Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to problem set 10
(due Thursday, April 26th, before class)

Problem 1 (Why monopolies exist)

Nuclear Power Plant - large fixed costs
Vista (Microsoft operating system) - patent
Casinos - a legal fiat
Niagara Falls State Park - a sole owner of the waterfalls

Problem 2 (Monopoly)

a) Total gains to trade are

$$GTT = \frac{1}{2} \times 100 \times 100 - F = 5000 - 1000 = 4000$$

Competitive producer sets the price to be equal to marginal cost $p = MC = 0$ and hence

$$CS = 5000$$

and

$$PS = -1000$$

Note that with price equal to zero firm has negative profit therefore it should exit the industry. The answer where the fixed cost was not subtracted is also correct. In such case F is considered as a sunk cost.

b) The total revenue is given by

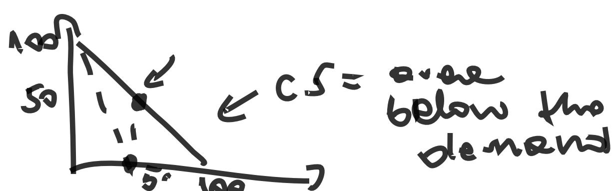
$$TR(y) = 100y - y^2$$

therefore marginal revenue is

$$MR(y) = 100 - 2y$$

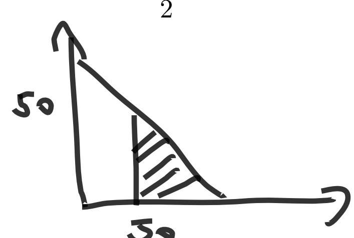
Since marginal cost is zero, optimal production is

$$\begin{aligned} y &= 50 \\ p &= 100 - 50 = 50 \\ \pi &= 50 \times 50 - 1000 = 1500 \end{aligned}$$



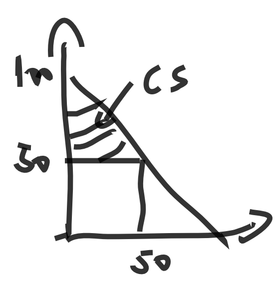
c) Such outcome is not Pareto efficient. DWL is

$$DWL = \frac{1}{2} \times 50 \times 50 = 1250$$

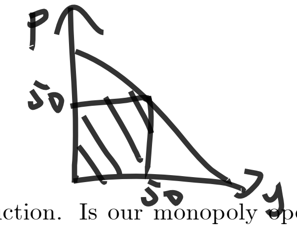


d) Consumer's surplus is given by

$$CS = \frac{1}{2} \times 50 \times 50 = 1250$$



and producer's surplus is



$$PS = 50 \times 50 - 1000 = 1500$$

Note that CS and PS sum up to a smaller number than TS

e) Find the elasticity of the demand at the optimal level of production. Is our monopoly operating on elastic or inelastic part of the demand?

Elasticity is defined as

$$\varepsilon = \frac{\Delta y}{y} / \frac{\Delta p}{p} = \frac{\Delta y}{\Delta p} \times \frac{p}{y}$$

where $\frac{\Delta y}{\Delta p}$ is a slope of the (not inverse!) demand function, which is equal to -1. In optimum $p = y = 50$ hence

$$\varepsilon = \frac{\Delta y}{\Delta p} \times \frac{p}{y} = -1 \times \frac{50}{50} = 1$$

>threshold between elastic and inelastic part

f) Markup: p/MC=p/MR=p/p[1+1/e]=1/[1+1/e]. Given e=-1 markup is infinity (marginal cost is zero)

Problem 3 (Monopoly and price discrimination)

a) In such case the producer's surplus coincides with the total gains to trade.

$$PS = \pi = 4000$$

b) The aggregate demand is

$$y(p) = y^I(p) + y^F(p) = 50 - \frac{4}{5}p + 50 - \frac{1}{5}p = 100 - p$$

hence inverse demand is

$$p(y) = 100 - y$$

Note that the aggregate demand is as in point a).

c) On the segment with individual buyers the inverse demand is

$$p^I = \frac{125}{2} - \frac{5}{4}y^I$$

and hence marginal revenue is

$$MR^I = 62.5 - \frac{5}{2}y^I$$

which implies that the level of production is

$$MR^I = 0 \Rightarrow y^I = \frac{2}{5} \times 62.5 = 25$$

and hence

$$p^I = 31\frac{1}{4}$$

Therefore the gross profit (ignoring fixed cost) on the individual segment is

$$\pi^I = 31\frac{1}{4} \times 25 = 781.25$$

and the consumer surplus on this market is

$$CS^I = 390.63$$

In case of firms' segment the demand is given by

$$p^F = 250 - 5y^F$$

and hence

$$MR^F = 250 - 10y^F = 0$$
$$y^F = 25$$

and price is equal to

$$p^F = 250 - 5 \times 25 = 125$$

and gross profit is

$$\pi^F = 125 \times 25 = 3125.0$$

which implies that producers surplus total (net) profit is

$$\pi = PS == \pi^I + \pi^F - F = 781.25 + 3125 - 1000 = 2906.3$$

Consumer surplus on this market is

$$CS^F = \frac{1}{2} \times 25 \times 125 = 1562.5$$

d) With uniform price we have the following values

$$\begin{aligned} CS &= 1250 \\ PS &= 1500 \end{aligned}$$

Given perfect discrimination

$$\begin{aligned} CS &= 0 \\ PS &= 4000 \end{aligned}$$

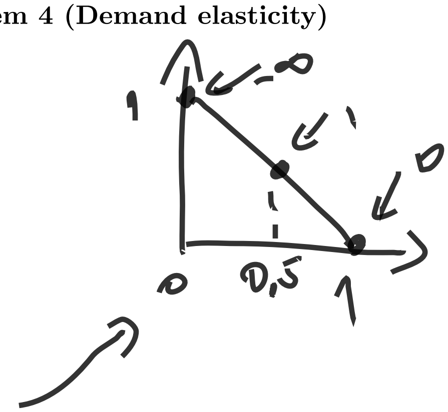
and with 3rd degree price discrimination

$$\begin{aligned} CS &= 1562.5 + 390.63 = 1953.1 \\ PS &= 2906.3 \end{aligned}$$

Note that producer's surplus is the greatest in 1st degree price discrimination and the lowest with uniform price

Problem 4 (Demand elasticity)

a)



b)

c) Secret of happiness requires

$$MR = MC$$

Since $MC = c > 0$ this implies that

$$MR > 0$$

MR can be written as

$$p\left[1+\frac{1}{\varepsilon}\right]>0$$

which, given $p>0$ implies that

$$1+\frac{1}{\varepsilon}>0$$

or

$$\frac{1}{\varepsilon}>-1$$

or

$$\varepsilon<-1$$

Therefore the optimal choice is on the elastic part of the demand.

d) find the markup over a marginal cost c.

$$\begin{cases} MR = MC \\ MR = p\left[1+\frac{1}{\varepsilon}\right] \Rightarrow \end{cases}$$

$$\Rightarrow p = \underbrace{\left[\frac{1}{1+1/\varepsilon}\right]}_{\text{markup}} MC$$

$\cancel{= \frac{5}{4}25 = 31.25}$

$$\cancel{p\frac{4}{5} = 50 - \frac{5}{4}y}$$