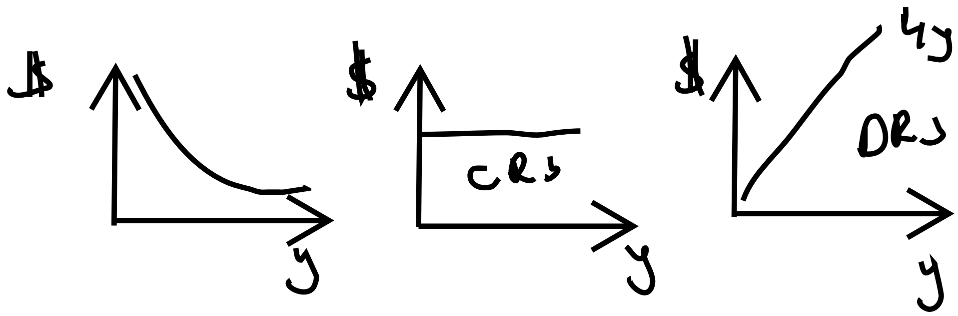
and marginal costs are

$$\begin{aligned} MC(y) &= \frac{1}{2} y^{-\frac{3}{4}} \\ MC(y) &= 1.9 \\ MC(y) &= 4y \end{aligned}$$



Problem 2 (Perfect Complements)

a) what are the returns to scale for each function?

$$\begin{aligned} F(\lambda K, \lambda L) &= \min(\lambda K, \lambda L) = \lambda \min(K, L) \;\; \text{(CRS)} \\ F(K, L) &= [\min(\lambda K, \lambda L)]^2 = \lambda^2 [\min(K, L)]^2 \;\; \text{(IRS)} \\ F(K, L) &= \sqrt{\min(\lambda K, \lambda L)} = \sqrt{\lambda}\sqrt{\min(K, L)} \text{(DRS)} \end{aligned}$$

b) Given the first production function $F(K,L) = \min(K,L)$ characterized by CRS, optimal proportion condition implies that $K = L$

$$y = \min(K, K) = K = L$$

and hence the cost function

$$C(y) = w_K K + w_L L = 1y + 1y = 2y$$

Also in the case of $y = [\min(K,L)]^2$ (IRS) optimal input proportion is $K = L$ and hence $y = [\min(K,K)]^2 = K^2$ and hence

$$K = L = \sqrt{y}$$

and hence the cost function

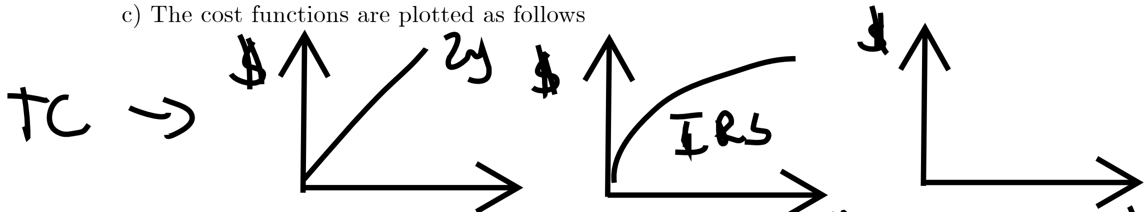
$$C(y) = w_K K + w_L L = 1 \times K + 1 \times L = \sqrt{y} + \sqrt{y} = 2\sqrt{y}$$

Finally for production function $F(K,L) = \sqrt{\min(K,L)}$ characterized by DRS again $K = L$ and hence

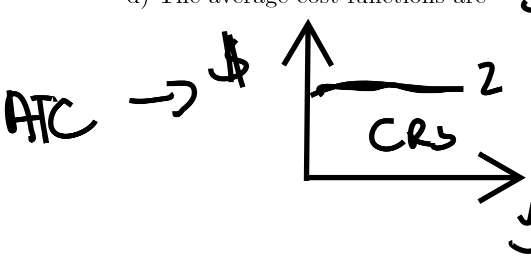
$$y = \sqrt{K} \Rightarrow K = L = y^2$$

$$C(y) = w_K K + w_L L = 1 \times K + 1 \times L = 2y^2$$

c) The cost functions are plotted as follows



d) The average cost functions are



$$\begin{aligned} AC(y) &= 2 \\ AC(y) &= 2\frac{1}{\sqrt{y}} \\ AC(y) &= 2y \end{aligned}$$