned}
F(\lambda K, \lambda L) &= \min(\lambda K, \lambda L) = \lambda \min(K, L) \text{ (CRS)} \\
F(K, L) &= [\min(\lambda K, \lambda L)]^2 = \lambda^2 [\min(K, L)]^2 \text{ (IRS)} \\
F(K, L) &= \sqrt{\min(\lambda K, \lambda L)} = \sqrt{\lambda}\sqrt{\min(K, L)} \text{(DRS)}
\end{aligned}
$$

b) Given the first production function $F(K, L) = \min(K, L)$ characterized by CRS, optimal proportion condition implies that $K = L$

$$y = \min(K, K) = K = L$$

and hence the cost function

$$C(y) = w_K K + w_L L = 1y + 1y = 2y$$

Also in the case of $y = [\min(K, L)]^2$ (IRS) optimal input proportion is $K = L$ and hence $y = [\min(K, K)]^2 = K^2$ and hence

$$K = L = \sqrt{y}$$

and hence the cost function

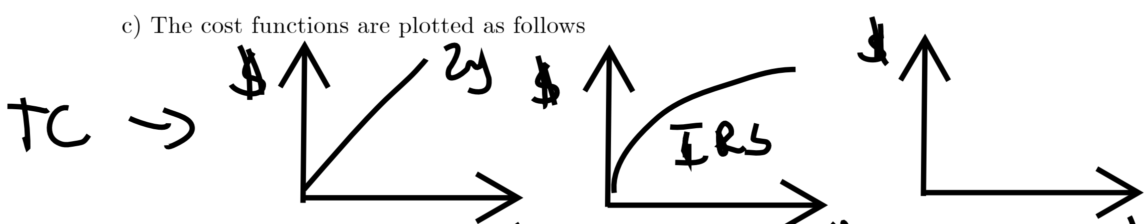
$$C(y) = w_K K + w_L L = 1 \times K + 1 \times L = \sqrt{y} + \sqrt{y} = 2\sqrt{y}$$

Finally for production function $F(K, L) = \sqrt{\min(K, L)}$ characterized by DRS again $K = L$ and hence

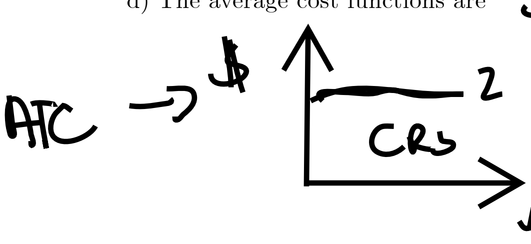
$$y = \sqrt{K} \Rightarrow K = L = y^2$$

$$C(y) = w_K K + w_L L = 1 \times K + 1 \times L = 2y^2$$

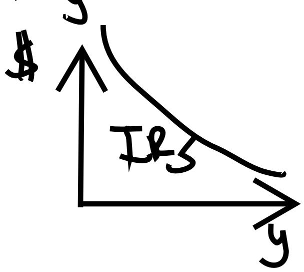
c) The cost functions are plotted as follows

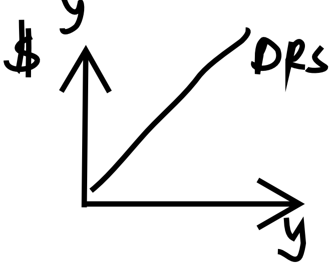


d) The average cost functions are



$$
\begin{aligned}
AC(y) &= 2 \\
AC(y) &= 2\frac{1}{\sqrt{y}} \\
AC(y) &= 2y
\end{aligned}
$$





and marginal costs are

$$\begin{aligned} MC(y) &= 2 \\ MC(y) &= 2\frac{1}{\sqrt{y}} \\ MC(y) &= 4y \end{aligned}$$

Problem 3 (Perfect Substitutes)

a) what are the returns to scale for each function?

$$\begin{aligned} F(\lambda K, \lambda L) &= \lambda K + \lambda 0.5L = \lambda\,(K + 0.5L) \text{ (CRS)} \\ F(K, L) &= (\lambda K + \lambda 0.5L)^2 = \lambda^2\,(K + 0.5L)^2 \text{ (IRS)} \\ F(K, L) &= \sqrt{(\lambda K + \lambda 0.5L)} = \sqrt{\lambda}\sqrt{(K + 0.5L)} \text{(DRS)} \end{aligned}$$

b) Given the first production function $F(K,L) = (K + 0.5L)$ characterized by CRS, capital is more productive and costs the same $L = 0$,

$$y = K$$

and hence the cost function

$$C(y) = w_K K + w_L L = 1y + 1 \times 0 = y$$

Also in the case of $y = [K + 0.5L]^2$ (IRS) again in optimum $L = 0$ and hence $y = K^2$and hence

$$K = \sqrt{y}$$

and hence the cost function

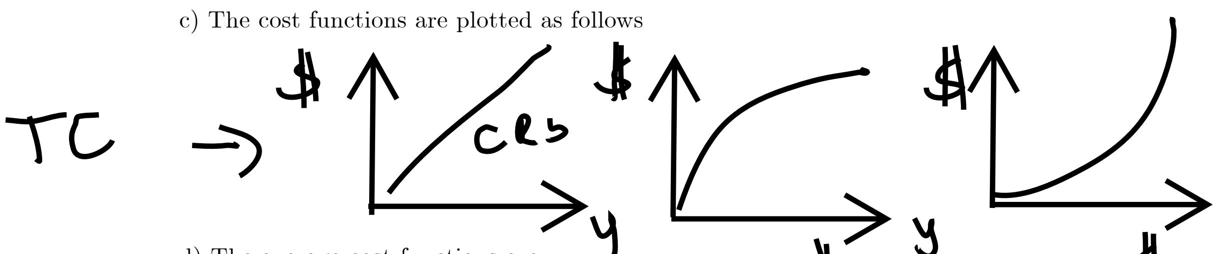
$$C(y) = w_K K + w_L L = 1 \times K = \sqrt{y} = \sqrt{y}$$

Finally for production function $F(K,L) = \sqrt{(K + 0.5L)}$characterized by DRS again $L = 0$ and hence

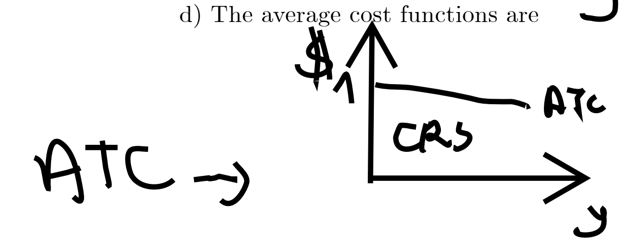
$$y = \sqrt{K} \Rightarrow K = y^2$$

$$C(y) = w_K K + w_L L = 1 \times K + 1 \times 0 = y^2$$

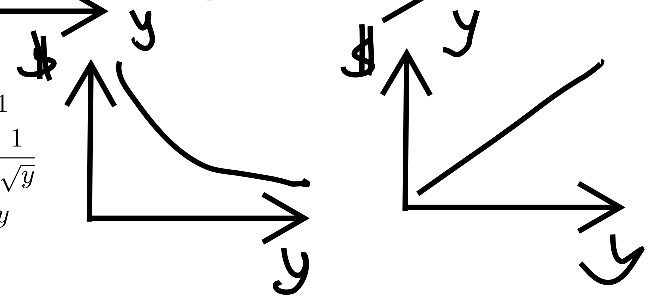
c) The cost functions are plotted as follows



d) The average cost functions are



$$\begin{aligned} AC(y) &= 1 \\ AC(y) &= \frac{1}{\sqrt{y}} \\ AC(y) &= y \end{aligned}$$



and marginal costs are

$$\begin{aligned} MC(y) &= 1 \\ MC(y) &= \frac{1}{2}\frac{1}{\sqrt{y}} \\ MC(y) &= 2y \end{aligned}$$