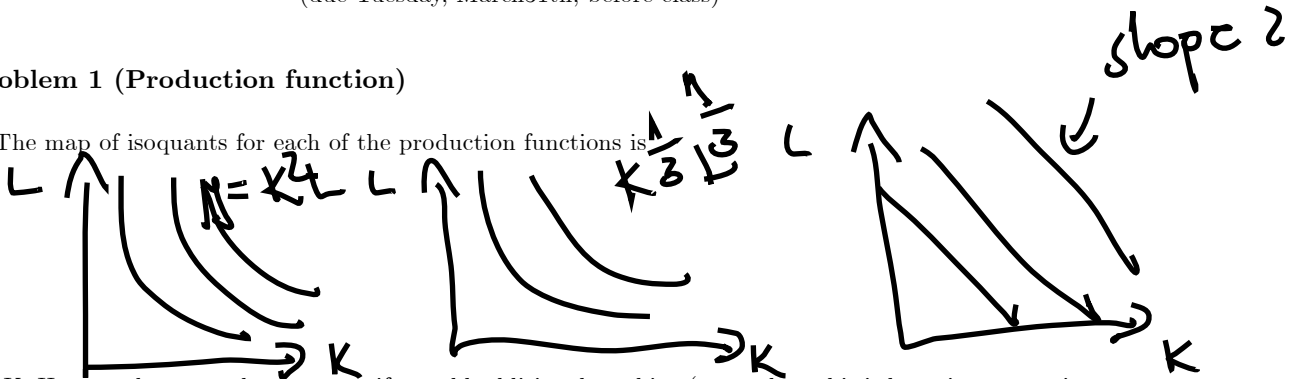


Solutions to problem set 7
 (due Tuesday, March 31st, before class)

Problem 1 (Production function)

a) The map of isoquants for each of the production functions is



b)

MPK- How much our product goes up if we add additional machine (remember, this is loose interpretation and only approximately true. Mathematically we require that the increase in K is very small)

MPL- How much our product goes up if we add additional worker. (again this is only loose interpretation)

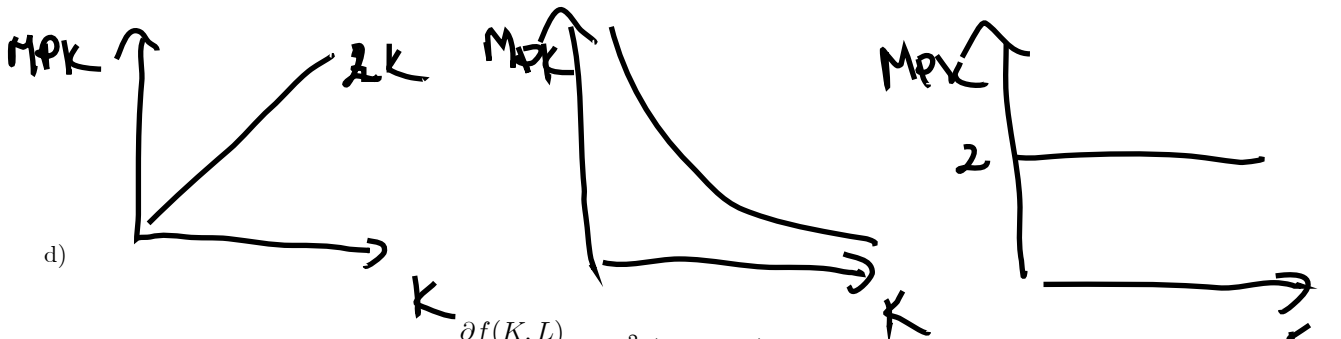
c)

$$MPK = \frac{\partial f(K, L)}{\partial K} = 2KL \text{ (increasing)}$$

$$MPK = \frac{\partial f(K, L)}{\partial K} = \frac{1}{3}K^{-2/3}L^{1/3} \text{ (decreasing)}$$

$$MPK = \frac{\partial f(K, L)}{\partial K} = 2 \text{ (constant)}$$

with $\bar{L} = 1$ their graphs are



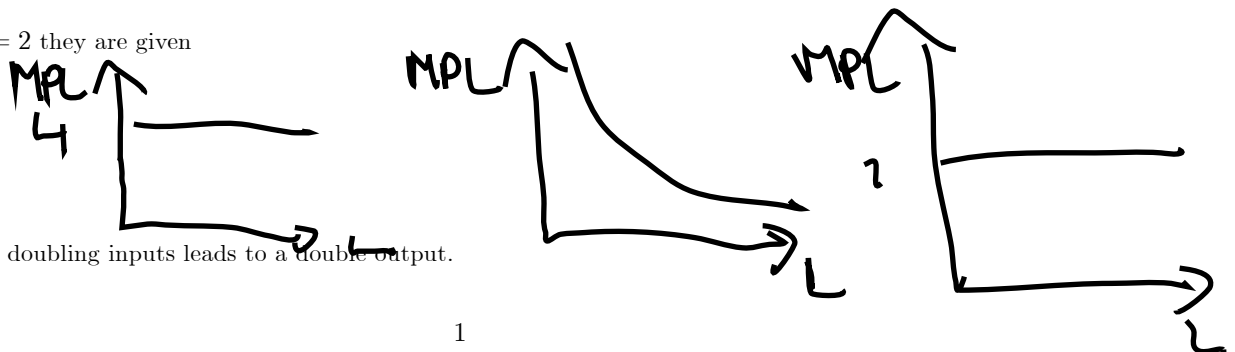
d)

$$MPL = \frac{\partial f(K, L)}{\partial L} = K^2 \text{ (constant)}$$

$$MPL = \frac{\partial f(K, L)}{\partial L} = \frac{1}{3}K^{1/3}L^{-2/3} \text{ (decreasing)}$$

$$MPL = \frac{\partial f(K, L)}{\partial L} = 1 \text{ (constant)}$$

For $\bar{K} = 2$ they are given



e) CRS: doubling inputs leads to a double output.

Example: restaurants

IRS: doubling inputs leads to more than a double output

Example: power plants, steel mills, oil tankers, networks

DRS: doubling inputs leads to less than a double output

Example: farming (inputs: capital and labor only)

f) Suppose $\lambda > 1$. Then

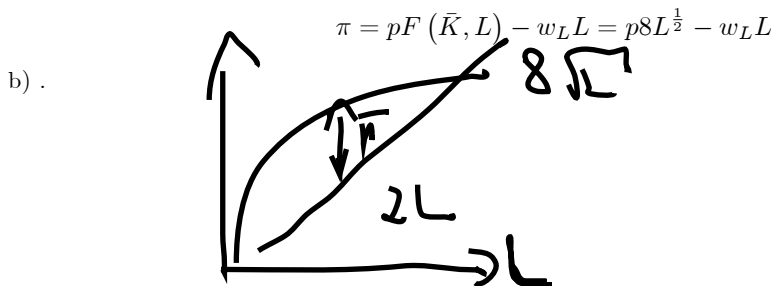
$$f(\lambda K, \lambda L) = (\lambda K)^2 \lambda L = \lambda^3 K^2 L = \lambda^3 f(K, L) > \lambda f(K, L) \text{ (IRS)}$$

$$f(\lambda K, \lambda L) = (\lambda K)^{\frac{1}{3}} (\lambda L)^{\frac{1}{3}} = \lambda^{\frac{1}{3} + \frac{1}{3}} K^{\frac{1}{3}} L^{\frac{1}{3}} = \lambda^{\frac{2}{3}} f(K, L) < \lambda f(K, L) \text{ (DRS)}$$

$$f(\lambda K, \lambda L) = 2\lambda K + \lambda L = \lambda(2K + L) = \lambda f(K, L) \text{ (CRS)}$$

Problem 2 (Profit Maximization- Short run)

a) Profit as a function of L is



c) The critical observation is that function $f(x)$ is flat at the point at which it attains maximum - hence its derivative is zero there. Note that by setting derivative to zero and solving for x we are finding all points at which our function is flat! Warning: the function is also flat when it attains minimum, therefore we should check whether actually our x is not minimizing the value of the function (second order condition). This however is not a problem in our application to maximization of profit function.

d) The derivative is given by

$$\frac{\partial \pi}{\partial L} = p \frac{\partial F(\bar{K}, L)}{\partial L} - w_L = 0 \Rightarrow MPL = \frac{w_L}{p}$$

The profit maximizing level of labor is such that the last worker "adds" the productions that exactly pays out his wage. All "earlier" workers produce more than their real wage, and all workers that would follow the optimal one would produce less than they would get!

e) The MPL is given by

$$MPL = 4L^{-\frac{1}{2}}$$

so optimality condition implies

$$4L^{-\frac{1}{2}} = \frac{w_L}{p} \Rightarrow L = \left(4 \frac{p}{w}\right)^2$$

For the considered values of w, p the labor supply is

$\frac{w}{p}$	8	4	2
L	$\frac{1}{4}$	1	4

The demand for labor can be plotted as

f) The maximal profit for each of the three values of $\frac{w}{p}$ from the table is

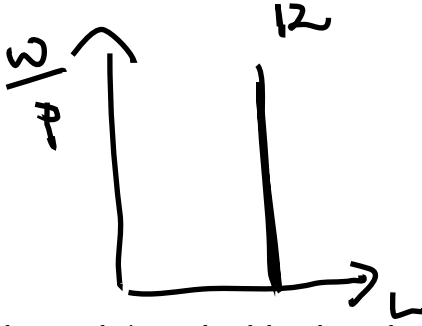
$$\pi = 8 \left(\frac{1}{4}\right)^{\frac{1}{2}} - 8 \frac{1}{4} = 4 - 2 = 2$$

$$\pi = 8(1)^{\frac{1}{2}} - 4 \times 1 = 8 - 4 = 4$$

$$\pi = 8(4)^{\frac{1}{2}} - 2 \times 4 = 16 - 8 = 8$$

Problem 3 (Labor Market)

a) Kate's labor supply is

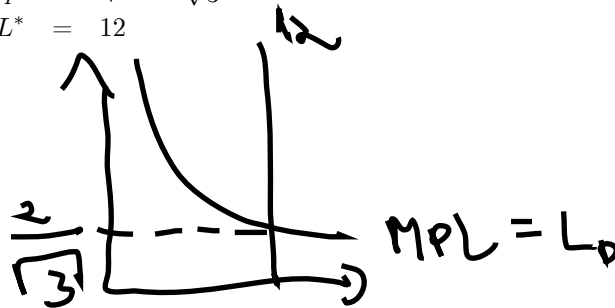


b) Equilibrium wage rate, where labor supply is equal to labor demand

$$L = \left(4 \frac{p}{w}\right)^2 = 12 \Rightarrow$$

$$\frac{w}{p} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = 1.1547$$

$$L^* = 12$$



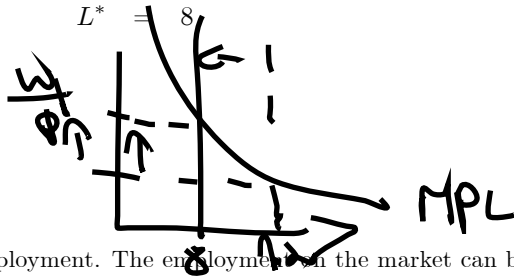
c) At a higher wage rate there are many "workers" who want to take a job but they cannot find it so would observe unemployment. The rational managers would reduce the wage rate, as they still can find workers willing to get a job, hence the wage should go down.

d) with lower willingness to work the equilibrium real wage and the level of labor is

$$L^* = \left(4 \frac{p}{w}\right)^2 = 8 \Rightarrow$$

$$\frac{w}{p} = \sqrt{2} = 1.4142$$

$$L^* = 8$$



e) Such policy results in unemployment. The employment in the market can be found as

$$L^* = \left(4 \frac{p}{w}\right)^2 = 4h$$

and hence the number of unemployed "hours" is $4h$. The unemployment rate is

$$U = \frac{8 - 4}{8} = 50\%$$

(the total number of unemployed divided by the number of those willing to take a job at the market wage rate)

Problem 4 (Long Run)

- a) This function exhibits DRS. (hence the profit maximization problem is well defined)
 b) Profit function is

$$\pi = pF(K, L) - w_L L - w_K K$$

- c) Setting partial derivatives of π with respect K and L to zero gives

$$MPK = \frac{w_K}{p} = 2 \text{ and } MPL = \frac{w_L}{p} = 1$$

The two conditions can be written as

$$\frac{1}{3}K^{-\frac{2}{3}}L^{\frac{1}{3}} = 2 \text{ and } \frac{1}{3}K^{\frac{1}{3}}L^{-\frac{2}{3}} = 1$$

Dividing one of the secrets of happiness by the other gives

$$\frac{\frac{1}{3}K^{-\frac{2}{3}}L^{\frac{1}{3}}}{\frac{1}{3}K^{\frac{1}{3}}L^{-\frac{2}{3}}} = \frac{2}{1} = 2 \Rightarrow \frac{L}{K} = 2 \Rightarrow L = 2K$$

Hence in optimum we will observe two inputs invested in proportion 1 : 2.

Using this fact in the previous two equations we get

$$\begin{aligned} \frac{1}{3}K^{-\frac{2}{3}}(2K)^{\frac{1}{3}} &= 2 \Rightarrow \frac{1}{3}K^{-\frac{2}{3}}(2K)^{\frac{1}{3}} = 2 \\ K &= \left(3 \times 2^{\frac{2}{3}}\right)^{-3} = \frac{1}{108} \\ L &= 2 \times K = \frac{2}{108} = \frac{1}{54} \end{aligned}$$

Given the two values, optimal value of product is

$$y = \left(\frac{1}{108}\right)^{\frac{1}{3}} \left(\frac{1}{54}\right)^{\frac{1}{3}} = \frac{1}{18}$$

and the profit is

$$\begin{aligned} \pi &= pF(K, L) - w_L L - w_K K = \\ &= \frac{1}{18} - 2 \times \frac{1}{108} - 1 \times \frac{1}{54} = \frac{1}{54} \end{aligned}$$

- d) The condition of minimization of the cost is

$$TRS = -\frac{w_K}{w_L} = -1$$

We find TRS using our shortcut formula

$$TRS = -\frac{MPK}{MPL} = -\frac{L}{K} = -\frac{\frac{1}{54}}{\frac{1}{108}} = -2 = -\frac{w_K}{w_L}$$

Hence in equilibrium the cost is minimized.