Econ 301
Intermediate Microeconomics
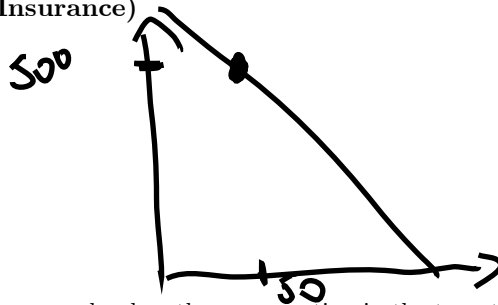**Prof. Marek Weretka**

Solutions to problem set 6

Problem 1 (Insurance)
a)

b) Given the insurance level x, the consumption in the two states of the world is

$$C_{NF} = \$500.000 - \frac{1}{10}x$$

$$C_F = \$50.000 + x - \frac{1}{10}x = \$50.000 + \frac{9}{10}x$$

Solving for x from the second equation gives

$$x = \frac{10}{9}(C_F - \$50.000)$$

and plugging it into the first one

$$C_{NF} = \$500.000 - 0.1x = \$500.000 + \frac{1}{9}\$50.000 - \frac{1}{9}C_F = 505.560 - \frac{1}{9}C_F$$

The two extreme points are $(0, 505.560)$ and (4550.000)

c) Consumer with such Bernoulli utility function is risk averse. To see consider two lotteries (4.0) and $(2, 2)$ and assume equally likely states. Observe that the two lotteries have the same average payoff, but the first one involves risk. We show on the graph that the risk makes it less attractive and hence agent is risk averse.

d) MRS is given by
$$MRS = -\frac{0.1\frac{1}{2\sqrt{C_F}}}{0.9\frac{1}{2\sqrt{C_F}}} = -\frac{1}{9}\frac{\sqrt{C_{NF}}}{\sqrt{C_F}} = -\frac{1}{9}\sqrt{\frac{C_{NF}}{C_F}}$$

and hence at the endowment point $\omega = (50.000, 500.000)$

$$MRS = -\frac{1}{9}\sqrt{\frac{C_{NF}}{C_F}} = -\frac{1}{9}\sqrt{10} = -0.35136$$

Note that the MRS is different from the slope of the budget set equal to $-1/9$

e) First we find optimal consumption levels: First secret of happiness implies that MRS is equal to the slope of the budget set. Therefore

$$-\frac{1}{9}\sqrt{\frac{C_{NF}}{C_F}} = -\frac{1}{9} \Rightarrow \sqrt{\frac{C_{NF}}{C_F}} = 1 \Rightarrow \frac{C_{NF}}{C_F} = 1 \Rightarrow C_{NF} = C_F$$

We can see that agent insures fully. Plugging it into the budget constraint

$$C_F = 505.560 - \frac{1}{9}C_F$$

and slowing for C_F gives

$$C_F = C_{NF} = \frac{9}{10}505.560 = 455.000$$

But then using the formula for x

$$\begin{aligned} x &= \frac{10}{9}(C_F - \$50.000) = \\ &= \frac{10}{9}(455.000 - \$50.000) = 450000 \end{aligned}$$

therefore he covers 450000

f) With such premium the slope of the budget set is

$$slope = -\frac{0.2}{1 - 0.2} = -\frac{1}{4}$$

hence budget line becomes steeper. Consequently optimal consumption is not on a 45° degree line.

Problem 2 (Risk aversion and certainty equivalence)
a) Yes, he is risk averse

b) The expected value from the lottery is

$$E(L) = \frac{1}{2}100 + \frac{1}{2}0 = 50$$

(see mark in graph above)

c) The expected utility from this lottery is

$$U(L) = \frac{1}{2}\sqrt{100} + \frac{1}{2}\sqrt{0} = 5$$

d) Certainty equivalent CE is the amount of "sure" cash that makes Frank indifferent to the lottery.

Lottery (CE, CE) is associated with utility

$$U(CE) = \frac{1}{2}\sqrt{CE} + \frac{1}{2}\sqrt{CE} = \sqrt{CE}$$

By definition it must be equal to the utility of the original lottery $U(L) = 5$ hence

$$\sqrt{CE} = 5 \Rightarrow CE = 25$$

Therefore Frank is indifferent between $25 for sure, and risky lottery ($100, $0)
  e) He should choose $40 or more generally anything exceeding $25
  f) Expected value of the lottery is unchanged

$$E(L) = \frac{1}{2}100 + \frac{1}{2}0 = 50$$

and expected utility from this lottery is

$$U(L) = \frac{1}{2}100 + \frac{1}{2}0 = 50$$

Utility from sure cash CE is given by
$$U(CE) = CE$$

which is in turn equal to 50. Consequently $CE = 50$ itself. Frank should take the lottery.
  g) Expected value of the lottery is 50 and expected utility is

$$U(L) = \frac{1}{2}(100)^2 + \frac{1}{2}0 = 5000$$

Utility from sure cash CE is
$$U(CE) = CE^2$$

hence
$$(CE)^2 = 5000 \Rightarrow CE = \sqrt{5000} = 70.7$$

Frank should take the lottery.

Problem 3 (Standard Edgeworth Box)
  a) The total resources are $\omega = (100, 10)$
  b) The allocation $\omega^E = (10, 10)$ and $\omega^M = (90, 0)$ is not Pareto efficient. One way to see it is that the endowment is off the contract curve (see point c)

c) In Pareto efficient allocation the slopes of indifference curves must coincide. Therefore

$$MRS^E = MRS^M$$

or

$$-\frac{x_2^M}{5x_1^M} = -\frac{x_2^E}{5x_1^E}$$

Elvis consumes $x_1^E = 100 - x_1^M$ and $x_2^E = 10 - x_2^M$ therefore we can write

$$\frac{x_2^M}{x_1^M} = \frac{10 - x_2^M}{100 - x_1^M}$$

Multiplying both sides by $x_1^M \left(100 - x_1^M\right)$ gives

$$x_2^M \left(100 - x_1^M\right) = x_1^M \left(10 - x_2^M\right)$$

which can be reduced to

$$100 x_2^M = 10 x_1^M$$

or finally

$$x_2^M = \frac{1}{10} x_1^M$$

Which defines a straight line in the Edgeworth box

d) Cefinition of Equilibrium: It is an allocation $\left(x_1^E, x_2^E\right)$,and $\left(x^M, x_2^M\right)$ and prices (p_1, p_2) such that
1) for each consumer $\left(x_1^i, x_2^i\right)$ is optimal given prices (p_1, p_2)
2) (p_1, p_2) are such that markets clear
Remark: Equilibrium is our prediction of what we will observe in markets, when they are not affected by any distortion. We find it in a different way than the Contract curve. Later we show, however that the equilibrium allocation, among many others, is Pareto efficient and hence it is located on a contrct curve.

Equilibrium determines only a relative price, therefore we can normalize $p_2 = 1$. We also focus on market for good 1. (You can verify that market for good 2 will clear automatically). With Cobb-Douglass utility functions, we find optimal choices using magic formula (shortcut) rather than by deriving it from secrets of happiness. The optimal demand for $MP3$ is given by

$$x_1^i = \frac{a}{a+b}\frac{m^i}{p_1}$$

where $a = 1$ and $b = 5$ (they are the same for both traders Elvis and Miriam). Income m^i is

$$\begin{aligned}m^E &= p_1 w_1^E + p_2 w_2^E = p_1 10 + 1 \times 10 = 10p_1 + 10 \\ m^M &= p_1 w_1^M + p_2 w_2^M = p_1 90 + 1 \times 0 = 90p_1\end{aligned}$$

Consequently the demands for x_1 are

$$\begin{aligned}x_1^E &= \frac{1}{6}\frac{10p_1 + 10}{p_1} \\ x_1^M &= \frac{1}{6}\frac{90p_1}{p_1}\end{aligned}$$

By the second equilibrium condition prices assure markets clearing, that is the total demand for MP3 , $x_1^E + x_1^M$ is equal to the total supply $w_1^E + w_1^M = 100$. More formally

$$100 = x_1^E + x_1^E = \frac{1}{6}\frac{10p_1 + 10}{p_1} + \frac{1}{6}\frac{90p_1}{p_1}$$

Multiplying both sides by $6p_1$

$$600p_1 = 10p_1 + 10 + 90p_1 = 10 + 100p_1$$

or

$$p_1 = \frac{1}{50}$$

At such prices the optimal consumption is

$$x_1^E = \frac{1}{6}\frac{10p_1 + 10}{p_1} = 85$$
$$x_1^M = \frac{1}{6}\frac{90p_1}{p_1} = 15$$

The consumption of DVD is (again we use magic formulas and equilibrium prices $p_1 = \frac{1}{50}$, $p_2 = 1$)

$$x_1^E = \frac{5}{6}\frac{10p_1 + 10}{p_2} = \frac{5}{6}(10p_1 + 10) = 8\frac{1}{2}$$
$$x_1^M = \frac{5}{6}\frac{90p_1}{p_2} = \frac{5}{6}\frac{90p_1}{p_2} = 1\frac{1}{2}$$

Hence allocation $x^E = (85, 8\frac{1}{2})$ and $x^M = (15, 1\frac{1}{2})$ and prices $(p_1, p_2) = (\frac{1}{50}, 1)$ is an equilibrium

e) Any price system that is associated with the relative price equal to $\frac{1}{50}$ supports the same allocation as

an equilibrium. For example
$$(p_1, p_2) = (1, 50) \text{ or } (p_1, p_2) = (2, 100)$$

f) Yes, they are efficient. To see it observe that MRS for Elvis and Miriam at equilibrium is equal to

$$MRS^E = \frac{x_2^E}{5x_1^E} = \frac{1}{50}$$
$$MRS^M = \frac{x_2^E}{5x_1^E} = \frac{1}{50}$$

and hence their indifference curves are tangent, so the allocation is Pareto efficient.

g) Remark: In this case all allocations in Edgeworth box are Pareto efficient.

Equilibrium: For any relative price
$$\frac{p_1}{p_2} < MRS^i = \frac{1}{5}$$

both trader spend their total income on x_1 and no income on x_2 - This results in excess demand for x_1. For

$$\frac{p_1}{p_2} > MRS^i = \frac{1}{5}$$

both traders spend their income on x_2 and hence there is excess demand for x_2. Therefore markets can clear only for relative price
$$\frac{p_1}{p_2} = \frac{1}{5}$$

But then all allocations on the budget set are optimal for each trader therefore any of them is an equilibrium allocation.

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Problem 4 (Uncertainty and Asset Pricing)

Endowment allocation is not Pareto efficient. It is also risky as it is associated with different payments in two states of the world.

b) Equilibrium: Given the Cobb -Douglass preferences the optimal demand of each of the traders is

$$x_1^J = \frac{a}{a+b}\frac{m^J}{p_1} = \frac{1}{2}\frac{p_1 \times 100 + p_2 \times 0}{p_1} = 50$$

$$x_1^B = \frac{a}{a+b}\frac{m^B}{p_1} = \frac{1}{2}\frac{p_1 \times 0 + p_2 \times 100}{p_1} = 50\frac{p_2}{p_1}$$

We normalize p_2 to one so that $p_2 = 1$. The market clearing on the market for x_1 requires that

$$x_1^J + x_1^B = 100$$

which gives a price

$$50 + 50\frac{p_2}{p_1} = 50 + 50\frac{1}{p_1} = 100 \Rightarrow p_1 = 1$$

Hence the equilibrium prices are $p_1 = p_2 = 1$. Given the prices the equilibrium allocations are

$$x_1^J = 50,$$
$$x_2^J = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{2}\frac{p_1 \times 100}{p_2} = 50$$

and hence consumption of Benjamin is what is left on the market

$$x_1^B = 100 - 50 = 50$$
$$x_2^J = 100 - 50 = 50$$

c) MRS for both traders is

$$MRS^J = \frac{\frac{1}{2}}{\frac{1}{2}}\frac{50}{50} = 1$$

$$MRS^B = \frac{\frac{1}{2}}{\frac{1}{2}}\frac{50}{50} = 1$$

hence indifference curves are tangent - the allocation is Pareto efficient. The equilibrium allocation is also not risky as the wealth is the same in the two states of the world (this is true for J and for B).

Problem 5 (Irving Fisher Determination of Interest Rate)

a) No, the initial allocation is not Pareto efficient.

b) Given Cobb -Douglass preferences, the optimal demand for consumption "today" is

$$C_1^J = \frac{a}{a+b}\frac{m^J}{p_1} = \frac{1}{1+\frac{1}{2}}\frac{p_1 \times 0 + p_2 \times 1000}{p_1} = \frac{2}{3}\frac{1000p_2}{p_1}$$

$$C_1^W = \frac{a}{a+b}\frac{m^B}{p_1} = \frac{1}{1+\frac{1}{2}}\frac{p_1 \times 1000 + p_2 \times 0}{p_1} = \frac{2}{3}1000$$

We normalize p_2 to one so that $p_2 = 1$. The market clearing for C_1 requires that

$$C_1^J + C_1^W = 1000$$

which gives a price

$$\frac{2}{3}\frac{1000}{p_1} + \frac{2}{3}1000 = 1000 \Rightarrow p_1 = 2$$

Hence the equilibrium prices are $p_1 = 2$ and $p_2 = 1$. Given the prices the equilibrium allocations are

$$C_1^J = \frac{2}{3}\frac{1000p_2}{p_1} = \frac{1000}{3} = 333.333$$

$$C_2^J = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{3}\frac{1000}{1} = 333.333$$

and hence consumption of William is

$$C_1^B = 1000 - 333.333 = 666.666$$
$$C_2^B = 1000 - 333.333 = 666.666$$

The interest rate is

$$1 + r = \frac{p_1}{p_2} = 2$$

hence

$$r = 100\%$$

c) Yes equilibrium allocation is Pareto efficient as

$$MRS^J = \frac{2C_2^J}{C_1^J} = \frac{2 \times 333.333}{333.333} = 2$$

$$MRS^B = \frac{2C_2^B}{C_1^B} = \frac{2 \times 666.666}{666.666} = 2$$

and hence the indifference curves of two traders are tangent.

d) Now the optimal consumptions are

$$C_1^J = \frac{a}{a+b}\frac{m^J}{p_1} = \frac{1}{1+1}\frac{p_1 \times 0 + p_2 \times 1000}{p_1} = \frac{1}{2}\frac{1000p_2}{p_1}$$

$$C_1^W = \frac{a}{a+b}\frac{m^B}{p_1} = \frac{1}{1+1}\frac{p_1 \times 1000 + p_2 \times 0}{p_1} = \frac{1}{2}1000$$

Again normalize p_2 to one so that $p_2 = 1$. The market clearing on the market for C_1 requires that

$$C_1^J + C_1^W = 1000$$

which gives a price

$$\frac{1}{2}\frac{1000}{p_1} + \frac{1}{2}1000 = 1000 \Rightarrow p_1 = 1$$

Hence the equilibrium prices are $p_1 = p_2 = 1$, therefore the interest rate is

$$1 + r = \frac{p_1}{p_2} = 1$$

hence
$$r = 0\%$$

With greater patients the willingness to save increases and the willingness to borrow goes down. In order to equilibrate the market the interest must go down.

e) The demands now are

$$
\begin{aligned}
C_1^J &= \frac{a}{a+b}\frac{m^J}{p_1} = \frac{1}{1+\frac{1}{2}}\frac{p_1 \times 0 + p_2 \times 2000}{p_1} = \frac{2}{3}\frac{2000 p_2}{p_1} \\
C_1^W &= \frac{a}{a+b}\frac{m^B}{p_1} = \frac{1}{1+\frac{1}{2}}\frac{p_1 \times 1000 + p_2 \times 0}{p_1} = \frac{2}{3}1000
\end{aligned}
$$

We normalize p_2 to one so that $p_2 = 1$. The market clearing on the market for C_1 requires that

$$C_1^J + C_1^W = 1000$$

which gives a price
$$\frac{2}{3}\frac{2000}{p_1} + \frac{2}{3}1000 = 1000 \Rightarrow p_1 = 4$$

Hence the equilibrium prices are $p_1 = 4$ and $p_2 = 1$. The interest rate is

$$1 + r = \frac{p_1}{p_2} = 4$$

hence
$$r = 300\%$$

Intuition: Jane's have large endowment tomorrow and therefore because she wants to use it today she needs to borrow more. In order to equilibrate the savings market interest must go up. This partially reduces her willingness to borrow and also encourages William to lend.