

Solutions to problem set 5

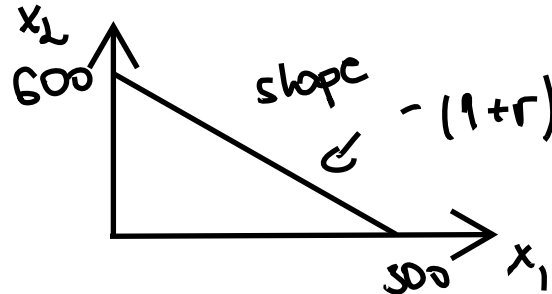
Problem 1 (Intertemporal Choice)

a) *PV* and *FV* of Gerald's income is

$$PV = \$200 + \frac{\$200}{1+r} = \$300$$

$$FV = (1+r) \times \$200 + \$200 = \$600$$

b)



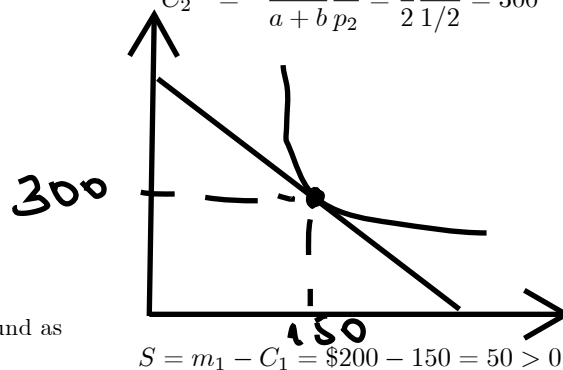
c) Extreme point (300, 0) : Borrow \$100 in period 1 and pay it back together with interests in period 2, $\$100 \times (1+r) = 200$ using your total income in period 2. Extreme (0, 600) : Save \$200 in period 1 and withdraw it together with interests $\$200 \times (1+r) = \400 in period 2. This, together with your income in period 2 gives 600!

d) This is a Cobb - Douglass utility function therefore we can use magic formula, where $a = b = 1$ and $p_1 = 1, p_2 = \frac{1}{1+r} = \frac{1}{2}$ and $m = PV = 300$. (alternatively you could use future value budget set with $p_1 = 1+r = 2, p_2 = 1$ and $m = FV = 600$)

$$C_1 = \frac{a}{a+b} \frac{m}{p_1} = \frac{1}{2} \frac{300}{1} = 150$$

$$C_2 = \frac{a}{a+b} \frac{m}{p_2} = \frac{1}{2} \frac{300}{1/2} = 300$$

On the graph



Savings can be found as

$$S = m_1 - C_1 = \$200 - 150 = 50 > 0$$

hence Gerald's saves \$50.

Problem 2 (Intertemporal choice)

a) Coefficient δ is a discount rate and determines how impatient the agent is. The greater δ the more impatient the consumer and the smaller weight is given to the utility in the future.

b) Since $a = 1, b = \frac{1}{1+\delta}, p_1 = 1, p_2 = \frac{1}{1+r}$ and income $m = PV = \frac{3000}{1+1} = 1500$, magic formula becomes

$$C_1 = \frac{1}{1 + \frac{1}{1+\delta}} PV = \frac{1+\delta}{2+\delta} PV = \frac{2}{3} 1500 = 1000$$

$$C_2 = \frac{\frac{1}{1+\delta}}{1 + \frac{1}{1+\delta} \frac{1}{1+r}} PV = \frac{1+r}{2+\delta} PV = \frac{2}{3} 1500 = 1000$$

the savings in the first period are

$$S = 0 - 1000 = -1000$$

and hence manager borrows \$1000.

c) Also in this case $a = 1$, $b = \frac{1}{1+\delta}$, $p_1 = 1$, $p_2 = \frac{1}{1+r}$ and income $m = PV = 1500$, and magic formulas are

$$C_1 = \frac{1}{1 + \frac{1}{1+\delta}} PV = \frac{1+\delta}{2+\delta} PV = \frac{2}{3} 1500 = 1000$$

$$C_2 = \frac{\frac{1}{1+\delta}}{1 + \frac{1}{1+\delta} \frac{1}{1+r}} PV = \frac{1+r}{2+\delta} PV = \frac{2}{3} 1500 = 1000$$

the savings in the first period are

$$S = 1500 - 1000 = 500$$

and hence sportsman saves \$500.

d) For both manager and sportsman $C_1 = C_2$ and hence they are perfectly smoothing their consumption over time.

e) From $MRS = -\frac{p_1}{p_2}$ we have that

$$-(1+\delta) \frac{C_2}{C_1} = -(1+r)$$

and hence

$$C_2 = \frac{(1+r)}{(1+\delta)} C_1$$

When $r < \delta$

$$\frac{(1+r)}{(1+\delta)} < 1$$

which implies that

$$C_2 < C_1$$

Problem 3 (Perpetuity)

a) Perpetuity gives x in each following period, and hence its present value is given by

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \dots$$

This equation can be written as

$$PV = \frac{x}{1+r} + \frac{1}{1+r} \left[\frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} \dots \right]$$

The sum of elements in the bracket is equal to the present value of perpetuity

$$PV = \frac{x}{1+r} + \frac{1}{1+r} [PV]$$

This equation says that present value of perpetuity coincides with present value of tomorrows payment plus discounted value of the console that with the first payment in the third period. Solving for PV gives

$$PV - \frac{1}{1+r} [PV] = \frac{x}{1+r}$$

so

$$\left(\frac{1+r}{1+r} - \frac{1}{1+r} \right) PV = \frac{r}{1+r} PV = \frac{x}{1+r}$$

hence

$$PV = \frac{x}{r}$$

b) The cashflow of the annuity differs from perpetuity in that it stops paying x after period T . The present value of the missing part of the cashflow in terms of dollars in T is $\frac{x}{r}$ (this is a perpetuity) and in terms of

zero dollars it is $\left(\frac{1}{1+r}\right)^T \frac{x}{r}$. This is by how much we have to reduce the PV of perpetuity to obtain PV of annuity. The PV of annuity is

$$PV = \frac{x}{r} - \left(\frac{1}{1+r}\right)^T \frac{x}{r} = \frac{x}{r} \left[1 - \left(\frac{1}{1+r}\right)^T\right]$$

Problem 4 (Present Value, use a calculator)

a) Since I am staying there forever the rent payment is a perpetuity hence

$$PV = \frac{x}{r} = \frac{500}{0.001} = 500,000$$

Since it is smaller than 600,000, I rent it.

b) Present value of the payment must coincide with the size of the loan, hence

$$\$4000 = PV = \frac{x}{0.005} \left(1 - \left(\frac{1}{1.005}\right)^{36}\right)$$

hence

$$x = \frac{4000 \times 0.005}{\left(1 - \left(\frac{1}{1.005}\right)^{36}\right)} = 121.69$$

c)

$$PV = \frac{100}{0.1} \left(1 - \frac{1}{(1.1)^9}\right) + \frac{1000}{(1.1)^{10}} = 961.45$$

Because PV is higher than the price, it is a good deal to purchase the bond. (we can always borrow to finance such purchase and then pay the loan back with the bonds payments and keeping the surplus for us)

Remark: if we assumed that the bond pays coupon in the last period then the value of the bond would be \$1000 (this is often assumed in finance though not in Varian textbook).

d) PV of consumption is

$$PV(Cons) = \frac{40}{0.05} \left(1 - \left(\frac{1}{1.05}\right)^{20}\right) \left(\frac{1}{1.05}\right)^{40} = 70.808$$

and PV of savings

$$PV(savings) = \frac{S}{0.05} \left(1 - \left(\frac{1}{1.05}\right)^{40}\right) = 17.159S$$

Equality of the two PV implies

$$S = \frac{70.808}{17.159} = 4.1266$$

And hence one must save 4.13 thousand dollars

e) PV of consumption is

$$PV(Cons) = \frac{C}{0.05} \left(1 - \left(\frac{1}{1.05}\right)^{20}\right) \left(\frac{1}{1.05}\right)^{40} = 1.7702C$$

and PV of savings

$$PV(savings) = \frac{20}{0.05} \left(1 - \left(\frac{1}{1.05}\right)^{40}\right) = 343.18$$

Equality of two PV implies

$$C = \frac{343.18}{1.7702} = 193.87$$

And hence consumption is 193.87 thousand dollars.

Problem 5 (Present Value, use a calculator)

a) PV of consumption is

$$PV(Cons) = \frac{C}{0.05} \left(1 - \left(\frac{1}{1.05} \right)^{60} \right) = 18.929C$$

and PV of income

$$PV(income) = \frac{200}{0.05} \left(1 - \left(\frac{1}{1.05} \right)^{40} \right) = 3431.8$$

Equality of two PV implies

$$C = \frac{3431.8}{18.929} = 181.30$$

The level of savings is $S_t = m_t - c = 200 - 181 = 19$ for $t = 21, 22, \dots, 60$ and $S_t = m_t - c = -181$ after retirement.

b)

$$PV(Cons) = \frac{C}{0.05} \left(1 - \left(\frac{1}{1.05} \right)^{60} \right) = 18.929C$$

and PV of income

$$PV(income) = \frac{200}{0.05} \left(1 - \left(\frac{1}{1.05} \right)^{40} \right) + 1000 = 3431.8 + 1000 = 4431.8$$

and consumption

$$C = \frac{4431.8}{18.929} = 234.13$$

The level of savings is $S_t = m_t - c = 200 - 234.13 = -34.13$ for $t = 21, 22, \dots, 60$ and $S_t = m_t - c = -234.13$ after retirement.

c)

$$PV(Cons) = \frac{C}{0.05} \left(1 - \left(\frac{1}{1.05} \right)^{60} \right) = 18.929C$$

and PV of income

$$PV(income) = \frac{200}{0.05} \left(1 - \left(\frac{1}{1.05} \right)^{40} \right) + 1000 - 1000 \left(\frac{1}{1.05} \right)^{60} = 3431.8 + 1000 - 1000 \left(\frac{1}{1.05} \right)^{60} = 4378.3$$

and consumption

$$C = \frac{4378.3}{18.929} = 231.3$$

The level of savings is $S_t = m_t - c = 200 - 231.3 = -31.3$ for $t = 21, 22, \dots, 60$ and $S_t = m_t - c = -231.3$ after retirement.