Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to problem set 12
(due Tuesday , May 4st, before class)

Problem 1 (Non-excludable and non-rival goods)

a) Nonexcludable good: good for which it is difficult to enforce property rights and hence prevent others from using it. Nonrival good - good for which consumption of one agent does not imply that the other can enjoy it as well.

b) See class slides

c) Pure public good= nonrival and nonexcludable

Problem 2 (Provision of public good)

a) Profit function is

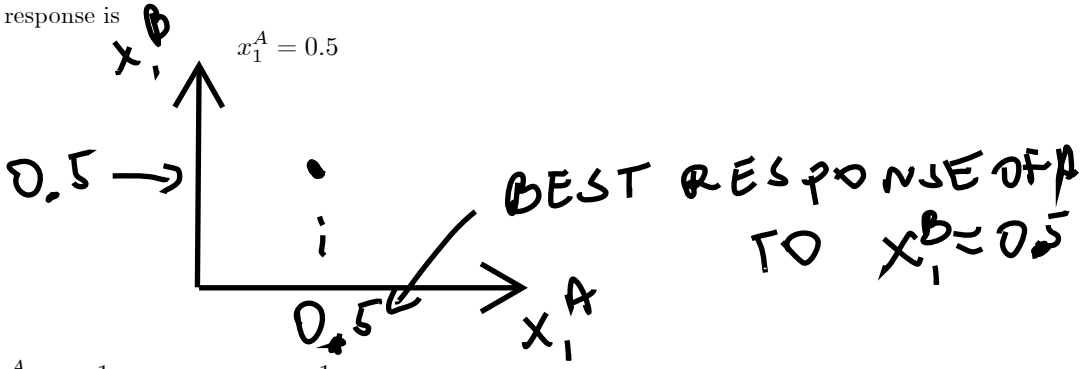
$$\pi^A = \ln\left(x_1^A + 0.5\right) + \ln\left(x_2^A\right) - x_1^A - x_2^A$$

and the first order condition is given by

$$\frac{\partial \pi^A}{\partial x_1^A} = \frac{1}{x_1^A + 0.5} - 1 = 0$$

which implies that the best response is

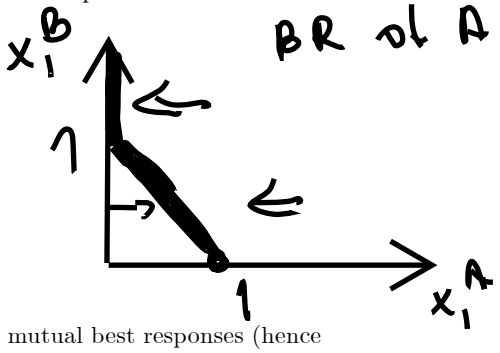
$$x_1^A = 0.5$$



b) Optimality condition

$$\frac{\partial \pi^A}{\partial x_1^A} = \frac{1}{x_1^A + 0.5} - 1 = 0 \Rightarrow \frac{1}{x_1^A + x_1^B} = 1 \Rightarrow x_1^A = 1 - x_1^B$$

When $x_1^B > 1$ then $x_1^A = 0$ as the negative investment in the common area is not possible.



c) The best response function for firm B is

$$\frac{\partial \pi^B}{\partial x_1^B} = 2\frac{1}{x_1^A + x_1^B} - 1 \Rightarrow x_1^B = 2 - x_1^A$$

d) Nash equilibrium is a point $\left(x_1^{*A}, x_1^{*B}\right)$ such that both contributions are mutual best responses (hence the point is located on both best response curves. There is only one such point and it is equal to $(0, 2)$ The total amount of cash spent on common area is 2.

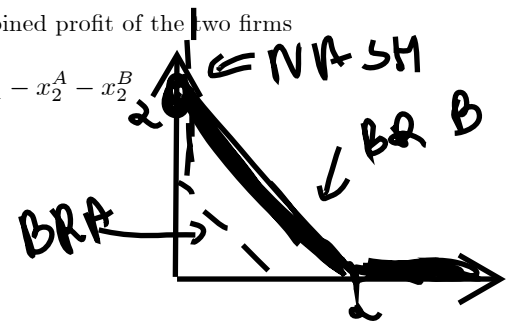
e) Firm A is free riding on the investment of firm B, Its customers are enjoying the common space (and hence the value of its property goes up) but its is not contributing anything to it.

f) Pareto efficient level of investment can be determined by maximizing a joined profit of the two firms

$$\pi = \pi^A + \pi^B = \ln(x_1) + \ln\left(x_2^A\right) + 2\ln(x_1) + \ln\left(x_2^B\right) - x_1 - x_2^A - x_2^B$$

The secret of happiness gives

$$\frac{\partial \pi}{\partial x_1} = \frac{1}{x_1} + \frac{2}{x_1} - 1 = 0 \Rightarrow x_1 = 3$$



The market outcome is below the desirable one. The public good is a special case of positive externality. In market interactions agents ignore the positive effects of their investment on profit on the other firm and hence they underinvestment

Problem 3 (Adverse Selection)

a) Gains-to-trade are 30 and 20 for lemons on plums respectively. Consequently if probabilities are $\frac{1}{2}$ the (expected) gains-to-trade are 25.

b) $p^l = 15$ and $p^p = 110$. The allocation is Pareto efficient as both lemons and plums end up is hands of traders with the highest valuation.

c) If the buyers cannot tell a lemon from a plum, then the expected value from buying a car is

$$EVB = \frac{1}{3} \times 30 + \frac{2}{3} \times 120 = 90$$

In such case the maximal price for a car (a lottery) cannot exceed 90.

d) In such case plums are not traded on the market. The price of a car is $p = 15$ and only lemons are sold on the market.

e) Outcome is not Pareto efficient as the plumes are not traded and hence the gains-to-trade of 20 are lost.

f) The probability can be found from the condition

$$EVB = \pi \times 30 + (1 - \pi) \times 120 \geq 100$$

which implies that

$$\pi \leq \frac{2}{9}$$

g) Yes, in pooling equilibrium the allocation is Pareto efficient. This is because all cars end up in hands of agents with the highest valuation.

h) Warranty

Problem 4 (Singaling)

a) In a pooling equilibrium the expected productivity of a worker is

$$EV = \frac{1}{2} \times 10 + \frac{1}{2} \times 4 = 7$$

and this is the wage offered to the worker (workers productivity is a lottery now). Both types of workers accept the job.

b) 2 passed tests is not a credible signal. This is because in separating equilibrium the benefit from being considered a workaholic is

$$w^h - w^l = 10 - 4 = 6$$

For lazybones workers the two passed tests are associated with cost \$4 and hence it pays out to take tests even though each test requires two approaches.

c) The credible number of passed tests makes the lazybones cost at least as high as the benefit from being considered a workaholic.

$$2e = 6 \Rightarrow e = 3$$

d) In a separating equilibrium the workaholics will be forced to take three tests. Because such tests do not improve the productivity of the workers, therefore from the social point of view they are associated with the waste of resources .- a Pareto inefficiency.