Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to problem set 3

(due Thursday, February 12th, before class)

Problem 1

For Cobb-Douglass utility functions $u(x_1, x_2) = x_1^a x_2^b$, fraction of income spent on $x_1 : \frac{a}{a+b}$ and on $x_2 : \frac{b}{a+b}$. Cash spent on $x_1 : \frac{a}{a+b}\ m$ and on $x_2 : \frac{b}{a+b}m$, Optimal quantities $x_1 = \frac{a}{a+b}\ \frac{m}{p_1}$ and $x_2 = \frac{b}{a+b}\ \frac{m}{p_2}$ and $MRS = -\frac{ax_2}{bx_1}$

a) Given $a = 4, b = 8, p_1 = 5, p_2 = 10$ and $m = 60$ we have: 1) $\frac{b}{a+b} = \frac{8}{12} = \frac{2}{3} = 66\%$ 2) $\frac{2}{3} \times 60\$ = 40$ 3) $x_1 = \frac{1}{3}\frac{60}{5} = 4$ 4) the optimal bundle is $x_1 = 4$, $x_2 = \frac{2}{3}\frac{60}{10} = 4$ and the slope of the indifference curve at this bundle is $MRS = -\frac{4}{8}\frac{4}{4} = -\frac{1}{2}$.

b) Given $a = \frac{1}{3}, b = \frac{1}{3}, p_1 = 4, p_2 = 1$ and $m = 12$ we have: 1) $\frac{b}{a+b} = \frac{1}{2} = 50\%$ 2) $\frac{1}{2} \times 12\$ = 6$ 3) $x_1 = \frac{1}{2}\frac{12}{4} = 1\frac{1}{2}$ 4) the optimal bundle is $x_1 = \frac{3}{2}$, $x_2 = 6$ and the slope of the indifference curve at this bundle is $MRS = -\frac{1/3}{1/3}\frac{6}{3/2} = -4$.

c) Given $a = \frac{1}{2}, b = \frac{3}{2}, p_1 = 5, p_2 = 1$ and $m = 20$ we have: 1) $\frac{b}{a+b} = \frac{3}{4} = 75\%$ 2) $\frac{3}{4} \times 20\$ = 15$ 3) $x_1 = \frac{1}{4}\frac{20}{5} = 1$ 4) the optimal bundle is $x_1 = 1$, $x_2 = 15$ and the slope of the indifference curve at this bundle is $MRS = -\frac{1}{3}\frac{15}{1} = -5$.

Problem 2

a) Marginal rate of substitution is given by

$$MRS = -\frac{MU_1}{MU_2} = -\frac{1/x_1}{1/x_2} = -\frac{x_2}{x_1}$$

Two secrets of happiness for well-behaved preferences are

$$p_1x_1 + p_2x_2 = m$$

and

$$MRS = -\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

From the second condition, $x_2 = \frac{p_1}{p_2}x_1$, which plugged in the budget constraint gives

$$p_1x_1 + p_2\frac{p_1}{p_2}x_1 = m$$

which in turn implies that

$$2p_1x_1 = m$$

and hence

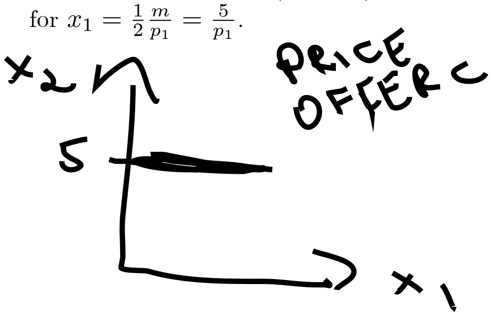
$$x_1 = \frac{1}{2}\frac{m}{p_1}$$

and

$$x_2 = \frac{p_1}{p_2}\frac{1}{2}\frac{m}{p_1} = \frac{1}{2}\frac{m}{p_2}$$

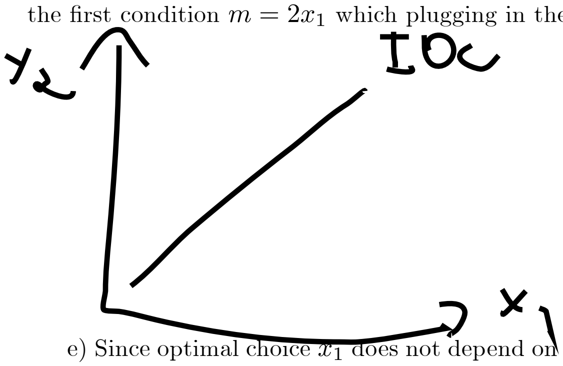
Note that the two formulas coincide with our magic formulas for $a = b = 1$.

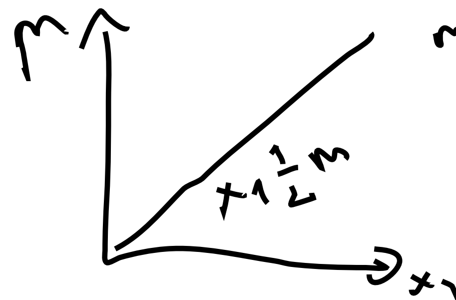
b) For $p_2 = 1$ and $m = 10$ the optimal consumption of $x_2 = 5$ regardless of p_1. Therefore the price offer curve will be constant (flat line) at the level of 5. Formula for the price offer curve is $x_2 = 5$ and for demand for $x_1 = \frac{1}{2}\frac{m}{p_1} = \frac{5}{p_1}$.

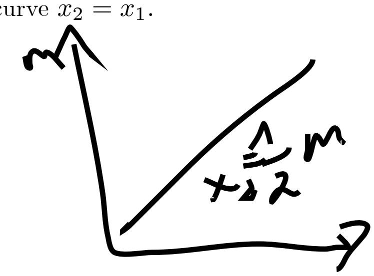


c) Demand for x_1 is decreasing in p_1 and hence this is an ordinary good.

d) Given $p_1 = p_2 = 1$ the optimal choices (defining Engel curves) are given by $x_1 = \frac{1}{2}m$, $x_2 = \frac{1}{2}m$..From the first condition $m = 2x_1$ which plugging in the second condition gives the income offer curve $x_2 = x_1$.







e) Since optimal choice x_1 does not depend on p_2 and x_2 does not depend on p_1, the two goods are neither complements nor substitutes.

Problem 3 Utility U(x1,x2)=min(2x1,x2)

a) Two secrets of happiness for well-behaved preferences are

$$p_1x_1 + p_2x_2 = m$$

and optimal proportion condition

$$x_2 = 2x_1$$

Plugging the second condition in budget constraint gives

$$p_1x_1 + 2p_2x_1 = m$$

which in turn implies that

$$x_1 (p_1 + 2p_2) = m$$

and hence

$$x_1 = \frac{m}{p_1 + 2p_2}$$

and

$$x_2 = \frac{2m}{p_1 + 2p_2}$$

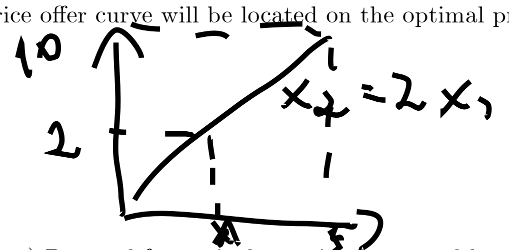
b) For $p_2 = 1$ and $m = 10$ the optimal choices

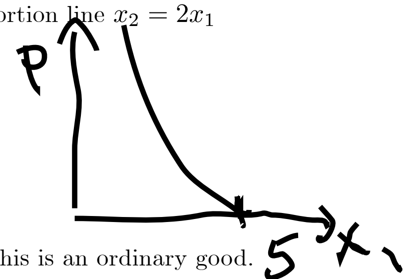
$$x_1 = \frac{10}{p_1 + 2}$$

which defines a demand for x_1 and

$$x_2 = \frac{20}{p_1 + 2}$$

Price offer curve will be located on the optimal proportion line $x_2 = 2x_1$





c) Demand for x_1 is decreasing in p_1 and hence this is an ordinary good.

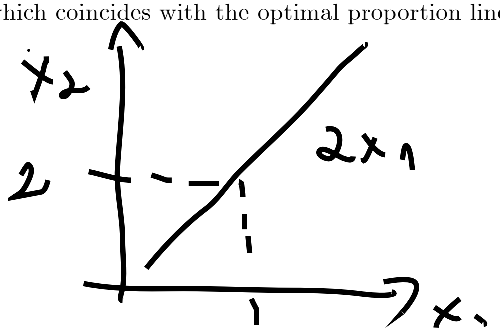
d) Given $p_1 = p_2 = 1$ the optimal choices (defining Engel curves) are given by

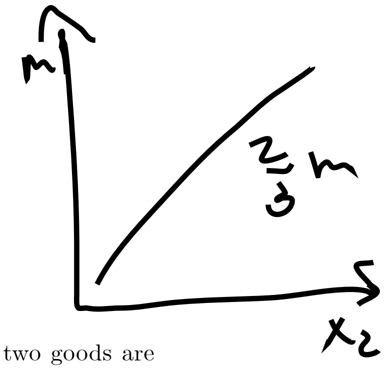
$$x_1 = \frac{1}{3}m$$

which defines a demand for x_1 and

$$x_2 = \frac{2}{3}m$$

From the first condition $m = 3x_1$ which plugging in the second condition gives the income offer curve $x_2 = 2x_1$ which coincides with the optimal proportion line.





e) Since optimal choice x_1 depends negatively on p_2 and x_2 is also decreasing in p_1, the two goods are gross complements.

Problem 4

a) Given utility function, $|MRS| = 2$ and hence the optimal demands are given by

$$\begin{aligned} x_1 &= \frac{m}{p_1} \\ x_2 &= 0 \end{aligned}$$

when $\frac{p_1}{p_2} < |MRS| = 2$ and

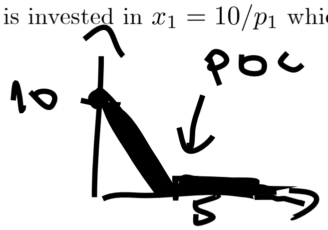
$$\begin{aligned} x_1 &= 0 \\ x_2 &= \frac{m}{p_2} \end{aligned}$$

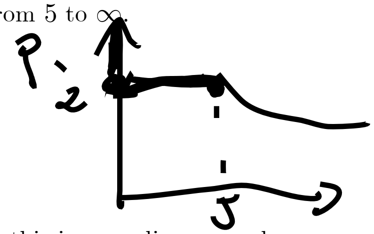
when $\frac{p_1}{p_2} > |MRS| = 2$. Finally any bundle satisfying

$$p_1x_1 + p_2x_2 = m$$

is optimal if $\frac{p_1}{p_2} = |MRS| = 2$

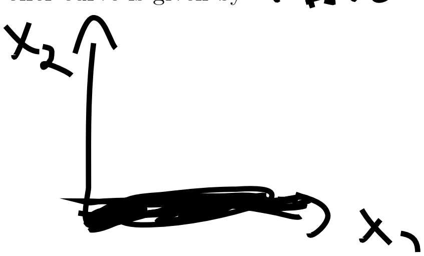
b) Price offer curve: For all $p_1 > 2$ relative price of x_1 is greater than $|MRS|$ and hence the whole income is invested in $x_2 = 10$. For $p_1 = 2$ any bundle on the budget line is equally good. and finally for $p_1 < 2$. whole income is invested in $x_1 = 10/p_1$ which varies from 5 to ∞.



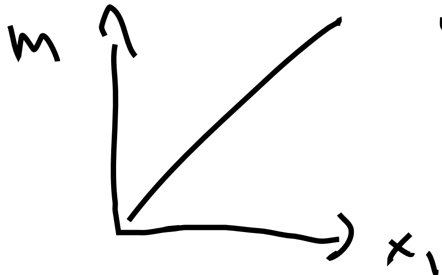


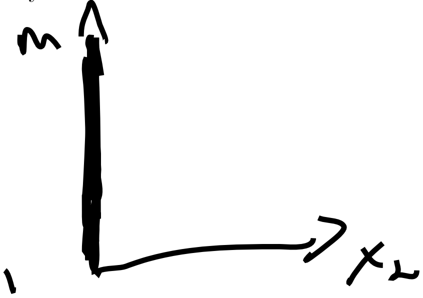
c) Demand for x_1 is not increasing in p_1 and hence this is an ordinary good.

d) Given $p_1 = p_2 = 1$ [redacted] KATE will always buy $x_1 = m$ and zero of the other commodity and hence the income offer curve is given by



and Engel curves are as follows





e) The demand for x_1 (discontinuously) increases in p_2, and hence the two goods are gross substitutes

Problem 5

a) Two secrets of happiness are

$$MRS = -\frac{1}{10 - x_2} = -\frac{p_1}{p_2} = -\frac{1}{2}$$

and

$$x_1 + 2x_2 = 10$$

From the first condition

$$x_2 = 8$$

which plugging to the budget constraint gives

$$x_1 = 10 - 16 = -6$$

Since consumption has to be non-negative, the optimal consumption is $x_1 = 0$ and $x_2 = 10/2 = 5$. Since one of the commodities is not consumed, this is corner solution

b) using the same conditions with new income $m = 20$, $x_2 = 8$ and $x_1 = 4$ is optimal.