Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to problem set 8

(due Tuesday, March 23th, before class)

Problem 1 (Cobb-Douglas)

a) what are the returns to scale for each function?

$$
\begin{aligned}
F(\lambda K, \lambda L) &= \lambda^2 K^2 \lambda^2 L^2 = \lambda^4 K^2 L^2 \text{ (IRS)} \\
F(\lambda K, \lambda L) &= \lambda^{\frac{1}{3}} K^{\frac{1}{3}} \lambda^{\frac{2}{3}} L^{\frac{2}{3}} = \lambda K^{\frac{1}{3}} L^{\frac{2}{3}} \text{(CRS)} \\
F(\lambda K, \lambda L) &= \lambda^{\frac{1}{4}} K^{\frac{1}{4}} \lambda^{\frac{1}{4}} L^{\frac{1}{4}} = \lambda^{\frac{1}{2}} K^{\frac{1}{4}} L^{\frac{1}{4}} \text{ (DRS)}
\end{aligned}
$$

b) Given the first production function $F(K, L) = K^2 L^2$ characterized by IRS, the first secret of happiness (for minimization of the cost) implies that

$$TRS = -\frac{w_K}{w_L} \Rightarrow -\frac{L}{K} = -1 \Rightarrow K = L$$

$$y = K^2 L^2 = K^2 K^2 = K^4 \Rightarrow K = y^{\frac{1}{4}} = L$$

and hence the cost function

$$C(y) = w_K K + w_L L = 1 \times K + 1 \times L = 2y^{\frac{1}{4}}$$

For production function $F(K, L) = K^{\frac{1}{3}} L^{\frac{2}{3}}$ characterized by CRS,

$$TRS = -\frac{w_K}{w_L} \Rightarrow -\frac{\frac{1}{3}}{\frac{2}{3}} \frac{L}{K} = -1 \Rightarrow L = 2K$$

$$
\begin{aligned}
y &= K^{\frac{1}{3}} L^{\frac{2}{3}} = K^{\frac{1}{3}} (2K)^{\frac{2}{3}} = 2^{\frac{2}{3}} K \Rightarrow \\
K &= 2^{-\frac{2}{3}} y = 0.629\,96y \\
L &= 2K = 2^{\frac{1}{3}} y = 1.259\,9y
\end{aligned}
$$

and hence the cost function

$$C(y) = w_K K + w_L L = 1 \times K + 1 \times L = 2^{-\frac{2}{3}} y + 2^{\frac{1}{3}} y = \left(2^{-\frac{2}{3}} + 2^{\frac{1}{3}}\right) y = 1.9y$$

Finally for production function $F(K, L) = K^{\frac{1}{4}} L^{\frac{1}{4}}$ characterized by DRS again

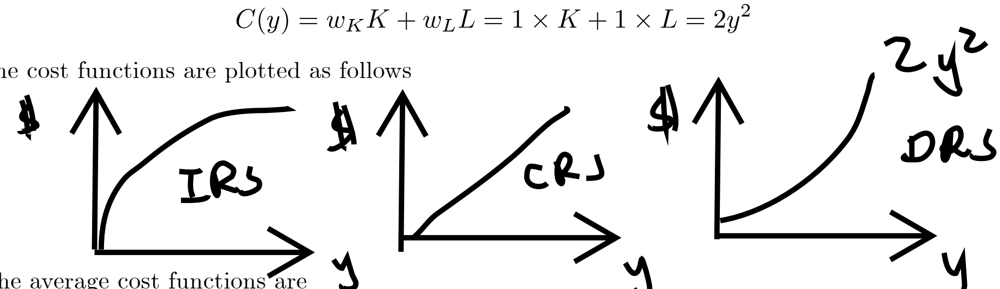
$$TRS = -\frac{w_K}{w_L} \Rightarrow -\frac{L}{K} = -1$$

$$y = K^{\frac{1}{4}} L^{\frac{1}{4}} = K^{\frac{1}{4}} K^{\frac{1}{4}} = K^{\frac{1}{2}} \Rightarrow K = y^2 = L$$

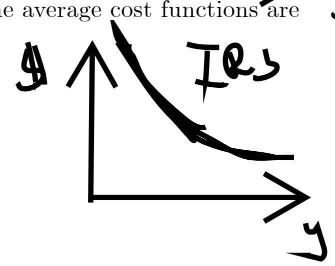
and hence the cost function

$$C(y) = w_K K + w_L L = 1 \times K + 1 \times L = 2y^2$$

c) The cost functions are plotted as follows



d) The average cost functions are



$$
\begin{aligned}
AC(y) &= 2y^{-\frac{3}{4}} \\
AC(y) &= 1.9 \\
AC(y) &= 2y
\end{aligned}
$$