

Problem set 12

(due Tuesday, May 4st, before class)

Problem 1 (Nonexcludable and non-rival goods)

- a) Explain what a nonexcludable and nonrival good is
- b) Give an example of
 - excludable and rival good
 - nonexcludable and rival good
 - excludable and nonrival good
 - non excludable and nonrival good

The examples should be different from the ones given in class. In each example explain why the good belongs to a given category

- c) Which of the four categories is called a pure public good?

Problem 2 (Provision of public good)

There are two major owners of real estate in Shorewood Hills area in Madison, WI, called Alfon Inc (A) and Beton Inc. (B). Both firms specialize in renting their apartments to UW faculty. Each year firm A and firm B decides how much to spend on the common areas in Shorewood Hills, such as playgrounds, bike paths. x_1^A and x_1^B are spendings for Alfon and Beton respectively and $x_1 = x_1^A + x_1^B$ is the total spending for both firms. The second type of cost is the maintainance of their individual properties (x_2^A and x_2^B). The common area is used by all members of the community, and hence x_1 is non-excludable. The market rent received by firm A depends on the quality of the property and its surrounding common area.

$$v^A(x_1, x_2^A) = \ln(x_1^A + x_1^B) + \ln(x_2^A)$$

The profit then is given by

$$\pi^A = v^A(x_1, x_2^A) - x_1^A - x_2^A$$

The customers of firm B value the common area more hence the value property is given by

$$v^B(x_1, x_2^B) = 2 \ln(x_1^A + x_1^B) + \ln(x_2^B)$$

The profit of firm B is them given by

$$\pi^B = v^B(x_1, x_2^B) - x_1^B - x_2^B$$

a) suppose $x_1^B = 0.5$. Find the optimal level of investment in a common area by firm A , x_1^A . Mark this point in space (x_1^A, x_1^B) .

b) Find analytically the best response function for firm A (an optimal investment x_1^A as a function of x_1^B) and plot the entire function in your graph from a) Make sure you also show the optimal x_1^A for $x_1^B > 1$.

c) Find the best response function for firm B . Add it to the graph.

d) Find in your graph the (Nash) equilibrium for two firms (x_1^{*A}, x_1^{*B}) . What is the total amount of money invested in common area?

e) Is the predicted outcome associated with free riding? If yes by which firm? Explain why this firm is free riding?

f) What is the Pareto efficient joint level of investment in the common area? Is it greater, smaller or equal to the one observed in the market (point d)? Explain intuitively why is it so?

Problem 3 (Adverse Selection)

Consider a second-hand car market. For each trader the value of a car depends on whether a trader is a buyer or a seller, and also on whether a car is a lemon or a plum. The values of cars are given by

	Lemon	Plum
Seller	0	100
Buyer	30	120

- a) What are the total gains-to-trade in the market for second-hand cars?
- b) Assuming perfect information and equal splitting of gains-to-trade between buyers and sellers on each segment of the market, give the two prices for lemons and plums. Is an allocation Pareto efficient if information is perfect?

Suppose the buyers cannot tell a lemon from a plum (asymmetric information).

- c) What is the expected value of a car for a buyer if $\frac{1}{3}$ of the cars are lemons? What is the maximal price that a buyer might pay?
- d) Shall we observe a pooling or separating equilibrium if $\frac{1}{3}$? Are plums traded in equilibrium?
- e) Is the outcome of market interactions in a separating equilibrium Pareto efficient? Why or why not?
- f) Find a threshold probability of a lemon for which we might observe a pooling equilibrium.
- g) Is the allocation in pooling equilibrium Pareto efficient? Why or why not?
- h) Propose a signal by which plum owners can differentiate themselves from the lemon owners.

Problem 4 (Signalling)

Consider a signalling model presented in the class. Suppose GMC is looking for new workers to its new factory in China (the same we considered in PS10). The pool of potential workers consists of two types workers: workaholics (w) and lazybones (l). The productivity of a workaholic is 10 cars, while a lazybones produces only 4 cars. Labor market is competitive - the wage is equal to the expected productivity of a worker (and one car costs one \$)

- a) Find a pooling equilibrium in which workers cannot credibly signal their true type.

Suppose now that before being hired by GMC workers can take a number of skill tests (the number of passed tests is e) that prove their abilities. The attempt to pass a test costs \$1. If they are workaholics, they pass each test in the first approach and hence e passed tests cost them $c^w(e) = e$ dollars. If they are lazybones, then it takes two approaches to pass one test. Consequently e passed tests cost $c^l(e) = 2e$

- b) Are two passed tests a credible signal that a worker is a workaholic? Why or why not?
- c) What is the minimal number of tests that constitutes a credible signal for the employer?
- d) Is signalling in form of taking tests efficient from the point of view of the society?