Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25, 20, 30, 25 points)+Just For Fun question.

Problem 1. (25p) (Intertemporal Choice)

Joseph earns m_1= \$10 when young and m_2= \$90 when old.

a) Write down Joseph's budget constraint (one inequality) and plot his budget set given interest rate $r = 200\%$ in the graph. Find PV and FV of income and mark it in your graph (give two numbers).

b) Joseph's utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_2)$ where discount rate is $\delta = 2$. Using magic formulas, find the optimal consumption plan and the optimal saving strategy (give three numbers C_1, C_2, S).

c) (Annuity) Joseph is contemplating leasing a car. Leasing would require three annual payments, each \$1600, starting next year (after which you can keep your car). Alternatively, he can buy a car for \$1,500 (payment this year). Which of the two alternatives should he choose if annual interest is $r = 100\%$? Why? (calculate PV of the two alternatives)

d) (Perpetuity) Derive the (general) formula for PV of perpetuity.

Problem 2. (20p) (Uncertainty and Insurance)

Trevor's motorbike is worth 16 (thousand \$). In an event (state of the world) of a crash, its value drops to 0 (and, hence, the bike is a lottery with $w = (w_1, w_2) = (16, 0)$). Assume that the probability of a crash is equal to $\frac{1}{2}$.

a) Find the expected value of the "bike" lottery $(16, 0)$ (one number) and its certainty equivalent CE, assuming Bernoulli utility function $u(y) = \sqrt{y}$ (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence) .

b) Write down the Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_1; C_2)$ for Bernoulli utility function $u(y) = \ln y$. Is Trevor risk averse, risk neutral or risk loving? (chose one+ one sentence)

c) Assume insurance premium $\gamma = \frac{1}{2}$ and the utility function as in point b), find the optimal consumption (C_1, C_2) and insurance coverage x (give three numbers, you can use Magic Formulas). Is Trevor fully insured? Why? (one sentence Hint: is insurance fair?)

Problem 3. (30p) (Edgeworth box and equilibrium)

Jayden and Olivia are consume two types of commodities, ice cream x_1 and burgers x_2. Jayden is initially endowed with $\omega^J = (9, 1)$ of the two goods and Olivia's endowment is $\omega^O = (1, 9)$. The utility function of Jayden and Olivia is the same and is given by

$$U(x_1, x_2) = \frac{1}{3}\ln x_1 + \frac{1}{3}\ln x_2.$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Find the competitive equilibrium (give six numbers) and depict the equilibrium in the Edgeworth box.

c) Verify that the allocation in the competitive equilibrium is Pareto efficient (compare two numbers).

d) Find analytically the contract curve (write down the condition for Pareto efficiency and solve for a contract curve line). Are the initial endowments located on this curve? (a yes or no answer)

Problem 4. (25p) (Producers)

A producer has the following technology

$$y = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

a) Is MPL increasing, decreasing or constant (choose one) What are the returns to scale? (choose: IRS, DRS or CRS)

b) Given short-run level of capital $\bar{K} = 1$ derive the labor demand (formula) of a competitive firm. Find the equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number).

c) Find the unemployment rate if the minimal (real) wage is $w^{min}/p = \frac{1}{2}$ (one number+graph).

d) Find analytically a cost function if $w_K = w_L = 2$ (formula). Plot the cost function in the graph.

Just For Fun

Give a definition of the Pareto efficient allocation. Prove that allocation is Pareto efficient if and only if "MRS" of consumers are equal (show the necessary and sufficient condition).

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Midterm 2 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25, 20, 30, 25 points)+Just For Fun question.

Problem 1. (25p) (Intertemporal Choice)

Joseph earns m_1= \$20 when young and m_2= \$180 when old.

a) Write down Joseph's budget constraint (one inequality) and plot his budget set given interest rate $r = 200\%$ in the graph. Find PV and FV of income and mark it in your graph (give two numbers).

b) Joseph's utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_2)$ where discount rate is $\delta = 2$. Using magic formulas, find the optimal consumption plan and the optimal saving strategy (give three numbers C_1, C_2, S).

c) (Annuity) Joseph is contemplating leasing a car. Leasing would require three annual payments, each \$8,000, starting next year (after which you can keep your car). Alternatively, he can buy a car for \$7,500 (payment this year). Which of the two alternatives should he choose if annual interest is $r = 100\%$? Why? (calculate PV of the two alternatives)

d) (Perpetuity) Derive the (general) formula for PV of perpetuity.

Problem 2. (20p) (Uncertainty and Insurance)

Trevor's motorbike is worth 36 (thousand \$). In an event (state of the world) of a crash, its value drops to 0 (and, hence, the bike is a lottery with $w = (w_1, w_2) = (36, 0)$). Assume that the probability of a crash is equal to $\frac{1}{2}$.

a) Find the expected value of the "bike" lottery $(36, 0)$ (one number) and its certainty equivalent CE, assuming Bernoulli utility function $u(y) = \sqrt{y}$ (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence).

b) Write down the Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_1; C_2)$ for Bernoulli utility function $u(y) = \ln y$. Is Trevor risk averse, risk neutral or risk loving? (chose one+ one sentence)

c) Assume insurance premium $\gamma = \frac{1}{2}$ and the utility function as in point b), find the optimal consumption (C_1, C_2) and insurance coverage x (give three numbers, you can use Magic Formulas). Is Trevor fully insured? Why? (one sentence Hint: is insurance fair?)

Problem 3. (30p) (Edgeworth box and equilibrium)

Jayden and Olivia are consume two types of commodities, ice cream x_1 and burgers x_2. Jayden is initially endowed with $\omega^J = (4, 6)$ of the two goods and Olivia's endowment is $\omega^O = (6, 4)$. The utility function of Jayden and Olivia is the same and is given by

$$U(x_1, x_2) = \frac{1}{5}\ln x_1 + \frac{1}{5}\ln x_2.$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Find the competitive equilibrium (give six numbers) and depict the equilibrium in the Edgeworth box.

c) Verify that the allocation in the competitive equilibrium is Pareto efficient (compare two numbers).

d) Find analytically the contract curve (write down the condition for Pareto efficiency and solve for a contract curve line). Are the initial endowments located on this curve? (a yes or no answer)

Problem 4. (25p) (Producers)

A producer has the following technology

$$y = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

a) Is MPL increasing, decreasing or constant (choose one) What are the returns to scale? (choose: IRS, DRS or CRS)

b) Given short-run level of capital $\bar{K} = 1$ derive the labor demand (formula) of a competitive firm. Find the equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number).

c) Find the unemployment rate if the minimal (real) wage is $w^{min}/p = \frac{1}{2}$ (one number+graph).

d) Find analytically a cost function if $w_K = w_L = 3$ (formula). Plot the cost function in the graph.

Just For Fun

Give a definition of the Pareto efficient allocation. Prove that allocation is Pareto efficient if and only if "MRS" of consumers are equal (show the necessary and sufficient condition).

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Midterm 2 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25, 20, 30, 25 points)+Just For Fun question.

Problem 1. (25p) (Intertemporal Choice)

Joseph earns m_1= \$10 when young and m_2= \$60 when old.

a) Write down Joseph's budget constraint (one inequality) and plot his budget set given interest rate $r = 100\%$ in the graph. Find PV and FV of income and mark it in your graph (give two numbers).

b) Joseph's utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_2)$ where discount rate is $\delta = 1$. Using magic formulas, find the optimal consumption plan and the optimal saving strategy (give three numbers C_1, C_2, S).

c) (Annuity) Joseph is contemplating leasing a car. Leasing would require three annual payments, each \$2400, starting next year (after which you can keep your car). Alternatively, he can buy a car for \$2,300 (payment this year). Which of the two alternatives should he choose if annual interest is $r = 100\%$? Why? (calculate PV of the two alternatives)

d) (Perpetuity) Derive the (general) formula for PV of perpetuity.

Problem 2. (20p) (Uncertainty and Insurance)

Trevor's motorbike is worth 64 (thousand \$). In an event (state of the world) of a crash, its value drops to 0 (and, hence, the bike is a lottery with $w = (w_1, w_2) = (64, 0)$). Assume that the probability of a crash is equal to $\frac{1}{2}$.

a) Find the expected value of the "bike" lottery $(64, 0)$ (one number) and its certainty equivalent CE, assuming Bernoulli utility function $u(y) = \sqrt{y}$ (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence) .

b) Write down the Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_1; C_2)$ for Bernoulli utility function $u(y) = \ln y$. Is Trevor risk averse, risk neutral or risk loving? (chose one+ one sentence)

c) Assume insurance premium $\gamma = \frac{1}{2}$ and the utility function as in point b), find the optimal consumption (C_1, C_2) and insurance coverage x (give three numbers, you can use Magic Formulas). Is Trevor fully insured? Why? (one sentence Hint: is insurance fair?)

Problem 3. (30p) (Edgeworth box and equilibrium)

Jayden and Olivia are consume two types of commodities, ice cream x_1 and burgers x_2. Jayden is initially endowed with $\omega^J = (6, 14)$ of the two goods and Olivia's endowment is $\omega^O = (14, 6)$. The utility function of Jayden and Olivia is the same and is given by

$$U(x_1, x_2) = 3\ln x_1 + 3\ln x_2.$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Find the competitive equilibrium (give six numbers) and depict the equilibrium in the Edgeworth box.

c) Verify that the allocation in the competitive equilibrium is Pareto efficient (compare two numbers).

d) Find analytically the contract curve (write down the condition for Pareto efficiency and solve for a contract curve line). Are the initial endowments located on this curve? (a yes or no answer)

Problem 4. (25p) (Producers)

A producer has the following technology

$$y = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

a) Is MPL increasing, decreasing or constant (choose one) What are the returns to scale? (choose: IRS, DRS or CRS)

b) Given short-run level of capital $\bar{K} = 1$ derive the labor demand (formula) of a competitive firm. Find the equilibrium real wage rate if labor supply is given by $L^s = 9$ (one number).

c) Find the unemployment rate if the minimal (real) wage is $w^{min}/p = \frac{1}{2}$ (one number+graph).

d) Find analytically a cost function if $w_K = w_L = 5$ (formula). Plot the cost function in the graph.

Just For Fun

Give a definition of the Pareto efficient allocation. Prove that allocation is Pareto efficient if and only if "MRS" of consumers are equal (show the necessary and sufficient condition).

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Midterm 2 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25, 20, 30, 25 points)+Just For Fun question.

Problem 1. (25p) (Intertemporal Choice)

Joseph earns m_1= \$10 when young and m_2= \$100 when old.

a) Write down Joseph's budget constraint (one inequality) and plot his budget set given interest rate $r = 100\%$ in the graph. Find PV and FV of income and mark it in your graph (give two numbers).

b) Joseph's utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_2)$ where discount rate is $\delta = 1$. Using magic formulas, find the optimal consumption plan and the optimal saving strategy (give three numbers C_1, C_2, S).

c) (Annuity) Joseph is contemplating leasing a car. Leasing would require three annual payments, each \$2400, starting next year (after which you can keep your car). Alternatively, he can buy a car for \$2,300 (payment this year). Which of the two alternatives should he choose if annual interest is $r = 100\%$? Why? (calculate PV of the two alternatives)

d) (Perpetuity) Derive the (general) formula for PV of perpetuity.

Problem 2. (20p) (Uncertainty and Insurance)

Trevor's motorbike is worth 100 (thousand \$). In an event (state of the world) of a crash, its value drops to 0 (and, hence, the bike is a lottery with $w = (w_1, w_2) = (100, 0)$). Assume that the probability of a crash is equal to $\frac{1}{2}$.

a) Find the expected value of the "bike" lottery $(100, 0)$ (one number) and its certainty equivalent CE, assuming Bernoulli utility function $u(y) = \sqrt{y}$ (one number). Is the certainty equivalent bigger or smaller than the expected value? Why? (one sentence) .

b) Write down the Von Neumann-Morgenstern (expected) utility function over lotteries $U(C_1; C_2)$ for Bernoulli utility function $u(y) = \ln y$. Is Trevor risk averse, risk neutral or risk loving? (chose one+ one sentence)

c) Assume insurance premium $\gamma = \frac{1}{2}$ and the utility function as in point b), find the optimal consumption (C_1, C_2) and insurance coverage x (give three numbers, you can use Magic Formulas). Is Trevor fully insured? Why? (one sentence Hint: is insurance fair?)

Problem 3. (30p) (Edgeworth box and equilibrium)

Jayden and Olivia are consume two types of commodities, ice cream x_1 and burgers x_2. Jayden is initially endowed with $\omega^J = (3, 17)$ of the two goods and Olivia's endowment is $\omega^O = (17, 3)$. The utility function of Jayden and Olivia is the same and is given by

$$U(x_1, x_2) = 5\ln x_1 + 5\ln x_2.$$

a) Plot an Edgeworth box and mark the initial endowments.

b) Find the competitive equilibrium (give six numbers) and depict the equilibrium in the Edgeworth box.

c) Verify that the allocation in the competitive equilibrium is Pareto efficient (compare two numbers).

d) Find analytically the contract curve (write down the condition for Pareto efficiency and solve for a contract curve line). Are the initial endowments located on this curve? (a yes or no answer)

Problem 4. (25p) (Producers)

A producer has the following technology

$$y = K^{\frac{1}{2}}L^{\frac{1}{2}}$$

a) Is MPL increasing, decreasing or constant (choose one) What are the returns to scale? (choose: IRS, DRS or CRS)

b) Given short-run level of capital $\bar{K} = 1$ derive the labor demand (formula) of a competitive firm. Find the equilibrium real wage rate $\frac{w_L}{p}$ if labor supply is given by $L^s = 9$ (one number).

c) Find the unemployment rate if the minimal (real) wage is $w^{min}/p = \frac{1}{2}$ (one number+graph).

d) Find analytically a cost function if $w_K = w_L = 8$ (formula). Plot the cost function in the graph.

Just For Fun

Give a definition of the Pareto efficient allocation. Prove that allocation is Pareto efficient if and only if "MRS" of consumers are equal (show the necessary and sufficient condition).

Second Midterm Solutions

Econ 301 - Spring 2012

Maximum Points per Question

1a. = 3	2a. = 7	3a. = 3	4a. = 4
1b. = 10	2b. = 4	3b. = 15	4b. = 8
1c. = 5	2c. = 9	3c. = 6	4c. = 5
1d. = 7		3d. = 6	4d. = 8

Group A

Problem #1

A) The budget constraint maps consumption between the two time periods. With the "young" period on the x-axis and the "old" period on the y-axis, the budget constraint appears as

$$
\begin{aligned}
C_1P_1 + C_2P_2 &\leq m_1 + m_2P_2 \\
P_2 &= \frac{1}{1+r} \\
C_1P_1 + C_2\frac{1}{3} &\leq \$10 + \$90\frac{1}{3} \\
\Longrightarrow C_1P_1 + C_2\frac{1}{3} &\leq \$40
\end{aligned}
$$

Future Value is the \$90 that Joseph gets when he is "old" in addition to the value of the \$10 he gets when he is "young" when he is old:

$$\$10(1+r) = \$30$$

This means that FV = \$120.

Present value is the \$10 Joseph has when he is young plus the value when he is "young" of the \$90 he will get when he is old:

$$\$90(\frac{1}{1+r}) + \$10 = \$40$$

This means that PV = \$40

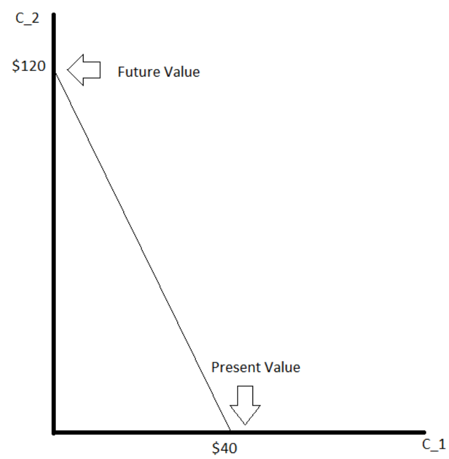


Figure 1: Budget Constraint

B) In terms of PV, the optimal demands for consumption are:

$$C_1 = \frac{1}{1+\frac{1}{1+\delta}} \times \frac{\$10P_1 + \$90P_2}{P_1}$$

$$\implies C_1 = \frac{3}{4} \times \frac{\$10 + \$90P_2}{1}$$

$$C_2 = \frac{\frac{1}{1+\delta}}{1+\frac{1}{1+\delta}} \times \frac{\$10P_1 + \$90P_2}{P_2}$$

$$\implies C_2 = \frac{1}{4} \times \frac{\$10 + \$90P_2}{P_2}$$

From the budget constraint we know that $C_1 + P_2C_2 \leq 40$, so combining this with the above equations we can confirm the value for P_2 we previously derived:

$$\begin{aligned}
\frac{3}{4} \times \frac{\$10 + \$90P_2}{1} + P_2(\frac{1}{4} \times \frac{\$10 + \$90P_2}{P_2}) &= \$40 \\
\$7.5 + \$67.5P_2 + \$2.5 + \$22.5P_2 &= \$40 \\
\$90P_2 &= \$30 \\
\Longrightarrow P_2 &= \frac{1}{3}
\end{aligned}$$

Plugging P_2 into the magic formulas for C_1 and C_2, we get:

$$C_1 = \frac{3}{4} \times \frac{\$10 + \$90\frac{1}{3}}{1} = \$30$$
$$C_2 = \frac{1}{4} \times \frac{\$10 + \$90\frac{1}{3}}{\frac{1}{3}} = \$30$$

Since Joseph is only endowed with $10 in the first period, but consumes $30, this means that he is borrowing (in present value) $20 from the future ($60 in FV). This makes his savings vector S = (-$20,0).

C) The formula for an annuity of $1,600 lasting for three periods, starting next period is:

$$\frac{\$1,600}{r}(1 - (\frac{1}{1+r}^3)) = \$1,400 < \$1,500$$

This means that the lease is a better option than the purchase since it costs less in terms of present value and available information (assuming you ignore the value of the car remaining after three years).

D) The formula of a stream of payments that never ends is:

$$\begin{aligned}
PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + ... \\
&= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ...] \\
&= \frac{1}{(1+r)}[x + PV]
\end{aligned}$$

so we can solve for PV to get a more concise solution:

$$\begin{aligned}(1 - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{1+r}{1+r} - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{r}{1+r})PV &= \frac{1}{1+r}x \\ PV &= \frac{x}{r}\end{aligned}$$

Problem 2

A) To calculate the expected value it is necessary to find the value in each state and multiply it by the probability that each state will occur. The summation is then:

$$EV = 0 * \pi + 16 * (1 - \pi) = 8$$

The certainty equivalent is the amount of money that Trevor would take *for certain* in order to avoid the gamble that would leave him with the *expectation* of \$8. To calculate this you must first find the utility that the gamble provides, and then find the amount of money that would provide the same utility with a 100% probability:

$$\begin{aligned}U(E(8)) &= \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{16} \\ &= 2 \\ U(CE) = 2 &\implies \sqrt{CE} = 2 \\ \implies CE &= 4\end{aligned}$$

Here we clearly see that the certainty equivalent, 4, is smaller than the expected payoff from the gamble, 8. This makes sense because the concave shape of Trevor's utility function reflects his risk-averse preferences - meaning he is willing to take a smaller payout for certain than the one he would get in expectation to avoid the possibility of being left with no money at all.

B) The expected utility representation is:

$$U(C_1, C_2) = \frac{1}{2}\ln C_1 + \frac{1}{2}\ln C_2$$

The natural log is a concave function, meaning that Trevor is risk averse.
C) First, we calculate the formulas for consumption in each state of the world:

$$C_1 = 16 - x\gamma$$
$$C_2 = (1-\gamma)x$$

Knowing that x, the amount of insurance, is the same in both states of the world, we can solve both of the above equations for x and set them equal to one another:

$$\begin{aligned} x &= \frac{16 - C_1}{\gamma} \\ x &= \frac{C_2}{(1-\gamma)} \end{aligned}$$
$$\implies \frac{16 - C_1}{\gamma} = \frac{C_2}{(1-\gamma)}$$
$$\implies C_1 = 16 - \frac{\gamma}{1-\gamma} C_2$$

This final equation is our budget constraint. Normalizing P_1 to 1, this makes $P_2 = 1$ as well. Plugging these values into the magic formulas for demand we get:

$$\begin{aligned} C_1 &= \frac{1}{2} \times \frac{16}{P_1} \\ C_2 &= \frac{1}{2} \times \frac{16}{P_2} \\ C_1 = C_2 &= 8 \end{aligned}$$

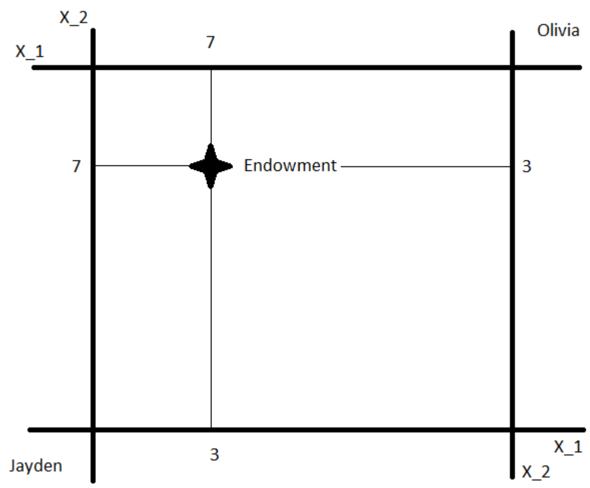
Going back to the first equation for C_1, we now have:

$$8 = 16 - \frac{1}{2}x$$
$$\implies x = 16$$

Consumption in both states of the world is 8, so that means that Trevor is fully insured. Full insurance means that your consumption does not depend on the state of the world. Trevor has chosen to fully insure in this case because the insurance premium, γ, is equal to the probability of loss, π.

Problem 3

A)



B) At an equilibrium, we need for MRS^J to equal MRS^O and also for MRS^J to be equal to $\frac{P_1}{P_2}$. We use the first identity to get the contract curve, the second to calculate the slope of the budget line. Given the endowment point, we can follow the budget line away from the endowment point to find its intersection with the contract curve, which is the equilibrium. Since the two utility functions are symmetrical, we can solve for Jayden and Olivia both by simply solving for Jayden. Analytically, this is done via the following equations: First, we normalize P_1 to equal 1 and use the Cobb-Douglas magic formulas to get the P_2:

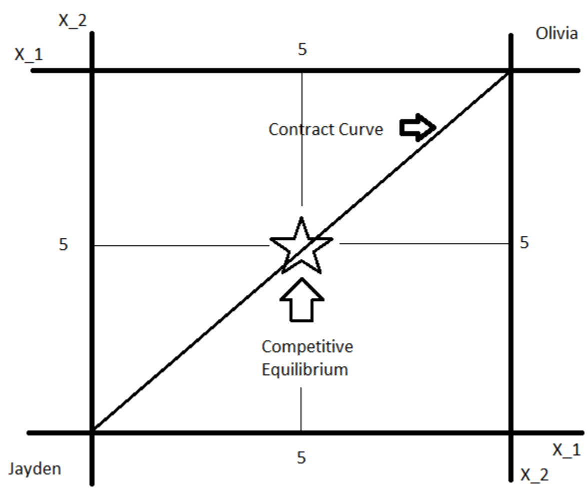
$$x_1 = \frac{1}{2} \times \frac{3P_1 + 7P_2}{P_1}$$
$$x_2 = \frac{1}{2} \times \frac{3P_1 + 7P_2}{P_2}$$
$$x_1 + x_2 = 3P_1 + 7P_2$$
$$\implies \frac{1}{2} \times \frac{3P_1 + 7P_2}{P_1} + \frac{1}{2} \times \frac{3P_1 + 7P_2}{P_2} = 3P_1 + 7P_2$$
$$5P_2 + 5 = 3 + 7P_2$$
$$P_2 = 1$$

$$
\begin{aligned}
MRS^J = \frac{MU_1^J}{MU_2^J} &= \frac{\frac{1}{3x_1}}{\frac{1}{3x_2}} = \frac{P_1}{P_2} \\
\implies \frac{x_2}{x_1} &= \frac{P_1}{P_2} \\
\implies x_2 &= x_1
\end{aligned}
$$

This last step, however, was extraneous expect to show that the contract curve is the set of points where $x_1 = x_2$. Thus, our solution must satisfy this identity. Anyway, revisiting the Cobb-Douglas demands with P_2 in hand, we get:

$$
\begin{aligned}
x_1 &= \frac{1}{2} \times \frac{3P_1 + 7P_2}{P_1} \\
&= 5
\end{aligned}
$$

$$\implies x_1^J = x_2^J = x_1^O = x_2^O = 5 \ \& \ P_1 = P_2 = 1$$



C) As we previously stated, at the competitive equilibrium, it is required that $MRS^J = MRS^O$ which means that $\frac{x_2^J}{x_1^J} = \frac{x_2^O}{x_1^O}$ which is satisfied when $x_1^J = x_2^J = x_1^O = x_2^O = 5$ since $\frac{5}{5} = \frac{5}{5}$

D) Solving analytically for the contract curve requires knowledge of the total endowment of each good in the economy. Here, we can see from the individual endowments that there are 10 units of each good in the economy and that the edgeworth box depicting it is square. To solve for the slope of the contract curve use the equations:

$$MRS^J = \frac{MU_1^J}{MU_2^J} = \frac{\frac{1}{3x_1^J}}{\frac{1}{3x_2^J}} = \frac{x_2^J}{x_1^J} = MRS^O = \frac{MU_1^O}{MU_2^O} = \frac{10 - x_2^J}{10 - x_1^J}$$
$$\implies x_2^J = x_1^J$$

By symmetry this is true for both individuals. Clearly the initial endowments are not located on the contract curve (no).

Problem 4

A) The MPL is decreasing and there are constant returns to scale exhibited by this production function.

B) Labor demand can be calculated by setting MPL = $\frac{w}{p}$. Given that K is fixed, this makes the equation:

$$\frac{1}{2}L^{-\frac{1}{2}} = \frac{w}{p}$$
$$\frac{1}{2\sqrt{L}} = \frac{w}{p}$$
$$\sqrt{L} = \frac{p}{2w}$$
$$L^* = (\frac{p}{2w})^2$$
$$9 = (\frac{p}{2w})^2$$
$$\frac{1}{6} = \frac{w}{p}$$

C)The unemployment rate is the ratio of the number of hours invidivduals are under-employed relative to the equilibrium level of employment in a free market. Here, when wages are free to adjust due to supply and demand the market clears at a wage real wage of $\frac{1}{6}$ and 9 hours of labor. When the wage is constrained to be $\frac{1}{2}$ the labor demand falls, while the supply remains constant at 9 hours. This yields the following unemployment rate:

$$\frac{1}{2\sqrt{L}} = \frac{1}{2}$$
$$2\sqrt{L} = 2$$
$$L = 1$$
$$\implies UnemploymentRate = \frac{(9-1)}{9} = \frac{8}{9}$$

D) First we must see that in order for the TRS = $\frac{W_K}{W_L}$ it must be the case that K = L:

$$TRS = \frac{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}}{\frac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}} = \frac{W_K}{W_L} = 1$$
$$\implies K = L$$

plugging this result back into the production function to get costs as a function of output, we see that:

$$\begin{aligned} y &= K^{\frac{1}{2}}(K)^{\frac{1}{2}} \\ &= K \\ \implies K = y \end{aligned}$$

thus, the cost function is given by:

$$\begin{aligned} C(y) &= W_K K + W_L L \\ &= 8K + 8L \\ &= 16K \end{aligned} \tag{1}$$

Since above we learned that $K = y$, is must be that case that costs, as a function of output, is given by the equation $C(y) = 16y$.

Group B

Problem #1

A) The budget constraint maps consumption between the two time periods. With the "young" period on the x-axis and the "old" period on the y-axis, the budget constraint appears as

$$\begin{aligned} C_1P_1 + C_2P_2 &\leq m_1 + m_2P_2 \\ P_2 &= \frac{1}{1+r} \\ C_1P_1 + C_2\frac{1}{3} &\leq \$20 + \$180\frac{1}{3} \\ \implies C_1P_1 + C_2\frac{1}{3} &\leq \$80 \end{aligned}$$

Future Value is the \$180 that Joseph gets when he is "old" in addition to the value of the \$20 he gets when he is "young" when he is old:

$$\$20(1+r) = \$60$$

This means that FV = \$240.

Present value is the \$20 Joseph has when he is young plus the value when he is "young" of the \$180 he will get when he is old:

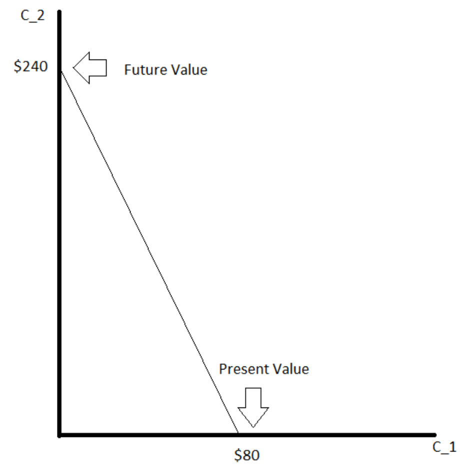
$$\$180(\frac{1}{1+r}) + \$20 = \$80$$

This means that PV = \$80

B) In terms of PV, the optimal demands for consumption are:

$$\begin{aligned} C_1 &= \frac{1}{1+\frac{1}{1+\delta}} \times \frac{\$20P_1 + \$180P_2}{P_1} \\ \implies C_1 &= \frac{3}{4} \times \frac{\$20 + \$180P_2}{1} \\ C_2 &= \frac{\frac{1}{1+\delta}}{1+\frac{1}{1+\delta}} \times \frac{\$20P_1 + \$180P_2}{P_2} \\ \implies C_2 &= \frac{1}{4} \times \frac{\$20 + \$180P_2}{P_2} \end{aligned}$$

Figure 2: Budget Constraint



From the budget constraint we know that $C_1 + P_2C_2 \leq 80$, so combining this with the above equations we can recover P_2:

$$\begin{aligned}
\frac{3}{4} \times \frac{\$20 + \$180P_2}{1} + P_2(\frac{1}{4} \times \frac{\$20 + \$180P_2}{P_2}) &= \$80 \\
\$15 + \$135P_2 + \$5 + \$45P_2 &= \$80 \\
\$180P_2 &= \$60 \\
\implies P_2 &= \frac{1}{3}
\end{aligned}$$

Plugging P_2 into the magic formulas for C_1 and C_2, we get:

$$\begin{aligned}
C_1 &= \frac{3}{4} \times \frac{\$20 + \$180\frac{1}{3}}{1} = \$60 \\
C_2 &= \frac{1}{4} \times \frac{\$20 + \$180\frac{1}{3}}{\frac{1}{3}} = \$60
\end{aligned}$$

Since Joseph is only endowed with \$20 in the first period, but consumes \$60, this means that he is borrowing (in present value) \$40 from the future (\$120 in FV). This makes his savings vector S = (-\$40,0).

C) The formula for an annuity of \$8,000 lasting for three periods, starting next period is:

$$\frac{\$8,000}{r}(1 - (\frac{1}{1+r}^{3})) = \$7,000 \leq \$7,500$$

This means that the lease is a better option than the purchase since it costs less in terms of present value and available information (assuming you ignore the value of the car remaining after three years).

D) The formula of a stream of payments that never ends is:

$$\begin{aligned} PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + ... \\ &= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ...] \\ &= \frac{1}{(1+r)}[x + PV] \end{aligned}$$

so we can solve for PV to get a more concise solution:

$$\begin{aligned} (1 - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{1+r}{1+r} - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{r}{1+r})PV &= \frac{1}{1+r}x \\ PV &= \frac{x}{r} \end{aligned}$$

Problem 2

A) To calculate the expected value it is necessary to find the value in each state and multiply it by the probability that each state will occur. The summation is then:

$$EV = 0 * \pi + 36 * (1 - \pi) = 18$$

The certainty equivalent is the amount of money that Trevor would take *for certain* in order to avoid the gamble that would leave him with the *expectation* of \$18. To calculate this you must first find the utility that the gamble provides, and then find the amount of money that would provide the same utility with a 100% probability:

$$\begin{aligned} U(E(18)) &= \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{36} \\ &= 3 \\ U(CE) = 3 &\implies \sqrt{CE} = 3 \\ \implies CE &= 9 \end{aligned}$$

Here we clearly see that the certainty equivalent, 9 is smaller than the expected payoff from the gamble, 18. This makes sense because the concave shape of Trevor's utility function reflects his risk-averse preferences - meaning he is willing to take a smaller payout for certain than the one he would get in expectation to avoid the possibility of being left with no money at all.

B) The expected utility representation is:

$$U(C_1, C_2) = \frac{1}{2}\ln C_1 + \frac{1}{2}\ln C_2$$

The natural log is a concave function, meaning that Trevor is risk averse.
C) First, we calculate the formulas for consumption in each state of the world:

$$C_1 = 36 - x\gamma$$
$$C_2 = (1-\gamma)x$$

Knowing that x, the amount of insurance, is the same in both states of the world, we can solve both of the above equations for x and set them equal to one another:

$$\begin{aligned} x &= \frac{36 - C_1}{\gamma} \\ x &= \frac{C_2}{(1-\gamma)} \end{aligned}$$
$$\implies \frac{36 - C_1}{\gamma} = \frac{C_2}{(1-\gamma)}$$
$$\implies C_1 = 36 - \frac{\gamma}{1-\gamma}C_2$$

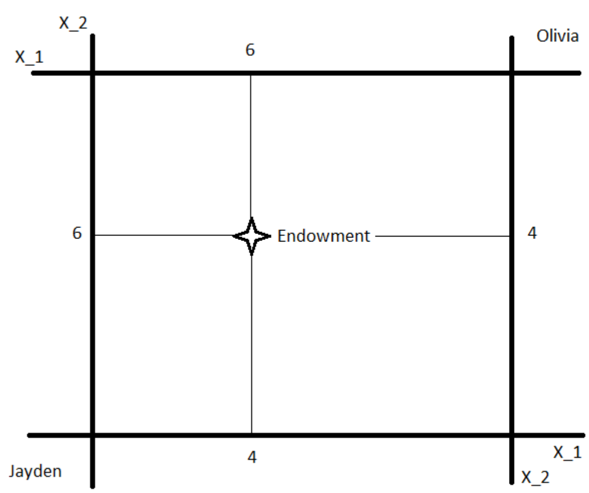
This final equation is our budget constraint. Normalizing P_1 to 1, this makes $P_2 = 1$ as well. Plugging these values into the magic formulas for demand we get:

$$\begin{aligned} C_1 &= \frac{1}{2} \times \frac{36}{P_1} \\ C_2 &= \frac{1}{2} \times \frac{36}{P_2} \\ C_1 = C_2 &= 18 \end{aligned}$$

Going back to the first equation for C_1, we now have:

$$18 = 36 - \frac{1}{2}x$$
$$\implies x = 36$$

Consumption in both states of the world is 18, so that means that Trevor is fully insured. Full insurance means that your consumption does not depend on the state of the world. Trevor has chosen to fully insure in this case because the insurance premium, γ, is equal to the probability of loss, π.
Problem 3
A)



B) At an equilibrium, we need for MRS^J to equal MRS^O and also for MRS^J to be equal to $\frac{P_1}{P_2}$. We use the first identity to get the contract curve, the second to calculate the slope of the budget line. Given the endowment point, we can follow the budget line away from the endowment point to find its intersection with the contract curve, which is the equilibrium. Since the two unility functions are symetrical, we can solve for Jayden and Olivia both by simply solving for Jayden. Analytically, this is done via the following equations: First, we normalize P_1 to equal 1 and use the Cobb-Douglas magic forumlas to get the P_2:

$$\begin{aligned} x_1 &= \frac{1}{2} \times \frac{4P_1 + 6P_2}{P_1} \\ x_2 &= \frac{1}{2} \times \frac{4P_1 + 6P_2}{P_2} \\ x_1 P_1 + x_2 P_2 &= 4P_1 + 6P_2 \\ \implies \frac{1}{2} \times \frac{4P_1 + 6P_2}{P_1} + P_2 \frac{1}{2} \times \frac{4P_1 + 6P_2}{P_2} &= 4P_1 + 6P_2 \\ 2 + 3P_2 + 2 + 3P_2 = 4 + 6P_2 &= 4 + 6P_2 \\ P_2 &= 1 \end{aligned}$$

$$\begin{aligned} MRS^J = \frac{MU_1^J}{MU_2^J} &= \frac{\frac{1}{5x_1}}{\frac{1}{5x_2}} = \frac{P_1}{P_2} \\ \implies \frac{x_2}{x_1} &= \frac{P_1}{P_2} \\ \implies x_2 &= x_1 \end{aligned}$$

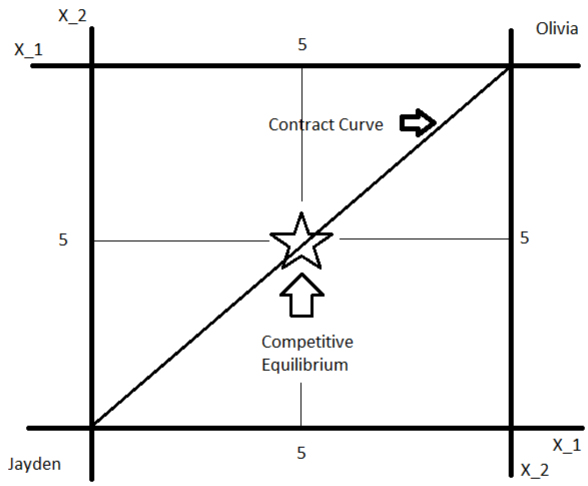
This last step, however, was extraneous expect to show that the contract curve is the set of points where $x_1 = x_2$. Thus, our solution must satisfy this identity. Anyway, revisiting the Cobb-Douglas demands with P_2 in hand, we get:

$$\begin{aligned} x_1 &= \frac{1}{2} \times \frac{4P_1 + 6P_2}{P_1} \\ &= 5 \end{aligned}$$

$$\implies x_1^J = x_2^J = x_1^O = x_2^O = 5 \;\&\; P_1 = P_2 = 1$$

C) As we previously stated, at the competitive equilibrium, it is required that $MRS^J = MRS^O$ which means that $\frac{x_2^J}{x_1^J} = \frac{x_2^O}{x_1^O}$ which is satisfied when $x_1^J = x_2^J = x_1^O = x_2^O = 5$ since $\frac{5}{5} = \frac{5}{5}$

D) Solving analytically for the contract curve requires knowledge of the total endowment of each good in the economy. Here, we can see from the individual endowments that there are 10 units of each good in the economy and that the edgeworth box depicting it is square. To solve for the slope of the contract



curve use the equations:

$$MRS^J = \frac{MU_1^J}{MU_2^J} = \frac{\frac{1}{5x_1^J}}{\frac{1}{5x_2^J}} = \frac{x_2^J}{x_1^J} = MRS^O = \frac{MU_1^O}{MU_2^O} = \frac{10 - x_2^J}{10 - x_1^J}$$
$$\implies x_2^J = x_1^J$$

By symmetry this is true for both individuals. Clearly the initial endowments are not located on the contract curve (no).

Problem 4

A) The MPL is decreasing and there are constant returns to scale exhibited by this production function.

B) Labor demand can be calculated by setting MPL = $\frac{w}{p}$. Given that K is fixed, this makes the equation:

$$\frac{1}{2}L^{-\frac{1}{2}} = \frac{w}{p}$$
$$\frac{1}{2\sqrt{L}} = \frac{w}{p}$$
$$\sqrt{L} = \frac{p}{2w}$$
$$L^* = (\frac{p}{2w})^2$$
$$9 = (\frac{p}{2w})^2$$
$$\frac{1}{6} = \frac{w}{p}$$

C)The unemployment rate is the ratio of the number of hours invidivduals are under-employed relative to the equilibrium level of employment in a free market. Here, when wages are free to adjust due to supply and demand the market clears at a wage real wage of $\frac{1}{6}$ and 9 hours of labor. When the wage is constrained to be $\frac{1}{2}$ the labor demand falls, while the supply remains constant at 9 hours. This yields the following unemployment rate:

$$\frac{1}{2\sqrt{L}} = \frac{1}{2}$$
$$2\sqrt{L} = 2$$
$$L = 1$$
$$\implies UnemploymentRate = \frac{(9-1)}{9} = \frac{8}{9}$$

D) First we must see that in order for the TRS = $\frac{W_K}{W_L}$ it must be the case that K = L:

$$TRS = \frac{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}}{\frac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}} = \frac{W_K}{W_L} = 1$$
$$\implies K = L$$

plugging this result back into the production function to get costs as a function of output, we see that:

$$\begin{aligned} y &= K^{\frac{1}{2}}(K)^{\frac{1}{2}} \\ &= K \\ \implies K = y \end{aligned}$$

thus, the cost function is given by:

$$\begin{aligned} C(y) &= W_K K + W_L L \\ &= 5K + 5L \\ &= 10K \end{aligned} \tag{2}$$

Since above we learned that $K = y$, is must be that case that costs, as a function of output, is given by the equation $C(y) = 10y$.

Group C

Problem #1

A) The budget constraint maps consumption between the two time periods. With the "young" period on the x-axis and the "old" period on the y-axis, the budget constraint appears as

$$\begin{aligned} C_1P_1 + C_2P_2 &\leq m_1 + m_2P_2 \\ P_2 &= \frac{1}{1+r} \\ C_1P_1 + C_2\frac{1}{2} &\leq \$10 + \$60\frac{1}{2} \\ \implies C_1P_1 + C_2\frac{1}{2} &\leq \$40 \end{aligned}$$

Future Value is the \$60 that Joseph gets when he is "old" in addition to the value of the \$10 he gets when he is "young" when he is old:

$$\$10(1+r)+\$60=\$80$$

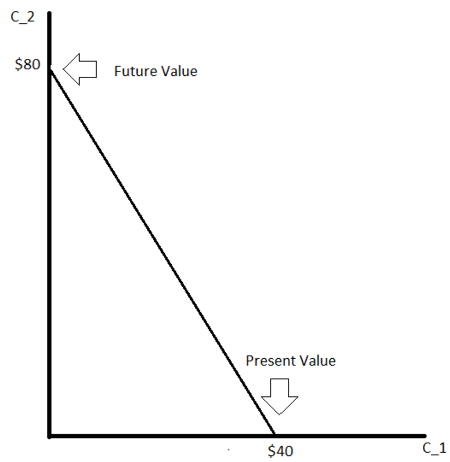
This means that FV = \$80.

Present value is the \$20 Joseph has when he is young plus the value when he is "young" of the \$60 he will get when he is old:

$$\$60(\frac{1}{1+r})+\$10=\$40$$

This means that PV = \$40

Figure 3: Budget Constraint



B) In terms of PV, the optimal demands for consumption are:

$$C_1=\frac{1}{1+\frac{1}{1+\delta}}\times\frac{\$10P_1+\$60P_2}{P_1}$$
$$\implies C_1=\frac{2}{3}\times\frac{\$10+\$60P_2}{1}$$
$$C_2=\frac{\frac{1}{1+\delta}}{1+\frac{1}{1+\delta}}\times\frac{\$10P_1+\$60P_2}{P_2}$$
$$\implies C_2=\frac{1}{3}\times\frac{\$10+\$60P_2}{P_2}$$

From the budget constraint we know that $C_1 + P_2 C_2 \leq 40$, so combining this with the above equations we can recover P_2:

$$\begin{aligned}
\frac{2}{3} \times \frac{\$10 + \$60P_2}{1} + P_2(\frac{1}{3} \times \frac{\$10 + \$60P_2}{P_2}) &= \$40 \\
\$6.66 + \$40P_2 + \$3.33 + \$20P_2 &= \$40 \\
\$60P_2 &= \$30 \\
\implies P_2 &= \frac{1}{2}
\end{aligned}$$

Plugging P_2 into the magic formulas for C_1 and C_2, we get:

$$C_1 = \frac{2}{3} \times \frac{\$10 + \$60\frac{1}{2}}{1} = \$\frac{80}{3}$$
$$C_2 = \frac{1}{3} \times \frac{\$10 + \$60\frac{1}{2}}{\frac{1}{2}} = \$\frac{80}{3}$$

Since Joseph is only endowed with \$10 in the first period, but consumes \$ $\frac{80}{3}$, this means that he is borrowing (in present value) $\$\frac{50}{3}$ from the future ($\$\frac{100}{3}$ in FV). This makes his savings vector S = (-$\$\frac{50}{3}$,0).

C) The formula for an annuity of \$2,400 lasting for three periods, starting next period is:

$$\frac{\$2,400}{r}(1 - (\frac{1}{1+r}^{3})) = \$2,100 < \$2,300$$

This means that the lease is a better option than the purchase since it costs less in terms of present value and available information (assuming you ignore the value of the car remaining after three years).

D) The formula of a stream of payments that never ends is:

$$\begin{aligned}
PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + ... \\
&= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ...] \\
&= \frac{1}{(1+r)}[x + PV]
\end{aligned}$$

so we can solve for PV to get a more concise solution:

$$(1 - \frac{1}{1+r})PV = \frac{1}{1+r}x$$
$$(\frac{1+r}{1+r} - \frac{1}{1+r})PV = \frac{1}{1+r}x$$
$$(\frac{r}{1+r})PV = \frac{1}{1+r}x$$
$$PV = \frac{x}{r}$$

Problem 2

A) To calculate the expected value it is necessary to find the value in each state and multiply it by the probability that each state will occur. The summation is then:

$$EV = 0 * \pi + 64 * (1 - \pi) = 32$$

The certainty equivalent is the amount of money that Trevor would take *for certain* in order to avoid the gamble that would leave him with the *expectation* of \$32. To calculate this you must first find the utility that the gamble provides, and then find the amount of money that would provide the same utility with a 100% probability:

$$\begin{aligned} U(E(18)) &= \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{64} \\ &= 4 \\ U(CE) = 4 &\implies \sqrt{CE} = 4 \\ \implies CE &= 16 \end{aligned}$$

Here we clearly see that the certainty equivalent, 16 is smaller than the expected payoff from the gamble, 32. This makes sense because the concave shape of Trevor's utility function reflects his risk-averse preferences - meaning he is willing to take a smaller payout for certain than the one he would get in expectation to avoid the possibility of being left with no money at all.

B) The expected utility representation is:

$$U(C_1, C_2) = \frac{1}{2}\ln C_1 + \frac{1}{2}\ln C_2$$

The natural log is a concave function, meaning that Trevor is risk averse.
C) First, we calculate the formulas for consumption in each state of the world:

$$C_1 = 64 - x\gamma$$
$$C_2 = (1-\gamma)x$$

Knowing that x, the amount of insurance, is the same in both states of the world, we can solve both of the above equations for x and set them equal to one another:

$$\begin{aligned} x &= \frac{64 - C_1}{\gamma} \\ x &= \frac{C_2}{(1-\gamma)} \end{aligned}$$
$$\implies \frac{64 - C_1}{\gamma} = \frac{C_2}{(1-\gamma)}$$
$$\implies C_1 = 64 - \frac{\gamma}{1-\gamma}C_2$$

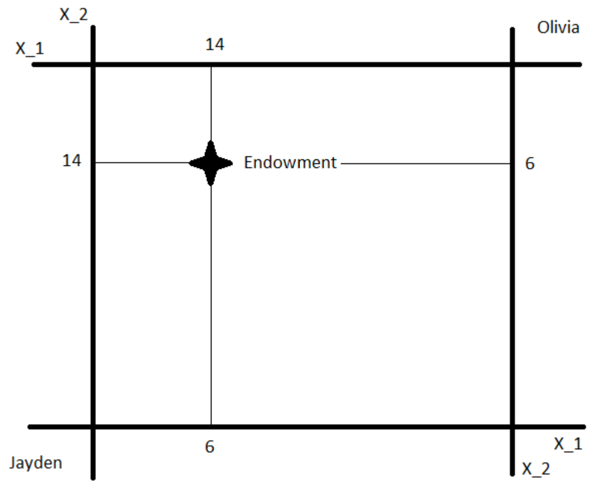
This final equation is our budget constraint. Normalizing P_1 to 1, this makes $P_2 = 1$ as well. Plugging these values into the magic formulas for demand we get:

$$\begin{aligned} C_1 &= \frac{1}{2} \times \frac{64}{P_1} \\ C_2 &= \frac{1}{2} \times \frac{64}{P_2} \\ C_1 = C_2 &= 32 \end{aligned}$$

Going back to the first equation for C_1, we now have:

$$32 = 64 - \frac{1}{2}x$$
$$\implies x = 64$$

Consumption in both states of the world is 32, so that means that Trevor is fully insured. Full insurance means that your consumption does not depend on the state of the world. Trevor has chosen to fully insure in this case



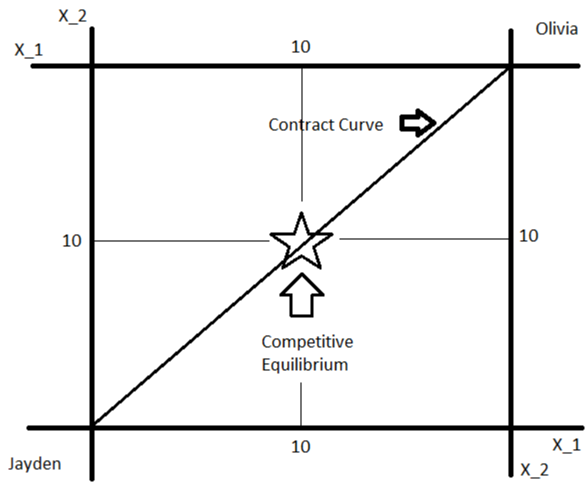
because the insurance premium, γ, is equal to the probability of loss, π.
Problem 3
A)
B) At an equilibrium, we need for MRS^J to equal MRS^O and also for MRS^J to be equal to $\frac{P_1}{P_2}$. We use the first identity to get the contract curve, the second to calculate the slope of the budget line. Given the endowment point, we can follow the budget line away from the endowment point to find its intersection with the contract curve, which is the equilibrium. Since the two unility functions are symetrical, we can solve for Jayden and Olivia both by simply solving for Jayden. Analytically, this is done via the following equations: First, we normalize P_1 to equal 1 and use the Cobb-Douglas magic forumlas to get the P_2:

$$x_1 = \frac{1}{2} \times \frac{6P_1 + 14P_2}{P_1}$$
$$x_2 = \frac{1}{2} \times \frac{6P_1 + 14P_2}{P_2}$$
$$x_1 P_1 + x_2 P_2 = 6P_1 + 14P_2$$
$$\implies \frac{1}{2} \times \frac{6P_1 + 14P_2}{P_1} + P_2 \frac{1}{2} \times \frac{6P_1 + 14P_2}{P_2} = 6P_1 + 14P_2$$
$$3 + 7P_2 + 3 + 7P_2 = 6 + 14P_2 = 6 + 14P_2$$
$$P_2 = 1$$

$$MRS^J = \frac{MU_1^J}{MU_2^J} = \frac{\frac{3}{x_1}}{\frac{3}{x_2}} = \frac{P_1}{P_2}$$
$$\implies \frac{x_2}{x_1} = \frac{P_1}{P_2}$$
$$\implies x_2 = x_1$$

This last step, however, was extraneous expect to show that the contract curve is the set of points where $x_1 = x_2$. Thus, our solution must satisfy this identity. Anyway, revisiting the Cobb-Douglas demands with P_2 in hand, we get:

$$x_1 = \frac{1}{2} \times \frac{6P_1 + 14P_2}{P_1}$$
$$= 10$$
$$\implies x_1^J = x_2^J = x_1^O = x_2^O = 10 \ \& \ P_1 = P_2 = 1$$



C) As we previously stated, at the competitive equilibrium, it is required that $MRS^J = MRS^O$ which means that $\frac{x_2^J}{x_1^J} = \frac{x_2^O}{x_1^O}$ which is satisfied when $x_1^J = x_2^J = x_1^O = x_2^O = 1$ since $\frac{10}{10} = \frac{10}{10}$

D) Solving analytically for the contract curve requires knowledge of the total endowment of each good in the economy. Here, we can see from the individual endowments that there are 10 units of each good in the economy and that the edgeworth box depicting it is square. To solve for the slope of the contract curve use the equations:

$$MRS^J = \frac{MU_1^J}{MU_2^J} = \frac{\frac{3}{x_1^J}}{\frac{3}{x_2^J}} = \frac{x_2^J}{x_1^J} = MRS^O = \frac{MU_1^O}{MU_2^O} = \frac{20 - x_2^J}{20 - x_1^J}$$
$$\implies x_2^J = x_1^J$$

By symmetry this is true for both individuals. Clearly the initial endowments are not located on the contract curve (no).

Problem 4

A) The MPL is decreasing and there are constant returns to scale exhibited by this production function.

B) Labor demand can be calculated by setting MPL = $\frac{w}{p}$. Given that K is fixed, this makes the equation:

$$\frac{1}{2}L^{-\frac{1}{2}} = \frac{w}{p}$$
$$\frac{1}{2\sqrt{L}} = \frac{w}{p}$$
$$\sqrt{L} = \frac{p}{2w}$$
$$L^* = (\frac{p}{2w})^2$$
$$9 = (\frac{p}{2w})^2$$
$$\frac{1}{6} = \frac{w}{p}$$

C)The unemployment rate is the ratio of the number of hours invidivduals are under-employed relative to the equilibrium level of employment in a free market. Here, when wages are free to adjust due to supply and demand the market clears at a wage real wage of $\frac{1}{6}$ and 9 hours of labor. When the wage is constrained to be $\frac{1}{2}$ the labor demand falls, while the supply remains constant at 9 hours. This yields the following unemployment rate:

$$\frac{1}{2\sqrt{L}} = \frac{1}{2}$$
$$2\sqrt{L} = 2$$
$$L = 1$$
$$\Longrightarrow UnemploymentRate = \frac{(9-1)}{9} = \frac{8}{9}$$

D) First we must see that in order for the TRS = $\frac{W_K}{W_L}$ it must be the case that K = L:

$$TRS = \frac{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}}{\frac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}} = \frac{W_K}{W_L} = 1$$
$$\Longrightarrow K = L$$

plugging this result back into the production function to get costs as a function of output, we see that:

$$\begin{aligned} y &= K^{\frac{1}{2}}(K)^{\frac{1}{2}} \\ &= K \\ \Longrightarrow K = y & \end{aligned}$$

thus, the cost function is given by:

$$\begin{aligned} C(y) &= W_K K + W_L L \\ &= 3K + 3L \\ &= 6K \end{aligned} \tag{3}$$

Since above we learned that $K = y$, is must be that case that costs, as a function of output, is given by the equation $C(y) = 16y$.

Group D

Problem #1

A) The budget constraint maps consumption between the two time periods. With the "young" period on the x-axis and the "old" period on the y-axis, the budget constraint appears as

$$\begin{aligned} C_1P_1 + C_2P_2 &\leq m_1 + m_2P_2 \\ P_2 &= \frac{1}{1+r} \\ C_1P_1 + C_2\frac{1}{2} &\leq \$10 + \$100\frac{1}{2} \\ \Longrightarrow C_1P_1 + C_2\frac{1}{2} &\leq \$60 \end{aligned}$$

Future Value is the \$100 that Joseph gets when he is "old" in addition to the value of the \$10 he gets when he is "young" when he is old:

$$\$10(1+r) + \$100 = \$120$$

This means that FV = \$120.

Present value is the \$10 Joseph has when he is young plus the value when he is "young" of the \$100 he will get when he is old:

$$\$100(\frac{1}{1+r}) + \$10 = \$60$$

This means that PV = \$60

B) In terms of PV, the optimal demands for consumption are:

$$\begin{aligned} C_1 &= \frac{1}{1+\frac{1}{1+\delta}} \times \frac{\$10P_1 + \$100P_2}{P_1} \\ \Longrightarrow C_1 &= \frac{2}{3} \times \frac{\$10 + \$100P_2}{1} \\ C_2 &= \frac{\frac{1}{1+\delta}}{1+\frac{1}{1+\delta}} \times \frac{\$10P_1 + \$100P_2}{P_2} \\ \Longrightarrow C_2 &= \frac{1}{3} \times \frac{\$10 + \$100P_2}{P_2} \end{aligned}$$

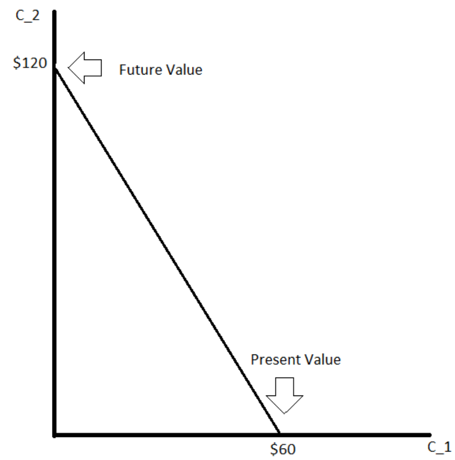


Figure 4: Budget Constraint

From the budget constraint we know that $C_1 + P_2C_2 \leq 60$, so combining this with the above equations we can recover P_2:

$$\begin{aligned}
\frac{2}{3} \times \frac{\$10 + \$100P_2}{1} + P_2(\frac{1}{3} \times \frac{\$10 + \$100P_2}{P_2}) &= \$60 \\
\$6.66 + \$66.6P_2 + \$3.33 + \$33.3P_2 &= \$60 \\
\$100P_2 &= \$50 \\
\implies P_2 &= \frac{1}{2}
\end{aligned}$$

Plugging P_2 into the magic formulas for C_1 and C_2, we get:

$$C_1 = \frac{2}{3} \times \frac{\$10 + \$100\frac{1}{2}}{1} = \$40$$

$$C_2 = \frac{1}{3} \times \frac{\$10 + \$100\frac{1}{2}}{\frac{1}{2}} = \$40$$

Since Joseph is only endowed with \$10 in the first period, but consumes \$ 40, this means that he is borrowing (in present value) \$30 from the future (\$60 in FV). This makes his savings vector S = (-\$30,0).

C) The formula for an annuity of \$2,400 lasting for three periods, starting next period is:

$$\frac{\$2,400}{r}(1 - (\frac{1}{1+r}^{3})) = \$2,100 < \$2,300$$

This means that the lease is a better option than the purchase since it costs less in terms of present value and available information (assuming you ignore the value of the car remaining after three years).

D) The formula of a stream of payments that never ends is:

$$\begin{aligned} PV &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + ... \\ &= \frac{1}{(1+r)}[x + \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + ...] \\ &= \frac{1}{(1+r)}[x + PV] \end{aligned}$$

so we can solve for PV to get a more concise solution:

$$\begin{aligned} (1 - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{1+r}{1+r} - \frac{1}{1+r})PV &= \frac{1}{1+r}x \\ (\frac{r}{1+r})PV &= \frac{1}{1+r}x \\ PV &= \frac{x}{r} \end{aligned}$$

Problem 2

A) To calculate the expected value it is necessary to find the value in each state and multiply it by the probability that each state will occur. The summation is then:

$$EV = 0 * \pi + 100 * (1 - \pi) = 50$$

The certainty equivalent is the amount of money that Trevor would take *for certain* in order to avoid the gamble that would leave him with the *expectation* of \$50. To calculate this you must first find the utility that the gamble provides, and then find the amount of money that would provide the same utility with a 100% probability:

$$\begin{aligned} U(E(50)) &= \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{100} \\ &= 5 \\ U(CE) = 5 &\implies \sqrt{CE} = 5 \\ \implies CE &= 25 \end{aligned}$$

Here we clearly see that the certainty equivalent, 25 is smaller than the expected payoff from the gamble, 50. This makes sense because the concave shape of Trevor's utility function reflects his risk-averse preferences - meaning he is willing to take a smaller payout for certain than the one he would get in expectation to avoid the possibility of being left with no money at all.

B) The expected utility representation is:

$$U(C_1, C_2) = \frac{1}{2}\ln C_1 + \frac{1}{2}\ln C_2$$

The natural log is a concave function, meaning that Trevor is risk averse.
C) First, we calculate the formulas for consumption in each state of the world:

$$C_1 = 100 - x\gamma$$
$$C_2 = (1-\gamma)x$$

Knowing that x, the amount of insurance, is the same in both states of the world, we can solve both of the above equations for x and set them equal to one another:

$$x = \frac{100 - C_1}{\gamma}$$
$$x = \frac{C_2}{(1-\gamma)}$$
$$\implies \frac{100 - C_1}{\gamma} = \frac{C_2}{(1-\gamma)}$$
$$\implies C_1 = 100 - \frac{\gamma}{1-\gamma}C_2$$

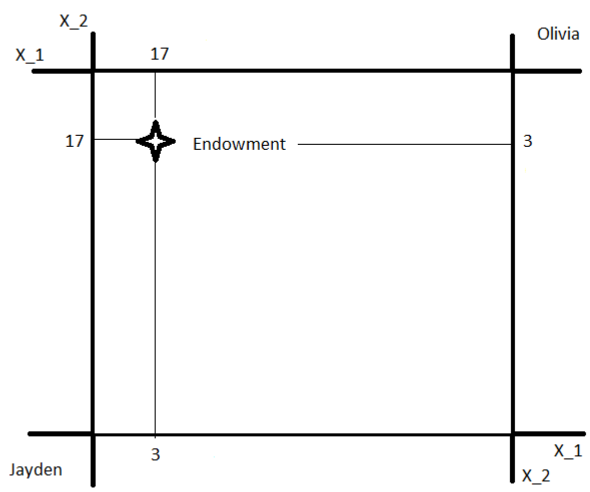
This final equation is our budget constraint. Normalizing P_1 to 1, this makes $P_2 = 1$ as well. Plugging these values into the magic formulas for demand we get:

$$C_1 = \frac{1}{2} \times \frac{100}{P_1}$$
$$C_2 = \frac{1}{2} \times \frac{100}{P_2}$$
$$C_1 = C_2 = 50$$

Going back to the first equation for C_1, we now have:

$$50 = 100 - \frac{1}{2}x$$
$$\implies x = 100$$

Consumption in both states of the world is 50, so that means that Trevor is fully insured. Full insurance means that your consumption does not depend on the state of the world. Trevor has chosen to fully insure in this case because the insurance premium, γ, is equal to the probability of loss, π.
Problem 3
A)



B) At an equilibrium, we need for MRS^J to equal MRS^O and also for MRS^J to be equal to $\frac{P_1}{P_2}$. We use the first identity to get the contract curve, the second to calculate the slope of the budget line. Given the endowment point, we can follow the budget line away from the endowment point to find its intersection with the contract curve, which is the equilibrium. Since the two unility functions are symetrical, we can solve for Jayden and Olivia both by simply solving for Jayden. Analytically, this is done via the following equations: First, we normalize P_1 to equal 1 and use the Cobb-Douglas magic forumlas to get the P_2:

$$\begin{aligned}
x_1 &= \frac{1}{2} \times \frac{3P_1 + 17P_2}{P_1} \\
x_2 &= \frac{1}{2} \times \frac{3P_1 + 17P_2}{P_2} \\
x_1P_1 + x_2P_2 &= 3P_1 + 17P_2 \\
\implies \frac{1}{2} \times \frac{3P_1 + 17P_2}{P_1} + P_2\frac{1}{2} \times \frac{3P_1 + 17P_2}{P_2} &= 3P_1 + 17P_2 \\
1.5 + 8.5P_2 + 1.5 + 8.5P_2 = 3 + 17P_2 &= 3 + 17P_2 \\
P_2 &= 1
\end{aligned}$$

$$\begin{aligned}
MRS^J = \frac{MU_1^J}{MU_2^J} &= \frac{\frac{5}{x_1}}{\frac{5}{x_2}} = \frac{P_1}{P_2} \\
\implies \frac{x_2}{x_1} &= \frac{P_1}{P_2} \\
\implies x_2 &= x_1
\end{aligned}$$

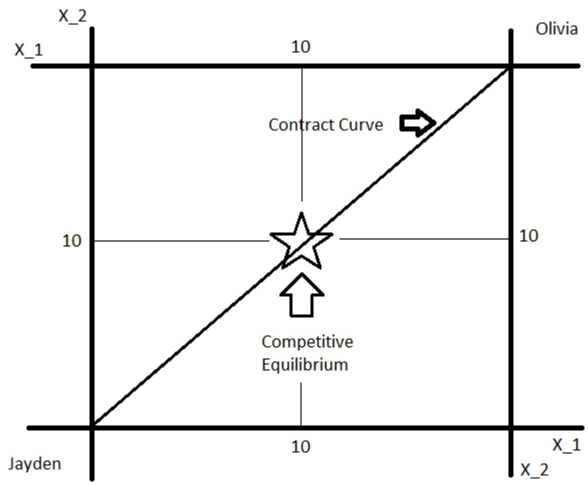
This last step, however, was extraneous expect to show that the contract curve is the set of points where $x_1 = x_2$. Thus, our solution must satisfy this identity. Anyway, revisiting the Cobb-Douglas demands with P_2 in hand, we get:

$$\begin{aligned}
x_1 &= \frac{1}{2} \times \frac{3P_1 + 17P_2}{P_1} \\
&= 10
\end{aligned}$$

$$\implies x_1^J = x_2^J = x_1^O = x_2^O = 10 \;\&\; P_1 = P_2 = 1$$

C) As we previously stated, at the competitive equilibrium, it is required that $MRS^J = MRS^O$ which means that $\frac{x_2^J}{x_1^J} = \frac{x_2^O}{x_1^O}$ which is satisfied when $x_1^J = x_2^J = x_1^O = x_2^O = 1$ since $\frac{10}{10} = \frac{10}{10}$

D) Solving analytically for the contract curve requires knowledge of the total endowment of each good in the economy. Here, we can see from the individual endowments that there are 10 units of each good in the economy and that the edgeworth box depicting it is square. To solve for the slope of the contract



curve use the equations:

$$MRS^J = \frac{MU_1^J}{MU_2^J} = \frac{\frac{5}{x_1^J}}{\frac{5}{x_2^J}} = \frac{x_2^J}{x_1^J} = MRS^O = \frac{MU_1^O}{MU_2^O} = \frac{20 - x_2^J}{20 - x_1^J}$$
$$\implies x_2^J = x_1^J$$

By symmetry this is true for both individuals. Clearly the initial endowments are not located on the contract curve (no).

Problem 4

A) The MPL is decreasing and there are constant returns to scale exhibited by this production function.

B) Labor demand can be calculated by setting MPL = $\frac{w}{p}$. Given that K is fixed, this makes the equation:

$$\frac{1}{2}L^{-\frac{1}{2}} = \frac{w}{p}$$
$$\frac{1}{2\sqrt{L}} = \frac{w}{p}$$
$$\sqrt{L} = \frac{p}{2w}$$
$$L^* = (\frac{p}{2w})^2$$
$$9 = (\frac{p}{2w})^2$$
$$\frac{1}{6} = \frac{w}{p}$$

C)The unemployment rate is the ratio of the number of hours invidivduals are under-employed relative to the equilibrium level of employment in a free market. Here, when wages are free to adjust due to supply and demand the market clears at a wage real wage of $\frac{1}{6}$ and 9 hours of labor. When the wage is constrained to be $\frac{1}{2}$ the labor demand falls, while the supply remains constant at 9 hours. This yields the following unemployment rate:

$$\frac{1}{2\sqrt{L}} = \frac{1}{2}$$
$$2\sqrt{L} = 2$$
$$L = 1$$
$$\implies UnemploymentRate = \frac{(9-1)}{9} = \frac{8}{9}$$

D) First we must see that in order for the TRS = $\frac{W_K}{W_L}$ it must be the case that K = L:

$$TRS = \frac{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{2}}}{\frac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}} = \frac{W_K}{W_L} = 1$$
$$\implies K = L$$

plugging this result back into the production function to get costs as a function of output, we see that:

$$\begin{aligned} y &= K^{\frac{1}{2}}(K)^{\frac{1}{2}} \\ &= K \\ \Longrightarrow K = y \end{aligned}$$

thus, the cost function is given by:

$$\begin{aligned} C(y) &= W_K K + W_L L \\ &= 2K + 2L \\ &= 4K \end{aligned} \tag{4}$$

Since above we learned that $K = y$, is must be that case that costs, as a function of output, is given by the equation $C(y) = 4y$.