

Problem set 6

(due Thursday, March 15th before class)

Problem 1 (Insurance)

Ben is a proud owner of a romantic mansion facing the lake Mendota worth \$500,000. The location attractive as it is, however has one drawback. Occasionally in the spring heavy rains raise the water level in the lake, flooding the house. When this happens, the value of the house drops to \$50,000. The flood occurs with probability $1/10$. Ben finds the situation too stressful and therefore he is going to sell the house in the summer (after the potential flood). By c_F and c_{NF} denote his wealth when there is, and there is no flood respectively.

- In the commodity space (c_F, c_{NF}) show the affordable bundle if there is no insurance (the endowment point).
- Ben can buy insurance that pays $\$x$ dollars when there is a flood, paying premium $0.1x$ (he can choose x)
 - Find analytically and show on the graph his budget constraint.
- Suppose Ben's Von Neumann Morgenstern utility function is given by

$$U(C_F, C_{NF}) = 0.1\sqrt{C_F} + 0.9\sqrt{C_{NF}}$$

Is Ben risk averse (argue using graph with his Bernoulli utility function)?

- What is his MRS at the endowment point (no insurance)?
- Find optimal level of insurance and the level of wealth under two contingencies. Does he insure fully?
- Show on the graph how your answer is changed when the insurance premium is $0.2x$?

Problem 2 (Risk aversion and certainty equivalence)

Frank McGambler needs our advice. He is thinking about a lottery ticket that gives \$100 with probability $\frac{1}{2}$ and zero otherwise (also with probability $\frac{1}{2}$). The alternative is \$40 for sure.

- Plot the Bernoulli utility function, given by

$$u(c) = \sqrt{c}$$

in the graph Is Frank risk averse?

- What is the expected value of the lottery? (Give the number and mark it on the graph).
- What is the expected utility from this lottery? (Give a number and mark it on the graph).
- What is the certainty equivalent of the lottery? Is it greater or smaller than the expected value of the lottery? Give economic interpretation of this number.
- Which Frank should chose, \$40 for sure or the lottery?
- Give answers to questions b-e, assuming Bernoulli utility function given by $u(c) = c$.
- Give answers to questions b-e, assuming Bernoulli utility function given by $u(c) = c^2$.

Problem 3 (Standard Edgeworth Box)

Elvis and Miriam both love listening to MP3s (x_1) and watching DVDs (x_2). Elvis has initially 10 DVDs and 10 MP3s, (hence $\omega^E = (10, 10)$). Miriam has 90 MP3s only, (so $\omega^M = (90, 0)$). Utility functions of Elvis and Miriam are the same and given by:

$$U^i(x_1, x_2) = \ln(x_1) + 5 \ln(x_2), \quad i \in \{\text{Elvis, Miriam}\}$$

- What are the total resources in the "economy" with Elvis and Miriam?
- Plot an Edgeworth box and mark the allocation corresponding to initial endowments. Argue that such an allocation is (or it is not) Pareto efficient.
- Find analytically the contract curve (collection of all Pareto efficient allocations) and plot it on your graph.
- Now let Elvis and Miriam trade. Find the equilibrium consumption of both commodities for Elvis and Miriam, and the equilibrium prices.

- e) Suggest another two prices that support the same allocation as an equilibrium.
- f) Are markets efficient in allocating the resources? (argue using the criterion of Pareto efficiency)
- g) Find geometrically (in the Edgeworth box) the equilibrium consumption and prices of both commodities for the new utility (perfect substitutes) given by:

$$U^i(x_1, x_2) = x_1 + 5x_2$$

Problem 4 (Uncertainty and Asset Pricing)

John and Benjamin are investors who trade shares of two companies: Rainalot Inc. (x_1) and HateRain Inc. (x_2). There are two equally likely states of the world in the future: rain and no rain, and the profits of two companies are risky. The dividend from one share of Rainalot Inc is \$1 when it rains and \$0 otherwise and dividends of HateRain Inc. are \$0 when it rains and \$1 otherwise. John initially has 100 shares of Rainalot Inc and no shares of HateRain Inc. ($\omega^J = (100, 0)$) and the endowment of Benjamin is $\omega^B = (0, 100)$. Both investors maximize Expected utility given by

$$U^i(x_1, x_2) = \frac{1}{2} \ln(x_1) + \frac{1}{2} \ln(x_2), \quad i \in \{\text{John, Benjamin}\}.$$

- a) Plot an Edgeworth box and mark the allocation corresponding to the initial endowments. Argue that such an allocation is (or it is not) Pareto efficient. Are the endowments risky?
- b) Find the equilibrium prices of the two companies' shares and their allocations, and show them on the graph.
- c) Is the equilibrium allocation efficient? Is it risky?

Problem 5 (Irving Fisher Determination of Interest Rate)

Consumption can take place two periods: today (C_1) and tomorrow (C_2). Jane's income is \$0 today (she is a student now). Tomorrow she is going to be a CEO with income \$1000 (hence her endowment is $\omega^J = (0, 1000)$). William is a sportsman with today's income \$1000 and tomorrow he will get \$0 ($\omega^W = (1000, 0)$). They both have the same utility function

$$U^i(C_1, C_2) = \ln(C_1) + \beta \ln(C_2), \quad i \in \{\text{Jane, William}\}.$$

where β is a discount factor that tells how impatient they are (the higher β , the more patient the consumer is since the value of utility tomorrow relative to utility today is higher).

- a) Plot an Edgeworth box and mark the allocation corresponding to the initial endowments. Is the allocation of initial endowments Pareto efficient?
- b) Assume $\beta = \frac{1}{2}$. Find the equilibrium interest rate and depict the equilibrium in the Edgeworth box. (Hint: Instead of working with a harder "intertemporal" model, you can first find equilibrium prices p_1 and p_2 similarly to the apple and orange model and then use that

$$\frac{p_1}{p_2} = (1 + r)$$

- c) Is the equilibrium allocation Pareto efficient?
- d) Assume now that consumers are more patient and $\beta = 1$. Repeat question b). How does your new interest rate compare to the one from point b)? Can you give some economic intuition?
- e) Now assume $\beta = 0.5$ and Jane's income tomorrow changes to 2000? ($\omega^J = (0, 2000)$). Is the interest rate higher or lower than the one in b)? Explain.