

Problem set 7

(due Thursday, March 22nd before class)

Problem 1 (Production function)

Consider the following three production functions

$$\begin{aligned} f(K, L) &= K^2 L \\ f(K, L) &= K^{\frac{1}{3}} L^{\frac{1}{3}} \\ f(K, L) &= 2K + L \end{aligned}$$

- in the (K, L) space, sketch the map of isoquants for each of them.
- give some economic interpretation for MPK and MPL (for an abstract production function).
- find analitically MPK for each production function above and plot it in the graph, with K on the horizontal axis and MPK on the vertical axis, assuming $\bar{L} = 1$. Is MPK increasing, decreasing or constant?
- find MPL for each of the four productions functions and plot it on the graph, with L on the horizontal axis and MPL on the vertical axis, assuming $\bar{K} = 2$. Is MPL increasing, decreasing or constant?
- Provide some economic intuition behind increasing, constant and decreasing returns to scale. Give an example of technology from real life that captures each of the three cases (one example per technology).
- For each of the three production functions above, show formally whether it exhibits increasing, decreasing or constant returns to scale.

Problem 2 (Profit Maximization- Short run)

GMC is producing cars using machines (K) and labor (L). The technology is capital intensive; the production function is given by

$$F(K, L) = K^{\frac{3}{4}} L^{\frac{1}{2}}$$

The value of GMC physical capital (machines, real estate etc.) is equal to $\bar{K} = \$16$ billion (in calculations ignore billions). We analyze the behavior of the firm in the short run, i.e. in the period in which \bar{K} cannot be changed. Suppose the price of a car is equal to p and the wage rate is w (parameters).

- write down the profit as a function of L .
- on a graph with L on the horizontal axis and \$ on the vertical axis, plot two components of profit function: total revenue $pF(\bar{K}, L)$ and labor cost wL (when drawing, assume $p = 1$ and $w = 2$). On the graph, mark the level of profit as the difference between the two lines (for some L).
- We know that in order to find x that maximizes some function $f(x)$, we take the first derivative of the function and set it equal to zero (we call it a first order condition). Please explain intuitively why this method allows us to find the optimum.
- Set the derivative of your profit function with respect to L equal to zero and derive the secret of happiness (the equation that tells that $MPL = \frac{w}{p}$). Explain the economic intuition behind the latter condition.
- find analitically the optimal level of labor L^* that maximizes the profit as a function of the real wage w/p . (we call it the firm's labor demand). Find the values of L for the following values of parameters

p	1	1	1
w	8	4	2

Plot your demand for labor on the graph with the real wage $\frac{w}{p}$ on the vertical axis and L on the horizontal one. Mark three points corresponding to the three values of $\frac{w}{p}$ from the table.

- What is the maximal profit for each of the three values of $\frac{w}{p}$ from the table?

Problem 3 (Labor Market)

a) review Problem 3 from PS4 (you will need it for the midterm). In that problem, we found that Kate's labor supply is inelastic and equal to $12h$ (per day). Plot her labor supply in a graph with the real wage ($\frac{w}{p}$) on the vertical axis and L on the horizontal one.

b) Suppose Kate works for GMC described in Problem 2 (in the present PS). Add the labor demand curve of this company to your graph from (a). Find the equilibrium real wage rate $\frac{w}{p}$ that clears the labor market (do it analytically and using the graph)

c) Suppose the wage rate is above the equilibrium level you found in (b). Do we observe unemployment or excess demand of labor on the market? What market forces drive the wage rate down (give a newspaper story; why will the wage go down?)

d) Suppose Kate's preferences change so that now she is willing to supply only $8h$. Show on the graph how this affects equilibrium real wage and the level of labor. Find the two values analytically.

e) Suppose the government passes the law requiring that the minimal (real) wage rate is equal to $w/p = 2$. How does this affect the equilibrium on the labor market (assume Kate's supply is $8h$)? Find the unemployment rate associated with such policy.

Problem 4 (Long Run)

Jimmy produces milk using milk-making machines (read: cows) (K) and labor (milkmen) (L). He has access to the technology given by

$$y = F(K, L) = K^{\frac{1}{3}} L^{\frac{1}{3}}$$

The price of a gallon of milk is $p = \$1$; the price of one machine (cow) is $w_k = \$2$ and the (milkmen) wage rate is $w_L = \$1$

a) does this function exhibit increasing, constant or decreasing returns to scale?

b) Write down the profit function - a function that depends on K and L :

c) Find the optimal level of inputs and output and the maximal profit.

(Hint: follow the steps shown in class - slides L14)

d) Argue that the optimal choice of inputs also minimizes the cost of production of the optimal level of production (from (c))

(show that the cost minimization condition holds.)