

Problem set 3

(due Thursday, February 16 before class)

Problem 1 (Cobb Douglas Utility function)

Tony likes nuts x_1 and berries x_2 , and his preferences are described by the following utility function $U(x_1, x_2) = x_1^a x_2^b$. Find the following variables:

- 1) the optimal fraction (percentage) of income spent on berries;
- 2) the optimal amount of total cash (dollars) spent on berries;
- 3) the optimal quantity of nuts consumed;
- 4) the slope of the indifference curve at the optimal bundle for the following values of parameters:
 - a) $a = 4, b = 8, p_1 = 5, p_2 = 10, m = 60$
 - b) $a = \frac{1}{3}, b = \frac{1}{3}, p_1 = 4, p_2 = 1, m = 12$
 - c) $a = \frac{1}{2}, b = \frac{3}{2}, p_1 = 5, p_2 = 1, m = 20$

Hint: Instead of calculating optimal choices using "two secrets of happiness," take advantage of the demand formulas for Cobb Douglas utility that we derived in the class.

Problem 2

Benjamin spends his time either watching movies (x_1) (he uses "on demand" option, cable TV) or listening to songs - MP3 downloaded from the Internet (x_2). His preferences are described by

$$U(x_1, x_2) = \ln x_1 + \ln x_2$$

Answer the following questions:

- a) Derive Benjamin's demand for movies and MP3 files as a function of prices p_1, p_2 and his income m . (do not use Cobb Douglas formula but rather derive demand using "two secrets of happiness").
- b) Fix the price of MP3 at $p_2 = 1$ and income on $m = 10$. Find the price offer curve (give an exact formula $x_2 = f(x_1)$) and plot it in the commodity space. Find the demand curve $x_1 = f(p_1)$ and plot it in the graph (with p_1 on vertical axis and x_1 on horizontal axis).
- c) Is x_1 an ordinary good or a Giffen good? Explain.
- d) Now fix $p_1 = 1$ and $p_2 = 1$. In the commodity space, plot the income offer curve. In addition, in two separate graphs, plot Engel curves for both movies and MP3 files. Argue that the two commodities are normal (not inferior).
- e) For the demand functions from point a), determine whether the two goods are gross complements, substitutes or neither.

Problem 3 (Perfect Complements)

Consider Trevor from our previous problem set who begins his day with strawberry milkshake. To prepare it, he mixes milk, x_1 , strawberries x_2 and does so always in his favorite: proportion 1 glass of milk to 2 strawberries. What is his utility function? Answer all the questions from Problem 2, from a) to e) using these preferences.

Problem 4 (Perfect Substitutes)

Kate's preferences are defined over consumption of two types of apples: Red Delicious (x_1) and Jonagold (x_2)

$$U(x_1, x_2) = 2x_1 + x_2.$$

Answer all the questions from Problem 2, starting from a) to e) using these preferences.

Problem 5 (Quasilinear preferences)

George is a stamp (x_1) collector, but he also likes fancy clothes (x_2). His utility function is given by

$$U(x_1, x_2) = x_1 + 10x_2 - \frac{1}{2}x_2^2.$$

Each stamp costs $p_1 = 1$ and a piece of his favorite clothing costs $p_2 = 2$

a) Assuming that his total income is given by $m = \$10$, find his optimal choice of x_1 and x_2 . (Is it interior?)

b) Suppose next year George's salary doubles, resulting in his higher income $m = \$20$. Find his new demanded quantities of stamps and clothes. (Is it interior?).

c) Harder: In point a) and b) what is the marginal utility from one dollar invested in stamps, and in clothing (at the optimal demand). Are they equal?

Hint: Unlike in a Cobb-Douglas utility function, with quasilinear preferences we might have corners!