

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (55+15+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (55p) (Well-behaved preferences)

Ava is a big fan of delicious gourmet steaks: their robust and hearty beef flavors is what she likes the most. Her two most favorite steaks are marinated ribeye (x_1) and top sirloin (x_2).

a) The price of a marinated ribeye steak is $p_1 = \$50$, the price of top sirloin is $p_2 = \$25$, and her income (spent entirely on steaks) is $m = \$500$. Show geometrically Ava's budget constraint. Mark the two extreme consumption bundles (give concrete values). Show on the graph how her budget set would be affected by inflation that increased the prices of both steaks by 100%, leaving m unchanged (draw a new budget line).

b) Ava's utility function over both types of steaks is $U(x_1, x_2) = x_1^5 x_2^5$. Find her marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give a formula for MRS). What is the value of MRS at bundle $(2, 4)$ (one number)? Which of the two steak types is more valuable to Ava?

d) Write down two secrets of happiness, assuming that p_1, p_2, m are parameters (two equations).

- Provide economic intuition behind the two conditions (two sentences for each).

- Derive optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation of magic formulas).

- Using magic formulas determine whether the solution is interior for all values of p_1, p_2, m (one sentence).

- What fraction of total income is spent on top sirloin?

e) Find the optimal consumption levels of two types of steak (x_1, x_2) for:

- $p_1 = \$50, p_2 = \25 and $m = \$500$ (give two numbers).

and after the price of ribeye decreased:

- for $p_1 = \$25, p_2 = \25 and $m = \$500$ (give two numbers).

What is the total change in consumption of marinated ribeye? (give a number). Illustrate the change on the graph. Is marinated ribeye an ordinary or a Giffen good? (Choose one + one sentence explaining why.)

f) Decompose the total change in consumption of x_1 from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

Problem 2 (15p) (Perfect complements)

Alex is a hotdog lover. In each hotdog bun (x_1) Alex always inserts two hotdogs (x_2): he hates hotdogs with ratios of buns to hotdogs different from 1:2.

a) Propose a utility function that captures Alex's preferences over (x_1) and (x_2) (function $U(x_1, x_2)$).

b) In the commodity space, carefully depict Alex's indifference curves (marking the optimal proportion line).

c) Assume prices $p_1 = 4$ and $p_2 = 3$ and income $m = 40$. Write down two secrets of happiness (two equations) and determine the optimal choice (two numbers). Is your solution interior (yes or no)?

d) Suppose price of a hotdog bun goes down to $p_1 = 2$, while price $p_2 = 3$ and income $m = 40$ are unchanged. Find optimal choice (two numbers). Without calculations, how much of this change can be attributed to a substitution effect (number + one sentence explaining the number).

Problem 3 (15p) (Perfect substitutes)

Pepsi x_1 and Coke x_2 both are goods that are indistinguishable.

a) Propose a utility function (the "craziest" function you can imagine), which captures preferences for such perfect substitutes.

b) Plot the budget line, assuming $p_1 = \$2, p_2 = \3 and $m = \$12$. In the same graph plot indifference curves.

- c) Find the optimal bundle for $U(x_1, x_2) = x_1 + x_2$. (give two numbers). Is your solution interior or corner? (one sentence)
- d) Plot income offer curve and Engel curve given fixed prices $p_1 = \$2, p_2 = \3 . (two graphs). Is Pepsi normal or inferior (choose one + one sentence)

Problem 4 (15p) (Quasilinear Preferences, Labor Supply)

Your best friend Aiden asked you to help him to determine how much time he should spend at work. His total available time (each day) is 24h and his only source of income is labor income given wage rate $w = \$10$. His spending on consumption good C is equal to what he earns. The price of such good is $p_c = 2$

- a) What is his real wage rate? (number+interpretation, one sentence)
- b) Plot his budget set on the graph.
- c) Aiden has quasilinear utility function $U(R, C) = \ln R + C$ where R is leisure (or relaxation time) and C is consumption of other goods. Write down two secrets of happiness that determine optimal choice of R, C for arbitrary w, p_c . Find optimal R as a function w, p_c (you can assume interior solution).
- d) What is Aiden's labor supply L if $w = \$1$ and $p_c = \$10$ (one number). How about $w = \$2$ and $p_c = \$10$ (one number). Is labor supply increasing in real wage rate, when preferences are quasilinear? (yes no answer)

Bonus question (Just for fun)

Let MRS^V and MRS^U be marginal rates of substitution of utility functions $V(x_1, x_2)$ and $U(x_1, x_2)$ respectively. Moreover, assume that $V(x_1, x_2) = f[U(x_1, x_2)]$ where $f[\cdot]$ is a strictly monotone transformation. Show that $MRS^V = MRS^U$. (Hint: Use formula for the derivative of a composite function)

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Midterm 1 (B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (55+15+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (55p) (Well-behaved preferences)

Ava is a big fan of delicious gourmet steaks: their robust and hearty beef flavors is what she likes the most. Her two most favorite steaks are marinated ribeye (x_1) and top sirloin (x_2).

a) The price of a marinated ribeye steak is $p_1 = \$10$, the price of top sirloin is $p_2 = \$5$, and her income (spent entirely on steaks) is $m = \$100$. Show geometrically Ava's budget constraint. Mark the two extreme consumption bundles (give concrete values). Show on the graph how her budget set would be affected by inflation that increased the prices of both steaks by 200%, leaving m unchanged (draw a new budget line).

b) Ava's utility function over both types of steaks is $U(x_1, x_2) = x_1^3 x_2^3$. Find her marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give a formula for MRS). What is the value of MRS at bundle $(3, 6)$ (one number)? Which of the two steak types is more valuable to Ava?

d) Write down two secrets of happiness, assuming that p_1, p_2, m are parameters (two equations).

- Provide economic intuition behind the two conditions (two sentences for each).

- Derive optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation of magic formulas).

- Using magic formulas determine whether the solution is interior for all values of p_1, p_2, m (one sentence).

- What fraction of total income is spent on top sirloin?

e) Find the optimal consumption levels of two types of steak (x_1, x_2) for:

- $p_1 = \$10, p_2 = \5 and $m = \$100$ (give two numbers).

and after the price of ribeye decreased:

- for $p_1 = \$5, p_2 = \5 and $m = \$100$ (give two numbers).

What is the total change in consumption of marinated ribeye? (give a number). Illustrate the change on the graph. Is marinated ribeye an ordinary or a Giffen good? (Choose one + one sentence explaining why.)

f) Decompose the total change in consumption of x_1 from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

Problem 2 (15p) (Perfect complements)

Alex is a hotdog lover. In each hotdog bun (x_1) Alex always inserts three hotdogs (x_2): he hates hotdogs with ratios of buns to hotdogs different from 1:3.

a) Propose a utility function that captures Alex's preferences over (x_1) and (x_2) (function $U(x_1, x_2)$).

b) In the commodity space, carefully depict Alex's indifference curves (marking the optimal proportion line).

c) Assume prices $p_1 = 4$ and $p_2 = 2$ and income $m = 40$. Write down two secrets of happiness (two equations) and determine the optimal choice (two numbers). Is your solution interior (yes or no)?

d) Suppose price of a hotdog bun goes down to $p_1 = 2$, while price $p_2 = 2$ income $m = 40$ are unchanged. Find optimal choice (two numbers). Without calculations, how much of this change can be attributed to a substitution effect (number + one sentence explaining the number).

Problem 3 (15p) (Perfect substitutes)

Pepsi x_1 and Coke x_2 both are goods that are indistinguishable.

a) Propose a utility function (the "craziest" function you can imagine), which captures preferences for such perfect substitutes.

b) Plot the budget line, assuming $p_1 = \$4, p_2 = \2 and $m = \$12$. In the same graph plot indifference curves.

- c) Find the optimal bundle for $U(x_1, x_2) = x_1 + x_2$. (give two numbers). Is your solution interior or corner? (one sentence)
- d) Plot income offer curve and Engel curve given fixed prices $p_1 = \$4, p_2 = \2 . (two graphs). Is Pepsi normal or inferior (choose one + one sentence)

Problem 4 (15p) (Quasilinear Preferences, Labor Supply)

Your best friend Aiden asked you to help him to determine how much time he should spend at work. His total available time (each day) is 24h and his only source of income is labor income given wage rate $w = \$10$. His spending on consumption good C is equal to what he earns. The price of such good is $p_c = 2$

- a) What is his real wage rate? (number+interpretation, one sentence)
- b) Plot his budget set on the graph.
- c) Aiden has quasilinear utility function $U(R, C) = \ln R + C$ where R is leisure (or relaxation time) and C is consumption of other goods. Write down two secrets of happiness that determine optimal choice of R, C for arbitrary w, p_c . Find optimal R as a function w, p_c (you can assume interior solution).
- d) What is Aiden's labor supply L if $w = \$1$ and $p_c = \$10$ (one number). How about $w = \$2$ and $p_c = \$10$ (one number). Is labor supply increasing in real wage rate, when preferences are quasilinear? (yes no answer)

Bonus question (Just for fun)

Let MRS^V and MRS^U be marginal rates of substitution of utility functions $V(x_1, x_2)$ and $U(x_1, x_2)$ respectively. Moreover, assume that $V(x_1, x_2) = f[U(x_1, x_2)]$ where $f[\cdot]$ is a strictly monotone transformation. Show that $MRS^V = MRS^U$. (Hint: Use formula for the derivative of a composite function)

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Midterm 1 (C)

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Problem 1 (55p) (Well-behaved preferences)

Ava is a big fan of delicious gourmet steaks: their robust and hearty beef flavors is what she likes the most. Her two most favorite steaks are marinated ribeye (x_1) and top sirloin (x_2).

a) The price of a marinated ribeye steak is $p_1 = \$10$, the price of top sirloin is $p_2 = \$5$, and her income (spent entirely on steaks) is $m = \$200$. Show geometrically Ava's budget constraint. Mark the two extreme consumption bundles (give concrete values). Show on the graph how her budget set would be affected by inflation that increased the prices of both steaks by 100%, leaving m unchanged (draw a new budget line).

b) Ava's utility function over both types of steaks is $U(x_1, x_2) = x_1^{\frac{1}{2}}x_2^{\frac{1}{2}}$. Find her marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give a formula for MRS). What is the value of MRS at bundle $(3, 6)$ (one number)? Which of the two steak types is more valuable to Ava?

d) Write down two secrets of happiness, assuming that p_1, p_2, m are parameters (two equations).

- Provide economic intuition behind the two conditions (two sentences for each).

- Derive optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation of magic formulas).

- Using magic formulas determine whether the solution is interior for all values of p_1, p_2, m (one sentence).

- What fraction of total income is spent on top sirloin?

e) Find the optimal consumption levels of two types of steak (x_1, x_2) for:

- $p_1 = \$10, p_2 = \5 and $m = \$200$ (give two numbers).

and after the price of ribeye decreased:

- for $p_1 = \$5, p_2 = \5 and $m = \$200$ (give two numbers).

What is the total change in consumption of marinated ribeye? (give a number). Illustrate the change on the graph. Is marinated ribeye an ordinary or a Giffen good? (Choose one + one sentence explaining why.)

f) Decompose the total change in consumption of x_1 from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

Problem 2 (15p) (Perfect complements)

Alex is a hotdog lover. In each hotdog bun (x_1) Alex always inserts four hotdogs (x_2): he hates hotdogs with ratios of buns to hotdogs different from 1:4.

a) Propose a utility function that captures Alex's preferences over (x_1) and (x_2) (function $U(x_1, x_2)$).

b) In the commodity space, carefully depict Alex's indifference curves (marking the optimal proportion line).

c) Assume prices $p_1 = 6$ and $p_2 = 1$ and income $m = 40$. Write down two secrets of happiness (two equations) and determine the optimal choice (two numbers). Is your solution interior (yes or no)?

d) Suppose price of a hotdog bun goes down to $p_1 = 4$, while price $p_2 = 1$ income $m = 40$ are unchanged. Find optimal choice (two numbers). Without calculations, how much of this change can be attributed to a substitution effect (number + one sentence explaining the number).

Problem 3 (15p) (Perfect substitutes)

Pepsi x_1 and Coke x_2 both are goods that are indistinguishable.

a) Propose a utility function (the "craziest" function you can imagine), which captures preferences for such perfect substitutes.

b) Plot the budget line, assuming $p_1 = \$6, p_2 = \2 and $m = \$12$. In the same graph plot indifference curves.

- c) Find the optimal bundle for $U(x_1, x_2) = x_1 + x_2$. (give two numbers). Is your solution interior or corner? (one sentence)
- d) Plot income offer curve and Engel curve given fixed prices $p_1 = \$6, p_2 = \2 . (two graphs). Is Pepsi normal or inferior (choose one + one sentence)

Problem 4 (15p) (Quasilinear Preferences, Labor Supply)

Your best friend Aiden asked you to help him to determine how much time he should spend at work. His total available time (each day) is 24h and his only source of income is labor income given wage rate $w = \$10$. His spending on consumption good C is equal to what he earns. The price of such good is $p_c = 2$

- a) What is his real wage rate? (number+interpretation, one sentence)
- b) Plot his budget set on the graph.
- c) Aiden has quasilinear utility function $U(R, C) = \ln R + C$ where R is leisure (or relaxation time) and C is consumption of other goods. Write down two secrets of happiness that determine optimal choice of R, C for arbitrary w, p_c . Find optimal R as a function w, p_c (you can assume interior solution).
- d) What is Aiden's labor supply L if $w = \$1$ and $p_c = \$10$ (one number). How about $w = \$2$ and $p_c = \$10$ (one number). Is labor supply increasing in real wage rate, when preferences are quasilinear? (yes no answer)

Bonus question (Just for fun)

Let MRS^V and MRS^U be marginal rates of substitution of utility functions $V(x_1, x_2)$ and $U(x_1, x_2)$ respectively. Moreover, assume that $V(x_1, x_2) = f[U(x_1, x_2)]$ where $f[\cdot]$ is a strictly monotone transformation. Show that $MRS^V = MRS^U$. (Hint: Use formula for the derivative of a composite function)

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Midterm 1 (D)

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Problem 1 (55p) (Well-behaved preferences)

Ava is a big fan of delicious gourmet steaks: their robust and hearty beef flavors is what she likes the most. Her two most favorite steaks are marinated ribeye (x_1) and top sirloin (x_2).

a) The price of a marinated ribeye steak is $p_1 = \$4$, the price of top sirloin is $p_2 = \$2$, and her income (spent entirely on steaks) is $m = \$80$. Show geometrically Ava's budget constraint. Mark the two extreme consumption bundles (give concrete values). Show on the graph how her budget set would be affected by inflation that increased the prices of both steaks by 100%, leaving m unchanged (draw a new budget line).

b) Ava's utility function over both types of steaks is $U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$. Find her marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give a formula for MRS). What is the value of MRS at bundle $(3, 6)$ (one number)? Which of the two steak types is more valuable to Ava?

d) Write down two secrets of happiness, assuming that p_1, p_2, m are parameters (two equations).

- Provide economic intuition behind the two conditions (two sentences for each).

- Derive optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation of magic formulas).

- Using magic formulas determine whether the solution is interior for all values of p_1, p_2, m (one sentence).

- What fraction of total income is spent on top sirloin?

e) Find the optimal consumption levels of two types of steak (x_1, x_2) for:

- $p_1 = \$4, p_2 = \2 and $m = \$80$ (give two numbers).

and after the price of ribeye decreased:

- for $p_1 = \$2, p_2 = \2 and $m = \$80$ (give two numbers).

What is the total change in consumption of marinated ribeye? (give a number). Illustrate the change on the graph. Is marinated ribeye an ordinary or a Giffen good? (Choose one + one sentence explaining why.)

f) Decompose the total change in consumption of x_1 from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

Problem 2 (15p) (Perfect complements)

Alex is a hotdog lover. In each hotdog bun (x_1) Alex always inserts two hotdogs (x_2): he hates hotdogs with ratios of buns to hotdogs different from 1:2.

a) Propose a utility function that captures Alex's preferences over (x_1) and (x_2) (function $U(x_1, x_2)$).

b) In the commodity space, carefully depict Alex's indifference curves (marking the optimal proportion line).

c) Assume prices $p_1 = 2$ and $p_2 = 2$ and income $m = 30$. Write down two secrets of happiness (two equations) and determine the optimal choice (two numbers). Is your solution interior (yes or no)?

d) Suppose price of a hotdog bun goes down to $p_1 = 1$, while price $p_2 = 2$ income $m = 30$ are unchanged. Find optimal choice (two numbers). Without calculations, how much of this change can be attributed to a substitution effect (number + one sentence explaining the number).

Problem 3 (15p) (Perfect substitutes)

Pepsi x_1 and Coke x_2 both are goods that are indistinguishable.

a) Propose a utility function (the "craziest" function you can imagine), which captures preferences for such perfect substitutes.

b) Plot the budget line, assuming $p_1 = \$2, p_2 = \6 and $m = \$12$. In the same graph plot indifference curves.

- c) Find the optimal bundle for $U(x_1, x_2) = x_1 + x_2$. (give two numbers). Is your solution interior or corner? (one sentence)
- d) Plot income offer curve and Engel curve given fixed prices $p_1 = \$2, p_2 = \6 . (two graphs). Is Pepsi normal or inferior (choose one + one sentence)

Problem 4 (15p) (Quasilinear Preferences, Labor Supply)

Your best friend Aiden asked you to help him to determine how much time he should spend at work. His total available time (each day) is 24h and his only source of income is labor income given wage rate $w = \$10$. His spending on consumption good C is equal to what he earns. The price of such good is $p_c = 2$

- a) What is his real wage rate? (number+interpretation, one sentence)
- b) Plot his budget set on the graph.
- c) Aiden has quasilinear utility function $U(R, C) = \ln R + C$ where R is leisure (or relaxation time) and C is consumption of other goods. Write down two secrets of happiness that determine optimal choice of R, C for arbitrary w, p_c . Find optimal R as a function w, p_c (you can assume interior solution).
- d) What is Aiden's labor supply L if $w = \$1$ and $p_c = \$10$ (one number). How about $w = \$2$ and $p_c = \$10$ (one number). Is labor supply increasing in real wage rate, when preferences are quasilinear? (yes no answer)

Bonus question (Just for fun)

Let MRS^V and MRS^U be marginal rates of substitution of utility functions $V(x_1, x_2)$ and $U(x_1, x_2)$ respectively. Moreover, assume that $V(x_1, x_2) = f[U(x_1, x_2)]$ where $f[\cdot]$ is a strictly monotone transformation. Show that $MRS^V = MRS^U$. (Hint: Use formula for the derivative of a composite function)

Midterm Exam 1 Solutions
(A) The Pink

Q1) (55 points)

a) Check figure 1 for the budget set (3 p)
If there is 100% inflation, the prices double.
The new budget set is also shown on Figure 1. (3 p)

b) $U(X_1, X_2) = X_1^5 X_2^5$

$$\frac{MRS^{12}}{X_2/X_1} = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2) = ((5X_1^4 X_2^5)/(5X_1^5 X_2^4)) =$$

(4 p)

at the point $x_1 = 2$ & $x_2 = 4$: $MRS^{12} = (4/2) = 2$ (2 p)

which means:

$MU_1 > MU_2$ So 1st good, ribeye, is more valuable than top sirloin to Ava
at the consumption level of

$x_1 = 2$ & $x_2 = 4$ (4 p)

d)

secret 1 : $p_1 x_1 + p_2 x_2 = m$ (3p)

so $50x_1 + 25x_2 = 500$

This means "spend all of your money". If the consumer does not spend all of his/her money s/he is wasting his/her opportunity to increase his/her utility since money does not have an effect on utility itself. (2p)

secret 2 : $MRS^{12} = (MU_1)/(MU_2) = P_1/P_2$ (3p)

$\rightarrow X_2/X_1 = P_1/P_2$

"the last spent on each good should give the same utility" OR "marginal utility of a \$ spent on each good should be equal". If this condition does not hold, let's say last \$ spent on good 1 brings more utility than the last \$ spent on good 2, then the consumer should buy less of the second good and buy more of the first good to increase his/her utility. (2p)

start with secret 2 : $X_2/X_1 = P_1/P_2 \rightarrow X_2 = ((X_1 P_1)/(P_2))$ (1p)

now plug this in secret 1 :

$p_1 x_1 + p_2 x_2 = m = P_1 x_1 + P_2 (X_1 P_1)/(P_2) = m = 2P_1 x_1$ (2p)

\rightarrow Then $X_1 = (m/(2p_1)) = (1/2)(m/p_1)$

Since $X_2 = ((X_1 P_1)/(P_2)) = ((m P_1)/(2P_1 P_2)) = (m/(2p_2))$
 $= (1/2)(m/(p_2)) = X_2$

$$X_1 = (m/(2p_1))$$

$$X_2 = (m/(2p_2)) \quad (2p)$$

The solution is interior since none of good's optimal consumption level is zero. Furthermore in a Cobb-Douglas case the solution has to be interior as long as income level (m) is larger than zero ! (we also assume positive prices)
(1p)

$$\text{Using the magic formula } X_2 = (m/(2p_2)) \rightarrow X_2 p_2 = m/2$$

note that the left hand side is the total money spent on good 2 (sirleon).
So half of the income is spent on sirleon. (2p)

$$\text{e) } X_1 = (m/(2p_1)) = ((500)/(2 * 50)) = 5 \quad (1p)$$

$$X_2 = (m/(2p_2)) = ((500)/(2 * 25)) = 10 \quad (1p)$$

$$X_1' = (m/(2p_1')) = ((500)/(2 * 25)) = 10 \quad (1p)$$

$$X_2' = (m/(2p_2)) = ((500)/(2 * 25)) = 10 \quad (1p)$$

$$\text{Total change in consumption of ribeye is } X_1' - X_1 = 10 - 5 = 5 \quad (1p)$$

Check figure 2 for the illustration of the change. (3p)

Ribeye is an ordinary good since it's consumption increases as it's price decrease. We can also see this fact from the magic formula . Since the price of ribeye is in the denominator in optimal consumption formula of ribeye, as the price goes down the consumption of it will increase ! (2p)

f) How much money does Ava need to consume the original bundle with the new prices ?

$$p_1' x_1 + p_2 x_2 = m' = (\$25 * 5) + (\$25 * 10) = \$375 \quad \text{is enough} \quad (3p)$$

Now calculating the optimal bundle with this imaginary income :

$$X_1^s = (m'/(2p_1)) = ((375)/(2 * 25)) = 15/2 \quad (2p)$$

$$X_2^s = (m'/(2p_2)) = ((375)/(2 * 25)) = 15/2$$

$$\text{So Substitution Effect (S.E.) is : } 15/2 - 5 = 5/2 \quad (2p)$$

$$T.E = S.E. + I.E. \quad \text{then : } I.E. = 5 - (5/2) = 5/2 \quad (1p)$$

Check figure 2 for the illustration. (3p)

Q 2) (15 points)

a) the bundle : $(1x_1 + 2x_2)$

A suitable utility function would be : $\min(2x_1, 1x_2)$ (1p)

b) Check figure 3 (2p)

c) Two secrets are : $(1.5p + 1.5p)$

$$\begin{aligned}2x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= m\end{aligned}$$

hence

$$4x_1 + 3(2x_1) = 40$$

$$x_1^* = \frac{m}{p_1 + 2p_2} \text{ and } x_2^* = 2\frac{m}{p_1 + 2p_2}$$

$$x_1^* = \frac{40}{4 + (2 * 3)} = 4$$

$$\text{and } x_2^* = 2\frac{40}{4 + (2 * 3)} = 8$$

(1.5p + 1.5p)

The solution is interior since none of the optimal consumption levels are zero. (1p)

d) Using the magic formulas :

$$x_1^* = \frac{40}{2 + (2 * 3)} = 5$$

$$\text{and } x_2^* = 2\frac{40}{2 + (2 * 3)} = 10$$

(1.5p + 1.5p)

S.E. is zero. In a perfect complements case there is no substitution effect related to a price change. (2p)

Q.3) (15 points)

a) $U(X_1, X_2) = 2X_1 + 2X_2$ can be a utility function for this kind of preferences. (2p)

b) Check the figure 4 (2p)

c) $MRS^{12} = ((MU_1)/(MU_2)) = 1/1 = 1$ given the utility function. (2p)
comparing MRS to the price ratio : $1 > \frac{2}{3} = \frac{P_1}{P_2}$

So only good 1 is consumed : $X_1^* = m/P_1 = 12/2 = 6$ and $X_2^* = 0$ (3p)

The solution is a corner one. (1p)

d) check figure 5 for the income-offer curve and the engel curve. (2p+2p)

Pepsi is normal as the consumption of it increases as the income increases.

(1p)

Q.4)

a) $\frac{10}{2} = 5$ is her real wage. (1p)

This number is her purchasing power in terms of consumption good. (1p)

b) check figure 6 (3p)

c)

$$\text{secret 1: } w * R + p_c C = 24 * w \quad (2p)$$

$$\text{or } 10R + 2C = 24 * 10 = 240$$

$$\text{secret 2 : } MRS^{RC} = ((MU_R)/(MU_C)) = w/P_c \rightarrow 1/R = w/P_c \quad (2p)$$

$$\rightarrow R = \frac{P_c}{w} \quad (2p)$$

d) If $w = \$1$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 10hr$ (1p)

If $w = \$2$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 5hr$ (1p)

The labor supply (24-R) is increasing in real wage ! (2p)

Bonus Question

$$V(x_1, x_2) = f[U(x_1, x_2)]$$

$$MRS^U = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2)$$

$$MRS^V = ((MV_1)/(MV_2)) = (\partial V/\partial X_1)/(\partial V/\partial X_2)$$

$$= \frac{\partial f[U(x_1, x_2)]/\partial X_1}{\partial f[U(x_1, x_2)]/\partial X_2} \quad \text{by chain rule :}$$

$$= \frac{f'(\cdot) * \partial U(x_1, x_2)/\partial X_1}{f'(\cdot) * \partial U(x_1, x_2)/\partial X_2} = \frac{\partial U(x_1, x_2)/\partial X_1}{\partial U(x_1, x_2)/\partial X_2} = MRS^U$$

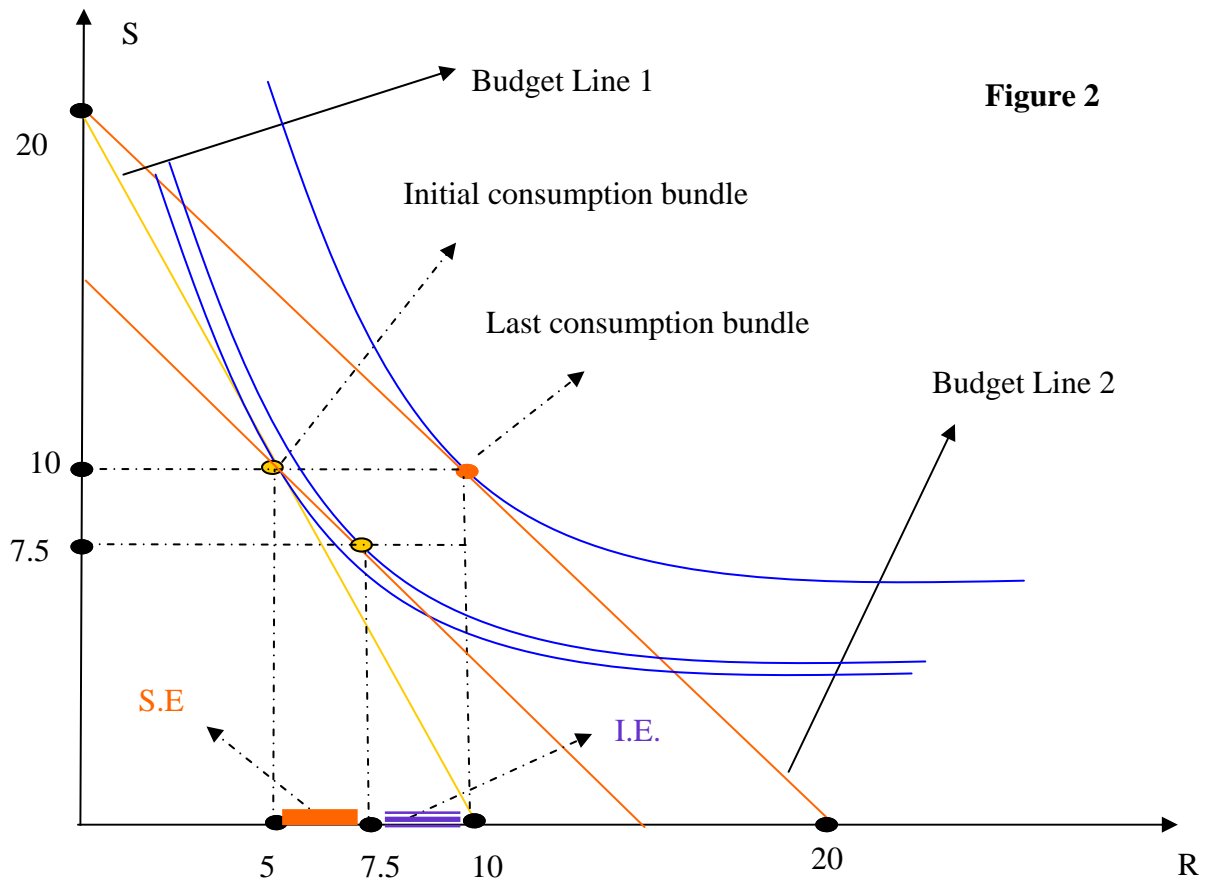
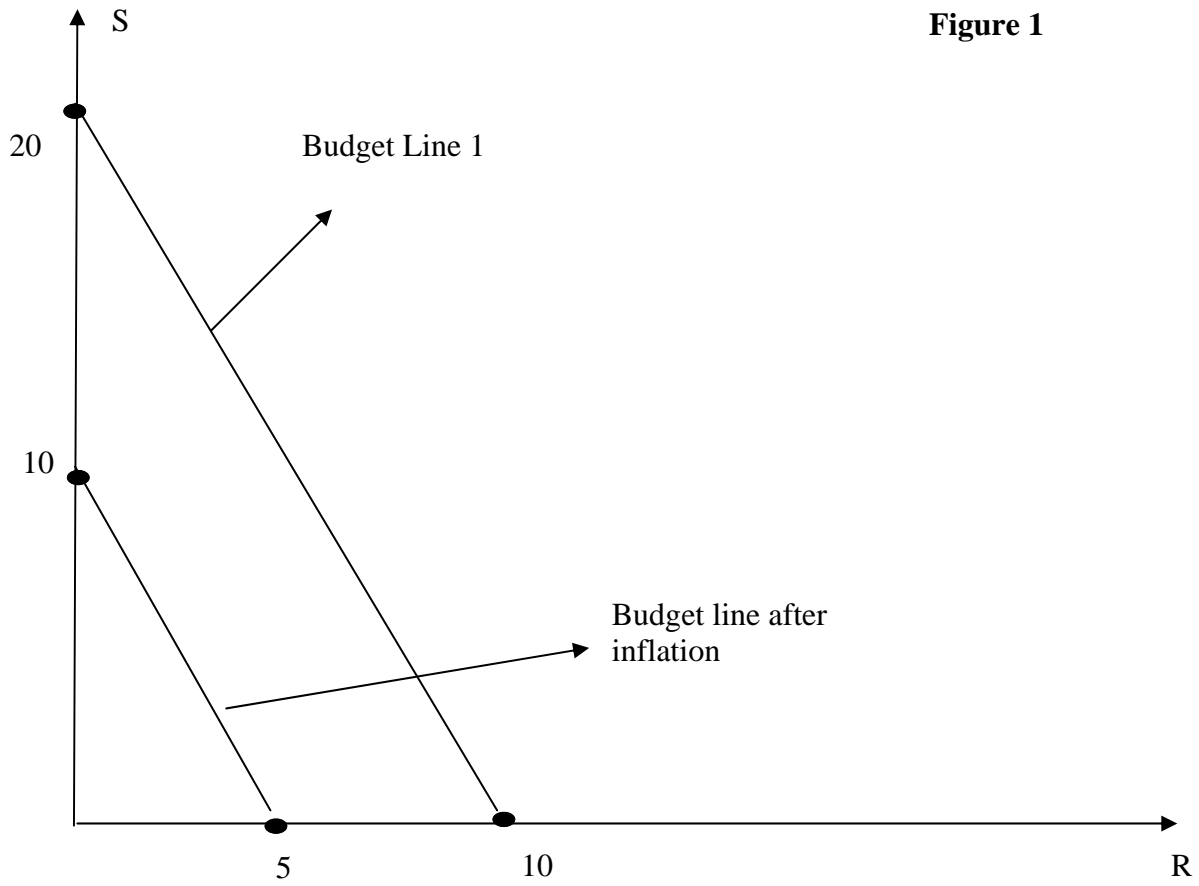


Figure 3

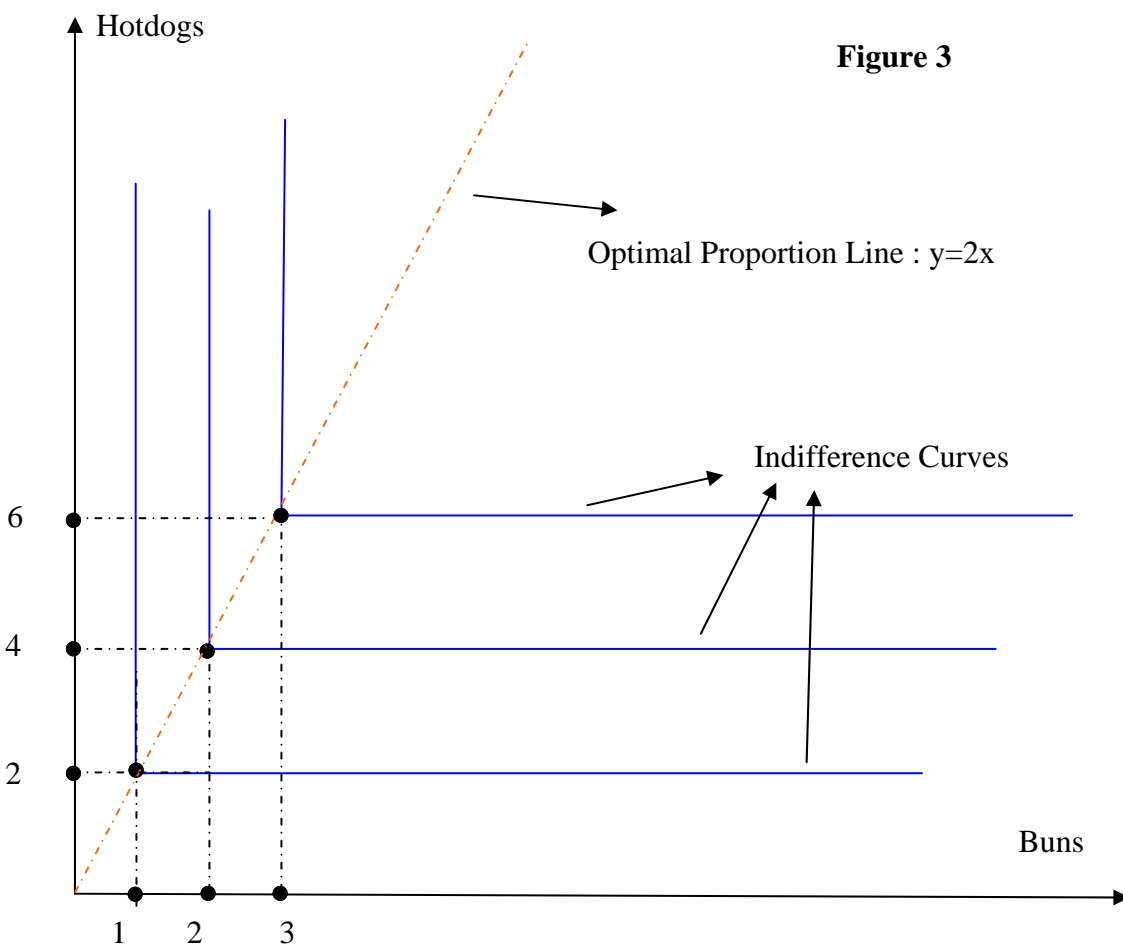
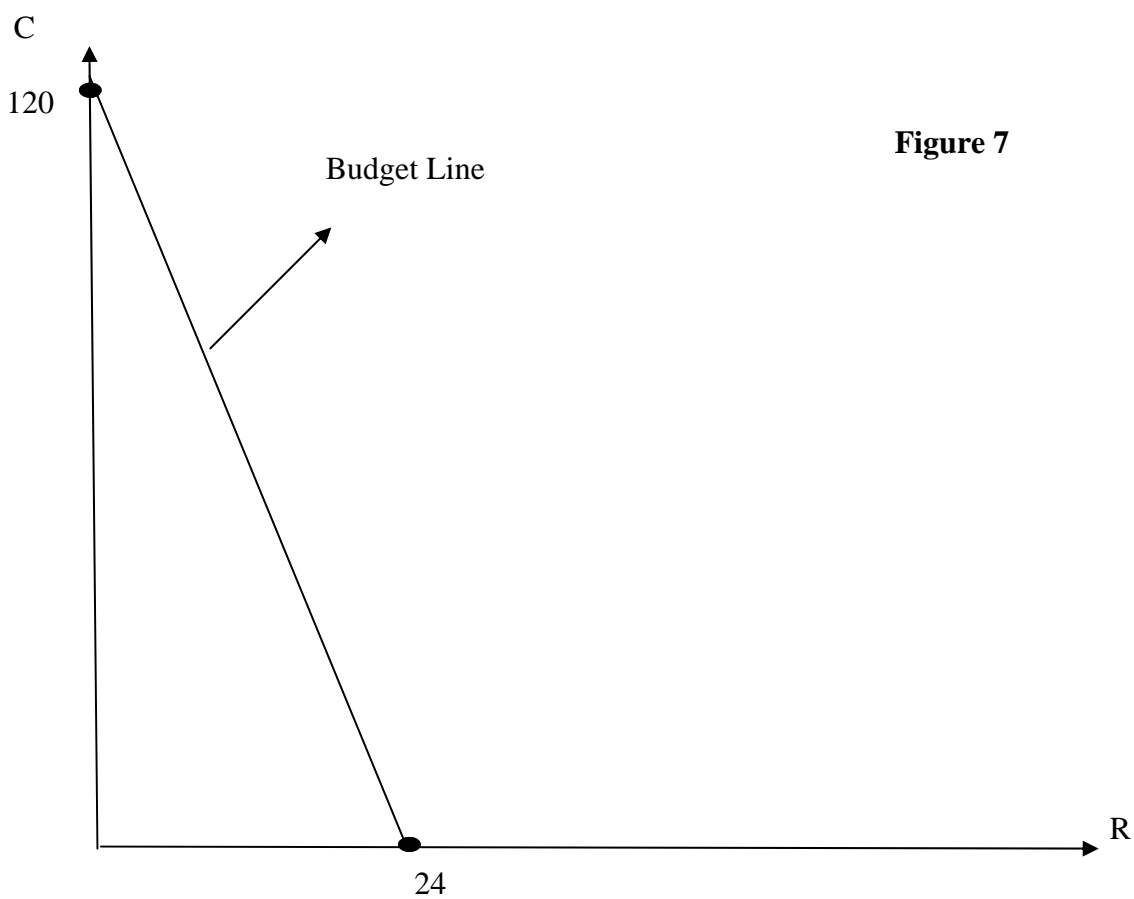


Figure 7



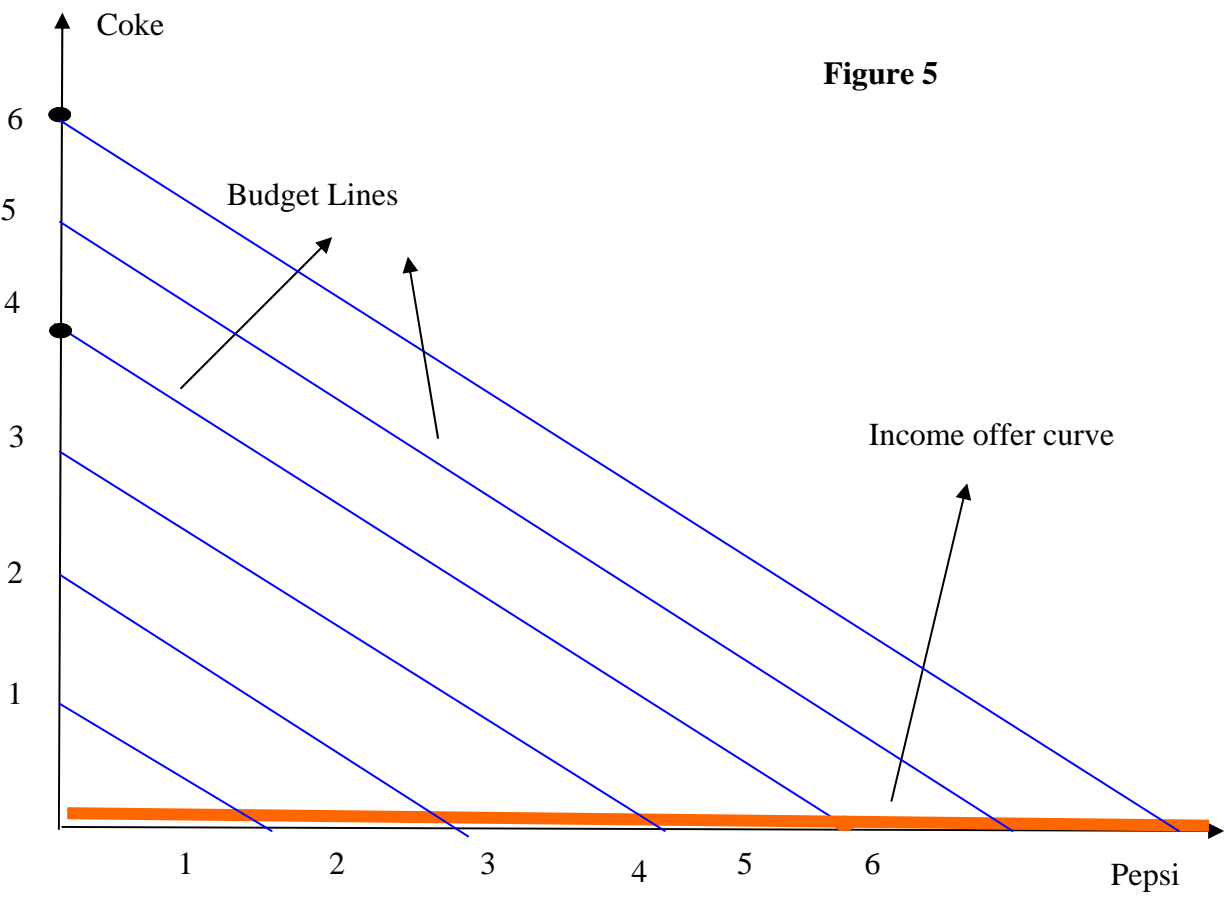
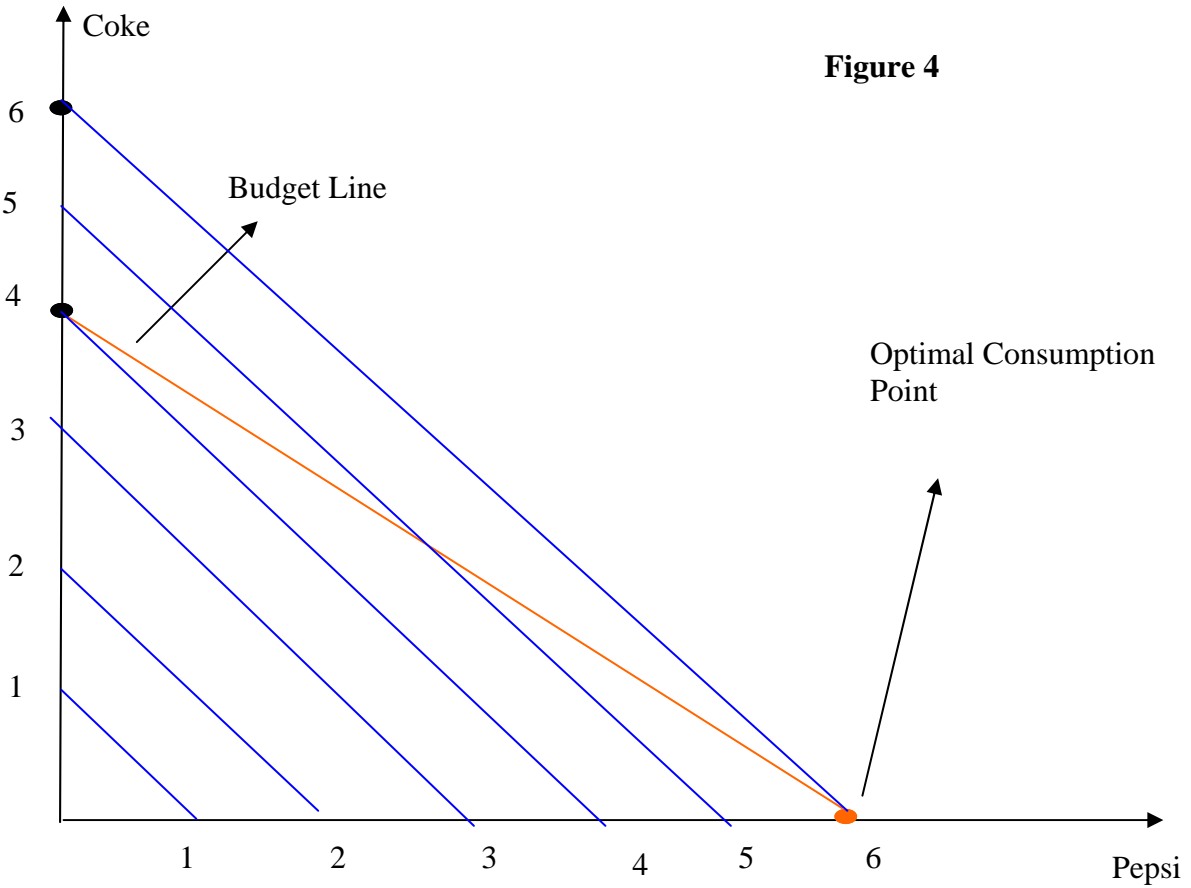
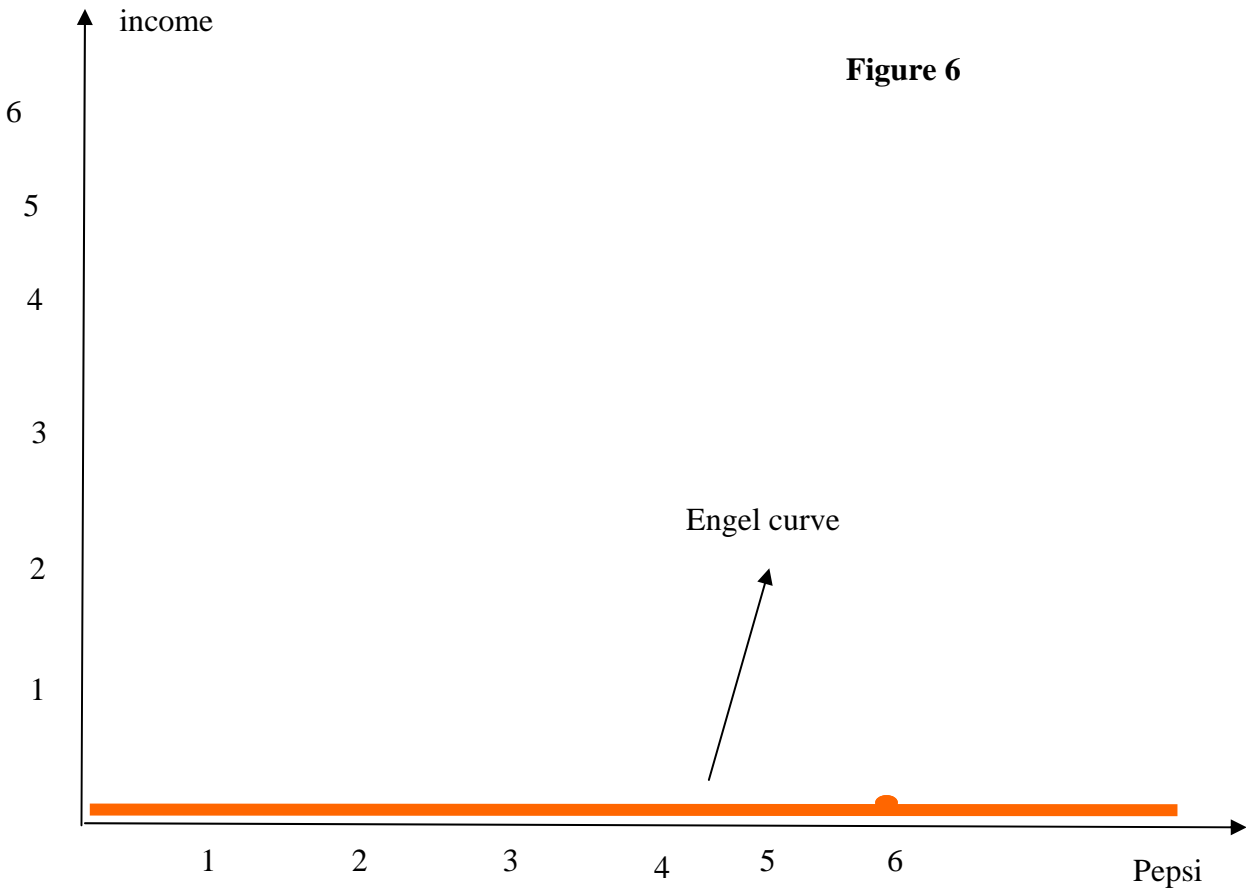


Figure 6



Midterm Exam 1 Solutions
(B) The Green

Q1) (55 points)

a) Check figure 1 for the budget set (3 p)
If there is 100% inflation, the prices double.
The new budget set is also shown on Figure 1. (3 p)

b) $U(X_1, X_2) = X_1^3 X_2^3$

$$MRS^{12} = ((MU_1)/(MU_2)) = -(\partial U/\partial X_1)/(\partial U/\partial X_2) = -(3X_1^2 X_2^3)/(3X_1^3 X_2^2) = -X_2/X_1 = MRS^{12}$$

(4 p)
at the point $x_1 = 3$ & $x_2 = 6$: $MRS^{12} = (-6/3) = -2$ (2 p)

which means:

$MU_1 > MU_2$ So 1st good, ribeye, is more valuable than top sirloin to Ava
at the consumption level of

$x_1 = 3$ & $x_2 = 6$ (4 p)

d)

secret 1 : $p_1 x_1 + p_2 x_2 = m$ (3p)

so $10x_1 + 5x_2 = 100$

This means "spend all of your money". If the consumer does not spend all of his/her money s/he is wasting his/her opportunity to increase his/her utility since money does not have an effect on utility itself. (2p)

secret 2 : $MRS^{12} = (MU_1)/(MU_2) = P_1/P_2$ (3p)

$\rightarrow X_2/X_1 = P_1/P_2$

"the last spent on each good should give the same utility" OR "marginal utility of a \$ spent on each good should be equal". If this condition does not hold, let's say last \$ spent on good 1 brings more utility than the last \$ spent on good 2, then the consumer should buy less of the second good and buy more of the first good to increase his/her utility. (2p)

start with secret 2 : $X_2/X_1 = P_1/P_2 \rightarrow X_2 = ((X_1 P_1)/(P_2))$ (1p)

now plug this in secret 1 :

$p_1 x_1 + p_2 x_2 = m = P_1 x_1 + P_2 (X_1 P_1)/(P_2) = m = 2P_1 x_1$ (2p)

\rightarrow Then $X_1 = (m/(2p_1)) = (1/2)(m/p_1)$

Since $X_2 = ((X_1 P_1)/(P_2)) = ((m P_1)/(2P_1 P_2)) = (m/(2p_2))$
 $= (1/2)(m/(p_2)) = X_2$

$$X_1 = (m/(2p_1))$$

$$X_2 = (m/(2p_2)) \quad (2p)$$

The solution is interior since none of good's optimal consumption level is zero. Furthermore in a Cobb-Douglas case the solution has to be interior as long as income level (m) is larger than zero ! (we also assume positive prices)
(1p)

$$\text{Using the magic formula } X_2 = (m/(2p_2)) \rightarrow X_2 p_2 = m/2$$

note that the left hand side is the total money spent on good 2 (sirleon).
So half of the income is spent on sirleon. (2p)

$$\text{e) } X_1 = (m/(2p_1)) = ((100)/(2 * 10)) = 5 \quad (1p)$$

$$X_2 = (m/(2p_2)) = ((100)/(2 * 5)) = 10 \quad (1p)$$

$$X'_1 = (m/(2p'_1)) = ((100)/(2 * 5)) = 10 \quad (1p)$$

$$X'_2 = (m/(2p_2)) = ((100)/(2 * 5)) = 10 \quad (1p)$$

$$\text{Total change in consumption of ribeye is } X'_1 - X_1 = 10 - 5 = 5 \quad (1p)$$

Check figure 2 for the illustration of the change. (3p)

Ribeye is an ordinary good since it's consumption increases as it's price decrease. We can also see this fact from the magic formula . Since the price of ribeye is in the denominator in optimal consumption formula of ribeye, as the price goes down the consumption of it will increase ! (2p)

f) How much money does Ava need to consume the original bundle with the new prices ?

$$p'_1 x_1 + p_2 x_2 = m' = (\$5 * 5) + (\$5 * 10) = \$75 \text{ is enough} \quad (3p)$$

Now calculating the optimal bundle with this imaginary income :

$$X_1^s = (m'/(2p_1)) = ((75)/(2 * 5)) = 15/2 \quad (2p)$$

$$X_2^s = (m'/(2p_2)) = ((75)/(2 * 5)) = 15/2$$

$$\text{So Substitution Effect (S.E.) is : } 15/2 - 5 = 5/2 \quad (2p)$$

$$T.E = S.E. + I.E. \quad \text{then : } I.E. = 5 - (5/2) = 5/2 \quad (1p)$$

Check figure 2 for the illustration. (3p)

Q 2) (15 points)

a) the bundle : $(1x_1 + 3x_2)$

A suitable utility function would be : $\min(3x_1, 1x_2)$ (1p)

b) Check figure 3 (2p)

c) Two secrets are : (1.5p + 1.5p)

$$\begin{aligned} 3x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= m \end{aligned}$$

hence

$$4x_1 + 2(3x_1) = 40$$

$$x_1^* = \frac{m}{p_1 + 3p_2} \text{ and } x_2^* = 3\frac{m}{p_1 + 3p_2}$$

$$\begin{aligned} x_1^* &= \frac{40}{4 + (2 * 3)} = 4 \\ \text{and } x_2^* &= 3\frac{40}{4 + (2 * 3)} = 12 \end{aligned}$$

(1.5p + 1.5p)

The solution is interior since none of the optimal consumption levels are zero. (1p)

d) Using the magic formulas :

$$\begin{aligned} x_1^* &= \frac{40}{2 + (2 * 3)} = 5 \\ \text{and } x_2^* &= 3\frac{40}{2 + (2 * 3)} = 15 \end{aligned}$$

(1.5p + 1.5p)

S.E. is zero. In a perfect complements case there is no substitution effect related to a price change. (2p)

Q.3) (15 points)

a) $U(X_1, X_2) = 2X_1 + 2X_2$ can be a utility function for this kind of preferences. (2p)

b) Check the figure 4 (2p)

c) $MRS^{12} = ((MU_1)/(MU_2)) = 1/1 = 1$ given the utility function. (2p)
comparing MRS to the price ratio : $1 < \frac{4}{2} = \frac{P_1}{P_2}$

So only good 2 is consumed : $X_2^* = m/P_2 = 12/2 = 6$ and $X_1^* = 0$ (3p)

The solution is a corner one. (1p)

d) check figure 5 & 6 for the income-offer curve and the engel curve. (2p+2p)

Pepsi is normal as the consumption of it does not decrease as the income increases. (1p)

(Answers of "inferior" and "ambiguous" are also accepted IF NECESSARY EXPLANATIONS ARE DONE!)

Q.4)

a) $\frac{10}{2} = 5$ is her real wage. (1p)

This number is her purchasing power in terms of consumption good. (1p)

b) check figure 7 (3p)

c)

$$\text{secret 1: } w * R + p_c C = 24 * w \quad (2p)$$

$$\text{or } 10R + 2C = 24 * 10 = 240$$

$$\text{secret 2 : } MRS^{RC} = ((MU_R)/(MU_C)) = w/P_c \rightarrow 1/R = w/P_c \quad (2p)$$

$$\rightarrow R = \frac{P_c}{w} \quad (2p)$$

d) If $w = \$1$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 10hr$ (1p)

If $w = \$2$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 5hr$ (1p)

The labor supply (24-R) is increasing in real wage ! (2p)

Bonus Question

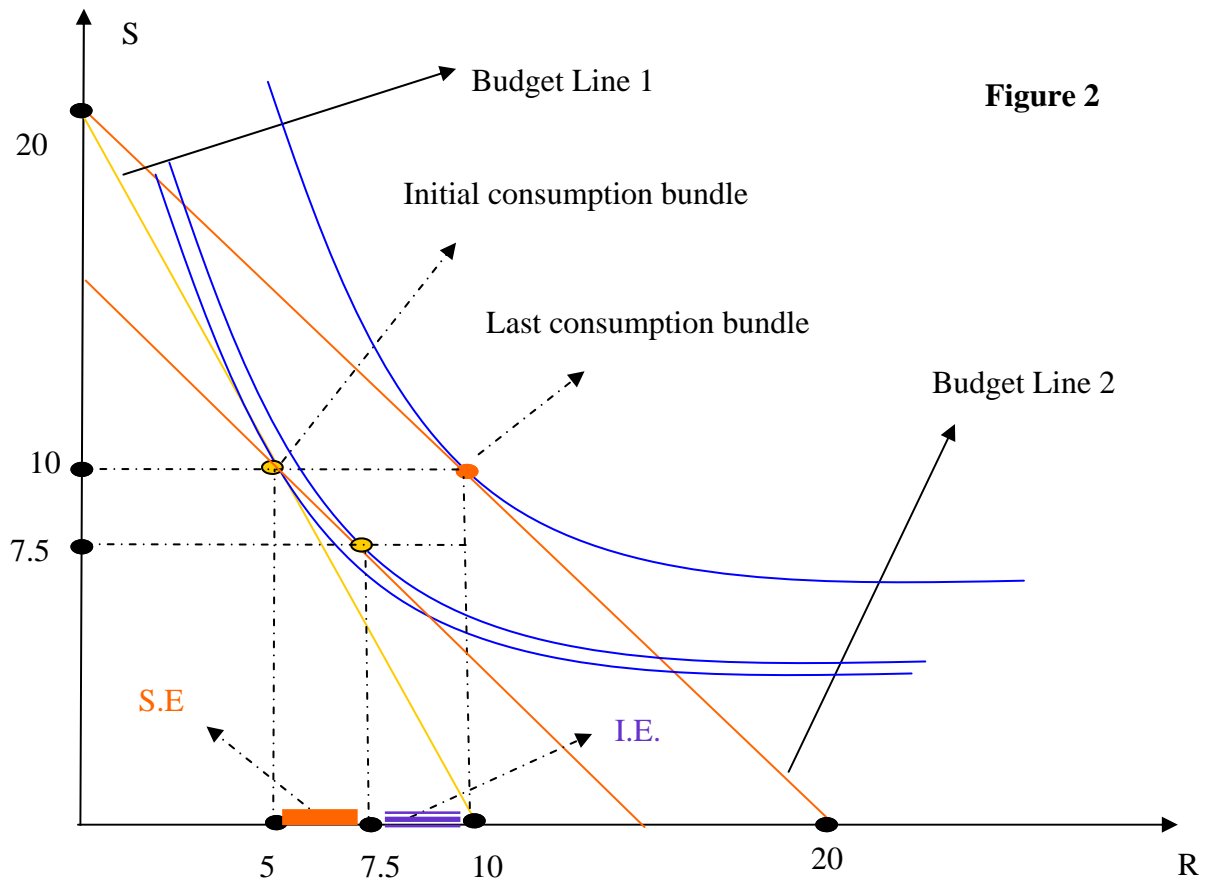
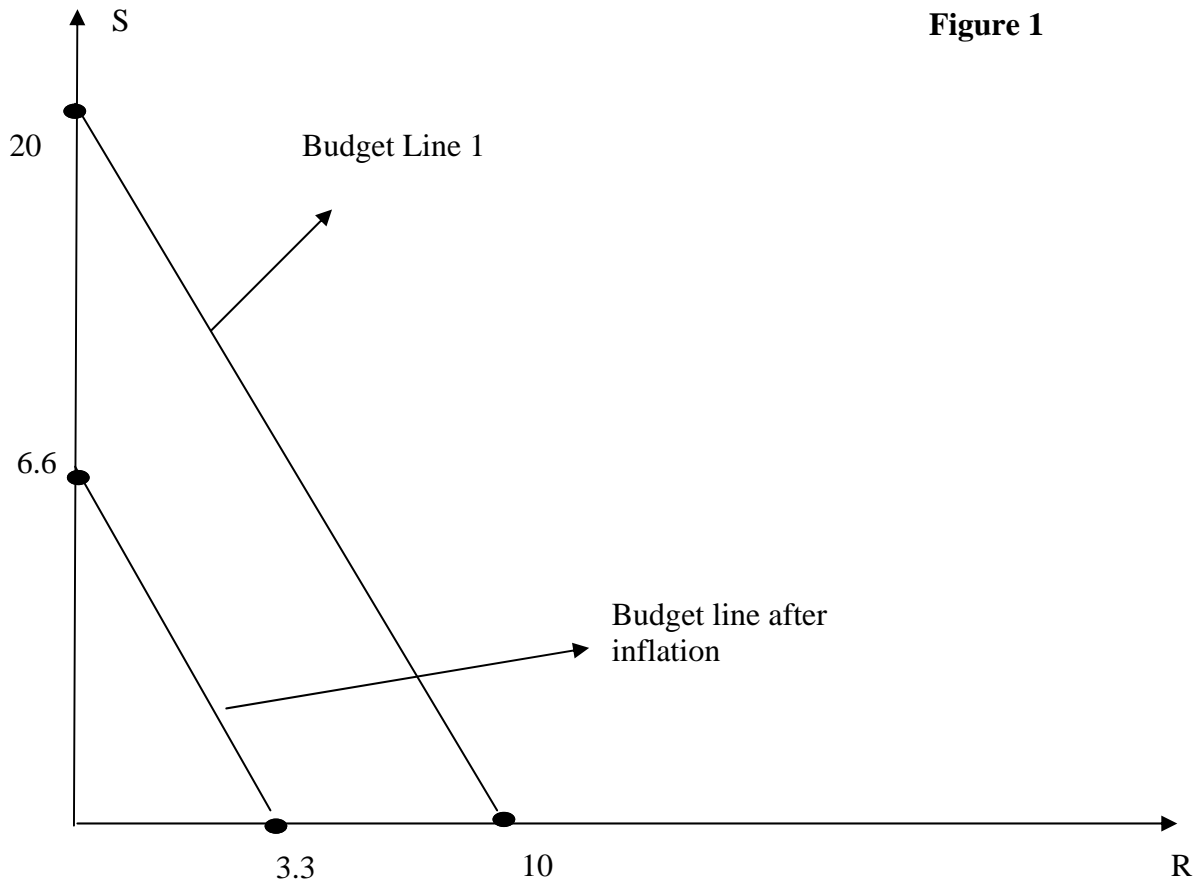
$$V(x_1, x_2) = f[U(x_1, x_2)]$$

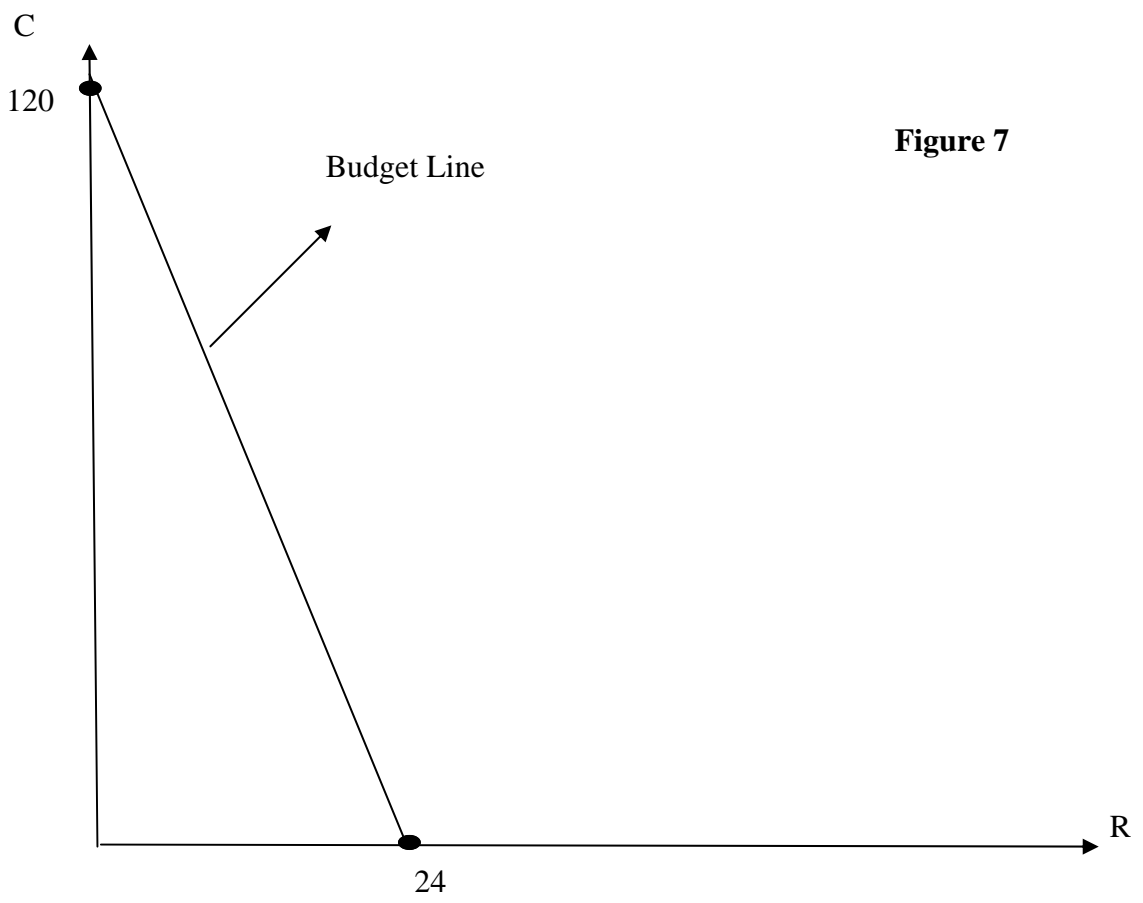
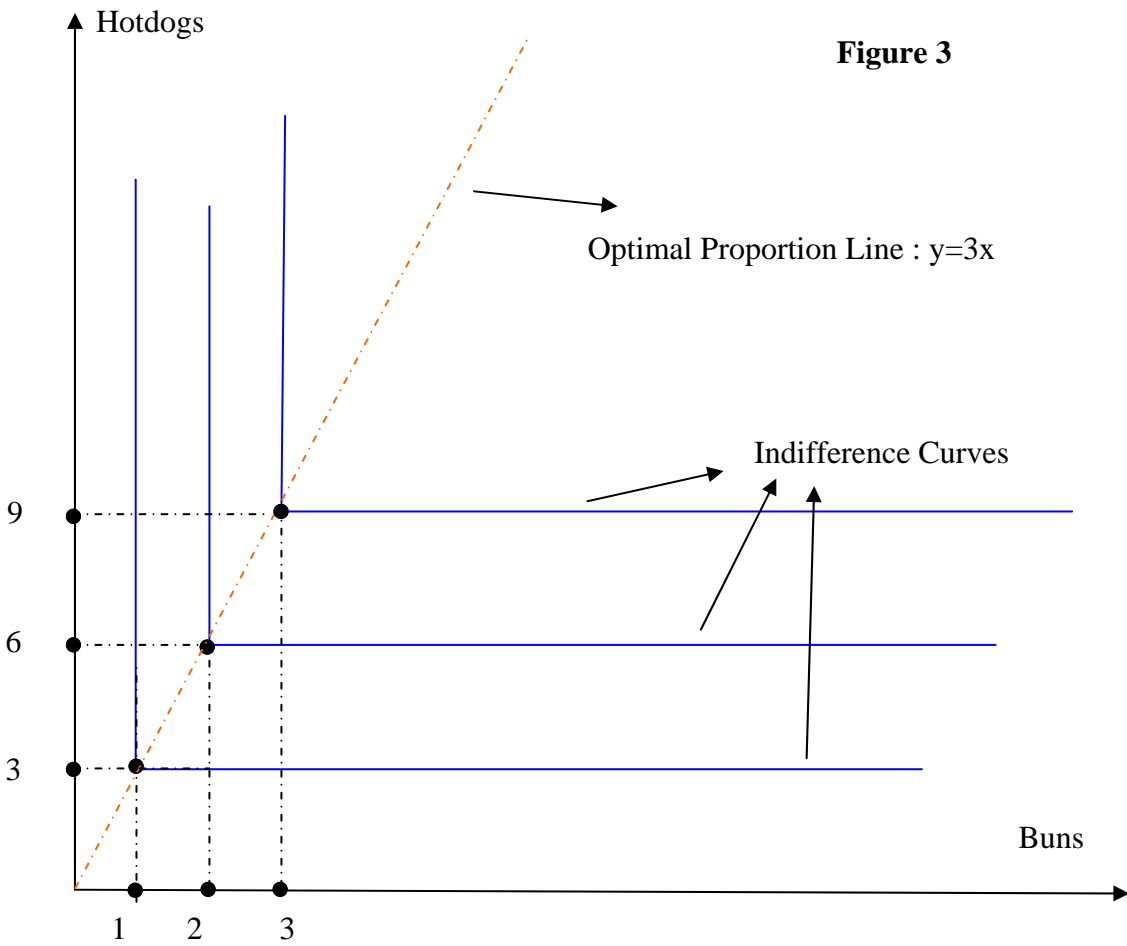
$$MRS^U = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2)$$

$$MRS^V = ((MV_1)/(MV_2)) = (\partial V/\partial X_1)/(\partial V/\partial X_2)$$

$$= \frac{\partial f[U(x_1, x_2)]/\partial X_1}{\partial f[U(x_1, x_2)]/\partial X_2} \quad \text{by chain rule :}$$

$$= \frac{f'(\cdot) * \partial U(x_1, x_2)/\partial X_1}{f'(\cdot) * \partial U(x_1, x_2)/\partial X_2} = \frac{\partial U(x_1, x_2)/\partial X_1}{\partial U(x_1, x_2)/\partial X_2} = MRS^U$$





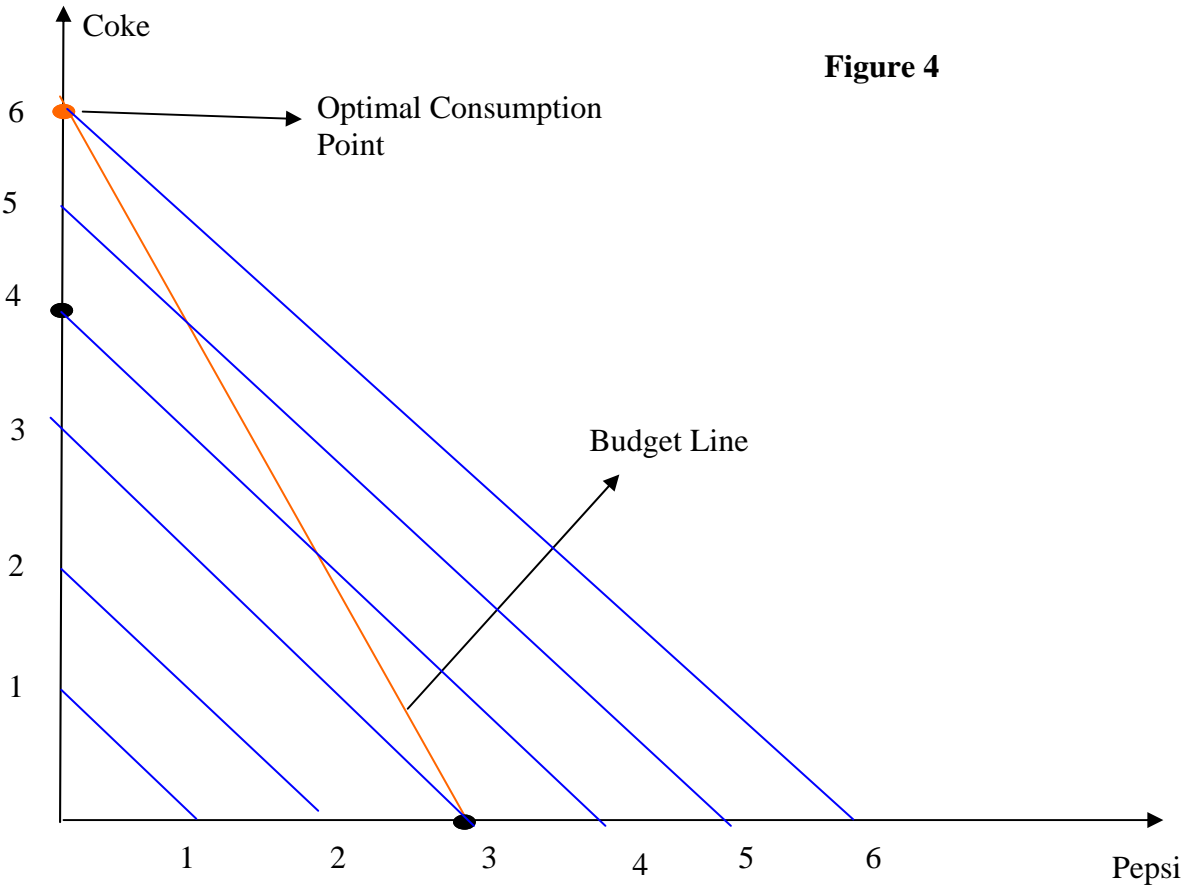


Figure 4

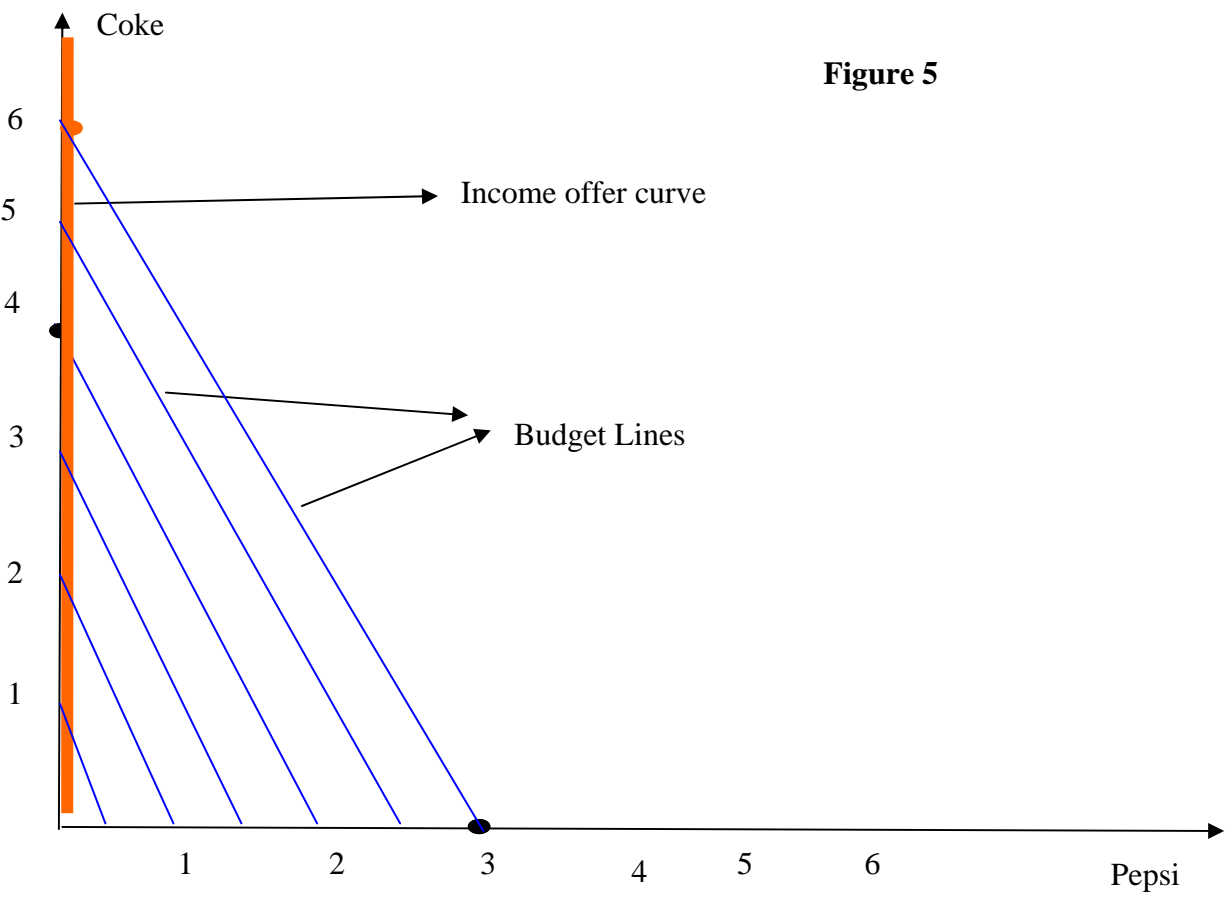


Figure 5

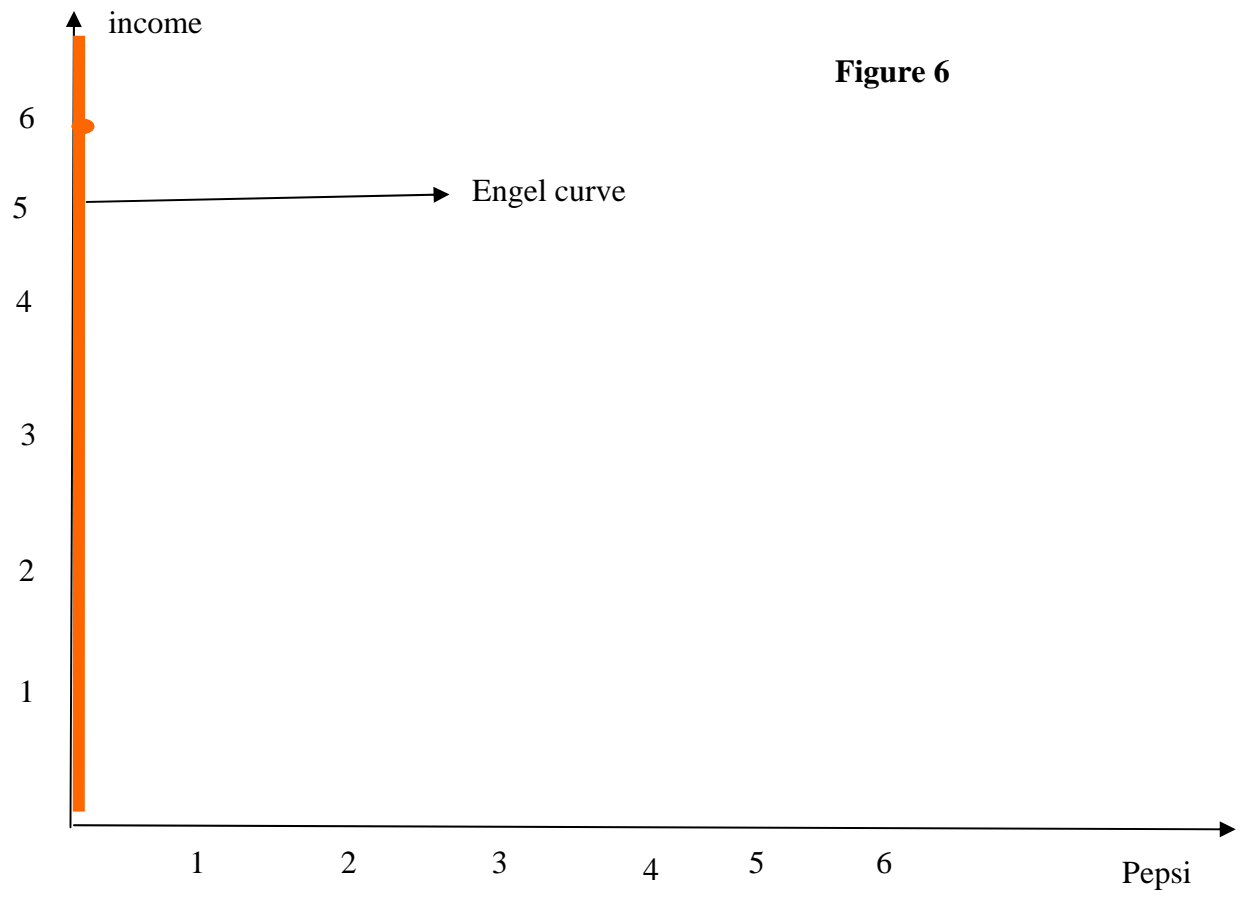


Figure 6

Midterm Exam 1 Solutions
(C) The Yellow

Q1) (55 points)

a) Check figure 1 for the budget set (3 p)
If there is 100% inflation, the prices double.
The new budget set is also shown on Figure 1. (3 p)

b) $U(X_1, X_2) = X_1^{1/2} X_2^{1/2}$

$$MRS^{12} = ((MU_1)/(MU_2)) = -(\partial U/\partial X_1)/(\partial U/\partial X_2) = -(\frac{1}{2}X_1^{-1/2}X_2^{1/2})/(\frac{1}{2}X_1^{1/2}X_2^{-1/2}) = -X_2/X_1 = MRS^{12}$$

(4 p)
at the point $x_1 = 3$ & $x_2 = 6$: $MRS^{12} = (-6/3) = -2$ (2 p)

which means:

$MU_1 > MU_2$ So 1st good, ribeye, is more valuable than top sirleon to Ava at the consumption level of

$x_1 = 3$ & $x_2 = 6$ (4 p)

d)

secret 1 : $p_1x_1 + p_2x_2 = m$ (3p)

so $10x_1 + 5x_2 = 200$

This means "spend all of your money". If the consumer does not spend all of his/her money s/he is wasting his/her opportunity to increase his/her utility since money does not have an effect on utility itself. (2p)

secret 2 : $MRS^{12} = (MU_1)/(MU_2) = P_1/P_2$ (3p)

$\rightarrow X_2/X_1 = P_1/P_2$

"the last spent on each good should give the same utility" OR "marginal utility of a \$ spent on each good should be equal ". If this condition does not hold, lets say last \$ spent on good 1 brings more utility than the last \$ spent on good 2 , then the consumer should buy less of the second good and buy more of the first good to increase his/her utility. (2p)

start with secret 2 : $X_2/X_1 = P_1/P_2 \rightarrow X_2 = ((X_1P_1)/(P_2))$ (1p)

now plug this in secret 1 :

$p_1x_1 + p_2x_2 = m = P_1x_1 + P_2(X_1P_1)/(P_2) = m = 2P_1x_1$ (2p)

\rightarrow Then $X_1 = (m/(2p_1)) = (1/2)(m/p_1)$

Since $X_2 = ((X_1P_1)/(P_2)) = ((mP_1)/(2P_1P_2)) = (m/(2p_2))$
 $= (1/2)(m/(p_2)) = X_2$

$$X_1 = (m/(2p_1))$$

$$X_2 = (m/(2p_2)) \quad (2p)$$

The solution is interior since none of good's optimal consumption level is zero. Furthermore in a Cobb-Douglas case the solution has to be interior as long as income level (m) is larger than zero ! (we also assume positive prices)
(1p)

Using the magic formula $X_2 = (m/(2p_2)) \rightarrow X_2 p_2 = m/2$

note that the left hand side is the total money spent on good 2 (sirleon).
So half of the income is spent on sirleon. (2p)

e) $X_1 = (m/(2p_1)) = ((200)/(2 * 10)) = 10 \quad (1p)$

$$X_2 = (m/(2p_2)) = ((200)/(2 * 5)) = 20 \quad (1p)$$

$$X'_1 = (m/(2p'_1)) = ((200)/(2 * 5)) = 20 \quad (1p)$$

$$X'_2 = (m/(2p_2)) = ((200)/(2 * 5)) = 20 \quad (1p)$$

Total change in consumption of ribeye is $X'_1 - X_1 = 20 - 10 = 10 \quad (1p)$

Check figure 2 for the illustration of the change. (3p)

Ribeye is an ordinary good since it's consumption increases as it's price decrease. We can also see this fact from the magic formula . Since the price of ribeye is in the denominator in optimal consumption formula of ribeye, as the price goes down the consumption of it will increase ! (2p)

f) How much money does Ava need to consume the original bundle with the new prices ?

$$p'_1 x_1 + p_2 x_2 = m' = (\$5 * 10) + (\$5 * 20) = \$150 \text{ is enough} \quad (3p)$$

Now calculating the optimal bundle with this imaginary income :

$$X_1^s = (m'/(2p_1)) = ((150)/(2 * 5)) = 15 \quad (2p)$$

$$X_2^s = (m'/(2p_2)) = ((150)/(2 * 5)) = 15$$

So Substitution Effect (S.E.) is : $15 - 10 = 5 \quad (2p)$

$T.E = S.E. + I.E.$ then : $I.E. = 10 - 5 = 5 \quad (1p)$

Check figure 2 for the illustration. (3p)

Q 2) (15 points)

a) the bundle : $(1x_1 + 4x_2)$

A suitable utility function would be : $\min(4x_1, 1x_2)$ (1p)

b) Check figure 3 (2p)

c) Two secrets are : $(1.5p + 1.5p)$

$$\begin{aligned}4x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= m\end{aligned}$$

hence

$$6x_1 + 1(4x_1) = 40$$

$$x_1^* = \frac{m}{p_1 + 4p_2} \text{ and } x_2^* = 4\frac{m}{p_1 + 4p_2}$$

$$\begin{aligned}x_1^* &= \frac{40}{6 + (1 * 4)} = 4 \\ \text{and } x_2^* &= 4\frac{40}{6 + (1 * 4)} = 16\end{aligned}$$

(1.5p + 1.5p)

The solution is interior since none of the optimal consumption levels are zero. (1p)

d) Using the magic formulas :

$$\begin{aligned}x_1^* &= \frac{40}{4 + (1 * 4)} = 5 \\ \text{and } x_2^* &= 4\frac{40}{4 + (1 * 4)} = 20\end{aligned}$$

(1.5p + 1.5p)

S.E. is zero. In a perfect complements case there is no substitution effect related to a price change. (2p)

Q.3) (15 points)

a) $U(X_1, X_2) = 2X_1 + 2X_2$ can be a utility function for this kind of preferences. (2p)

b) Check the figure 4 (2p)

c) $MRS^{12} = ((MU_1)/(MU_2)) = 1/1 = 1$ given the utility function. (2p)
comparing MRS to the price ratio : $1 < \frac{6}{2} = \frac{P_1}{P_2}$

So only good 2 is consumed : $X_2^* = m/P_2 = 12/2 = 6$ and $X_1^* = 0$ (3p)

The solution is a corner one. (1p)

d) check figure 5 & 6 for the income-offer curve and the engel curve. (2p+2p)

Pepsi is normal as the consumption of it does not decrease as the income increases. (1p)

(Answers of "inferior" and "ambiguous" are also accepted IF NECESSARY EXPLANATIONS ARE DONE!)

Q.4)

a) $\frac{10}{2} = 5$ is her real wage. (1p)

This number is her purchasing power in terms of consumption good. (1p)

b) check figure 7 (3p)

c)

$$\text{secret 1: } w * R + p_c C = 24 * w \quad (2p)$$

$$\text{or } 10R + 2C = 24 * 10 = 240$$

$$\text{secret 2 : } MRS^{RC} = ((MU_R)/(MU_C)) = w/P_c \rightarrow 1/R = w/P_c \quad (2p)$$

$$\rightarrow R = \frac{P_c}{w} \quad (2p)$$

d) If $w = \$1$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 10hr$ (1p)

If $w = \$2$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 5hr$ (1p)

The labor supply (24-R) is increasing in real wage ! (2p)

Bonus Question

$$V(x_1, x_2) = f[U(x_1, x_2)]$$

$$MRS^U = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2)$$

$$MRS^V = ((MV_1)/(MV_2)) = (\partial V/\partial X_1)/(\partial V/\partial X_2)$$

$$= \frac{\partial f[U(x_1, x_2)]/\partial X_1}{\partial f[U(x_1, x_2)]/\partial X_2} \quad \text{by chain rule :}$$

$$= \frac{f'(\cdot) * \partial U(x_1, x_2)/\partial X_1}{f'(\cdot) * \partial U(x_1, x_2)/\partial X_2} = \frac{\partial U(x_1, x_2)/\partial X_1}{\partial U(x_1, x_2)/\partial X_2} = MRS^U$$

Figure 1

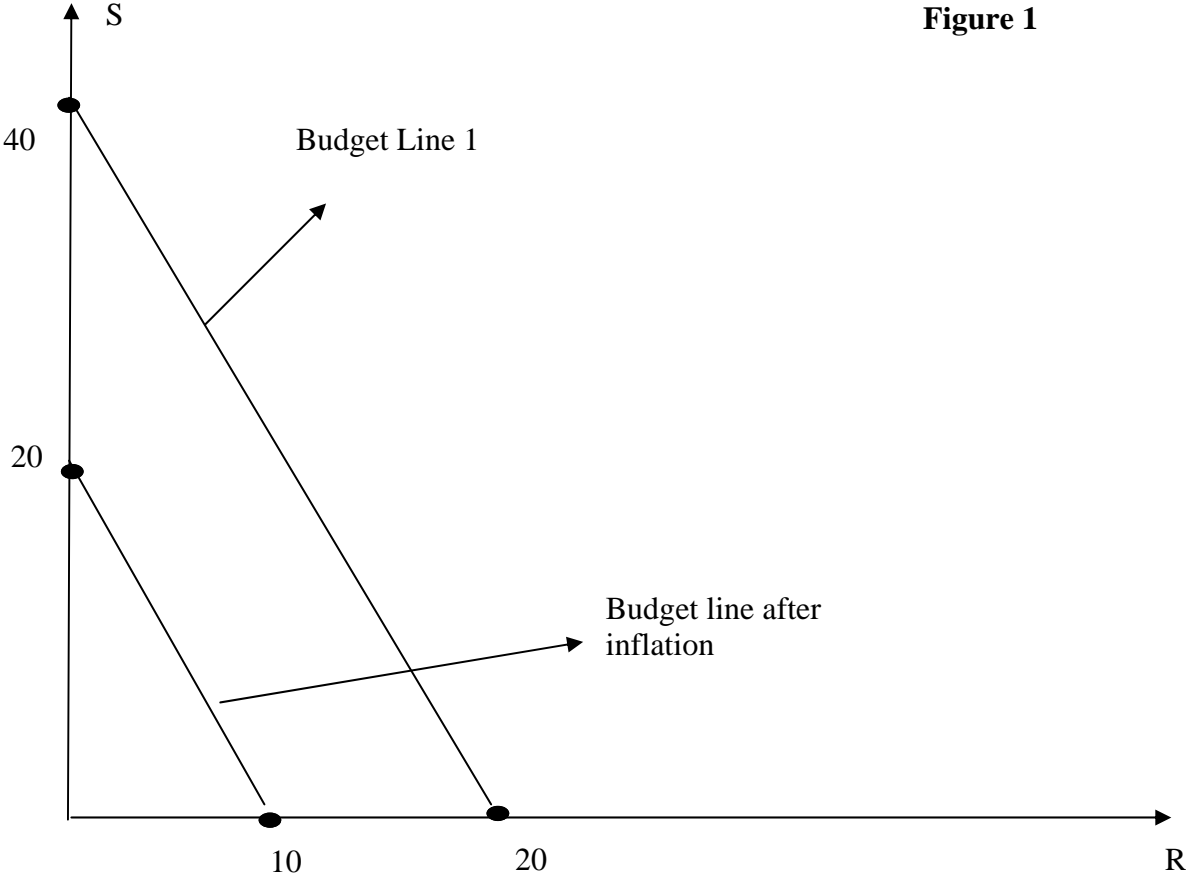
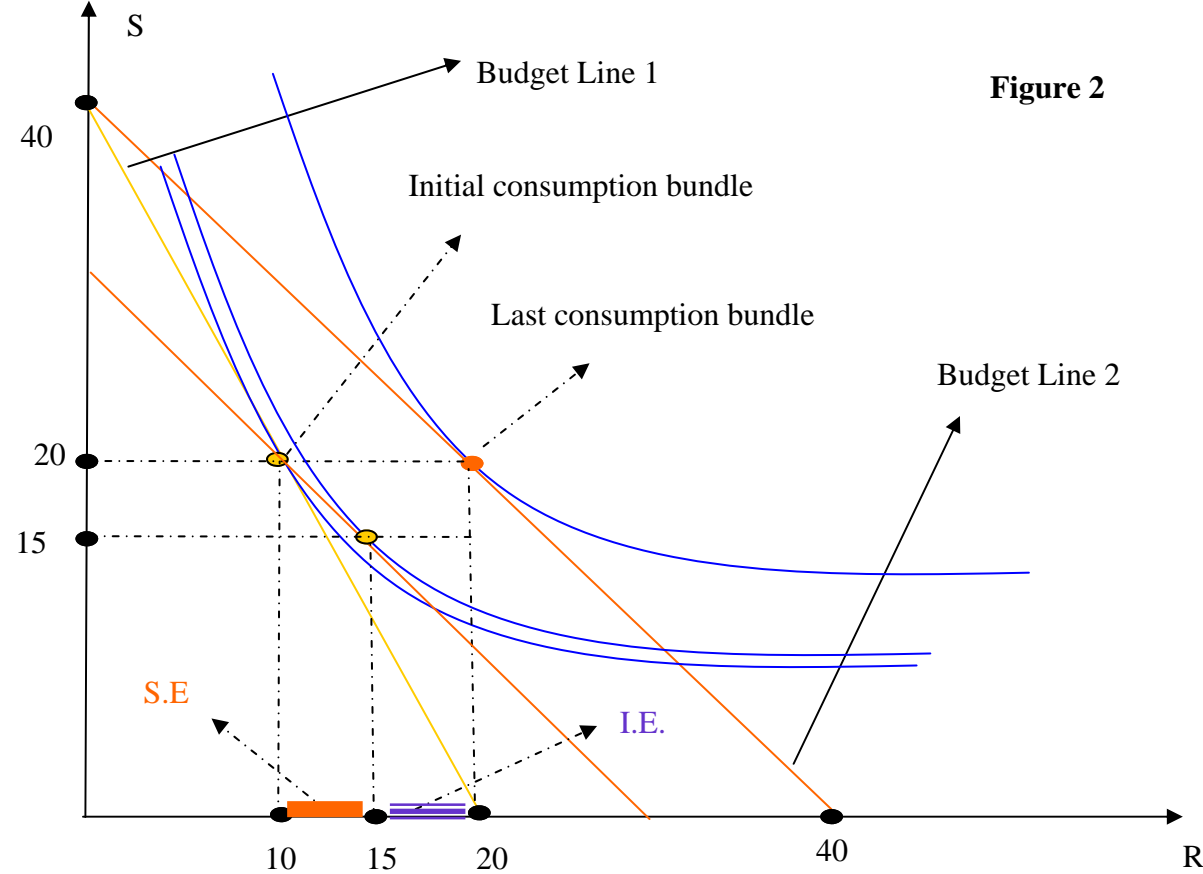
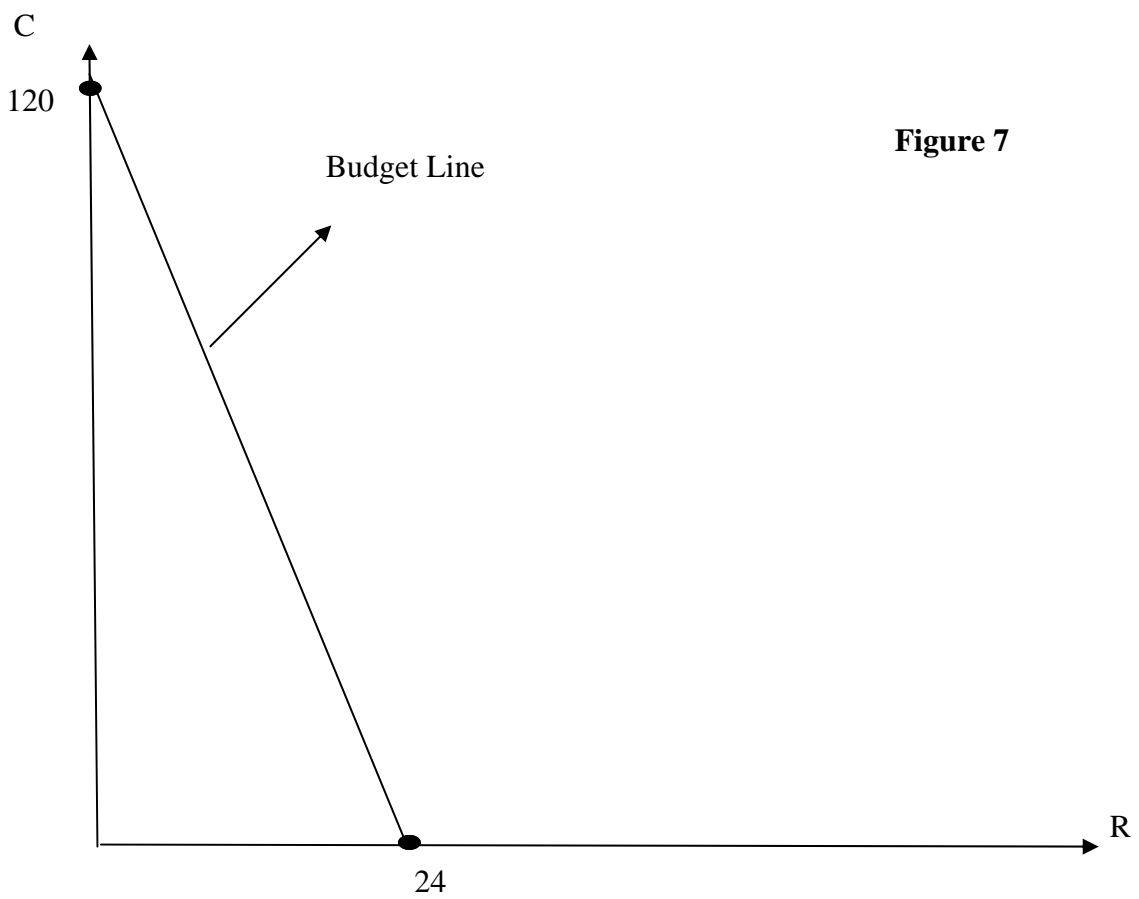
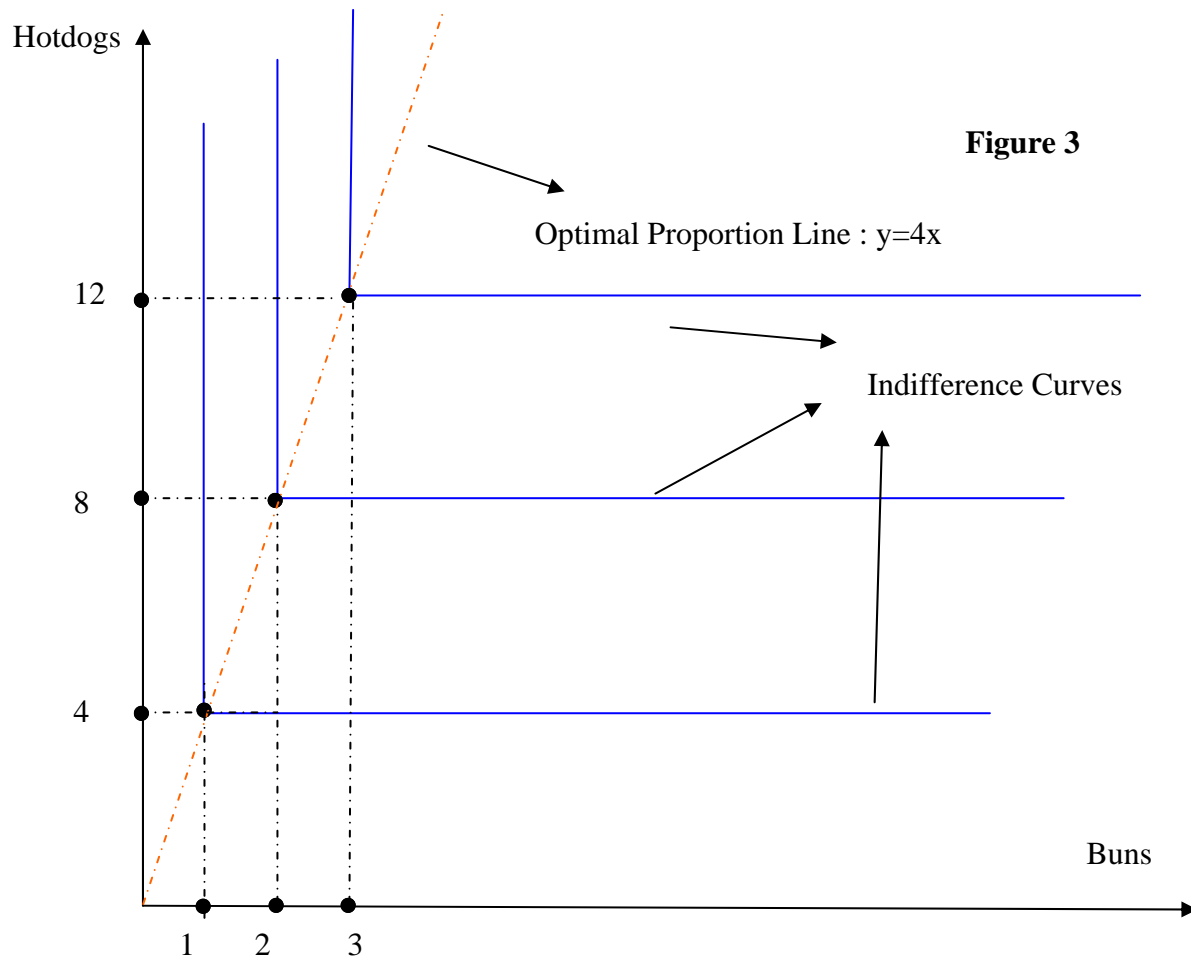


Figure 2





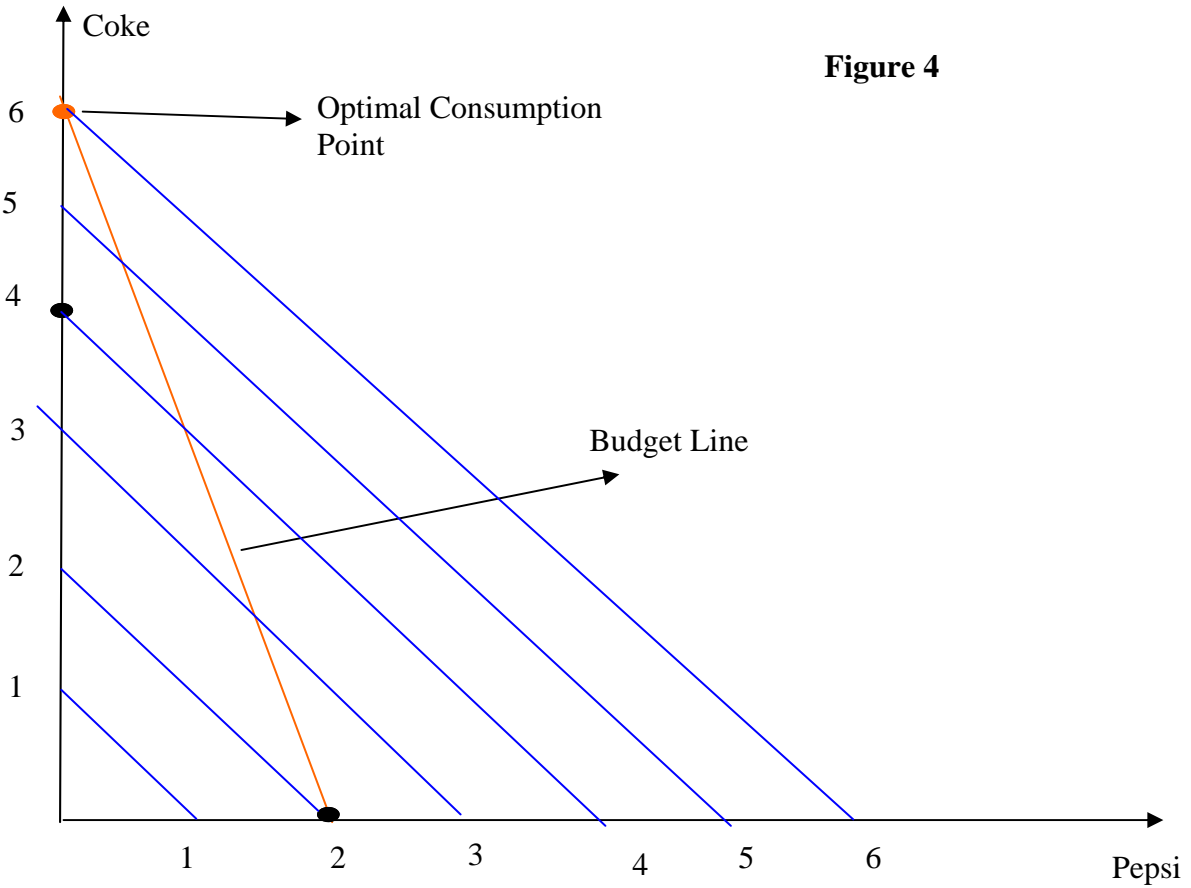


Figure 4

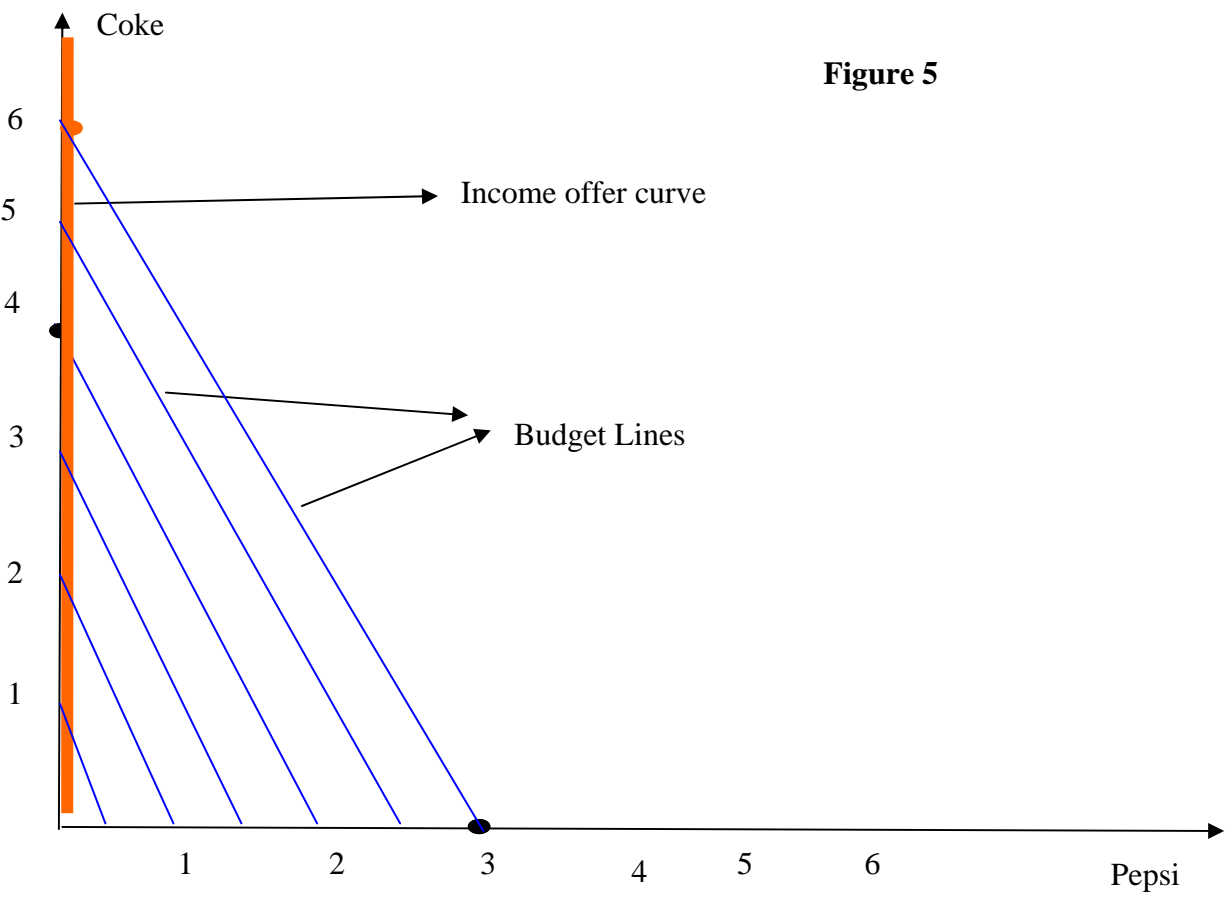


Figure 5

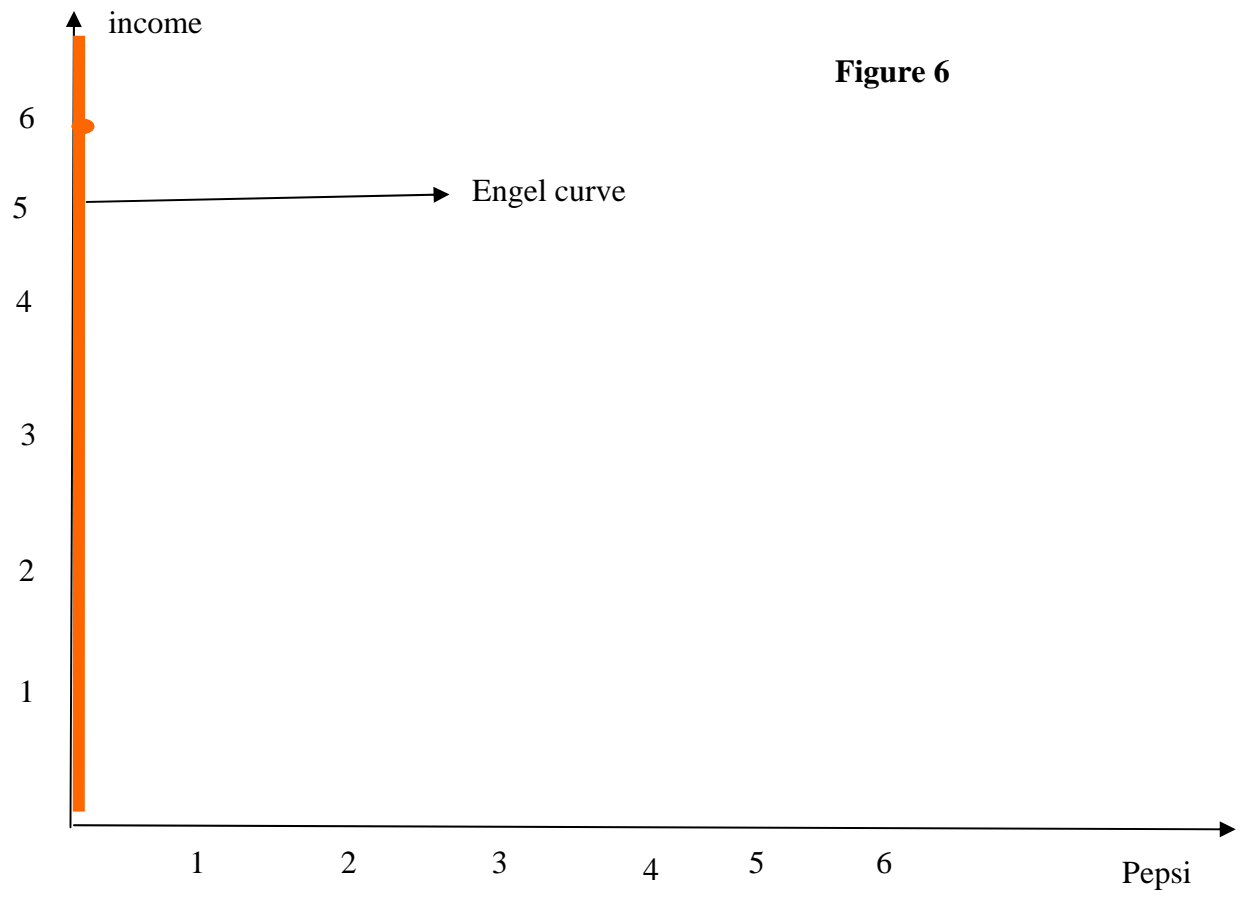


Figure 6

Midterm Exam 1 Solutions

(D) The Blue

Q1) (55 points)

a) Check figure 1 for the budget set (3 p)

If there is 100% inflation, the prices double.

The new budget set is also shown on Figure 1. (3 p)

b) $U(X_1, X_2) = X_1^{1/3} X_2^{1/3}$

$$MRS^{12} = ((MU_1)/(MU_2)) = -(\partial U/\partial X_1)/(\partial U/\partial X_2) = -(\frac{1}{3}X_1^{-2/3}X_2^{1/3})/(\frac{1}{3}X_1^{1/3}X_2^{-2/3}) = -X_2/X_1 = MRS^{12}$$

(4 p)

at the point $x_1 = 3$ & $x_2 = 6$: $MRS^{12} = (-6/3) = -2$ (2 p)

which means:

$MU_1 > MU_2$ So 1st good, ribeye, is more valuable than top sirloin to Ava at the consumption level of

$x_1 = 3$ & $x_2 = 6$ (4 p)

d)

secret 1 : $p_1x_1 + p_2x_2 = m$ (3p)

so $4x_1 + 2x_2 = 80$

This means "spend all of your money". If the consumer does not spend all of his/her money s/he is wasting his/her opportunity to increase his/her utility since money does not have an effect on utility itself. (2p)

secret 2 : $MRS^{12} = (MU_1)/(MU_2) = P_1/P_2$ (3p)

$\rightarrow X_2/X_1 = P_1/P_2$

"the last spent on each good should give the same utility" OR "marginal utility of a \$ spent on each good should be equal". If this condition does not hold, let's say last \$ spent on good 1 brings more utility than the last \$ spent on good 2, then the consumer should buy less of the second good and buy more of the first good to increase his/her utility. (2p)

start with secret 2 : $X_2/X_1 = P_1/P_2 \rightarrow X_2 = ((X_1P_1)/(P_2))$ (1p)

now plug this in secret 1 :

$p_1x_1 + p_2x_2 = m = P_1x_1 + P_2(X_1P_1)/(P_2) = m = 2P_1x_1$ (2p)

\rightarrow Then $X_1 = (m/(2p_1)) = (1/2)(m/p_1)$

Since $X_2 = ((X_1P_1)/(P_2)) = ((mP_1)/(2P_1P_2)) = (m/(2p_2))$
 $= (1/2)(m/(p_2)) = X_2$

$$X_1 = (m/(2p_1))$$

$$X_2 = (m/(2p_2)) \quad (2p)$$

The solution is interior since none of good's optimal consumption level is zero. Furthermore in a Cobb-Douglas case the solution has to be interior as long as income level (m) is larger than zero ! (we also assume positive prices)
(1p)

$$\text{Using the magic formula } X_2 = (m/(2p_2)) \rightarrow X_2 p_2 = m/2$$

note that the left hand side is the total money spent on good 2 (sirleon).
So half of the income is spent on sirleon. (2p)

$$\text{e) } X_1 = (m/(2p_1)) = ((80)/(2 * 4)) = 10 \quad (1p)$$

$$X_2 = (m/(2p_2)) = ((80)/(2 * 2)) = 20 \quad (1p)$$

$$X'_1 = (m/(2p'_1)) = ((80)/(2 * 2)) = 20 \quad (1p)$$

$$X'_2 = (m/(2p_2)) = ((80)/(2 * 2)) = 20 \quad (1p)$$

$$\text{Total change in consumption of ribeye is } X'_1 - X_1 = 20 - 10 = 10 \quad (1p)$$

Check figure 2 for the illustration of the change. (3p)

Ribeye is an ordinary good since it's consumption increases as it's price decrease. We can also see this fact from the magic formula . Since the price of ribeye is in the denominator in optimal consumption formula of ribeye, as the price goes down the consumption of it will increase ! (2p)

f) How much money does Ava need to consume the original bundle with the new prices ?

$$p'_1 x_1 + p_2 x_2 = m' = (\$2 * 10) + (\$2 * 20) = \$60 \quad \text{is enough} \quad (3p)$$

Now calculating the optimal bundle with this imaginary income :

$$X_1^s = (m'/(2p_1)) = ((60)/(2 * 2)) = 15 \quad (2p)$$

$$X_2^s = (m'/(2p_2)) = ((60)/(2 * 2)) = 15$$

$$\text{So Substitution Effect (S.E.) is : } 15 - 10 = 5 \quad (2p)$$

$$T.E = S.E. + I.E. \quad \text{then : } I.E. = 10 - 5 = 5 \quad (1p)$$

Check figure 2 for the illustration. (3p)

Q 2) (15 points)

a) the bundle : $(1x_1 + 2x_2)$

A suitable utility function would be : $\min(2x_1, 1x_2)$ (1p)

b) Check figure 3 (2p)

c) Two secrets are : $(1.5p + 1.5p)$

$$\begin{aligned} 2x_1 &= x_2 \\ p_1x_1 + p_2x_2 &= m \end{aligned}$$

hence

$$2x_1 + 2(2x_1) = 30$$

$$x_1^* = \frac{m}{p_1 + 2p_2} \text{ and } x_2^* = 2\frac{m}{p_1 + 2p_2}$$

$$\begin{aligned} x_1^* &= \frac{30}{2 + (2 * 2)} = 5 \\ \text{and } x_2^* &= 2\frac{30}{2 + (2 * 2)} = 10 \end{aligned}$$

(1.5p + 1.5p)

The solution is interior since none of the optimal consumption levels are zero. (1p)

d) Using the magic formulas :

$$\begin{aligned} x_1^* &= \frac{30}{1 + (2 * 2)} = 6 \\ \text{and } x_2^* &= 2\frac{30}{1 + (2 * 2)} = 12 \end{aligned}$$

(1.5p + 1.5p)

S.E. is zero. In a perfect complements case there is no substitution effect related to a price change. (2p)

Q.3) (15 points)

a) $U(X_1, X_2) = 2X_1 + 2X_2$ can be a utility function for this kind of preferences. (2p)

b) Check the figure 4 (2p)

c) $MRS^{12} = ((MU_1)/(MU_2)) = 1/1 = 1$ given the utility function. (2p)
comparing MRS to the price ratio : $1 > \frac{2}{6} = \frac{P_1}{P_2}$

So only good 1 is consumed : $X_1^* = m/P_1 = 12/2 = 6$ and $X_2^* = 0$ (3p)

The solution is a corner one. (1p)

d) check figure 5 & 6 for the income-offer curve and the engel curve. (2p+2p)
Pepsi is normal as the consumption of it increases as the income increases.

(1p)

Q.4)

a) $\frac{10}{2} = 5$ is her real wage. (1p)

This number is her purchasing power in terms of consumption good. (1p)

b) check figure 7 (3p)

c)

$$\text{secret 1: } w * R + p_c C = 24 * w \quad (2p)$$

$$\text{or } 10R + 2C = 24 * 10 = 240$$

$$\text{secret 2 : } MRS^{RC} = ((MU_R)/(MU_C)) = w/P_c \rightarrow 1/R = w/P_c \quad (2p)$$

$$\rightarrow R = \frac{P_c}{w} \quad (2p)$$

d) If $w = \$1$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 10hr$ (1p)

If $w = \$2$ and $P_c = \$10$ the labor supply : $R = \frac{P_c}{w} = 5hr$ (1p)

The labor supply (24-R) is increasing in real wage ! (2p)

Bonus Question

$$V(x_1, x_2) = f[U(x_1, x_2)]$$

$$MRS^U = ((MU_1)/(MU_2)) = (\partial U/\partial X_1)/(\partial U/\partial X_2)$$

$$MRS^V = ((MV_1)/(MV_2)) = (\partial V/\partial X_1)/(\partial V/\partial X_2)$$

$$= \frac{\partial f[U(x_1, x_2)]/\partial X_1}{\partial f[U(x_1, x_2)]/\partial X_2} \quad \text{by chain rule :}$$

$$= \frac{f'(\cdot) * \partial U(x_1, x_2)/\partial X_1}{f'(\cdot) * \partial U(x_1, x_2)/\partial X_2} = \frac{\partial U(x_1, x_2)/\partial X_1}{\partial U(x_1, x_2)/\partial X_2} = MRS^U$$

Figure 1

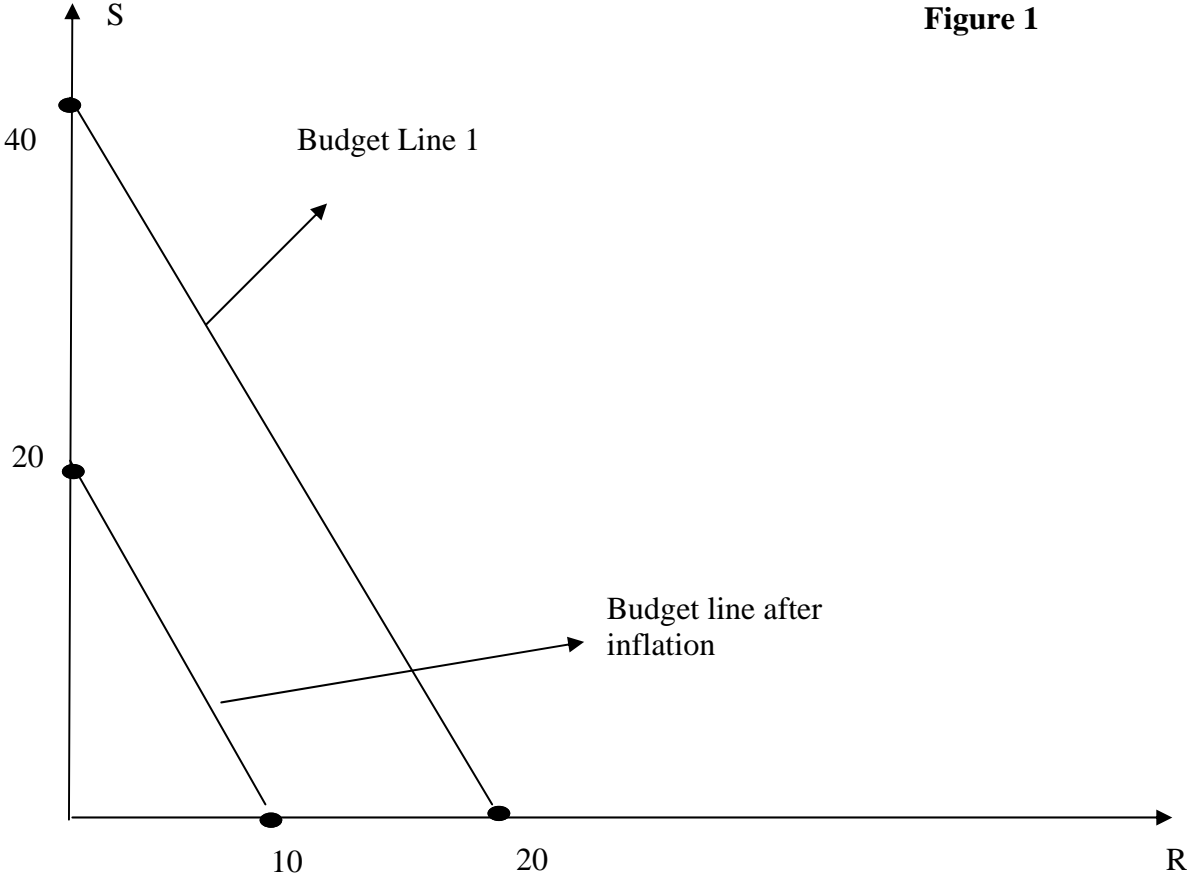
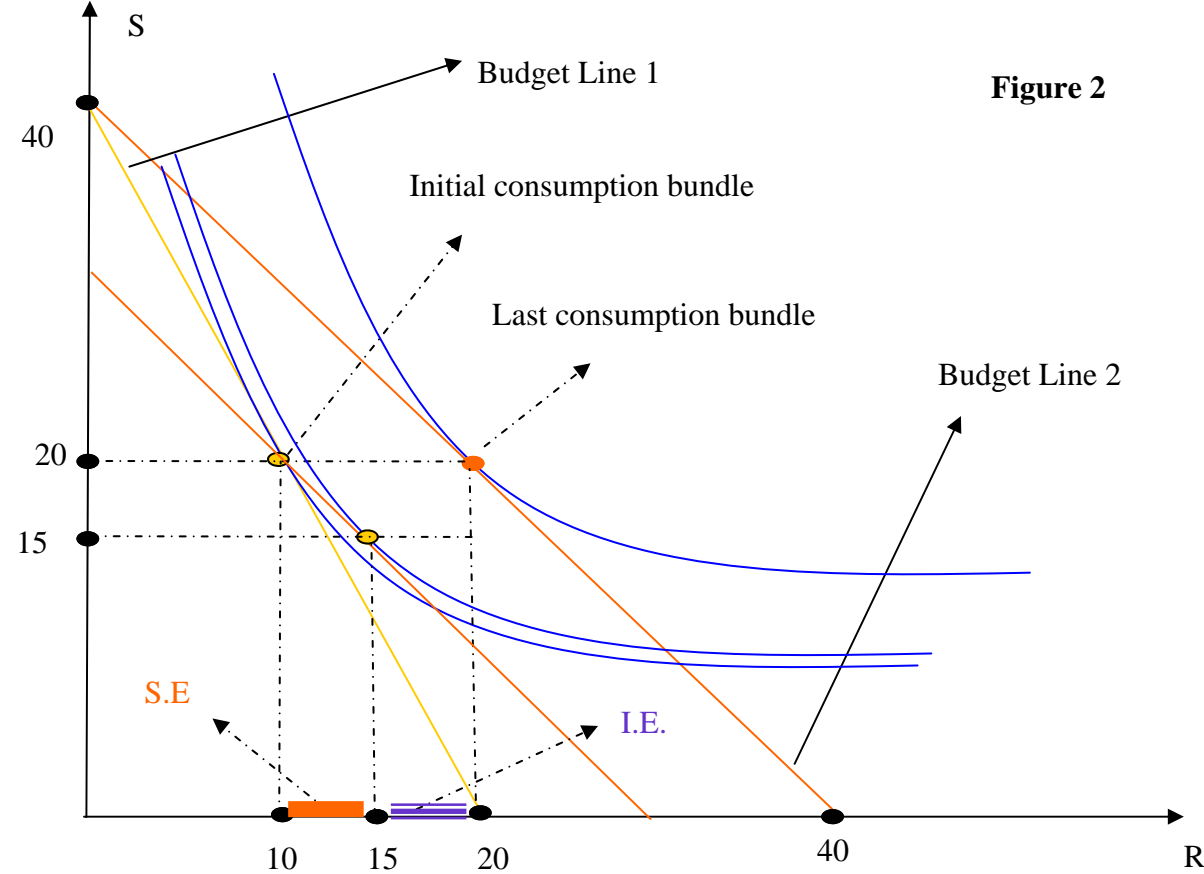
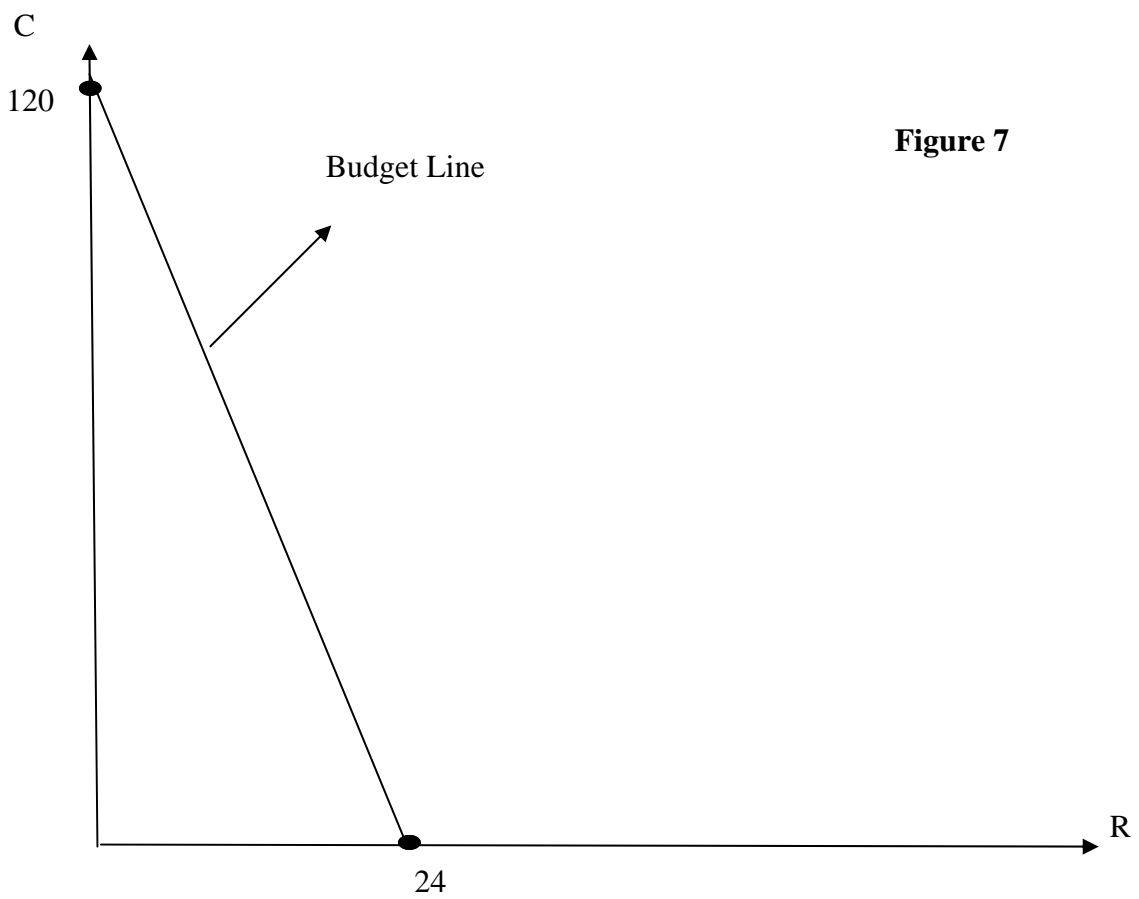
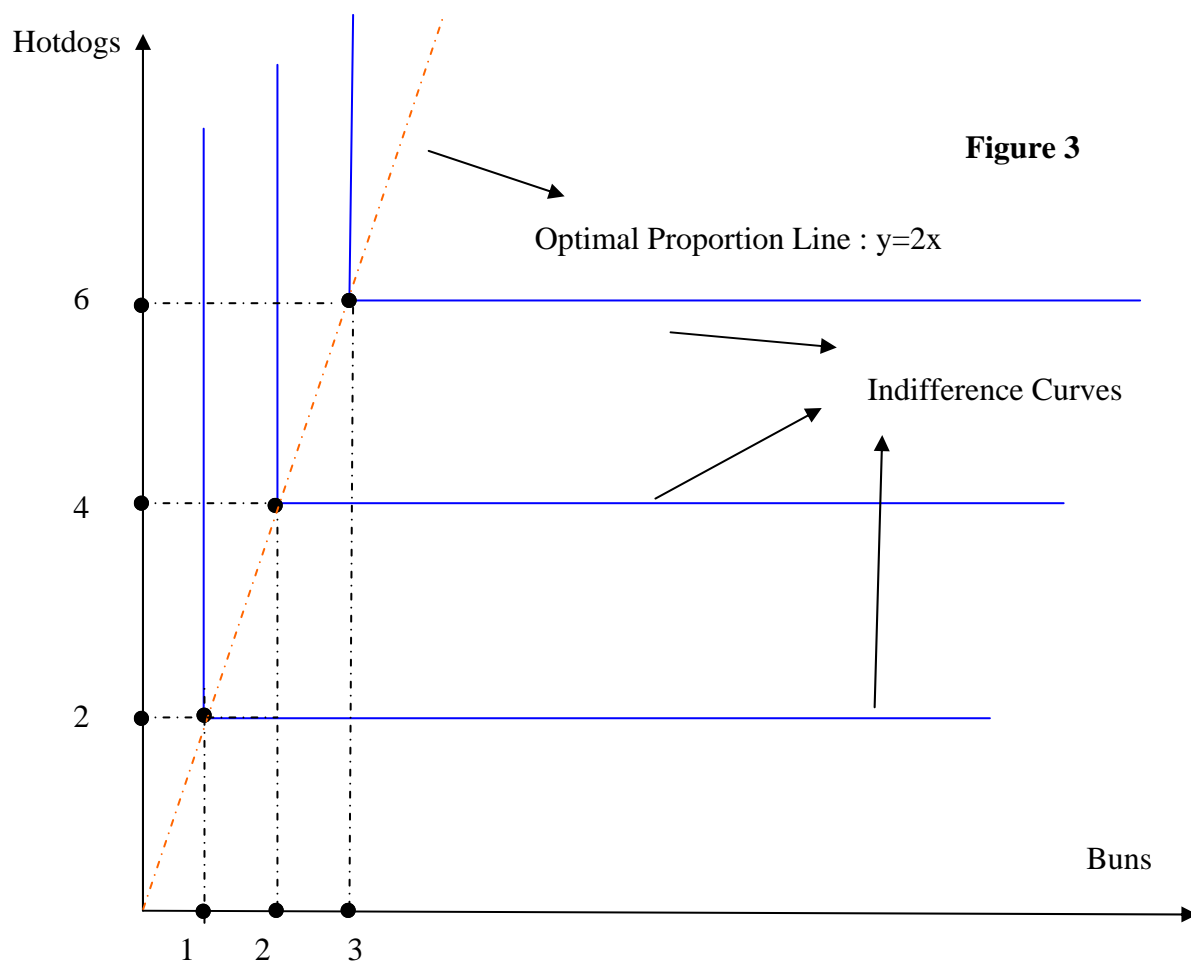
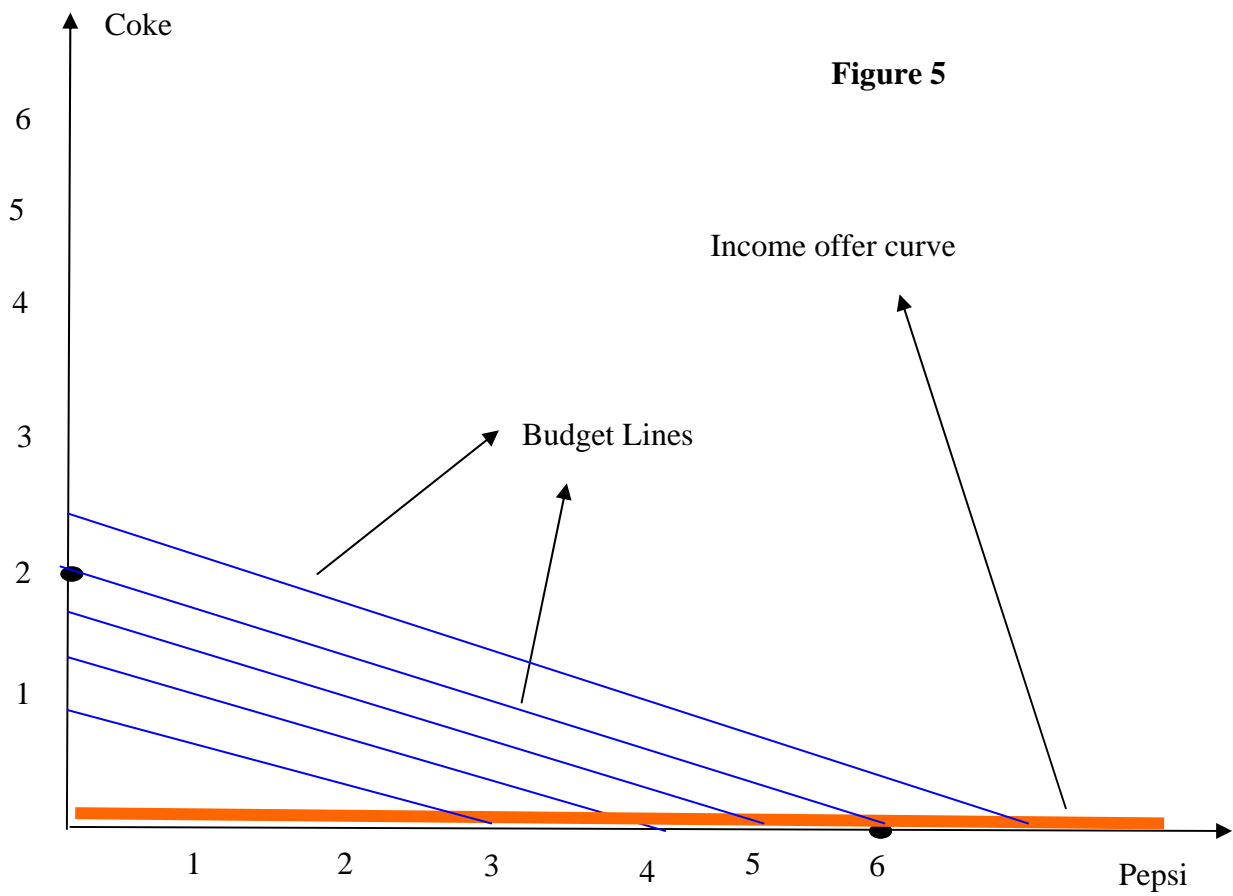
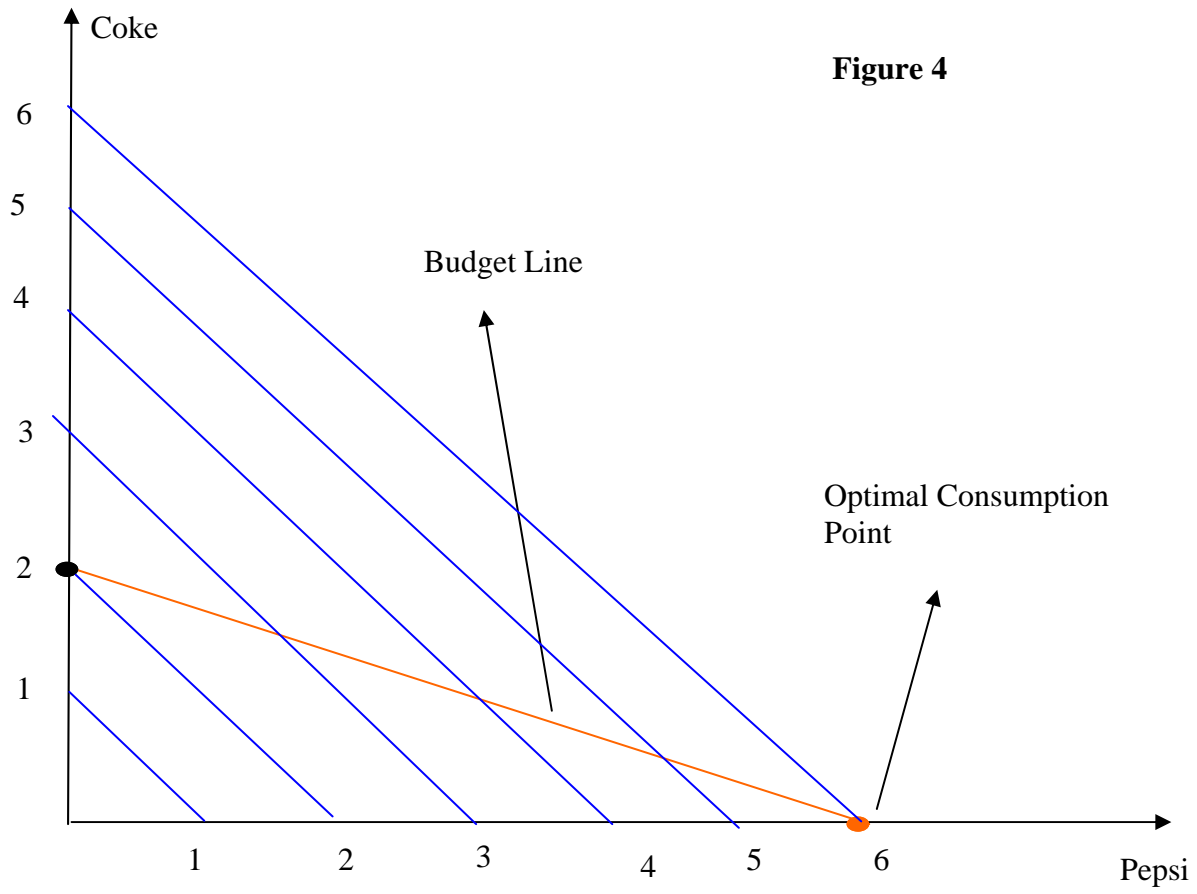


Figure 2







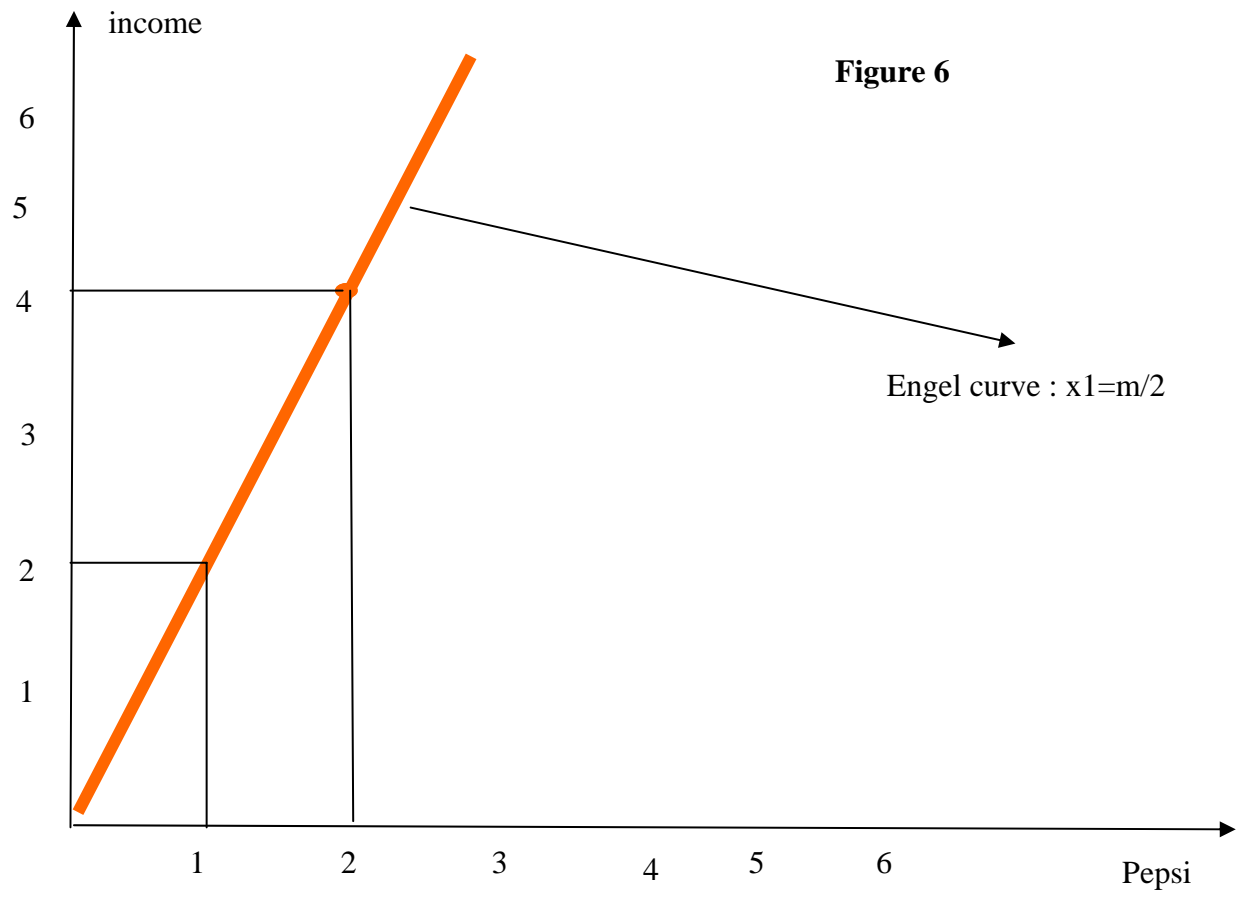


Figure 6