Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to problem set 4

(due Tuesday, February 16th, before class)

Problem 1

The magic formula for Cobb Douglass demands is

$$x_1 = \frac{a}{a+b}\frac{m}{p_1} = \frac{4}{5}\frac{m}{p_1} \text{ and } x_2 = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{5}\frac{m}{p_2}$$

a)
START $(p_1 = 10, p_2 = 1, m = 100)$

$$x_1 = \frac{4}{5}\frac{m}{p_1} = 8 \quad \text{and } x_2 = \frac{1}{5}\frac{m}{p_2} = \frac{1}{5}\frac{100}{1} = 20$$

END $(p_1 = 5, p_2 = 1, m = 100)$

$$x_1 = \frac{4}{5}\frac{m}{p_1} = \frac{4}{5}\frac{100}{5} = 16 \quad \text{and } x_2 = \frac{1}{5}\frac{m}{p_1} = \frac{1}{5}\frac{100}{1} = 20$$

Change in consumption of x_1

$$\Delta x_1 = 16 - 8 = 8$$

b) they are ordinary goods as the demand goes up after price drop (downwarslopping demand)
c) Auxiliary income

$$m' = 5 \times 8 + 1 \times 20 = 60$$

and hence the optimal demand with income $m' = 60$ and new prices is

$$x_1 = \frac{4}{5}\frac{m'}{p_1} = \frac{48}{5} = 9\frac{3}{5}$$

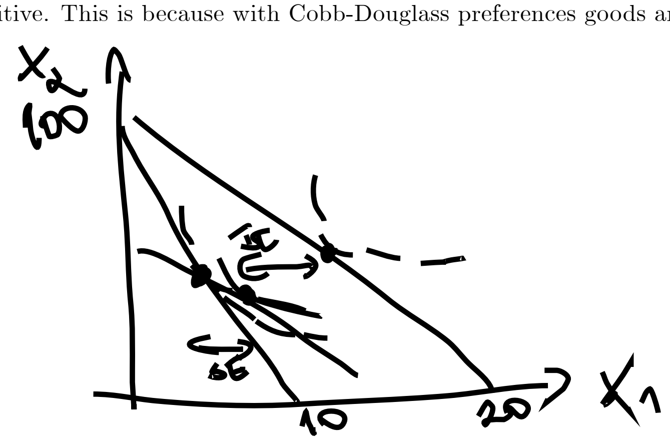
This implies that substitution effect is

$$SE = 9\frac{3}{5} - 8 = 1\frac{3}{5}$$

d) Income effect is "the rest" of the change

$$IE = \Delta x_1 - SE = 8 - 1\frac{3}{5} = 6\frac{2}{5}$$

e) Income effect is positive. This is because with Cobb-Douglass preferences goods are normal.
f)



Problem 2

Remark: in the solutions I will use proportion 1 milk to 5 strawberries (we will accept also also 5 strawberries to 1 milk as in PS4)

a) Two secrets of happiness (for perfect complements!) are

$$\begin{aligned} x_1 &= \frac{1}{5}x_2 \\ p_1x_1 + p_2x_2 &= m \end{aligned}$$

hence

$$x_1 = \frac{m}{p_1 + 5p_2} \text{ and } x_2 = 5\frac{m}{p_1 + 5p_2},$$

START ($p_1 = 15, p_2 = 1, m = 200$)

$$x_1 = \frac{200}{15 + 5 \times 1} = 10 \text{ and } x_2 = \frac{5 \times 200}{15 + 5 \times 1} = 50$$

END ($p_1 = 5, p_2 = 1, m = 200$)

$$x_1 = \frac{200}{5 + 5 \times 1} = 20 \text{ and } x_2 = \frac{5 \times 200}{5 + 5 \times 1} = 100$$

Change in consumption of x_1

$$\Delta x_1 = 20 - 10 = 10$$

b) Substitution effect is zero $SE = 0$.(with perfect complements substitution effect is always zero

c) Income effect is equal to total change $IE = \Delta x_1 = 10$.

Problem 3

a)

$$MRS = \frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = \frac{5}{x_1}$$

Two secret of happiness are, $MRS = \frac{p_1}{p_2}$ and $p_1x_1 + p_2x_2 = m$.

b) assuming interior solution we can use two secret of happiness to get the optimal solution directly:

$$\frac{5}{x_1} = P_1 \qquad (1)$$

$$p_1x_1 + x_2 = 10 \qquad (2)$$

So optimal consumption $x_1^* = \frac{5}{p_1}$ and $x_2^* = 5$

c) First when $p_1 = 5$ we have $x_1^* = 1$ and $x_2^* = 5$. When $p_1 = 1$ we have $x_1''^* = 5$ and $x_2''^* = 5$.

To find the substitution effect we need to find the income level, m', such that the original (when $p_1 = 5$) optimal bundle is just affordable under the new prices.

So we have $m' = 1 * 1 + 1 * 5 = 5$, under this income and the new prices level we can conclude that the optimal consumption $x_1'^* = 5$. (since this is affordable when $m' = 5$ and $p_1 = 1$ so it is indeed our optimal choice).

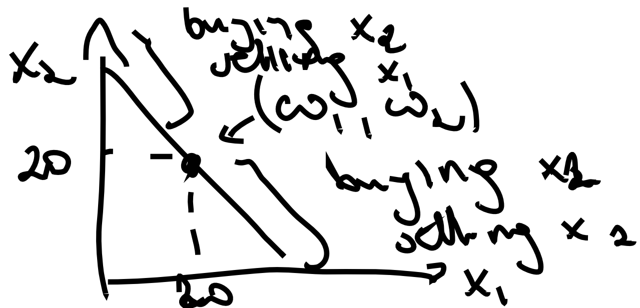
Now we have all we need to find substitution effect,

$$S.E = x_1'^* - x_1^* = 5 - 1 = 4$$

d) $I.E = x_1''^* - x_1'^* = 0$ (this is generaly the case with quasilinear utilities that income effect is zer

Problem 4

a) Let x_1 be the consumption of apples and x_2 be the consumption of oranges. Then we have budget constraint: $2x_1 + 2x_2 \leq 2 * 20 + 2 * 20 = 80$



b) $MRS = -\frac{x_2}{x_1}$ so at the endowment point we have $MRS = -1$. Utility at the endowment point is given by $u(20, 20) = 400$. Thus the bundles that are infdifferent and hence give the same value of utility are given by $u(x_1, x_2) = 400 = x_2 x_1$ which implies that $x_2 = 400/x_1$

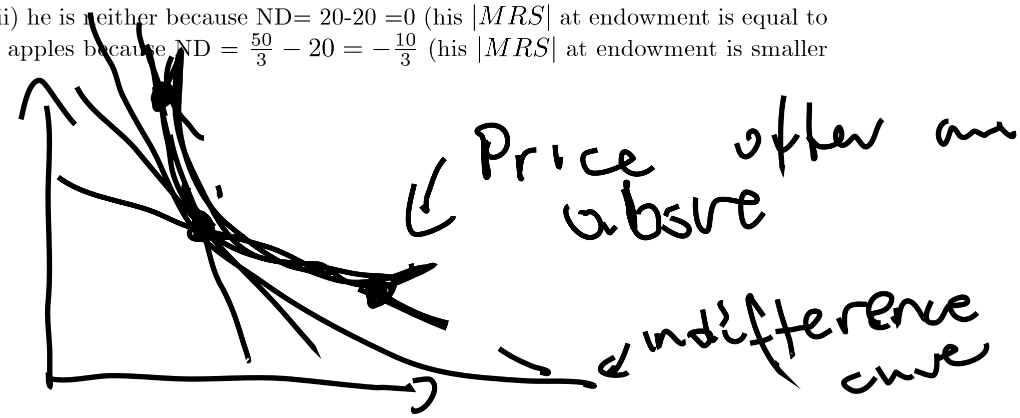
c) Since we have Cobb-Douglas utility function, $u(x_1, x_2) = x_1 x_2$, so the optimal consumptions are:

$$x_1 = \frac{1}{2}\frac{m}{p_1}, x_2 = \frac{1}{2}\frac{m}{p_2}$$

To find optimal consumption we just need to know prices and income.
Given $P_2 = 2$,

1. i when $P_1 = 1$ we have $m = 20 + 40 = 60$ so $x_1^* = \frac{1}{2}\frac{60}{1} = 30$ and $x_2^* = \frac{1}{2}\frac{60}{2} = 15$.
   ii when $P_1 = 2$ we have $m = 40 + 40 = 80$ so $x_1^* = \frac{1}{2}\frac{80}{2} = 20$ and $x_2^* = \frac{1}{2}\frac{80}{2} = 20$.
   iii when $P_1 = 3$ we have $m = 60 + 40 = 100$ so $x_1^* = \frac{1}{2}\frac{100}{3} = \frac{50}{3}$ and $x_2^* = \frac{1}{2}\frac{100}{2} = 25$.

d) In case (i) Dave is buying apples because net demand of apple = 30 - 20 =10 (his $|MRS|$ at endowment is greater than price ratio). In (ii) he is neither because ND= 20-20 =0 (his $|MRS|$ at endowment is equal to price ratio). In (iii) he is selling apples because ND = $\frac{50}{3} - 20 = -\frac{10}{3}$ (his $|MRS|$ at endowment is smaller than price ratio).



Problem 5

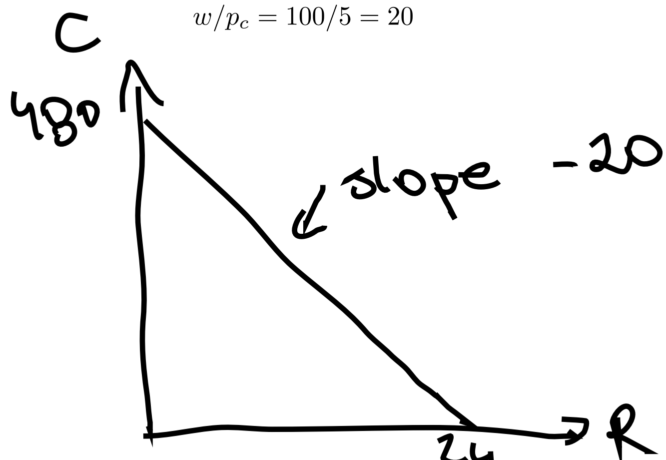
The variables in the problem are

$$x_1 = R, x_2 = C, p_1 = w, p_2 = p_c, m = 24w$$

a) real wage is

$$w/p_c = 100/5 = 20$$

b)



c) Given utility function $a = b = 1$ and the magic formula for $R(= x_1)$ is given by

$$R = \frac{1}{2}\frac{m}{w} = \frac{1}{2}\frac{24w}{w} = 12$$

and for bananas

$$C = \frac{1}{2}\frac{24w}{p_c} = \frac{1}{2}\frac{24 \times 100}{5} = 240$$

and the labor supply is

$$LS = 24 - R = 12$$

d) with $w = 200$ the leisure and labor supply is still $R = 12$ and $LS = 12$ and consumption of bananas is $C = 480$. The labor supply does not change because the substitution effect (higher wage encourages to work more and buy more commodities) is offset by income effect that makes leisure more attractive.