Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to problem set 11

(due Tuesday, April 29 1st, before class)

Problem 1 (Oligopolistic Industry)

a) Big four index (concentration ratio) of the beer industry in the USA is given by

$$CR = 36.8\% + 19.1\% + 18.5\% + 9.2\% = 83.6\%$$

b) HHI index is given by

$$HHI = 36^2+19^2+18^2+9^2+6^2+3^2+2^2+2^2+1+1 = 2117$$

c) Such industry is highly concentrated as $HHI > 1800$

c)Trade Commission is likely to block the merger to protect the consumers from the monopolistic behavior of the beer producers.

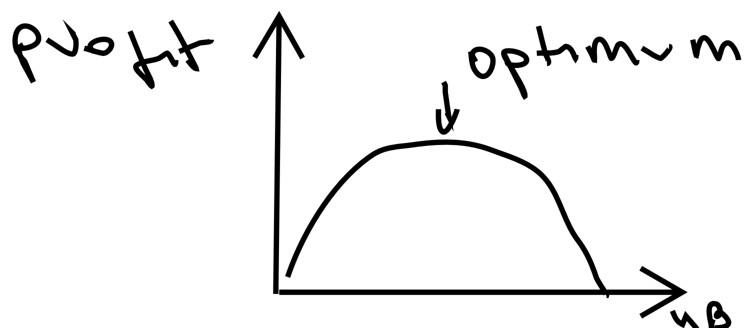
Problem 2 (Aircraft industry)

a) Profit function is given by

$$\pi_B(y_B) = (200 - y_A - y_B)\, y_B - 20 y_B$$

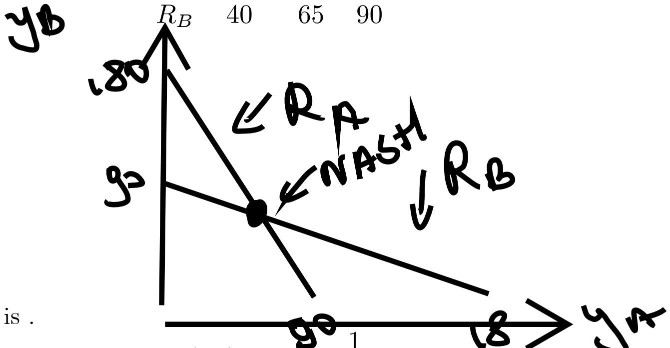
and hence it has a shape of a parabola

$$\pi_B(y_B) = 80 y_B - (y_B)^2$$



b) No, given $y_A = 100$, the optimal production is $y_B = 40$ jets. The best response of Boeing to Aribus production is

y^A	100	50	0
R_B	40	65	90



c) Best response function is .

$$R_B(y_A) \;= 90 - \frac{1}{2} y_A$$

d) The best response function for Airbus is symmetric, and is given by

$$R_A(y_B) \;= 90 - \frac{1}{2} y_B$$

e) The optimal production solves the two best response functions simultaneously

$$y_A = y_B = 60$$

$y = y_A + y_B = 120$

$$p(120) = 200 - 120 = 80$$

and the individual profits in equilibrium are

$$\pi^A = \pi^B = 80 \times 60 - 20 \times 60 = 60 \times 60 = 3600$$

f) The deadweight loss (DWL) associated with oligopolistic trading by the two firms is

$$\begin{aligned} DWL &= \frac{1}{2}(80 - 20)(180 - 120) = \\ &= \frac{1}{2} 60 \times 60 = 1800 \end{aligned}$$

g) If A and B form a cartel selling y jets, the profit function will become

$$\pi^{Cartel} = (200 - y)y - 20y$$

and the optimal level of production is

$$y = 90$$

and the cartel price is

$$p(90) = 200 - 90 = 110$$

Profit per firm in cartel is

$$\pi^A = \pi^B = \frac{\pi^{Cartel}}{2} = \frac{110 \times 90 - 20 \times 90}{2} = \frac{90 \times 90}{2} = 4050$$

Therefore it is beneficial to form a cartel for both firms.
h) A DWL in the case of collusion is

$$DWL^{Cartel} = \frac{1}{2} \times (110 - 20) \times (180 - 90) = 4050 > DWL^{Duopoly} = 1800$$

and it is significantly greater than then one for duopoly. This is because collusion increases the market power of the two traders.

i) If the interactions are only in a short run, cartel is not sustainable as each firm has incentives to "cheat" the other firm by unilaterally increasing a production above 45 jets. This is, because given the other firm produces 45 jets, producing more leads to the profit that is even higher than in the case of the cartel. When interactions are repeated, the loss of reputation associated with cheating "today" makes the cooperation of the two firms tomorrow impossible. Such cost of cheating, might make the cooperation today sustainable.

Problem 3 (Accounting & Audit services in the USA)

a) Monopoly profit is

$$\pi^{Monopoly} = (1000 - y)y - 2y^i$$

and optimal production is

$$y = 495$$

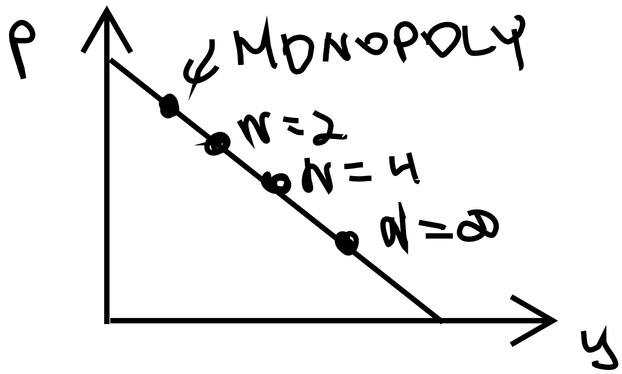
and the monopoly price is

$$p = 1000 - 495 = 505$$

Competitive firms supply can be found from the condition $p = MC$ and hence

$$\begin{aligned} p^{Competitive} &= 10 \\ y^{Competitive} &= 1000 - 10 = 990 \end{aligned}$$

b)



c) Consider firm i (you should think of i as a number identifying one firm) The profit of such firm is

$$\pi^i = (1000 - y^i - \sum_{j \neq i} y^j)y^i - 10y^i$$

where symbol

$$\sum_{j \neq i} y^j = y^1 + y^2 + ... + y^{i-1} + y^{i+1} + ...y^N$$

denotes the level of production by all other firms but i.

The optimality condition gives

$$1000 - 2y^i - \sum_{j \neq i} y^j - 10 = 0$$

or

$$y^i = \frac{990 - \sum_{j \neq i} y^j}{2}$$

Firms are symmetric therefore $y^i = y^j$ for any j and hence we can write it as

$$y^i = \frac{990 - (N-1)\, y^j}{2}$$

so the individual production is

$$y^i = \frac{990}{N+1}$$

and the price and aggregate level of production is

$$\begin{aligned} y &= N \times y^i = \frac{N}{N+1} 990 \\ p &= 1000 - Ny^i = 1000 - \frac{N}{N+1} \times 990 \end{aligned}$$

Finally the deadweight loss is

$$\begin{aligned} DWL &= \frac{1}{2} \times \left(1000 - \frac{N}{N+1} \times 990 - 10\right) \times (990 - \frac{N}{N+1} 990) \\ &= \frac{(990)^2}{2} \times \left(\frac{1}{N+1}\right)^2 \end{aligned}$$

d) The values of aggregate production y and p for $N = 2,\ 5$ and 10 are

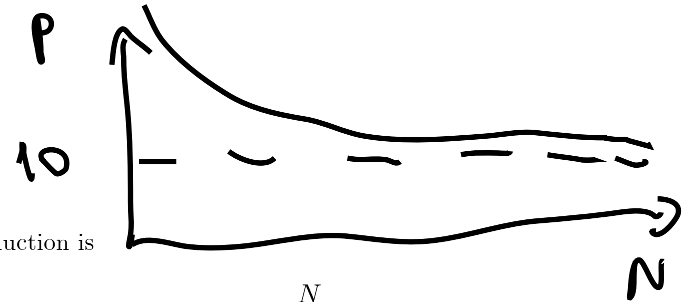
N	2	5	10
y	660	825	900
p	340	175	100

e) The price goes converges to a competitive one as the number of firms goes up. This is because

$$\lim_{N\to\infty}\frac{N}{N+1}=1$$

and hence

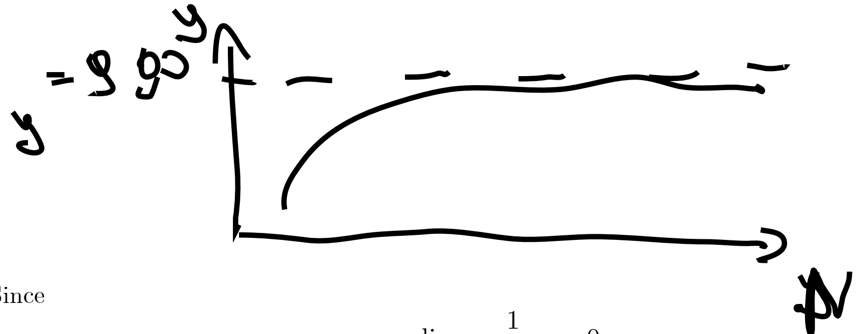
$$\lim_{N\to\infty}p=1000-\lim_{N\to\infty}\frac{N}{N+1}\times 990=1000-1\times 990=10$$



f) The limit production is

$$\lim_{N\to\infty}y=\lim_{N\to\infty}\frac{N}{N+1}\times 990=1\times 990=990$$

Hence output also converges to its competitive limit.



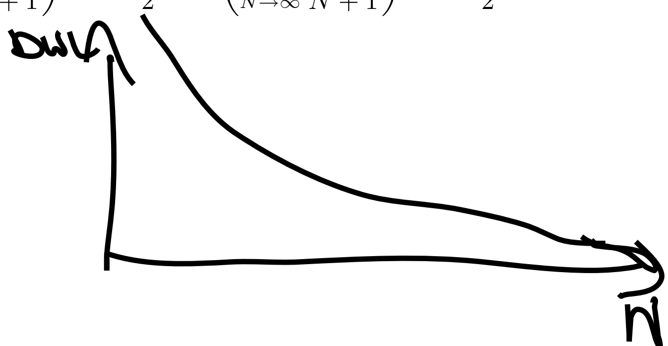
g) Since

$$\lim_{N\to\infty}\frac{1}{N+1}=0$$

therefore

$$\lim_{N\to\infty}DWL=\frac{(990)^2}{2}\times\lim_{N\to\infty}\left(\frac{1}{N+1}\right)^2=\frac{(990)^2}{2}\times\left(\lim_{N\to\infty}\frac{1}{N+1}\right)^2=\frac{(990)^2}{2}\times(0)^2=0$$

hence DWL disappears.



Problem 4 **(Externality)**

a)

Positive:

Participation in a cellular network

R&D by Google

Negative

Neighbor singing in the bathroom

Too many cars in NYC (congestion)

b) Without regulation the outcome is likely to be Pareto inefficient. In case of positive externality in market interactions we will observe too little activity, as the positive effect on others utility/profit is not internalized. In case of negative externality we observe too much activity.

c) In case of positive externality one could introduce a subsidy encouraging the action, while in case of negative one we could tax the activity.

Problem 5 (Positive externality)

a) Positive

b) Given the price of dynamite is equal to one, a profit function of the dynamite producer is given by

$$\pi_d=d-TC_d\,(d,x)$$

The two secrets of happiness (first order conditions) are given by

$$
\begin{aligned}
\frac{\partial \pi_d}{\partial d} &= 1 - \frac{\partial TC_d(d,x)}{\partial d} = 0 \\
\frac{\partial \pi_d}{\partial x} &= -\frac{\partial TC_d(d,x)}{\partial x} = 0
\end{aligned}
$$

hence

$$
\begin{aligned}
\frac{\partial TC_d(d,x)}{\partial d} &= 1 \Rightarrow d = 1 \\
-\frac{\partial TC_d(d,x)}{\partial x} &= 0 \Rightarrow x = 2
\end{aligned}
$$

The first condition implies that the optimal level of dynamite production is

$$\frac{\partial TC_d(d,x)}{\partial d} = 1$$

and the optimal level of intensity is

$$-\frac{\partial TC_d(d,x)}{\partial x} = -2(x-2) = 0 \Rightarrow x = 2$$

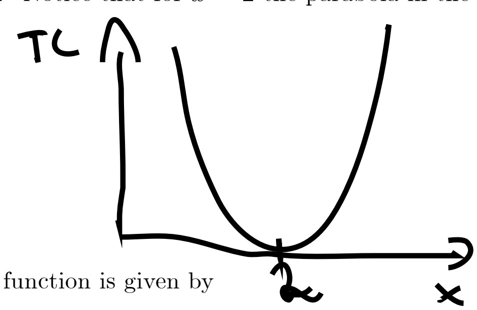
The maximal level of profit is

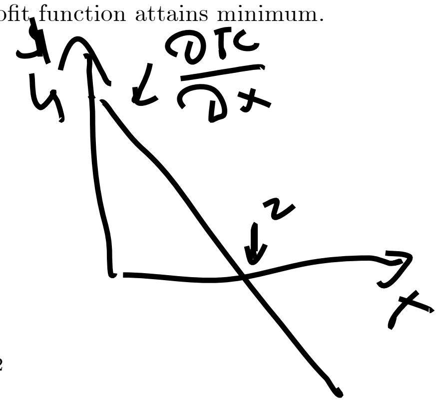
$$\pi_d = d - TC_d(d,x) = 1 - \frac{1}{2} = \frac{1}{2}$$

c) The marginal benefit is given by

$$-\frac{\partial TC_d(d,x)}{\partial x} = -2(x-2)$$

Given optimal level of $x = 2$, such benefit is equal to zero. Intuitively our produces increases the level of intensity x as long as the marginal revenue is positive as it increases its profit. It stops when the last unit x does not add to the profit. Notice that for $x = 2$ the parabola in the profit function attains minimum.





d) The farmer's profit function is given by

$$\pi_t = t - \left(\frac{1}{2}t^2 + 2t - 2t\right) = t - \frac{1}{2}t^2$$

and the secret of happiness is

$$\frac{\partial \pi_t}{\partial x} = 1 - t = 0 \Rightarrow t = 1$$

and the maximal profit of the farmer is

$$\pi_t = 1 - \frac{1}{2} \times 1 = \frac{1}{2}$$

e) The joint profit is

$$\pi_d + \pi_t = 1$$

f) Find the Pareto efficient level of production of d, t and use of nitrogen x. Compare these values to the ones obtained in points b and d.

Joint profit is given

$$\pi = d - \left(\frac{1}{2}d^2 + (x-2)^2\right) + t - \left(\frac{1}{2}t^2 + 2t - xt\right)$$

The three conditions for Pareto efficiency imply that

$$\begin{aligned}\frac{\partial \pi}{\partial d} &= 0 \Rightarrow d = 1 \\ \frac{\partial \pi}{\partial x} &= 0 \Rightarrow 4 - 2x + t = 0 \\ \frac{\partial \pi}{\partial t} &= 0 \Rightarrow 1 - t - 2 + x = 0\end{aligned}$$

From the second equation we get $t = 2x - 4$ and we plug it into the third one

$$1 + 4 - 2x - 2 + x = 0 \Rightarrow x = 3$$

and

$$t = 2$$

In social optimum the intensity of used nitrogen is hither than the market outcome and the production of tomatoes is higher. The reason for that is the producer when choosing the optimal level of nitrogen is ignoring positive effect on the production of tomatoes. Such effect is taken into account by social planner.

g) The marginal benefit is

$$-\frac{\partial TC_d\left(d, x\right)}{\partial x} = -2\left(x-2\right) = -2$$

and it is negative. The reason for that is that the negative marginal benefit for the dynamite producer is compensated by the positive marginal benefit on the production of tomatoes.

h) Yes, the market outcome is associated with x that is too small from the social point of view.