Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to problem set 2

(due Tuesday, February 5th, before class)

Problem 1

a) Fill out the following table

$U(x_1, x_2)$	$\frac{\partial U}{\partial x_1}()$	$\frac{\partial U}{\partial x_2}()$	$MRS(x_1, x_2)$	$MRS(2,3)$
$U() = x_1 x_2$	x_2	x_1	$-\frac{x_2}{x_1}$	$-\frac{3}{2}$
$U() = (x_1)^3 (x_2)^5$	$3(x_1)^2 (x_2)^5$	$5(x_1)^3 (x_2)^4$	$-\frac{3x_2}{5x_1}$	$-\frac{9}{10}$
$U() = 3\ln x_1 + 5\ln x_2$	$3/x_1$	$5/x_2$	$-\frac{3x_2}{5x_1}$	$-\frac{9}{10}$

b) MRS at $(2,3)$ is given by -0.9. Interpretation: Consumer needs to get 0.9 of good two after loosing one unit of good one. Good two is more valuable. One must get 0.000009. of good 2, to be indifferent.

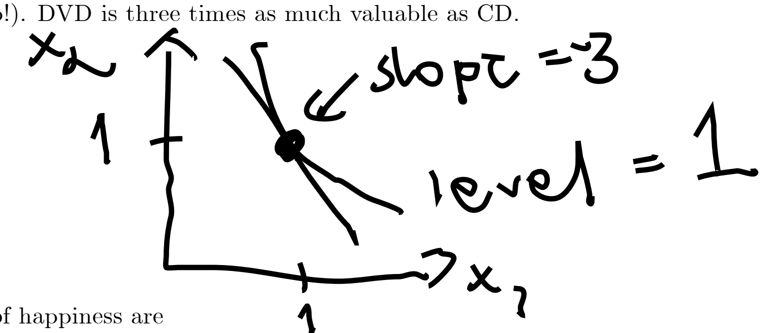
c) $U() = 3\ln x_1 + 5\ln x_2$ is a monotone transformation of $U() = (x_1)^3 (x_2)^5$ and hence the two functions define the same preferences. Since MRS is the slope of indifference curves it must coincide for two functions.

Problem 2

a) Logarithmic transformation of the utility function is $U() = 3\ln x_1 + \ln x_2$ and hence $MU_1 = 3/x_1$ and $MU_2 = 1/x_2$.Alicia's MRS is given by

$$MRS = -\frac{3x_2}{x_1}$$

b) Utility at $(1,1)$ is given by $U(1,1) = 1$.The slope of indifference curve at this point is given by $MRS = -3$. (steep!). DVD is three times as much valuable as CD.



c) Two secrets of happiness are

1) Exhaust your income (monotone preferences)

$$\begin{aligned} p_1 x_1 + p_2 x_2 &= m \\ 40x_1 + 20x_2 &= 800 \end{aligned}$$

2) marginal dollar adds the same utility when invested in CD and DVD

$$\begin{aligned} MRS &= -\frac{p_1}{p_2} \\ -\frac{3x_2}{x_1} &= -\frac{40}{20} = -2 \end{aligned}$$

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d) From the second condition we get

$$x_2 = \frac{2}{3}x_1$$

Plugging it to a Budget line condition

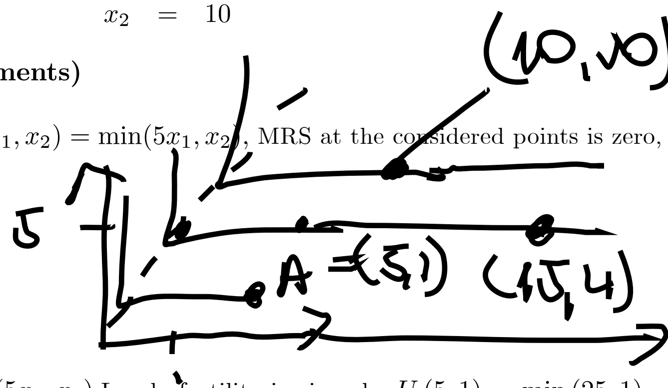
$$40x_1 + \frac{40}{3}x_1 = 800$$

which gives

$$\begin{aligned} x_1 &= 15 \\ x_2 &= 10 \end{aligned}$$

Problem 3 (Perfect Complements)

a) Utility function is given by $U(x_1, x_2) = \min(5x_1, x_2)$, MRS at the considered points is zero, and the preferences are depicted below



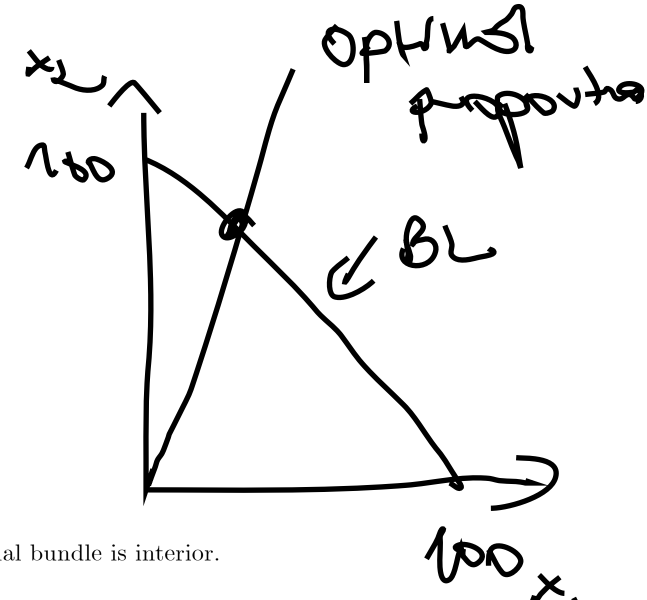
b) For example $U(x_1, x_2) = \min(5x_1, x_2)$.Level of utility is given by $U(5,1) = \min(25,1) = 1$, and $U(10,10) = \min(50,10) = 10$ and $U(15,4) = \min(75,4) = 4$.

c) $V(x_1, x_2) = 10\min(5x_1, x_2) + 2$. Since this is a monotone transformation, indifference curves are the same. The level of utility however has changed $V(5,1) = 12$, $V(10,10) = 102$ and V(74,4)=10*min(74,4)+2=42

d) Secrets of happiness for perfect complements:

1) Exhaust your income (monotone preferences)

$$\begin{aligned} p_1x_1 + p_2x_2 &= m \\ x_1 + x_2 &= 100 \end{aligned}$$



2) consume optimal proportion

$$5x_1 = x_2$$

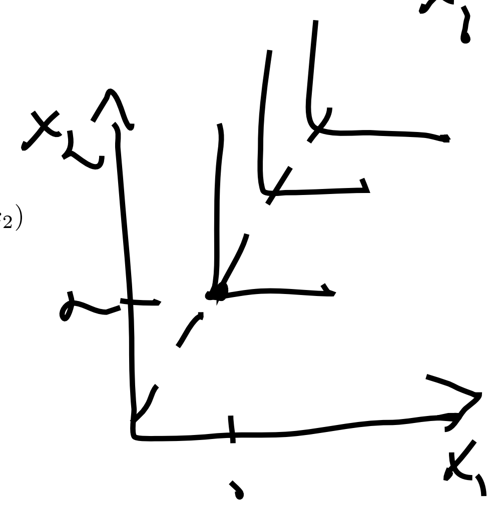
and hence the optimal bundle gives

$$x_1 + 5x_1 = 100$$

which implies that

$$\begin{aligned} x_1 &= \frac{100}{6} \\ x_2 &= \frac{500}{6} \end{aligned}$$

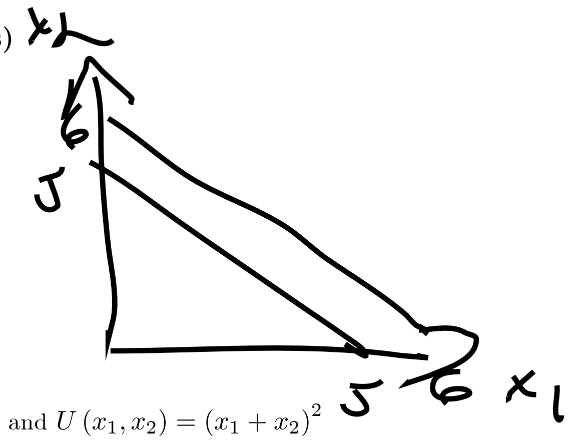
Since Trevor consumes positive quantities of both goods the optimal bundle is interior.



e) Utility function with genetically modified strawberries $U() = \min(2x_1, x_2)$

Problem 4 (Perfect Substitutes)

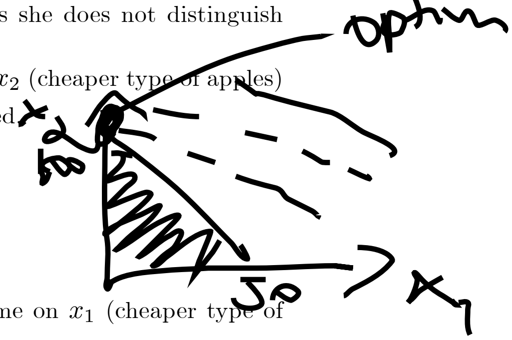
a) Preferences are given by



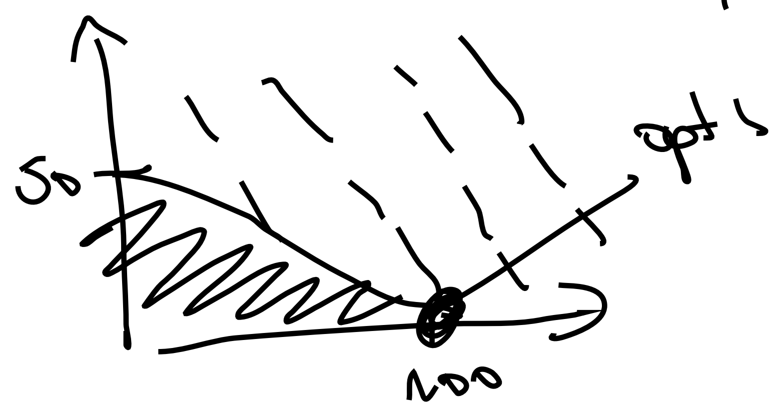
b) For example $U(x_1, x_2) = x_1 + x_2$ and $U(x_1, x_2) = (x_1 + x_2)^2$

c) $MRS = -1$ and it does not depend on the bundle. This is because no matter what bundle is consumed Kate is willing to substitute one type apple with one apple of the other type, as she does not distinguish between the two.

d) Since $|MRS| = 1 < \frac{p_1}{p_2} = 2$ and hence Kate will spend her total income on x_2 (cheaper type of apples) $x_1 = 0$, $x_2 = \frac{m}{p_2} = \frac{100}{1} = 100$. This is corner solution as one good is not consumed.



e) Since $|MRS| = 1 > \frac{p_1}{p_2} = 1/2$ and hence Kate will spend her total income on x_1 (cheaper type of apples) $x_1 = 100$, $x_2 = 0$



f) Any bundle on the budget set is equally good.