Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (50p) (Standard choice)

Jim has two pleasures in his life, drinking Port wine (x_1) and reading books (x_2).

a) Port wine x_1 costs $p_1 = \$20$ a bottle and each book x_2, $p_2 = \$20$, and his daily budget is $m = \$600$. Show geometrically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set would be affected by introduction of ad valorem tax on imported wine (but not books) assuming tax rate $\tau = 100\%$.

b) Jim seems to be a fairly sophisticated fellow with quite a complicated utility function

$$U(x_1, x_2) = 123 \times \log\left[\exp\left[(6\ln x_1 + 3\ln x_2)^2\right] + 10\right]^{322}$$

Argue that in fact Jim is not really that sophisticated as his preferences can represented by a significantly simpler utility function. (one sentence + simpler utility function). If you are unable to answer point b) in the remaining part of Problem 1, you can assume $U(x_1, x_2) = x_1^2 x_2$.

c) Find Jim's marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give a formula for MRS). What is the value of MRS at consumption bundle $(1, 2)$ (give a number)? Which of the two commodities is more precious to Jim?

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).
- Describe how the two "secrets of happiness" can be seen in the graph.(graph + 2 sentences)
- Derive the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation).

e) Using your formula from d), find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- $p_1 = \$40, p_2 = \$20$ and $m = \$600$ (give two numbers).

and after the price of Port wine decreased:

- for $p_1 = \$20, p_2 = \$20$ and $m = \$600$ (give two numbers).

What is the total change in consumption of Port wine? (give a number). Illustrate the change on the graph. Is Port wine an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of x_1 from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

Problem 2 (20p) (Intertemporal choice)

Marky Mark is a rap singer who earns $m_1 = \$100$ when young and $m_2 = \$300$ when old.

a) What is the present value (in terms of \$ from period one) of Marky Mark's income (give one number).

b) Write down Marky Mark's budget constraint (one inequality) and plot budget set given interest rate $r = 200\%$ in the graph. Mark all consumption plans on the budget line that require borrowing and the ones that require saving.

c) Marky Mark's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$ and discount rate is $\delta = 2$. Provide an economic interpretation of parameter δ (one sentence). Using magic formulas, find the optimal consumption plan and the optimal saving strategy. (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes no + one sentence) Is Sam tilting his consumption? (yes no + one sentence)

d) (Perpetuity) Your sister has just promised to send you pocket money of \$500 each month starting next month and she will keep doing it forever. What is the present value of "having such a sister" if monthly interest rate is equal to 1% (one number).

e) (Annuity) You are going to work for 45 years (from 20 to 65) earning income of \$1000,000 a year. Then you are going to retire and going to live for another 35 years (till you are 100). Assume that the annual interest is given by $r = 5\%$. Write down equation that would allow you to determine the maximal constant level of consumption C throughout your whole life (80 years). (write down equation that determines PV of income and consumption, but you need not calculate C).

Problem 3 (15p) (Perfect complements)

The old recipe for Pierogies (Polish dumplings) requires that sauerkraut x_1 is mixed with portabella mushrooms x_2 in a fixed (and sacred) proportion of 4:1.

a) Propose a utility function over sauerkraut and mushrooms (function $U(x_1, x_2)$) assuming that pierogies is a good.

b) In the commodity space, carefully depict indifference curves (marking the optimal proportion line).

c) Assume that $p_1 = 2$ and $p_2 = 2$ and income is $m = 40$. Write down two secrets of happiness (give two equations) that determine the optimal choice (two numbers). Explain the economic intuition behind the conditions (one sentence for each secret). Is your solution interior (yes or no)?

d) Without any calculations, in two separate graphs plot the price offer curve and income offer curve (just plot two curves).

Problem 4 (15p) (Perfect substitutes)

Sam does not have any money but is endowed with $\omega_1 = 10$ apples and $\omega_1 = 30$ oranges and prices of the two are $p_1 = p_2 = 1$. His utility function is given by $U(x_1, x_2) = 2x_1 + x_2$

a) Find Sam's budget line, marking the endowment point.

b) Find the optimal choice of the two commodities if his utility is given by $U(x_1, x_2) = 2x_1 + x_2$ (give two numbers). Is your solution interior?

c) Find net demands in optimum (give two numbers). Is Sam a net buyer or a net seller of apples. How about oranges? (chose one for each commodity)

d) In mathematics, the solution to a problem with perfect substitutes is called "bang, bang" solution. Provide intuition why (one sentence).

Bonus question (Just for fun)

Derive PV formula for annuity and perpetuity.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (50p) (Standard choice)

Jim has two pleasures in his life, drinking Port wine (x_1) and reading books (x_2).

a) Port wine x_1 costs $p_1 = \$10$ a bottle and each book x_2, $p_2 = \$10$, and his daily budget is $m = \$120$. Show geometrically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set would be affected by introduction of ad valorem tax on imported wine (but not books) assuming tax rate $\tau = 100\%$.

b) Jim seems to be a fairly sophisticated fellow with quite a complicated utility function

$$U(x_1, x_2) = 12 \times \log\left[\exp\left[\left(4\ln x_1 + 2\ln x_2\right)^2\right] + 10\right]^{322}$$

Argue that in fact Jim is not really that sophisticated as his preferences can represented by a significantly simpler utility function. (one sentence + simpler utility function). If you are unable to answer point b) in the remaining part of Problem 1, you can assume $U(x_1, x_2) = x_1^2 x_2^1$.

c) Find Jim's marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give a formula for MRS). What is the value of MRS at consumption bundle $(1, 2)$ (give a number)? Which of the two commodities is more precious to Jim?

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).
- Describe how the two "secrets of happiness" can be seen in the graph.(graph + 2 sentences)
- Derive the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation).

e) Using your formula from d), find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- $p_1 = \$20, p_2 = \$10$ and $m = \$120$ (give two numbers).

and after the price of Port wine decreased:

- for $p_1 = \$10, p_2 = \$10$ and $m = \$120$ (give two numbers).

What is the total change in consumption of Port wine? (give a number). Illustrate the change on the graph. Is Port wine an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of x_1 from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

Problem 2 (20p) (Intertemporal choice)

Marky Mark is a rap singer who earns $m_1 = \$20$ when young and $m_2 = \$60$ when old.

a) What is the present value (in terms of \$ from period one) of Marky Mark's income (give one number).

b) Write down Marky Mark's budget constraint (one inequality) and plot budget set given interest rate $r = 200\%$ in the graph. Mark all consumption plans on the budget line that require borrowing and the ones that require saving.

c) Marky Mark's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$ and discount rate is $\delta = 2$. Provide an economic interpretation of parameter δ (one sentence). Using magic formulas, find the optimal consumption plan and the optimal saving strategy. (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes no + one sentence) Is Sam tilting his consumption? (yes no + one sentence)

d) (Perpetuity) Your sister has just promised to send you pocket money of \$500 each month starting next month and she will keep doing it forever. What is the present value of "having such a sister" if monthly interest rate is equal to 10% (one number).

e) (Annuity) You are going to work for 45 years (from 20 to 65) earning income of \$500,000 a year. Then you are going to retire and going to live for another 35 years (till you are 100). Assume that the annual interest is given by $r = 1\%$. Write down equation that would allow you to determine the maximal constant level of consumption C throughout your whole life (80 years). (write down equation that determines PV of income and consumption, but you need not calculate C).

Problem 3 (15p) (Perfect complements)

The old recipe for Pierogies (Polish dumplings) requires that sauerkraut x_1 is mixed with portabella mushrooms x_2 in a fixed (and sacred) proportion of 1:4.

a) Propose a utility function over sauerkraut and mushrooms (function $U(x_1, x_2)$) assuming that pierogies is a good.

b) In the commodity space, carefully depict indifference curves (marking the optimal proportion line).

c) Assume that $p_1 = 2$ and $p_2 = 2$ and income is $m = 40$. Write down two secrets of happiness (give two equations) that determine the optimal choice (two numbers). Explain the economic intuition behind the conditions (one sentence for each secret). Is your solution interior (yes or no)?

d) Without any calculations, in two separate graphs plot the price offer curve and income offer curve (just plot two curves).

Problem 4 (15p) (Perfect substitutes)

Sam does not have any money but is endowed with $\omega_1 = 100$ apples and $\omega_1 = 200$ oranges and prices of the two are $p_1 = p_2 = 1$. His utility function is given by $U(x_1, x_2) = x_1 + 3x_2$

a) Find Sam's budget line, marking the endowment point.

b) Find the optimal choice of the two commodities if his utility is given by $U(x_1, x_2) = x_1 + 3x_2$ (give two numbers). Is your solution interior?

c) Find net demands in optimum (give two numbers). Is Sam a net buyer or a net seller of apples. How about oranges? (chose one for each commodity)

d) In mathematics, the solution to a problem with perfect substitutes is called "bang, bang" solution. Provide intuition why (one sentence).

Bonus question (Just for fun)

Derive PV formula for annuity and perpetuity.

Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (50p) (Standard choice)

Jim has two pleasures in his life, drinking Port wine (x_1) and reading books (x_2).

a) Port wine x_1 costs $p_1 = \$5$ a bottle and each book x_2, $p_2 = \$5$, and his daily budget is $m = \$60$. Show geometrically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set would be affected by introduction of ad valorem tax on imported wine (but not books) assuming tax rate $\tau = 100\%$.

b) Jim seems to be a fairly sophisticated fellow with quite a complicated utility function

$$U(x_1, x_2) = 12 \times \log\left[\exp\left[(10\ln x_1 + 5\ln x_2)^2\right] + 10\right]^{322}$$

Argue that in fact Jim is not really that sophisticated as his preferences can represented by a significantly simpler utility function. (one sentence + simpler utility function). If you are unable to answer point b) in the remaining part of Problem 1, you can assume $U(x_1, x_2) = x_1^2 x_2^1$.

c) Find Jim's marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give a formula for MRS). What is the value of MRS at consumption bundle $(1, 2)$ (give a number)? Which of the two commodities is more precious to Jim?

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).
- Describe how the two "secrets of happiness" can be seen in the graph.(graph + 2 sentences)
- Derive the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation).

e) Using your formula from d), find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- $p_1 = \$10, p_2 = \$5$ and $m = \$60$ (give two numbers).

and after the price of Port wine decreased:

- for $p_1 = \$5, p_2 = \$5$ and $m = \$60$ (give two numbers).

What is the total change in consumption of Port wine? (give a number). Illustrate the change on the graph. Is Port wine an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of x_1 from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

Problem 2 (20p) (Intertemporal choice)

Marky Mark is a rap singer who earns $m_1 = \$40$ when young and $m_2 = \$120$ when old.

a) What is the present value (in terms of \$ from period one) of Marky Mark's income (give one number).

b) Write down Marky Mark's budget constraint (one inequality) and plot budget set given interest rate $r = 200\%$ in the graph. Mark all consumption plans on the budget line that require borrowing and the ones that require saving.

c) Marky Mark's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$ and discount rate is $\delta = 2$. Provide an economic interpretation of parameter δ (one sentence). Using magic formulas, find the optimal consumption plan and the optimal saving strategy. (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes no + one sentence) Is Sam tilting his consumption? (yes no + one sentence)

d) (Perpetuity) Your sister has just promised to send you pocket money of \$1000 each month starting next month and she will keep doing it forever. What is the present value of "having such a sister" if monthly interest rate is equal to 2% (one number).

e) (Annuity) You are going to work for 40 years (from 20 to 60) earning income of \$100,000 a year. Then you are going to retire and going to live for another 30 years (till you are 90). Assume that the annual

interest is given by $r = 1\%$. Write down equation that would allow you to determine the maximal constant level of consumption C throughout your whole adult life (70 years). (write down equation that determines PV of income and consumption, but you need not calculate C).

Problem 3 (15p) (Perfect complements)

The old recipe for Pierogies (Polish dumplings) requires that sauerkraut x_1 is mixed with portabella mushrooms x_2 in a fixed (and sacred) proportion of 1:3.

a) Propose a utility function over sauerkraut and mushrooms (function $U(x_1, x_2)$) assuming that pierogies is a good.

b) In the commodity space, carefully depict indifference curves (marking the optimal proportion line).

c) Assume that $p_1 = 2$ and $p_2 = 2$ and income is $m = 80$. Write down two secrets of happiness (give two equations) that determine the optimal choice (two numbers). Explain the economic intuition behind the conditions (one sentence for each secret). Is your solution interior (yes or no)?

d) Without any calculations, in two separate graphs plot the price offer curve and income offer curve (just plot two curves).

Problem 4 (15p) (Perfect substitutes)

Sam does not have any money but is endowed with $\omega_1 = 100$ apples and $\omega_1 = 200$ oranges and prices of the two are $p_1 = p_2 = 1$. His utility function is given by $U(x_1, x_2) = x_1 + 4x_2$

a) Find Sam's budget line, marking the endowment point.

b) Find the optimal choice of the two commodities if his utility is given by $U(x_1, x_2) = x_1 + 4x_2$ (give two numbers). Is your solution interior?

c) Find net demands in optimum (give two numbers). Is Sam a net buyer or a net seller of apples. How about oranges? (chose one for each commodity)

d) In mathematics, the solution to a problem with perfect substitutes is called "bang, bang" solution. Provide intuition why (one sentence).

Bonus question (Just for fun)

Derive PV formula for annuity and perpetuity.

Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (just for fun). Make sure you answer the first four questions before working on the bonus one!

Problem 1 (50p) (Standard choice)

Jim has two pleasures in his life, drinking Port wine (x_1) and reading books (x_2).

a) Port wine x_1 costs $p_1 = \$20$ a bottle and each book x_2, $p_2 = \$20$, and his daily budget is $m = \$240$. Show geometrically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set would be affected by introduction of ad valorem tax on imported wine (but not books) assuming tax rate $\tau = 100\%$.

b) Jim seems to be a fairly sophisticated fellow with quite a complicated utility function

$$U(x_1, x_2) = 12 \times \sqrt{\left[\ln\left[\left(20\ln x_1 + 10\ln x_2\right)^2\right] + 10\right]^{322}}$$

Argue that in fact Jim is not really that sophisticated as his preferences can represented by a significantly simpler utility function. (one sentence + simpler utility function). If you are unable to answer point b) in the remaining part of Problem 1, you can assume $U(x_1, x_2) = x_1^2 x_2^1$.

c) Find Jim's marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give a formula for MRS). What is the value of MRS at consumption bundle $(1, 2)$ (give a number)? Which of the two commodities is more precious to Jim?

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).
- Describe how the two "secrets of happiness" can be seen in the graph.(graph + 2 sentences)
- Derive the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation).

e) Using your formula from d), find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- $p_1 = \$40, p_2 = \$20$ and $m = \$240$ (give two numbers).

and after the price of Port wine decreased:

- for $p_1 = \$20, p_2 = \$20$ and $m = \$240$ (give two numbers).

What is the total change in consumption of Port wine? (give a number). Illustrate the change on the graph. Is Port wine an ordinary of a Giffen good? (Chose one + one sentence explaining why.)

f) Decompose the total change in consumption of x_1 from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

Problem 2 (20p) (Intertemporal choice)

Marky Mark is a rap singer who earns $m_1 = \$80$ when young and $m_2 = \$240$ when old.

a) What is the present value (in terms of \$ from period one) of Marky Mark's income (give one number).

b) Write down Marky Mark's budget constraint (one inequality) and plot budget set given interest rate $r = 200\%$ in the graph. Mark all consumption plans on the budget line that require borrowing and the ones that require saving.

c) Marky Mark's intertemporal utility is given by $U(C_1, C_2) = \ln(c_1) + \frac{1}{1+\delta}\ln(c_1)$ and discount rate is $\delta = 2$. Provide an economic interpretation of parameter δ (one sentence). Using magic formulas, find the optimal consumption plan and the optimal saving strategy. (give three numbers C_1, C_2, S). Does Sam smooth his consumption? (yes no + one sentence) Is Sam tilting his consumption? (yes no + one sentence)

d) (Perpetuity) Your sister has just promised to send you pocket money of \$200 each month starting next month and she will keep doing it forever. What is the present value of "having such a sister" if monthly interest rate is equal to 10% (one number).

e) (Annuity) You are going to work for 40 years (from 20 to 60) earning income of \$200,000 a year. Then you are going to retire and going to live for another 30 years (till you are 90). Assume that the annual interest is given by $r = 1\%$. Write down equation that would allow you to determine the maximal constant level of consumption C throughout your whole adult life (70 years). (write down equation that determines PV of income and consumption, but you need not calculate C).

Problem 3 (15p) (Perfect complements)

The old recipe for Pierogies (Polish dumplings) requires that sauerkraut x_1 is mixed with portabella mushrooms x_2 in a fixed (and sacred) proportion of 3:1.

a) Propose a utility function over sauerkraut and mushrooms (function $U(x_1, x_2)$) assuming that pierogies is a good.

b) In the commodity space, carefully depict indifference curves (marking the optimal proportion line).

c) Assume that $p_1 = 2$ and $p_2 = 2$ and income is $m = 160$. Write down two secrets of happiness (give two equations) that determine the optimal choice (two numbers). Explain the economic intuition behind the conditions (one sentence for each secret). Is your solution interior (yes or no)?

d) Without any calculations, in two separate graphs plot the price offer curve and income offer curve (just plot two curves).

Problem 4 (15p) (Perfect substitutes)

Sam does not have any money but is endowed with $\omega_1 = 100$ apples and $\omega_1 = 200$ oranges and prices of the two are $p_1 = p_2 = 1$. His utility function is given by $U(x_1, x_2) = 2x_1 + x_2$

a) Find Sam's budget line, marking the endowment point.

b) Find the optimal choice of the two commodities if his utility is given by $U(x_1, x_2) = 2x_1 + x_2$ (give two numbers). Is your solution interior?

c) Find net demands in optimum (give two numbers). Is Sam a net buyer or a net seller of apples. How about oranges? (chose one for each commodity)

d) In mathematics, the solution to a problem with perfect substitutes is called "bang, bang" solution. Provide intuition why (one sentence).

Bonus question (Just for fun)

Derive PV formula for annuity and perpetuity.

Problem 1 (50pts)

(a) (5pts) The budget constraint is given by $20x_1 + 20x_2 = 600$. With the tax $\tau = 100\%$, it becomes $20(1+\tau)x_1 + 20x_2 = 40x_1 + 20x_2 = 600$. Since it plays a role in increasing p_1 by 100%, the budget line must shift in as displayed in Figure 1.

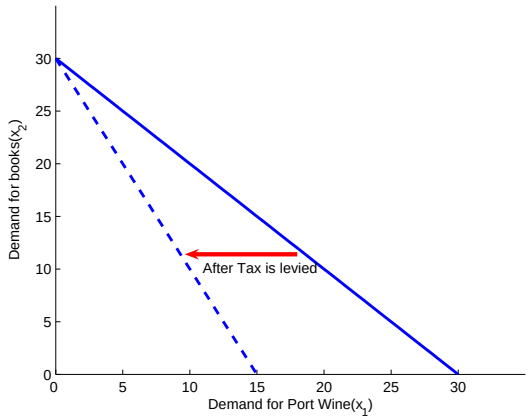


Figure 1: The Budget Set

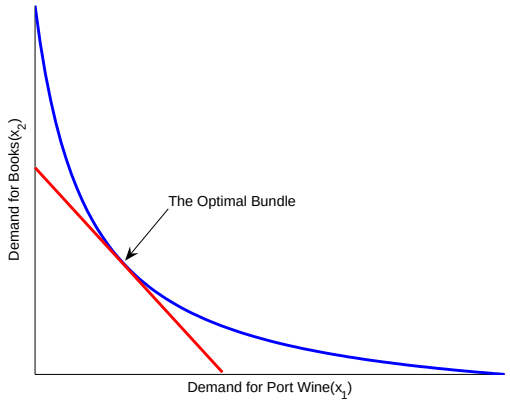


Figure 2: Two Secrets of Happiness

(b) (5pts) His sophisticated utility function is simply a monotone transformation of $2\ln x_1 + \ln x_2$.

(c) (5pts) From the previous part, we shall write his utility function $U(x_1, x_2) = 2\ln x_1 + \ln x_2$ without loss of generality. Differentiating with respect to x_1 and x_2 gives us

$$\text{MU}_1 = \frac{2}{x_1} \quad \text{and} \quad \text{MU}_2 = \frac{1}{x_2}$$

Hence the marginal rate of substitution, defined as the (negative) ratio of two marginal utilities, is given by $\text{MRS} = -\dfrac{2/x_1}{1/x_2} = -\dfrac{2x_2}{x_1}$. At the bundle $(1,2)$, it takes $-\dfrac{2\cdot 2}{1} = -4$. Since its absolute value is greater than 1, the port wine is more valuable from his perspective.

(d) (10pts) Two secrets of happiness state

$$(i)\ \ p_1x_1 + p_2x_2 = m$$
$$(ii)\ \ MRS\left(= -\frac{2x_2}{x_1}\right) = -\frac{p_1}{p_2}$$

The first condition indicates Jim's budget line; he has to utilize all his monetary resource to maximize utility. And the second condition requires the MRS coincide with the relative price. It implies that the consumer's subjective exchange ratio(MRS) must be the same as the market's objective exchange ratio at the optimal bundle. Figure 2 illustrates it in the graph. For a bundle to be optimal, it must be on the budget line and the indifference curve through it must be tangent to the budget line.

Note that solving (ii) for x_1 gives $x_1 = 2\dfrac{p_2}{p_1}x_2$. Plugging it in (i) for x_1 gives

$$p_1\left(2\frac{p_2}{p_1}x_2\right) + p_2x_2 = m \rightarrow x_2 = \frac{m}{3p_2}$$

Consequently, $x_1 = 2\dfrac{p_2}{p_1}\dfrac{m}{3p_2} = \dfrac{2m}{3p_1}$.

(e) (15pts) When $p_1 = 40$, $p_2 = 20$, $m = 600$,

$$x_1 = \frac{2 \cdot 600}{3 \cdot 40} = 10 \quad \text{and} \quad x_2 = \frac{600}{3 \cdot 20} = 10.$$

When p_1 falls down to $p_1 = 20$,

$$x_1 = \frac{2 \cdot 600}{3 \cdot 20} = 20 \quad \text{and} \quad x_2 = \frac{600}{3 \cdot 20} = 10.$$

The total change in x_1 is $20 - 10 = 10$. Note that the demand for Port wine is increasing as p_1 decreases. It is an ordinary good. Figure 3 illustrate the total change in x_1.

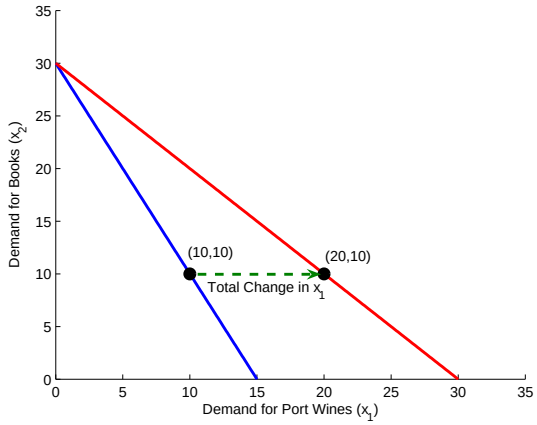


Figure 3: Total Change in x_1

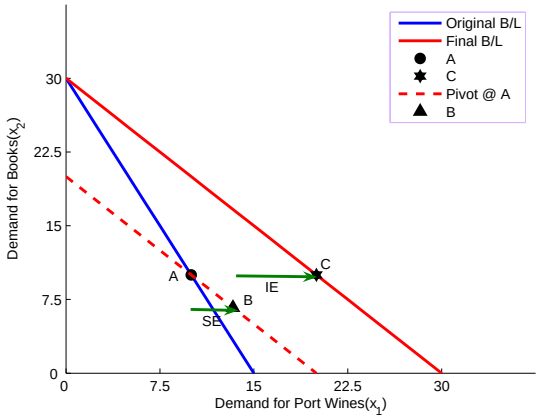


Figure 4: Substitution and Income Effect

(f) (10pts) In order to decompose it, we have to adjust money income m', which represents the amount of money income that is necessary to make the original bundle(i.e. before p_1 changes it was (10,10)) affordable at the new price.

$$m' = 20 \cdot 10 + 20 \cdot 10 = 400.$$

After p_1 changes, Jim needs $m' = 400$ to purchase the original commodity bundle. Then we need figure out what is the optimal choice for Port wine at the new price and this m'. Plugging $p_1 = 20$ and $m = 400$ into the formula $x_1 = \dfrac{2m}{3p_1}$ gives us $x_1 = \dfrac{40}{3}$. The substitution effect is given by the difference between the new demand $\frac{40}{3}$ and the original one 10

$$\text{SE} = \frac{40}{3} - 10 = \frac{10}{3}$$

The associated income effect immediately follows from Slutsky's equation, which tells us that the total change can be decomposed into the substitution and income effect.

$$\text{IE} = \text{Total Change} - \text{SE} = 10 - \frac{10}{3} = \frac{20}{3}$$

In figure 4, I denote by A,B and C the original optimal choice, the final optimal choice, and the intermediate optimal choice we obtained from m', respectively. The substitution effect is the gap between A and C on the horizontal axis while the income effect is between C and B.

Problem 2 (20pts)

(a) (2pts) $\text{PV} = 100 + \dfrac{300}{1+r} = 100 + \dfrac{300}{3} = 200$

(b) (3pts) The budget constraint is written by

$$C_1 + \frac{1}{3}C_2 = 200$$

To wit briefly, the left-hand side is the present value of consumption stream (C_1, C_2). Observe that I discounted C_2 by $\frac{1}{1+r} = \frac{1}{3}$ to express it in terms of present value. The right-hand side simply represents the present value of his income we got in part (a). The budget line tells us that they should be balanced. See figure 5.

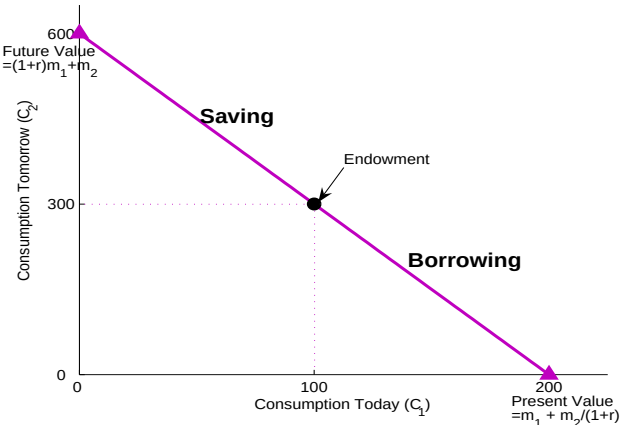


Figure 5: Intertemporal Choice Model

(c) (7pts) For $\delta = 2$, observe that the utility function comes down to

$$U(C_1, C_2) = \ln C_1 + \frac{1}{3}\ln C_2$$

which is a Cobb-Douglas function. Using its magic formula, we obtain

$$C_1 = \frac{a}{a+b}\cdot\frac{m}{p_1} = \frac{1}{1+\frac{1}{3}}\cdot\frac{200}{1} = 150$$

$$C_2 = \frac{b}{a+b}\cdot\frac{m}{p_2} = \frac{\frac{1}{3}}{1+\frac{1}{3}}\cdot\frac{200}{\frac{1}{3}} = 150.$$

To consume 150 today, he must save $100 - 150 = -50$, that is, he must borrow 50. He does smooth his consumption plan because $C_1 = C_2 = 150$.

(d) (3pts) $\text{PV} = \dfrac{x}{r} = \dfrac{500}{0.01} = 50,000.$

(e) (5pts) The characterization equation is given by

$$\frac{1,000,000}{0.05}\left[1 - \frac{1}{(1+0.05)^{45}}\right] = \frac{C}{0.05}\left[1 - \frac{1}{(1+0.05)^{80}}\right]$$

The left-hand side and the right-hand side represents the present value of earnings for next 45 years and the present value of consumptions for next 80 years, respectively. And the constant level of consumption C comes from the equation in which these two values are balanced.

Problem 3 (15pts)

(a) (2pts) $U(x_1, x_2) = \min\{x_1, 4x_2\}$

(b) (3pts) See figure 6.

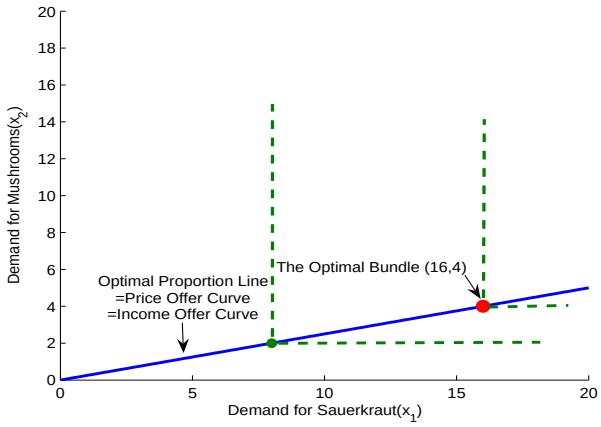


Figure 6: Perfect Complements

(c) (6pts) The associated "two secrets of happiness" with perfect complements is

(i) $2x_1 + 2x_2 = 40$

(ii) $x_1 = 4x_2$.

The first condition indicates the consumer's balanced budget, and its economic intuition is the same as before: All income must be exhausted. The second condition represents the optimal proportion $(4:1)$ between quantities demanded for sauerkraut and mushrooms.

To figure out the optimal choice, we plug in $4x_2$ for x_1 in the budget line and obtain

$$2(4x_2) + 2x_2 = 10x_2 = 40 \Rightarrow x_2 = 4.$$

Substituting into $x_1 = 4x_2$ we get $x_1 = 16$. The solution is interior as x_1 and x_2 are strictly positive.

(d) (4pts) Since the two goods are perfect complements, both the price offer curve and the income offer curve would coincide with the optimal proportion line. Refer Figure 6.

Problem 4 (15pts)

(a) (4pts) $x_1 + x_2 = 40$. The value of endowment, the total resources available for consumption, is $p_1\omega_1 + p_2\omega_2 = 10 + 30 = 40$. See Figure 7.

(b) (4pts) Since $p_1 = p_2 = 1$ and the two goods are perfect substitutes, Sam is willing to consume the good with more marginal utilities. In this example, it follows from the utility function that apples(good 1) yield more marginal utilities. Therefore, $x_1 = 40$ and $x_2 = 0$. Since $x_2 = 0$, the solution is not interior but it is at the corner.

(c) (4pts) The net demand for apples is $x_1 - \omega_1 = 40 - 10 = 30$, which is positive. It means that Sam is a net buyer of apples. The net demand for oranges, on the other hand, is $x_2 - \omega_2 = 0 - 30 = -30 < 0$. Hence he is a net seller for oranges.

(d) (3pts) It is because apples and oranges, in this example, are perfectly substitutable. Hence the consumer would choose either one good only, depending on the marginal utility from a dollar.

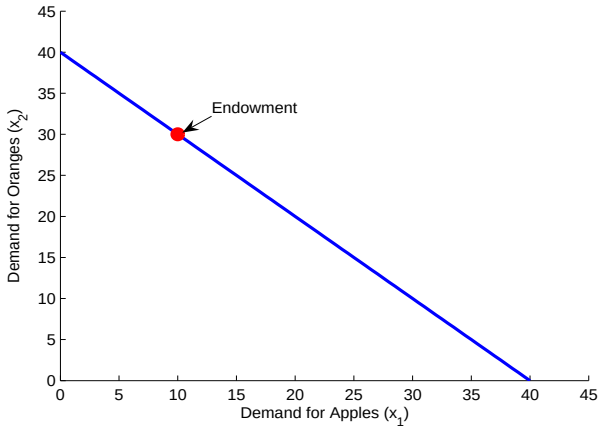


Figure 7: Budget Line and Endowment Point

Bonus Problem

Perpetuity

A perpetuity in an annuity that has no definite end, or a stream of cash payments, say x, that continues indefinitely. With the (risk-free) interest rate r, we can write its present value as

$$\begin{aligned}
\text{PV} &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots \\
&= \frac{x}{(1+r)} + \left[\frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots\right] \\
&= \frac{x}{(1+r)} + \frac{1}{(1+r)} \underbrace{\left[\frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \cdots\right]}_{=\text{PV}}
\end{aligned}$$

In the second step, I just partitioned the sum into the first term and the remainder using the parenthesis. The key step is the next one, factoring $\frac{1}{1+r}$ out of the remainder. Then we come up with the same sequence as the original PV. It leads to the following simple equation

$$\text{PV} = \frac{x}{(1+r)} + \frac{1}{(1+r)} \cdot \text{PV}$$

Solving for PV gives us the desired formula, $\text{PV} = \frac{x}{r}$.

Annuity

The annuity is an asset that promises a terminating stream of fixed payments over a prespecified period of time. Its value is closely linked to *"No Arbitrage Principle"*, the most fundamental principle in finance. It states that "two assets with identical cash flows must trade at the same price." With this principle in hand, consider one annuity which guarantees to annually give us payment x over T years. Now it is the key idea that with the two perpetuities we can replicate the identical cash flows as the annuity.

- Perpetuity 1 - It promises x payment forever from now on.
- Perpetuity 2 - It promises x payment forever but $T+1$ years from now.

Observe that the cash flow of the annuity is the same as the cash flow of "Perpetuity 1 − Perpetuity 2". By no arbitrage condition, then, it must be the case that

$$\begin{aligned}
\text{PV(Annuity)} &= \text{PV(Perpetuity} \quad 1) - \text{PV(Perpetuity} \quad 2) \\
&= \frac{x}{r} - \frac{x}{r} \cdot \frac{1}{(1+r)^T} \\
&= \frac{x}{r}\left[1 - \frac{1}{(1+r)^T}\right].
\end{aligned}$$

Problem 1 (50pts)

(a) (5pts) The budget constraint is given by $10x_1 + 10x_2 = 120$. With the tax $\tau = 100\%$, it becomes $10(1+\tau)x_1 + 10x_2 = 20x_1 + 10x_2 = 120$. Since it plays a role in increasing p_1 by 100%, the budget line must shift in as displayed in Figure 1.

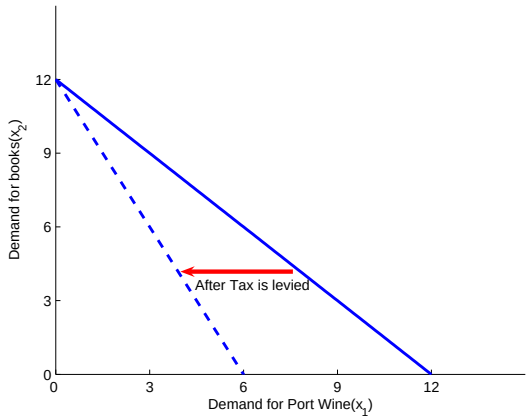


Figure 1: The Budget Set

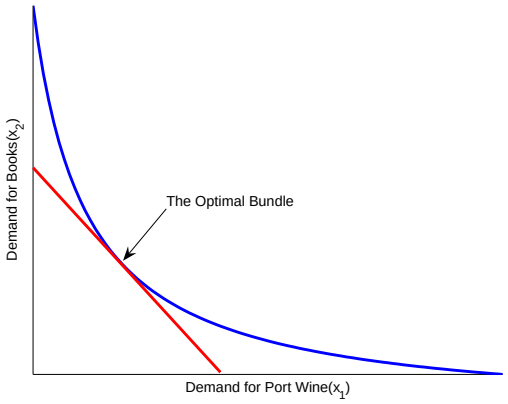


Figure 2: Two Secrets of Happiness

(b) (5pts) His sophisticated utility function is simply a monotone transformation of $2\ln x_1 + \ln x_2$.

(c) (5pts) From the previous part, we shall write his utility function $U(x_1, x_2) = 2\ln x_1 + \ln x_2$ without loss of generality. Differentiating with respect to x_1 and x_2 gives us

$$\text{MU}_1 = \frac{2}{x_1} \quad \text{and} \quad \text{MU}_2 = \frac{1}{x_2}$$

Hence the marginal rate of substitution, defined as the (negative) ratio of two marginal utilities, is given by $\text{MRS} = -\dfrac{2/x_1}{1/x_2} = -\dfrac{2x_2}{x_1}$. At the bundle $(1,2)$, it takes $-\dfrac{2\cdot 2}{1} = -4$. Since its absolute value is greater than 1, the port wine is more valuable from his perspective.

(d) (10pts) Two secrets of happiness state

$$(i)\ \ p_1x_1 + p_2x_2 = m$$
$$(ii)\ \ MRS\left(= -\frac{2x_2}{x_1}\right) = -\frac{p_1}{p_2}$$

The first condition indicates Jim's budget line; he has to utilize all his monetary resource to maximize utility. And the second condition requires the MRS coincide with the relative price. It implies that the consumer's subjective exchange ratio(MRS) must be the same as the market's objective exchange ratio at the optimal bundle. Figure 2 illustrates it in the graph. For a bundle to be optimal, it must be on the budget line and the indifference curve through it must be tangent to the budget line.

Note that solving (ii) for x_1 gives $x_1 = 2\dfrac{p_2}{p_1}x_2$. Plugging it in (i) for x_1 gives

$$p_1\left(2\frac{p_2}{p_1}x_2\right) + p_2x_2 = m \rightarrow x_2 = \frac{m}{3p_2}$$

Consequently, $x_1 = 2\dfrac{p_2}{p_1}\dfrac{m}{3p_2} = \dfrac{2m}{3p_1}$.

(e) (15pts) When $p_1 = 20$, $p_2 = 10$, $m = 120$,

$$x_1 = \frac{2 \cdot 120}{3 \cdot 20} = 4 \quad \text{and} \quad x_2 = \frac{120}{3 \cdot 10} = 4.$$

When p_1 falls down to $p_1 = 10$,

$$x_1 = \frac{2 \cdot 120}{3 \cdot 10} = 8 \quad \text{and} \quad x_2 = \frac{120}{3 \cdot 10} = 4.$$

The total change in x_1 is $8 - 4 = 4$. Note that the demand for Port wine is increasing as p_1 decreases. It is an ordinary good. Figure 3 illustrate the total change in x_1.

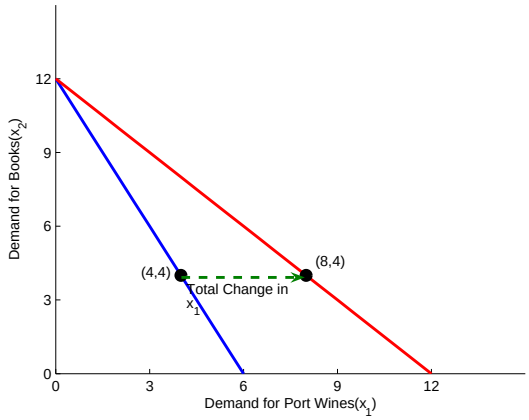


Figure 3: Total Change in x_1

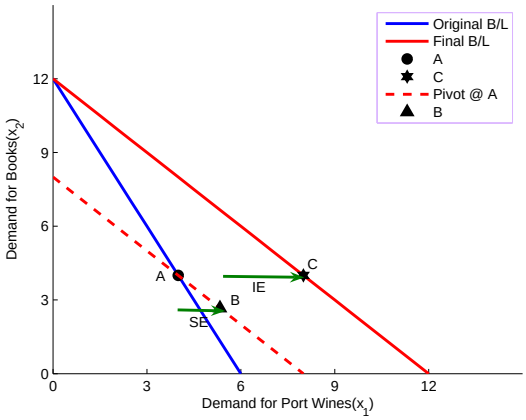


Figure 4: Substitution and Income Effect

(f) (10pts) In order to decompose it, we have to adjust money income m', which represents the amount of money income that is necessary to make the original bundle(i.e. before p_1 changes it was (4,4)) affordable at the new price.

$$m' = 4 \cdot 10 + 4 \cdot 10 = 80.$$

After p_1 changes, Jim needs $m' = 80$ to purchase the original commodity bundle. Then we need figure out what is the optimal choice for Port wine at the new price and this m'. Plugging $p_1 = 10$ and $m = 80$ into the formula $x_1 = \dfrac{2m}{3p_1}$ gives us $x_1 = \dfrac{16}{3}$. The substitution effect is given by the difference between the new demand $\frac{16}{3}$ and the original one 4

$$\text{SE} = \frac{16}{3} - 4 = \frac{4}{3}$$

The associated income effect immediately follows from Slutsky's equation, which tells us that the total change can be decomposed into the substitution and income effect.

$$\text{IE} = \text{Total Change} - \text{SE} = 4 - \frac{4}{3} = \frac{8}{3}$$

In figure 4, I denote by A,B and C the original optimal choice, the final optimal choice, and the intermediate optimal choice we obtained from m', respectively. The substitution effect is the gap between A and C on the horizontal axis while the income effect is between C and B.

Problem 2 (20pts)

(a) (2pts) $\text{PV} = 20 + \dfrac{60}{1+r} = 20 + \dfrac{60}{3} = 40$

(b) (3pts) The budget constraint is written by

$$C_1 + \frac{1}{3}C_2 = 40$$

To wit briefly, the left-hand side is the present value of consumption stream (C_1, C_2). Observe that I discounted C_2 by $\frac{1}{1+r} = \frac{1}{3}$ to express it in terms of present value. The right-hand side simply represents the present value of his income we got in part (a). The budget line tells us that they should be balanced. See figure 5.

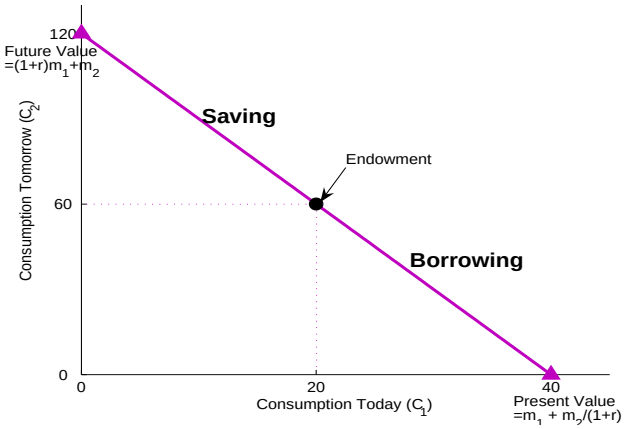


Figure 5: Intertemporal Choice Model

(c) (7pts) For $\delta = 2$, observe that the utility function comes down to

$$U(C_1, C_2) = \ln C_1 + \frac{1}{3}\ln C_2$$

which is a Cobb-Douglas function. Using its magic formula, we obtain

$$C_1 = \frac{a}{a+b}\cdot\frac{m}{p_1} = \frac{1}{1+\frac{1}{3}}\cdot\frac{40}{1} = 30$$

$$C_2 = \frac{b}{a+b}\cdot\frac{m}{p_2} = \frac{\frac{1}{3}}{1+\frac{1}{3}}\cdot\frac{40}{\frac{1}{3}} = 30.$$

To consume 30 today, he must save $20 - 30 = -10$, that is, he must borrow 10. He does smooth his consumption plan because $C_1 = C_2 = 30$.

(d) (3pts) $\text{PV} = \dfrac{x}{r} = \dfrac{500}{0.1} = 5,000.$

(e) (5pts) The characterization equation is given by

$$\frac{500,000}{0.01}\left[1 - \frac{1}{(1+0.01)^{45}}\right] = \frac{C}{0.01}\left[1 - \frac{1}{(1+0.01)^{80}}\right]$$

The left-hand side and the right-hand side represents the present value of earnings for next 45 years and the present value of consumptions for next 80 years, respectively. And the constant level of consumption C comes from the equation in which these two values are balanced.

Problem 3 (15pts)

(a) (2pts) $U(x_1, x_2) = \min\{4x_1, x_2\}$

(b) (3pts) See figure 6.

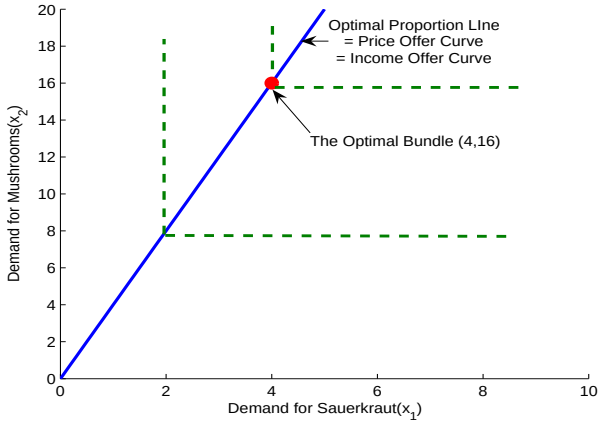


Figure 6: Perfect Complements

(c) (6pts) The associated "two secrets of happiness" with perfect complements is

(i) $2x_1 + 2x_2 = 40$

(ii) $4x_1 = x_2$.

The first condition indicates the consumer's balanced budget, and its economic intuition is the same as before: All income must be exhausted. The second condition represents the optimal proportion $(1:4)$ between quantities demanded for sauerkraut and mushrooms.

To figure out the optimal choice, we plug in $4x_1$ for x_2 in the budget line and obtain

$$2x_1 + 2(4x_1) = 10x_1 = 40. \Rightarrow x_1 = 4.$$

Substituting into $x_2 = 4x_1$ we get $x_2 = 16$. The solution is interior as x_1 and x_2 are strictly positive.

(d) (4pts) Since the two goods are perfect complements, both the price offer curve and the income offer curve would coincide with the optimal proportion line. Refer Figure 6.

Problem 4 (15pts)

(a) (4pts) $x_1 + x_2 = 300$. The value of endowment, the total resources available for consumption, is $p_1\omega_1 + p_2\omega_2 = 100 + 200 = 300$. See Figure 7.

(b) (4pts) Since $p_1 = p_2 = 1$ and the two goods are perfect substitutes, Sam is willing to consume the good with more marginal utilities. In this example, it follows from the utility function that oranges(good 2) yield more marginal utilities. Therefore, $x_2 = 300$ and $x_1 = 0$. Since $x_1 = 0$, the solution is not interior but it is at the corner.

(c) (4pts) The net demand for apples is $x_1 - \omega_1 = 0 - 100 = -100$, which is negative. It means that Sam is a net seller of apples. The net demand for oranges, on the other hand, is $x_2 - \omega_2 = 300 - 200 = 100 > 0$. Hence he is a net buyer for oranges.

(d) (3pts) It is because apples and oranges, in this example, are perfectly substitutable. Hence the consumer would choose either one good only, depending on the marginal utility from a dollar.

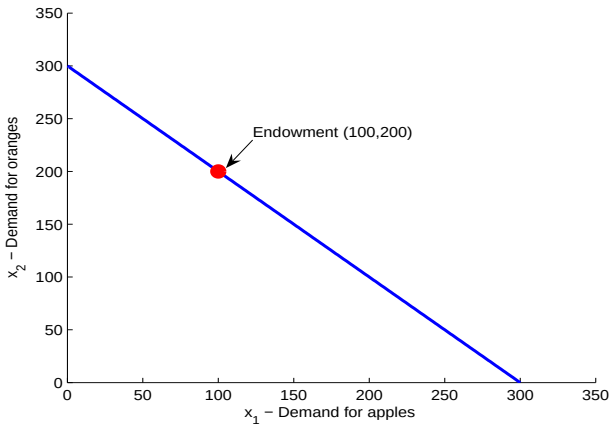


Figure 7: Budget Line and Endowment Point

Bonus Problem

Perpetuity

A perpetuity in an annuity that has no definite end, or a stream of cash payments, say x, that continues indefinitely. With the (risk-free) interest rate r, we can write its present value as

$$\begin{aligned}
\text{PV} &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots \\
&= \frac{x}{(1+r)} + \left[\frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots\right] \\
&= \frac{x}{(1+r)} + \frac{1}{(1+r)} \underbrace{\left[\frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \cdots\right]}_{=\text{PV}}
\end{aligned}$$

In the second step, I just partitioned the sum into the first term and the remainder using the parenthesis. The key step is the next one, factoring $\frac{1}{1+r}$ out of the remainder. Then we come up with the same sequence as the original PV. It leads to the following simple equation

$$\text{PV} = \frac{x}{(1+r)} + \frac{1}{(1+r)} \cdot \text{PV}$$

Solving for PV gives us the desired formula, $\text{PV} = \frac{x}{r}$.

Annuity

The annuity is an asset that promises a terminating stream of fixed payments over a prespecified period of time. Its value is closely linked to *"No Arbitrage Principle"*, the most fundamental principle in finance. It states that "two assets with identical cash flows must trade at the same price." With this principle in hand, consider one annuity which guarantees to annually give us payment x over T years. Now it is the key idea that with the two perpetuities we can replicate the identical cash flows as the annuity.

- Perpetuity 1 - It promises x payment forever from now on.
- Perpetuity 2 - It promises x payment forever but $T+1$ years from now.

Observe that the cash flow of the annuity is the same as the cash flow of "Perpetuity 1 − Perpetuity 2". By no arbitrage condition, then, it must be the case that

$$\begin{aligned}
\text{PV(Annuity)} &= \text{PV(Perpetuity} \quad 1) - \text{PV(Perpetuity} \quad 2) \\
&= \frac{x}{r} - \frac{x}{r} \cdot \frac{1}{(1+r)^T} \\
&= \frac{x}{r}\left[1 - \frac{1}{(1+r)^T}\right].
\end{aligned}$$

Problem 1 (50pts)

(a) (5pts) The budget constraint is given by $5x_1 + 5x_2 = 60$. With the tax $\tau = 100\%$, it becomes $5(1+\tau)x_1 + 5x_2 = 10x_1 + 5x_2 = 60$. Since it plays a role in increasing p_1 by 100%, the budget line must shift in as displayed in Figure 1.

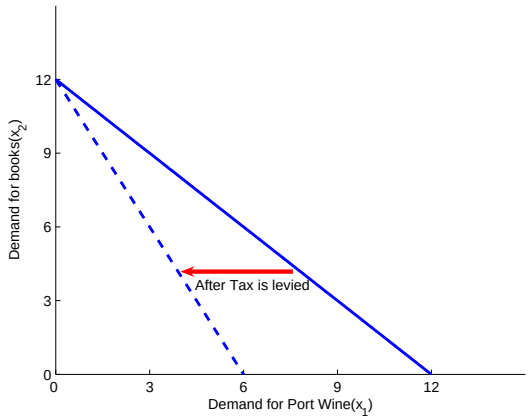


Figure 1: The Budget Set

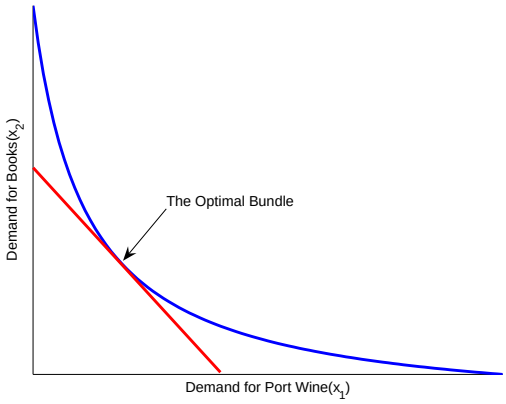


Figure 2: Two Secrets of Happiness

(b) (5pts) His sophisticated utility function is simply a monotone transformation of $2\ln x_1 + \ln x_2$.

(c) (5pts) From the previous part, we shall write his utility function $U(x_1, x_2) = 2\ln x_1 + \ln x_2$ without loss of generality. Differentiating with respect to x_1 and x_2 gives us

$$\text{MU}_1 = \frac{2}{x_1} \quad \text{and} \quad \text{MU}_2 = \frac{1}{x_2}$$

Hence the marginal rate of substitution, defined as the (negative) ratio of two marginal utilities, is given by $\text{MRS} = -\dfrac{2/x_1}{1/x_2} = -\dfrac{2x_2}{x_1}$. At the bundle $(1,2)$, it takes $-\dfrac{2\cdot 2}{1} = -4$. Since its absolute value is greater than 1, the port wine is more valuable from his perspective.

(d) (10pts) Two secrets of happiness state

$$(i)\ \ p_1x_1 + p_2x_2 = m$$
$$(ii)\ \ MRS\left(= -\frac{2x_2}{x_1}\right) = -\frac{p_1}{p_2}$$

The first condition indicates Jim's budget line; he has to utilize all his monetary resource to maximize utility. And the second condition requires the MRS coincide with the relative price. It implies that the consumer's subjective exchange ratio(MRS) must be the same as the market's objective exchange ratio at the optimal bundle. Figure 2 illustrates it in the graph. For a bundle to be optimal, it must be on the budget line and the indifference curve through it must be tangent to the budget line.

Note that solving (ii) for x_1 gives $x_1 = 2\dfrac{p_2}{p_1}x_2$. Plugging it in (i) for x_1 gives

$$p_1\left(2\frac{p_2}{p_1}x_2\right) + p_2x_2 = m \rightarrow x_2 = \frac{m}{3p_2}$$

Consequently, $x_1 = 2\dfrac{p_2}{p_1}\dfrac{m}{3p_2} = \dfrac{2m}{3p_1}$.

(e) (15pts) When $p_1 = 10$, $p_2 = 5$, $m = 60$,

$$x_1 = \frac{2 \cdot 60}{3 \cdot 10} = 4 \quad \text{and} \quad x_2 = \frac{60}{3 \cdot 5} = 4.$$

When p_1 falls down to $p_1 = 5$,

$$x_1 = \frac{2 \cdot 60}{3 \cdot 5} = 8 \quad \text{and} \quad x_2 = \frac{60}{3 \cdot 5} = 4.$$

The total change in x_1 is $8 - 4 = 4$. Note that the demand for Port wine is increasing as p_1 decreases. It is an ordinary good. Figure 3 illustrate the total change in x_1.

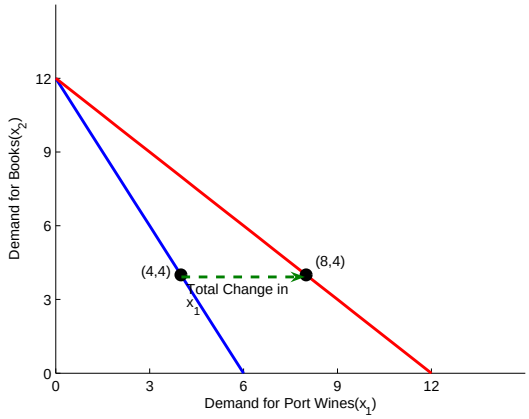


Figure 3: Total Change in x_1

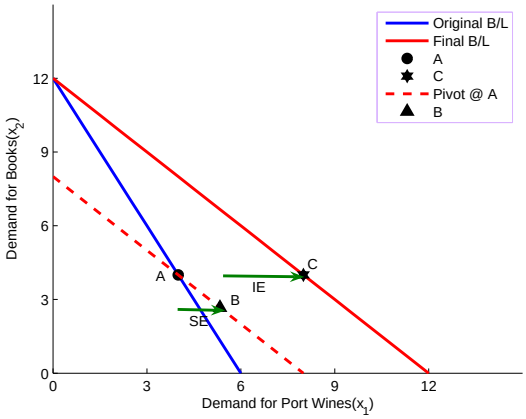


Figure 4: Substitution and Income Effect

(f) (10pts) In order to decompose it, we have to adjust money income m', which represents the amount of money income that is necessary to make the original bundle(i.e. before p_1 changes it was (4,4)) affordable at the new price.

$$m' = 4 \cdot 5 + 4 \cdot 5 = 40.$$

After p_1 changes, Jim needs $m' = 40$ to purchase the original commodity bundle. Then we need figure out what is the optimal choice for Port wine at the new price and this m'. Plugging $p_1 = 5$ and $m = 40$ into the formula $x_1 = \dfrac{2m}{3p_1}$ gives us $x_1 = \dfrac{16}{3}$. The substitution effect is given by the difference between the new demand $\frac{16}{3}$ and the original one 4

$$\text{SE} = \frac{16}{3} - 4 = \frac{4}{3}$$

The associated income effect immediately follows from Slutsky's equation, which tells us that the total change can be decomposed into the substitution and income effect.

$$\text{IE} = \text{Total Change} - \text{SE} = 4 - \frac{4}{3} = \frac{8}{3}$$

In figure 4, I denote by A,B and C the original optimal choice, the final optimal choice, and the intermediate optimal choice we obtained from m', respectively. The substitution effect is the gap between A and C on the horizontal axis while the income effect is between C and B.

Problem 2 (20pts)

(a) (2pts) $\text{PV} = 40 + \dfrac{120}{1+r} = 40 + \dfrac{120}{3} = 80$

(b) (3pts) The budget constraint is written by

$$C_1 + \frac{1}{3}C_2 = 80$$

To wit briefly, the left-hand side is the present value of consumption stream (C_1, C_2). Observe that I discounted C_2 by $\frac{1}{1+r} = \frac{1}{3}$ to express it in terms of present value. The right-hand side simply represents the present value of his income we got in part (a). The budget line tells us that they should be balanced. See figure 5.

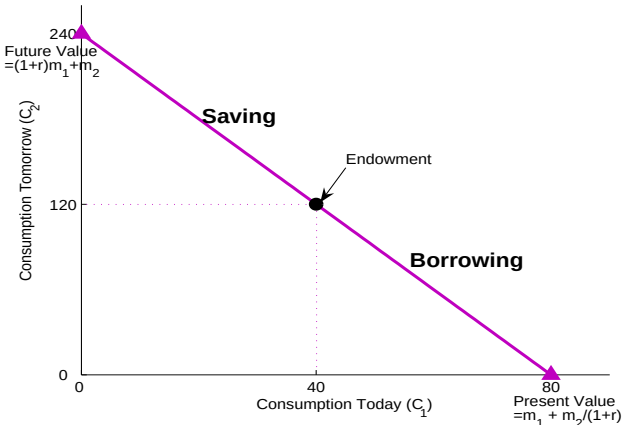


Figure 5: Intertemporal Choice Model

(c) (7pts) For $\delta = 2$, observe that the utility function comes down to

$$U(C_1, C_2) = \ln C_1 + \frac{1}{3}\ln C_2$$

which is a Cobb-Douglas function. Using its magic formula, we obtain

$$C_1 = \frac{a}{a+b}\cdot\frac{m}{p_1} = \frac{1}{1+\frac{1}{3}}\cdot\frac{80}{1} = 60$$

$$C_2 = \frac{b}{a+b}\cdot\frac{m}{p_2} = \frac{\frac{1}{3}}{1+\frac{1}{3}}\cdot\frac{80}{\frac{1}{3}} = 60.$$

To consume 60 today, he must save $60 - 40 = -20$, that is, he must borrow 20. He does smooth his consumption plan because $C_1 = C_2 = 60$.

(d) (3pts) $\text{PV} = \dfrac{x}{r} = \dfrac{1000}{0.02} = 50,000.$

(e) (5pts) The characterization equation is given by

$$\frac{100,000}{0.01}\left[1 - \frac{1}{(1+0.01)^{40}}\right] = \frac{C}{0.01}\left[1 - \frac{1}{(1+0.01)^{70}}\right]$$

The left-hand side and the right-hand side represents the present value of earnings for next 40 years and the present value of consumptions for next 70 years, respectively. And the constant level of consumption C comes from the equation in which these two values are balanced.

Problem 3 (15pts)

(a) (2pts) $U(x_1, x_2) = \min\{3x_1, x_2\}$

(b) (3pts) See figure 6.

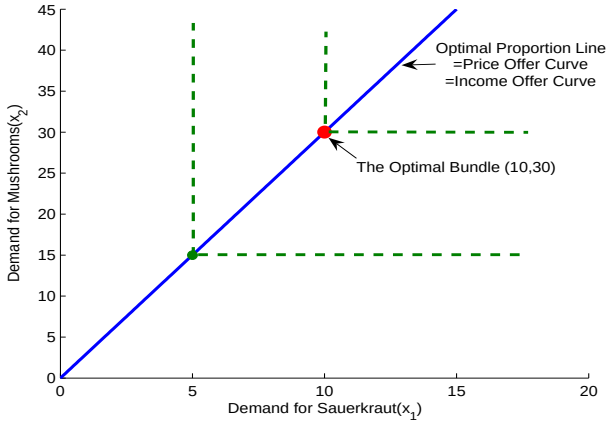


Figure 6: Perfect Complements

(c) (6pts) The associated "two secrets of happiness" with perfect complements is

(i) $2x_1 + 2x_2 = 80$

(ii) $3x_1 = x_2$.

The first condition indicates the consumer's balanced budget, and its economic intuition is the same as before: All income must be exhausted. The second condition represents the optimal proportion $(1:3)$ between quantities demanded for sauerkraut and mushrooms.

To figure out the optimal choice, we plug in $3x_1$ for x_2 in the budget line and obtain

$$2x_1 + 2(3x_1) = 8x_1 = 80 \Rightarrow x_1 = 8.$$

Substituting into $x_2 = 3x_1$ we get $x_2 = 24$. The solution is interior as x_1 and x_2 are strictly positive.

(d) (4pts) Since the two goods are perfect complements, both the price offer curve and the income offer curve would coincide with the optimal proportion line. Refer Figure 6.

Problem 4 (15pts)

(a) (4pts) $x_1 + x_2 = 300$. The value of endowment, the total resources available for consumption, is $p_1\omega_1 + p_2\omega_2 = 100 + 200 = 300$. See Figure 7.

(b) (4pts) Since $p_1 = p_2 = 1$ and the two goods are perfect substitutes, Sam is willing to consume the good with more marginal utilities. In this example, it follows from the utility function that oranges(good 2) yield more marginal utilities. Therefore, $x_2 = 300$ and $x_1 = 0$. Since $x_1 = 0$, the solution is not interior but it is at the corner.

(c) (4pts) The net demand for apples is $x_1 - \omega_1 = 0 - 100 = -100$, which is negative. It means that Sam is a net seller of apples. The net demand for oranges, on the other hand, is $x_2 - \omega_2 = 300 - 200 = 100 > 0$. Hence he is a net buyer for oranges.

(d) (3pts) It is because apples and oranges, in this example, are perfectly substitutable. Hence the consumer would choose either one good only, depending on the marginal utility from a dollar.

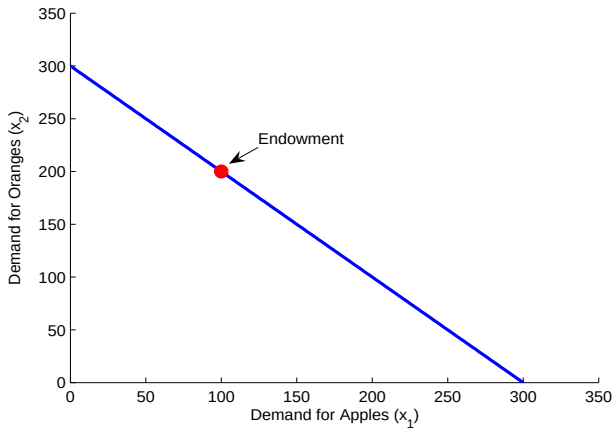


Figure 7: Budget Line and Endowment Point

Bonus Problem

Perpetuity

A perpetuity in an annuity that has no definite end, or a stream of cash payments, say x, that continues indefinitely. With the (risk-free) interest rate r, we can write its present value as

$$\begin{aligned}
\text{PV} &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots \\
&= \frac{x}{(1+r)} + \left[\frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots\right] \\
&= \frac{x}{(1+r)} + \frac{1}{(1+r)} \underbrace{\left[\frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \cdots\right]}_{=\text{PV}}
\end{aligned}$$

In the second step, I just partitioned the sum into the first term and the remainder using the parenthesis. The key step is the next one, factoring $\frac{1}{1+r}$ out of the remainder. Then we come up with the same sequence as the original PV. It leads to the following simple equation

$$\text{PV} = \frac{x}{(1+r)} + \frac{1}{(1+r)} \cdot \text{PV}$$

Solving for PV gives us the desired formula, $\text{PV} = \frac{x}{r}$.

Annuity

The annuity is an asset that promises a terminating stream of fixed payments over a prespecified period of time. Its value is closely linked to *"No Arbitrage Principle"*, the most fundamental principle in finance. It states that "two assets with identical cash flows must trade at the same price." With this principle in hand, consider one annuity which guarantees to annually give us payment x over T years. Now it is the key idea that with the two perpetuities we can replicate the identical cash flows as the annuity.

- Perpetuity 1 - It promises x payment forever from now on.
- Perpetuity 2 - It promises x payment forever but $T+1$ years from now.

Observe that the cash flow of the annuity is the same as the cash flow of "Perpetuity 1 − Perpetuity 2". By no arbitrage condition, then, it must be the case that

$$\begin{aligned}
\text{PV(Annuity)} &= \text{PV(Perpetuity} \quad 1) - \text{PV(Perpetuity} \quad 2) \\
&= \frac{x}{r} - \frac{x}{r} \cdot \frac{1}{(1+r)^T} \\
&= \frac{x}{r}\left[1 - \frac{1}{(1+r)^T}\right].
\end{aligned}$$

Problem 1 (50pts)

(a) (5pts) The budget constraint is given by $20x_1 + 20x_2 = 240$. With the tax $\tau = 100\%$, it becomes $20(1+\tau)x_1 + 20x_2 = 40x_1 + 20x_2 = 240$. Since it plays a role in increasing p_1 by 100%, the budget line must shift in as displayed in Figure 1.

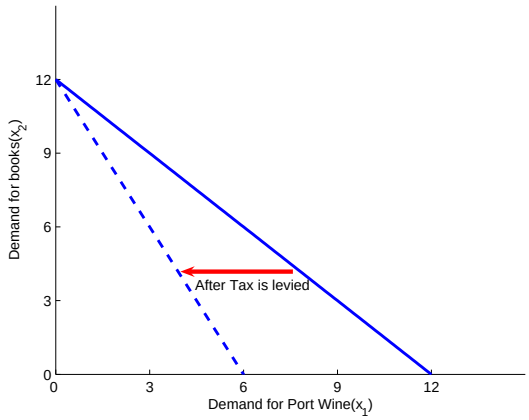


Figure 1: The Budget Set

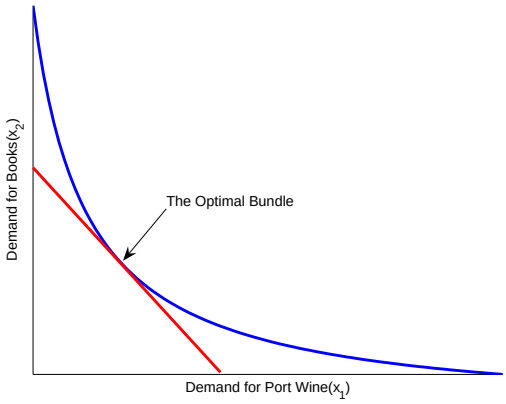


Figure 2: Two Secrets of Happiness

(b) (5pts) His sophisticated utility function is simply a monotone transformation of $2\ln x_1 + \ln x_2$.

(c) (5pts) From the previous part, we shall write his utility function $U(x_1, x_2) = 2\ln x_1 + \ln x_2$ without loss of generality. Differentiating with respect to x_1 and x_2 gives us

$$\text{MU}_1 = \frac{2}{x_1} \quad \text{and} \quad \text{MU}_2 = \frac{1}{x_2}$$

Hence the marginal rate of substitution, defined as the (negative) ratio of two marginal utilities, is given by $\text{MRS} = -\dfrac{2/x_1}{1/x_2} = -\dfrac{2x_2}{x_1}$. At the bundle $(1,2)$, it takes $-\dfrac{2\cdot 2}{1} = -4$. Since its absolute value is greater than 1, the port wine is more valuable from his perspective.

(d) (10pts) Two secrets of happiness state

$$(i)\ \ p_1x_1 + p_2x_2 = m$$
$$(ii)\ \ MRS\left(= -\frac{2x_2}{x_1}\right) = -\frac{p_1}{p_2}$$

The first condition indicates Jim's budget line; he has to utilize all his monetary resource to maximize utility. And the second condition requires the MRS coincide with the relative price. It implies that the consumer's subjective exchange ratio(MRS) must be the same as the market's objective exchange ratio at the optimal bundle. Figure 2 illustrates it in the graph. For a bundle to be optimal, it must be on the budget line and the indifference curve through it must be tangent to the budget line.

Note that solving (ii) for x_1 gives $x_1 = 2\dfrac{p_2}{p_1}x_2$. Plugging it in (i) for x_1 gives

$$p_1\left(2\frac{p_2}{p_1}x_2\right) + p_2x_2 = m \rightarrow x_2 = \frac{m}{3p_2}$$

Consequently, $x_1 = 2\frac{p_2}{p_1}\frac{m}{3p_2} = \frac{2m}{3p_1}$.

(e) (15pts) When $p_1 = 40$, $p_2 = 20$, $m = 240$,

$$x_1 = \frac{2 \cdot 240}{3 \cdot 40} = 4 \quad \text{and} \quad x_2 = \frac{240}{3 \cdot 20} = 4.$$

When p_1 falls down to $p_1 = 20$,

$$x_1 = \frac{2 \cdot 240}{3 \cdot 20} = 8 \quad \text{and} \quad x_2 = \frac{240}{3 \cdot 20} = 4.$$

The total change in x_1 is $8 - 4 = 4$. Note that the demand for Port wine is increasing as p_1 decreases. It is an ordinary good. Figure 3 illustrate the total change in x_1.

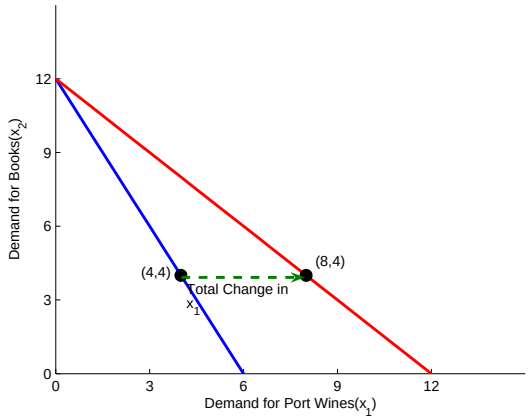


Figure 3: Total Change in x_1

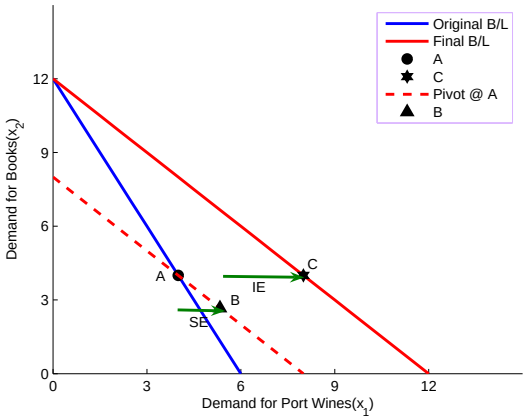


Figure 4: Substitution and Income Effect

(f) (10pts) In order to decompose it, we have to adjust money income m', which represents the amount of money income that is necessary to make the original bundle(i.e. before p_1 changes it was (4,4)) affordable at the new price.

$$m' = 4 \cdot 20 + 4 \cdot 20 = 160.$$

After p_1 changes, Jim needs $m' = 160$ to purchase the original commodity bundle. Then we need figure out what is the optimal choice for Port wine at the new price and this m'. Plugging $p_1 = 20$ and $m = 160$ into the formula $x_1 = \frac{2m}{3p_1}$ gives us $x_1 = \frac{16}{3}$. The substitution effect is given by the difference between the new demand $\frac{16}{3}$ and the original one 4

$$\text{SE} = \frac{16}{3} - 4 = \frac{4}{3}$$

The associated income effect immediately follows from Slutsky's equation, which tells us that the total change can be decomposed into the substitution and income effect.

$$\text{IE} = \text{Total Change} - \text{SE} = 4 - \frac{4}{3} = \frac{8}{3}$$

In figure 4, I denote by A,B and C the original optimal choice, the final optimal choice, and the intermediate optimal choice we obtained from m', respectively. The substitution effect is the gap between A and C on the horizontal axis while the income effect is between C and B.

Problem 2 (20pts)

(a) (2pts) $\text{PV} = 80 + \dfrac{240}{1+r} = 80 + \dfrac{240}{3} = 160$

(b) (3pts) The budget constraint is written by

$$C_1 + \frac{1}{3}C_2 = 160$$

To wit briefly, the left-hand side is the present value of consumption stream (C_1, C_2). Observe that I discounted C_2 by $\frac{1}{1+r} = \frac{1}{3}$ to express it in terms of present value. The right-hand side simply represents the present value of his income we got in part (a). The budget line tells us that they should be balanced. See figure 5.

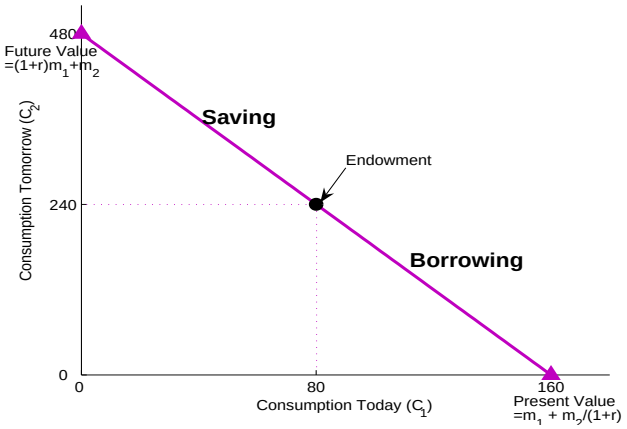


Figure 5: Intertemporal Choice Model

(c) (7pts) For $\delta = 2$, observe that the utility function comes down to

$$U(C_1, C_2) = \ln C_1 + \frac{1}{3}\ln C_2$$

which is a Cobb-Douglas function. Using its magic formula, we obtain

$$C_1 = \frac{a}{a+b}\cdot\frac{m}{p_1} = \frac{1}{1+\frac{1}{3}}\cdot\frac{160}{1} = 120$$

$$C_2 = \frac{b}{a+b}\cdot\frac{m}{p_2} = \frac{\frac{1}{3}}{1+\frac{1}{3}}\cdot\frac{160}{\frac{1}{3}} = 120.$$

To consume 120 today, he must save $80 - 120 = -40$, that is, he must borrow 40. He does smooth his consumption plan because $C_1 = C_2 = 120$.

(d) (3pts) $\text{PV} = \dfrac{x}{r} = \dfrac{200}{0.1} = 2,000.$

(e) (5pts) The characterization equation is given by

$$\frac{200,000}{0.01}\left[1 - \frac{1}{(1+0.01)^{40}}\right] = \frac{C}{0.01}\left[1 - \frac{1}{(1+0.01)^{70}}\right]$$

The left-hand side and the right-hand side represents the present value of earnings for next 40 years and the present value of consumptions for next 80 years, respectively. And the constant level of consumption C comes from the equation in which these two values are balanced.

Problem 3 (15pts)

(a) (2pts) $U(x_1, x_2) = \min\{x_1, 3x_2\}$

(b) (3pts) See figure 6.

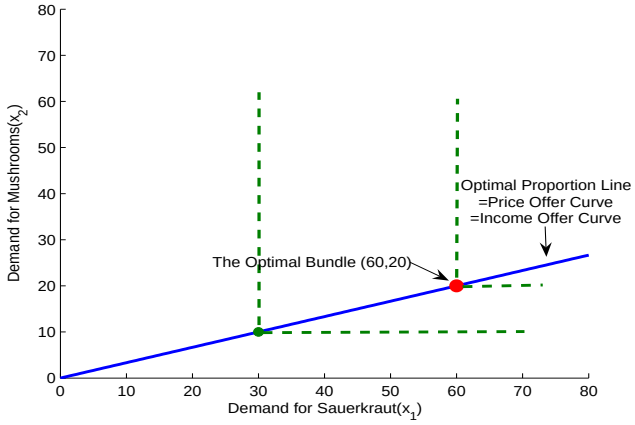


Figure 6: Perfect Complements

(c) (6pts) The associated "two secrets of happiness" with perfect complements is

(i) $2x_1 + 2x_2 = 160$

(ii) $x_1 = 3x_2$.

The first condition indicates the consumer's balanced budget, and its economic intuition is the same as before: All income must be exhausted. The second condition represents the optimal proportion $(3:1)$ between quantities demanded for sauerkraut and mushrooms.

To figure out the optimal choice, we plug in $3x_2$ for x_1 in the budget line and obtain

$$2(3x_2) + 2x_2 = 8x_2 = 160 \Rightarrow x_2 = 20.$$

Substituting into $x_1 = 3x_2$ we get $x_1 = 60$. The solution is interior as x_1 and x_2 are strictly positive.

(d) (4pts) Since the two goods are perfect complements, both the price offer curve and the income offer curve would coincide with the optimal proportion line. Refer Figure 6.

Problem 4 (15pts)

(a) (4pts) $x_1 + x_2 = 300$. The value of endowment, the total resources available for consumption, is $p_1\omega_1 + p_2\omega_2 = 100 + 200 = 300$. See Figure 7.

(b) (4pts) Since $p_1 = p_2 = 1$ and the two goods are perfect substitutes, Sam is willing to consume the good with more marginal utilities. In this example, it follows from the utility function that apples(good 1) yield more marginal utilities. Therefore, $x_1 = 300$ and $x_2 = 0$. Since $x_2 = 0$, the solution is not interior but it is at the corner.

(c) (4pts) The net demand for apples is $x_1 - \omega_1 = 300 - 100 = 200$, which is positive. It means that Sam is a net buyer of apples. The net demand for oranges, on the other hand, is $x_2 - \omega_2 = 0 - 200 = -200 < 0$. Hence he is a net seller for oranges.

(d) (3pts) It is because apples and oranges, in this example, are perfectly substitutable. Hence the consumer would choose either one good only, depending on the marginal utility from a dollar.

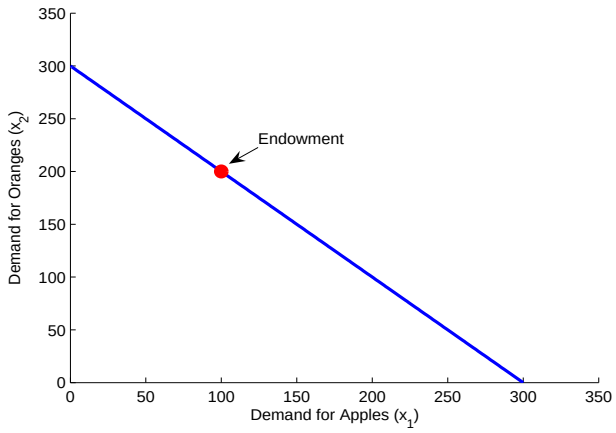


Figure 7: Budget Line and Endowment Point

Bonus Problem

Perpetuity

A perpetuity in an annuity that has no definite end, or a stream of cash payments, say x, that continues indefinitely. With the (risk-free) interest rate r, we can write its present value as

$$\begin{aligned}
\text{PV} &= \frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots \\
&= \frac{x}{(1+r)} + \left[\frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \frac{x}{(1+r)^4} + \cdots\right] \\
&= \frac{x}{(1+r)} + \frac{1}{(1+r)} \underbrace{\left[\frac{x}{(1+r)} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \cdots\right]}_{=\text{PV}}
\end{aligned}$$

In the second step, I just partitioned the sum into the first term and the remainder using the parenthesis. The key step is the next one, factoring $\frac{1}{1+r}$ out of the remainder. Then we come up with the same sequence as the original PV. It leads to the following simple equation

$$\text{PV} = \frac{x}{(1+r)} + \frac{1}{(1+r)} \cdot \text{PV}$$

Solving for PV gives us the desired formula, $\text{PV} = \frac{x}{r}$.

Annuity

The annuity is an asset that promises a terminating stream of fixed payments over a prespecified period of time. Its value is closely linked to *"No Arbitrage Principle"*, the most fundamental principle in finance. It states that "two assets with identical cash flows must trade at the same price." With this principle in hand, consider one annuity which guarantees to annually give us payment x over T years. Now it is the key idea that with the two perpetuities we can replicate the identical cash flows as the annuity.

- Perpetuity 1 - It promises x payment forever from now on.
- Perpetuity 2 - It promises x payment forever but $T+1$ years from now.

Observe that the cash flow of the annuity is the same as the cash flow of "Perpetuity 1 − Perpetuity 2". By no arbitrage condition, then, it must be the case that

$$\begin{aligned}
\text{PV(Annuity)} &= \text{PV(Perpetuity} \quad 1) - \text{PV(Perpetuity} \quad 2) \\
&= \frac{x}{r} - \frac{x}{r} \cdot \frac{1}{(1+r)^T} \\
&= \frac{x}{r}\left[1 - \frac{1}{(1+r)^T}\right].
\end{aligned}$$

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (10 extra "e" points). Make sure you answer the first four questions before working on the bonus one!

Problem 1. (50 points) To reenergize for Econ 301 class, in the morning, Tony always drinks Mountain Dew (x_1) and eats Burritos (x_2).

a) Suppose Mountain Dew costs $p_1 = \$2$, burrito costs $p_2 = \$10$, and his daily budget is $m = \$40$. Show graphically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set is affected by inflation of 100% that affects prices of both commodities but does not affect income (so his income stays the same $m = \$40$)?

b) Tony's preferences are given by the following utility function

$$U(x_1, x_2) = x_1^7 x_2^7.$$

Find Tony's marginal rate of substitution (MRS) as a function of x_1, x_2 (give a formula for MRS).

- What is the value of MRS at consumption bundle $(2, 1)$ (give a number)?

- Which of the two goods is more valuable, soda or burrito, if Tony drinks two Mountain Dews and consumes one burrito?

- Depict his indifference curve map in a commodity space. Mark the slope of the indifference curve at the bundle $(2, 1)$.

c) In the commodity space (x_1, x_2), find (geometrically) Tony's optimal choice,.assuming pre-inflation prices $p_1 = \$2$, and $p_2 = \$10$. Describe how the two properties of the optimal bundle, known as two "secrets of happiness" (two short sentences) can be seen in the graph.

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).

- Derive the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation).

- What fraction of income is spent on burritos (give the percentage)?

- Find analytically and geometrically the demand curve for Mountain Dew (given $p_2 = \$10$ and $m = \$40$) and Engel curve (given $p_1 = \$2$, and $p_2 = \$10$)

- Are they Giffen goods? Why? (yes/no answer + one sentence).

e) Using your formula from d) find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- $p_1 = \$2, p_2 = \$10$ and $m = \$40$ (give two numbers).

and after the price of Mountain Dew decreased:

- for $p_1 = \$1, p_2 = \$10$ and $m = \$40$ (give two numbers).

What is the total change in consumption of Mountain Dew? (give a number). Illustrate the change on the graph.

f) decompose the total change in consumption of x_1 from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

Problem 2. (20 points) Bill is a wild-animal lover. From his recent trip to Galapagos Islands he brought a small Iguana. His new pet has only three legs: one left and two right. (Iguanas use magma heated soil to warm their eggs and his favorite pet lost one left leg during the last volcano eruption). To survive the famous Madisonian winter, the iguana has to wear shoes,.left (x_1) and right ones (x_2).

a) Write down Bill's utility function representing his preferences over right and left shoe (function $U(x_1, x_2)$).

b) In the commodity space (x_1, x_2), carefully depict Bill's indifference curves.

c) Find analytically Bill's demand for shoes if $p_1 = \$6$ and $p_2 = \$2$ and Bill's budget for iguana shoes is $m = \$40$. Is the solution interior? (give two numbers and a yes/no answer).

d) Illustrate Bill's optimal choice on the graph including the indifference curves and the budget set.

e) Suppose the price of left shoe goes down to $p_1 = \$1$. Find Bill's new demand for shoes. What can you say about the substitution effect? How about the income effect? (Answer the latter question without any calculations, using only a graph).

Problem 3. (15 points) Ramon decides about his new collections of postage stamps. He is interested in two themes: "the birds of the world", x_1, (measures the number of stamps in the subcollection with birds) and "the famous mathematicians", x_2. The utility derived from the collection is given by

$$U(x_1, x_2) = x_1 + 100 \times \ln x_2.$$

a) What is the optimal collection of stamps if the prices are $p_1 = 1$ and $p_2 = 1$ and $m = 200$. (find two numbers x_1 and x_2). Is your solution interior, or corner?

b) Find the optimal collection if the prices are still $p_1 = 1$ and $p_2 = 1$, but the income is only $m = 50$. Depict Ramon's optimal choice in the commodity space.

Problem 4. (15 points) Jacob can use his 24h for leisure R, or work. The hourly wage rate is $w = 10$. Jacob spends all his money on cheese curds C.

a) Draw Jacob's budget set, given the price of cheese is $p_c = \$5$ (mark the endowment point).

Let the utility function be given by

$$U(x_1, x_2) = R + C.$$

b) Are leisure and cheese curds perfect complements, perfect substitutes or none of them?

c) What is the optimal choice of leisure, cheese curds and labor supply? (Find geometrically and give three numbers).

d) Harder: Plot a graph with labor supply (horizontal axis) and wage rate (vertical one) assuming $p_c = \$5$ (hint: for what value of w do we go from one "bang" to the other "bang" solution?)

Bonus Problem. (extra 10 points) Let $U(x_1, x_2) = x_1^3 x_2$, and $V(x_1, x_2) = 3 \ln x_1 + \ln x_2$ be two utility functions.

a) Show that $U(\cdot)$ is a monotone transformation of $V(\cdot)$, and hence they define the same preferences.

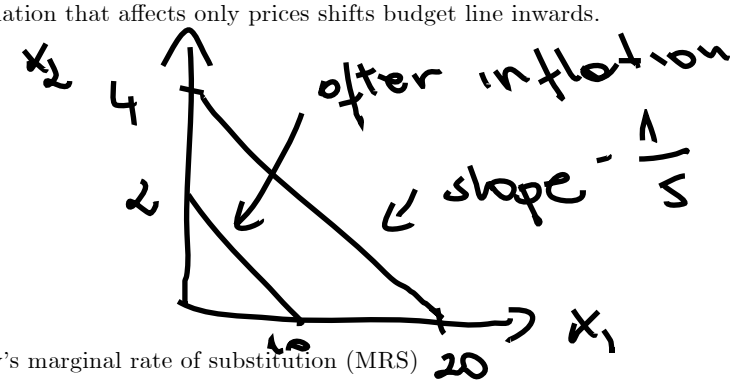
b) Derive MRS for each of the two functions. Using the two formulas for MRS, argue that the functions define the same indifference curve maps.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to midterm 1 (Group A)

Problem 1. (50 points)

a) With $p_1 = \$2$, $p_2 = \$10$ and $m = \$40$ the budget set is (two extreme consumption bundles are 20 and 4). Inflation that affects only prices shifts budget line inwards.



b) Tony's marginal rate of substitution (MRS)

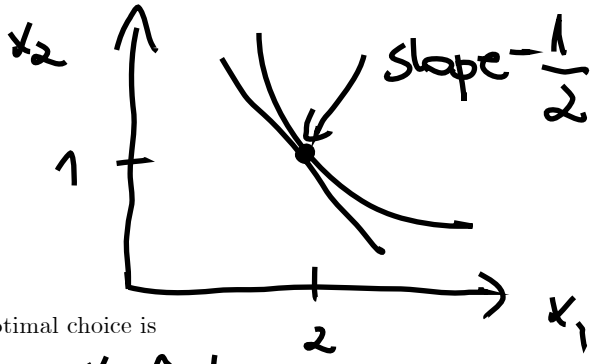
$$MRS = -\frac{MU_1}{MU_2} = -\frac{x_2}{x_1}$$

- The value of MRS at consumption bundle $(2, 1)$ is

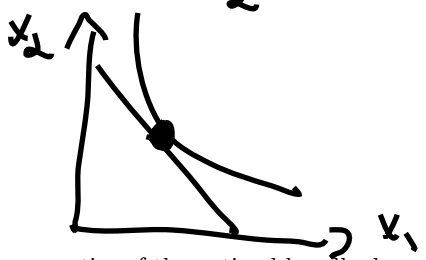
$$|MRS| = |-\frac{1}{2}| = \frac{1}{2}$$

- Burrito (x_2) is more valuable than Mountain Dew (x_1)

- Tony's indifference curve map is. (the slope of her indifferent curve that passes through bundle $(2, 1)$ is $-\frac{1}{2}$.



c) Tony's optimal choice is



- the two geometric properties of the optimal bundle, known as two "secrets of happiness" are:

1. At the optimal bundle, the indifference curve is tangent to a budget set
2. The optimal bundle is located on budget line

d) mathematically the two secrets of happiness, are

$$\begin{cases} MRS = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

- the economic intuition behind the two conditions is:
The individual value of x_1 in terms of x_2 coincides with the market value
The income of a consumer is exhausted
- the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m can be found as follows
From the MRS condition

$$MRS = -\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

hence

$$x_2 = \frac{p_1}{p_2} x_1$$

plugging in budget constraint

$$p_1 x_1 + p_2 \left(\frac{p_1}{p_2} x_1 \right) = m$$

Solving for x_1 gives

$$x_1 = \frac{1}{2} \frac{m}{p_1}$$

Plugging in

$$x_2 = \frac{p_1}{p_2} \left(\frac{1}{2} \frac{m}{p_1} \right) = \frac{1}{2} \frac{m}{p_2}$$

- the fraction of income spent on burritos is

$$\frac{p_1 x_1}{m} = \frac{1}{2} = 50\%$$

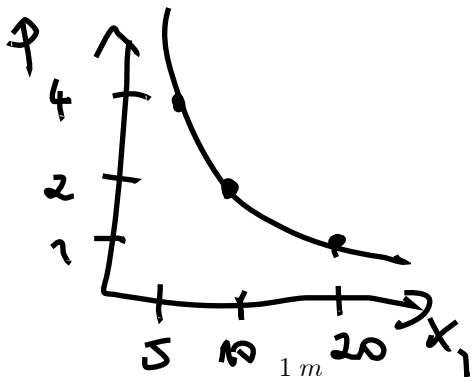
- and the demand curve for burritos book (given $p_2 = \$10$, and $m = \$40$) and Engel curve (given $p_1 = \$2$, and $p_2 = \$10$)
Demand curve

$$x_1 = \frac{1}{2} \frac{m}{p_1} = \frac{1}{2} \frac{40}{p_1} = \frac{20}{p_1}$$

and hence inverse demand is

$$p_1 (x_1) = \frac{20}{x_1}$$

Geometrically



Engel curve: Since

$$x_1 = \frac{1}{2} \frac{m}{p_1}$$

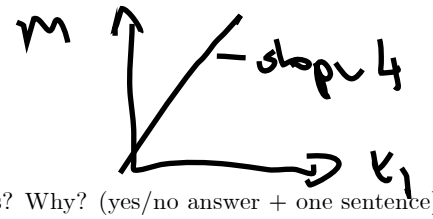
at $p_1 = \$2$

$$x_1 = \frac{1}{2} \frac{m}{2} = \frac{1}{4} m$$

hence

$$m (x_1) = 4 x_1$$

Geometrically



- are they Giffen goods? Why? (yes/no answer + one sentence).

No, because the demand curve is downwardslopping on the whole domain.

e) The optimal consumption levels for (x_1, x_2).

- at $p_1 = \$2, p_2 = \$10$ and $m = \$40$

$$x_1 = \frac{1}{2}\frac{m}{p_1} = \frac{1}{2}\frac{40}{2} = 10$$

and

$$x_2 = \frac{1}{2}\frac{m}{p_2} = \frac{1}{2}\frac{40}{10} = 2$$

and after the price of science-fiction book decreased, for $p_1 = \$1, p_2 = \$10$ and $m = \$40$

$$x_1 = \frac{1}{2}\frac{m}{p_1} = 20$$

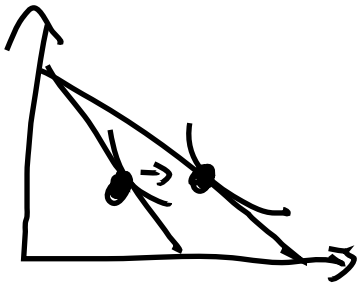
and

$$x_2 = \frac{1}{2}\frac{m}{p_2} = \frac{1}{2}\frac{40}{10} = 2$$

Hence the total change in consumption of x_1 is

$$\Delta x_1 = 20 - 10 = 10$$

Geometrically



f) Substitution effect: auxiliary budget

$$m' = 10 \times 1 + 10 \times 2 = 30$$

and hence

$$x_1 = \frac{1}{2}\frac{30}{1} = 15$$

so SE is equal to

$$SE = 15 - 10 = 5$$

and income effect is

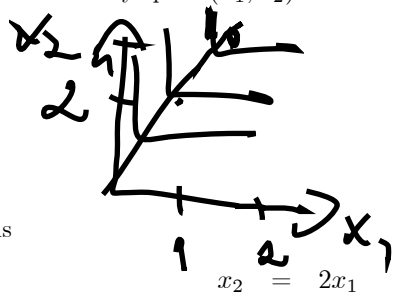
$$IE = 10 - 5 = 5$$

Problem 2. .

a) Bill's utility function is

$$U(x_1, x_2) = \min(2x_1, x_2)$$

b) indifference curves.in the commodity space (x_1, x_2) are



c) Bill's demand for shoes is

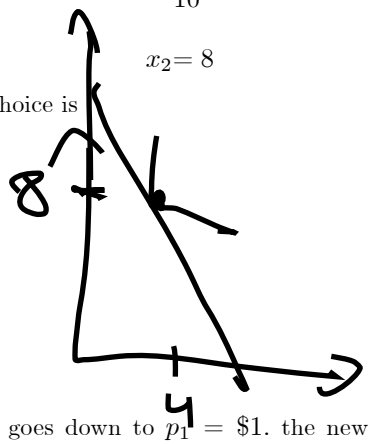
$$\begin{aligned} x_2 &= 2x_1 \\ 6x_1 + 2x_2 &= 40 \end{aligned}$$

$$6x_1 + 2(2x_1) = 40$$

$$x_1 = \frac{40}{10} = 4$$

$$x_2 = 8$$

d) geometrically Bills's optimal choice is



e) when the price of a left shoe goes down to $p_1 = \$1$. the new demand is given by the system of equations

$$\begin{aligned} x_2 &= 2x_1 \\ x_1 + 2x_2 &= 40 \end{aligned}$$

and hence demand is

$$\begin{aligned} x_1 &= 8 \\ x_2 &= 16 \end{aligned}$$

The substitution effect is zero (perfect complements) and the income effect is 4.

Problem 3.

a) the two secrets of happiness are

$$\begin{aligned} -\frac{x_2}{100} &= -1 \\ x_1 + x_2 &= 200 \end{aligned}$$

and hence $x_2 = 100$ and $x_1 = 100$. Since both are positive, this is interior solution.

b) the two secrets of happiness are

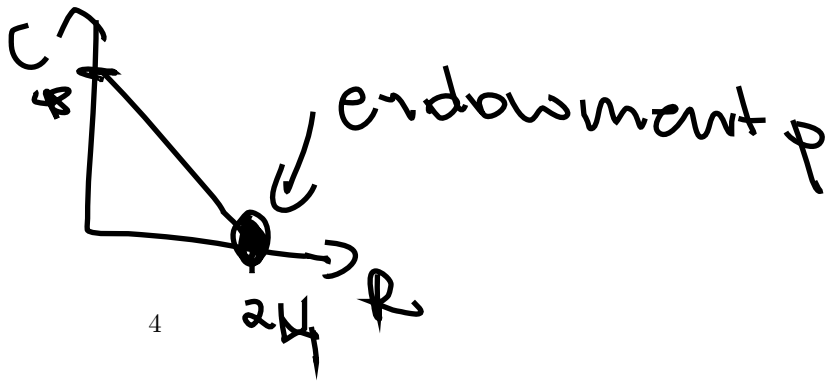
$$\begin{aligned} -\frac{x_2}{100} &= -1 \\ x_1 + x_2 &= 50 \end{aligned}$$

and hence secrets of happiness give $x_2 = 100$ and $x_1 = -50$. Since consumption must be non-negative the optimal consumption is $x_1 = 0$ and $x_2 = 50$, which is a cornet solution.

Problem 4.

a) Jacob's budget set, with $w = \$10$ and $p_c = \$5$ is

Income is $m = 10 \times 24 = \$240$

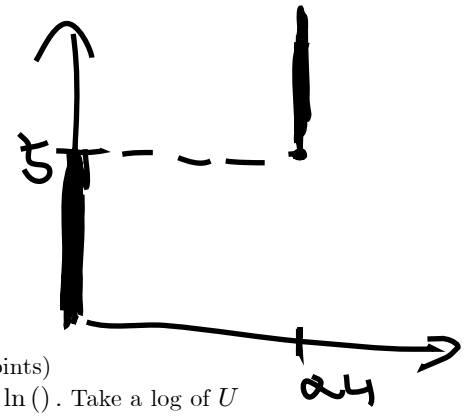


b) They are perfect substitutes

c) $|MRS| = 1 < \frac{w}{p_c} = 2$ which implies that Jacob cares less about leisure than consumption,. therefore he will spend the whole day at work

$$R = 0, LS = 24 \text{ and } C = 24\frac{10}{5} = 48$$

d)



Bonus Problem. (extra 10 points)

a) Monotone transformation $\ln()$. Take a log of U

$$\ln U() = \ln x_1^3 x_2 = \ln x_1^3 + \ln x_2 = 3\ln x_1 + \ln x_2 = V()$$

where we used two properties of ln function.

b) For $U()$, marginal rate of substitution is

$$MRS = -\frac{MU_1}{MU_2} = -\frac{3x_1^2 x_2}{x_1^3} = -\frac{3x_2}{x_1}$$

and for $V()$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{3/x_1}{1/x_2} = -\frac{3x_2}{x_1}$$

and hence MRS coincides for all (x_1, x_2).It follows that the slopes of indifference curves are the same at any point and hence they must be the same.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 3 questions (60+25+15=100 points) + bonus (10 "e" points). Make sure you answer the first three questions before working on the bonus one!

Problem 1. (60 points) Maggie likes to read science fiction (x_1) and romance (x_2) novels. Her preferences over the two types of books are represented by a utility function

$$U(x_1, x_2) = (x_1)^{10} (x_2)^{20}$$

a) Find Maggie's marginal rate of substitution (MRS) as a function of x_1, x_2 (give a formula).

- what is the value of MRS at consumption bundle $(2, 2)$ (give a number).
- complete the sentence: "The Marginal Rate of Substitution is a (marginal) value of a ... in terms of ..."
- how much one must compensate Maggie in terms of romance books, after taking away 0.00001 of a science-fiction book, in order to keep her indifferent? (give a number, assume she consumes bundle $(2, 2)$).
- depict her indifference curve map in a commodity space. Mark the slope of her indifferent curve that passes through bundle $(2, 2)$.

b) Suppose the price of a science-fiction book is $p_1 = \$10$, a romance book costs $p_2 = \$5$ and her total monthly spending on books is $m = \$300$. Show graphically her budget constraint in the commodity space. Mark the two extreme consumption bundles (give values). On the same graph, show how the budget set would be affected by the introduction of an ad valorem tax on romance books (x_2) at rate 100%?

c) In the commodity space (x_1, x_2), find (geometrically) Maggie's optimal choice.

- describe the two properties of the optimal bundle, known as two "secrets of happiness" (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values).

- provide some economic intuition behind the two conditions (ca. two sentences for each).
- derive the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation).
- what fraction of income is spent on science - fiction novels (give the percentage).
- find analytically and geometrically the demand curve for science finction book (given $p_2 = \$5$, and $m = \$300$) and Engel curve (given $p_1 = \$10$, and $p_2 = \$5$)
- are science-fiction books normal goods? Why? (yes/no answer + one sentence).
- are they Giffen goods? Why? (yes/no answer + one sentence).

e) Using your formula from d) find the optimal consumption levels for both types of books (x_1, x_2).

- for $p_1 = \$10, p_2 = \$5$ and $m = \$300$ (give two numbers).

and after the price of science-fiction book decreased:

- for $p_1 = \$5, p_2 = \$5$ and $m = \$300$ (give two numbers).

What is the total change in consumption of x_1? (give one number). Illustrate this change on the graph.

f) decompose the total change in x_1 from f) into a substitution and income effect. (Calculate the two numbers, show how you found them on the graph.) Complete the two sentences:

"The substitution effect is attributed to the pure change in .... induced by the decrease of nominal price p_1"

"The income effect can be attribute to the pure change in .... induced by the decrease of nominal price p_1"

g) which of the following alternative utility functions represents Maggie's preferences (there are two such

functions)?

$$\begin{aligned} V(x_1, x_2) &= 30\left(x_1^{10} \times x_2^{20}\right) + 3 \\ V(x_1, x_2) &= 10x_1 \times 20x_2 \\ V(x_1, x_2) &= 10\ln x_1 \times 20\ln x_2 + 2 \\ V(x_1, x_2) &= 10\ln x_1 + 20\ln x_2 + 7 \end{aligned}$$

Explain why the utility functions you have selected represent Maggie's preferences (one sentence). Suggest the transformation of $U()$ function that makes the two $V()$ functions equivalent.

Problem 2. (20 points) Jimmy's favorite hobby is slot car racing. He assembles slot cars from parts, by adding four wheels (x_1) to an engine (x_2) (these are supertrucks, with five wheels on each side). He purchases the parts on the market.

a) write down Jimmy's utility function representing his preferences over wheels and engines (function $U(x_1, x_2)$).

b) in the commodity space (x_1, x_2), carefully depict Jimmy's indifference curves.

c) find analytically Jimmy's demand for parts if one wheel costs $p_1 = \$50$, an engine is $p_2 = \$100$ and Jimmy's budget for slot cars is $m = \$600$. Is the solution interior (give two numbers and yes/no answer).

d) illustrate Jimmy's optimal choice on the graph including the indifference curves and the budget set.

e) suppose the price of one wheel goes down to $p_1 = \$25$. Find Jimmy's new demand for the parts. What can you say about the substitution effect? How about the income effect? (you can answer the last question without any calculations, using only a graph).

Problem 3. (20 points) Ramon Gonzales M. Panetelas is a specialist in Habanos cigars (famous Cuban cigars). Cuban cigars are sold either in 10 cigar packs (x_1), or in singles (x_2). Ramon has no income. Instead he is initially endowed with $\omega_1 = 5$ packs and $\omega_2 = 50$ cigars.

a) draw Ramon's budget set, given the price of a pack is equal to $p_1 = \$5$ and a single cigar is $p_2 = \$1$ (mark the endowment point).

b) Illustrate geometrically Ramon's optimal demand for packs and single cigars, given his utility function

$$U(x_1, x_2) = 10x_1 + x_2$$

(Give two numbers (x_1, x_2), and mark them on the graph, including budget set and the indifference curves.)

c) What is your answer to b) when prices are $p_1 = 20$ and $p_2 = 1$. (Give two numbers (x_1, x_2), and plot the graph.)

d) Harder: Give the formula for the demands x_1, x_2 as a function of p_1, p_2 and endowments ω_1 and ω_2. Show the demand curve for x_1 on the graph, assuming $p_2 = 1$, $\omega_1 = 5$ and $\omega_2 = 50$.

Bonus Problem. (extra 10 points) Depict a map of indifference curves that is consistent with

a) inferior goods

b) Giffen goods

(Make sure you explain why these graphs represent the respective preferences.)

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to midterm 1 (Group A)

Problem 1. (60 points)

a) Maggie's marginal rate of substitution (MRS)

$$MRS = -\frac{MU_1}{MU_2} = -\frac{1}{2}\frac{x_2}{x_1}$$

- the value of MRS at consumption bundle $(2,2)$ is

$$MRS = -\frac{1}{2}$$

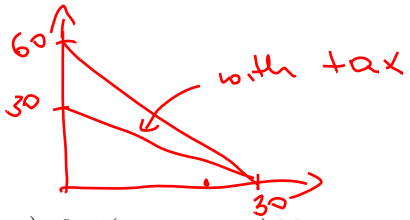
- "The Marginal Rate of Substitution is a (marginal) value of a science finction books in terms of romance novels"

- after taking away 0.00001 of a science-fiction book, to keep her indifferent one must compensate Maggie in terms of romance books

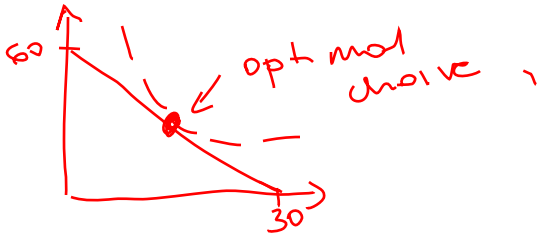
$$0.00001 \times \frac{1}{2} = 0.000005$$

- her indifference curve map is. (the slope of her indifferent curve that passes through bundle $(2,2)$ is $-\frac{1}{2}$.

b) With $p_1 = \$10$, $p_2 = \$5$ and $m = \$300$ the budget set is (two extreme consumption bundles are 30 and 60). The budget set with ad valorem tax shifts inwards.



c) In the commodity space (x_1, x_2), find (geometrically) Maggie's optimal choice.



- the two geometric properties of the optimal bundle, known as two "secrets of happiness" are:

1. At the optimal bundle, the indifference curve is tangent to a budget set
2. The oprimal bundle is locaded on budget line

d) mathematically the two secrets of happiness, are

$$\begin{cases} MRS = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

- the economic intuition behind the two conditions is:

The individual value of x_1 in temrms of x_2 coincides with the market value

The income of a consumer is exchausted

- the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m can be found as follows

From the MRS condition

$$MRS = -\frac{1}{2}\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

hence

$$x_2 = 2\frac{p_1}{p_2}x_1$$

plugging in budget constraint

$$p_1 x_1 + p_2\left(2\frac{p_1}{p_2}x_1\right) = m$$

Solving for x_1 gives

$$x_1 = \frac{1}{3}\frac{m}{p_1}$$

Pluging in

$$x_2 = 2\frac{p_1}{p_2}\left(\frac{1}{3}\frac{m}{p_1}\right) = \frac{2}{3}\frac{m}{p_2}$$

- the fraction of income spent on science - fiction novels is

$$\frac{p_1 x_1}{m} = \frac{1}{3} = 33\%$$

- and the demand curve for science finction book (given $p_2 = \$5$, and $m = \$300$) and Engel curve (given $p_1 = \$10$, and $p_2 = \$5$)

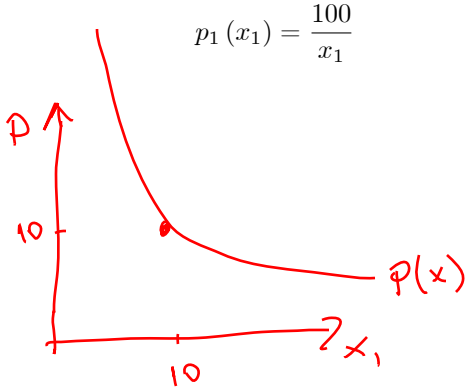
Demand curve

$$x_1 = \frac{1}{3}\frac{m}{p_1} = \frac{100}{p_1}$$

and hence invese demand is

$$p_1(x_1) = \frac{100}{x_1}$$

Geometrically



Engel curve: Since

$$x_1 = \frac{1}{3}\frac{m}{p_1}$$

at $p_1 = \$10$

$$x_1 = \frac{1}{3}\frac{m}{10} = \frac{1}{30}m$$

hence

$$m(x_1) = 30x_1$$

Geomettrically

- are science-fiction books normal goods? Why? (yes/no answer + one sentence).

Yes, because their demand increases in income

- are they Giffen goods? Why? (yes/no answer + one sentence).

No, becase the demand curve is downwardslopping on the whole domain.

e) The optimal consumption levels for both types of books (x_1, x_2).

- for $p_1 = \$10, p_2 = \$5$ and $m = \$300$

$$x_1 = \frac{1}{3}\frac{m}{p_1} = \frac{1}{3}\frac{300}{10} = 10$$

and

$$x_2 = \frac{2}{3}\frac{m}{p_2} = \frac{2}{3}\frac{300}{5} = 40$$

and after the price of science-fiction book decreased, for $p_1 = \$5, p_2 = \$5$ and $m = \$300$

$$x_1 = \frac{1}{3}\frac{m}{p_1} = \frac{1}{3}\frac{300}{5} = 20$$

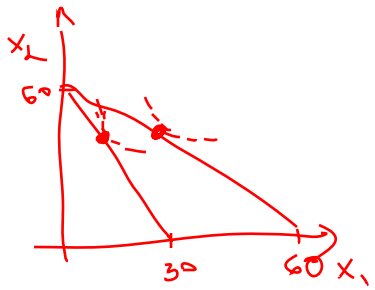
and

$$x_2 = \frac{2}{3}\frac{m}{p_2} = \frac{2}{3}\frac{300}{5} = 40$$

Hence the total change in consumption of x_1 is

$$\Delta x_1 = 20 - 10 = 10$$

Geometrically



f) Substitution effect: auxiliary budget

$$m' = 10 \times 5 + 5 \times 40 = 250$$

and hence

$$x_1 = \frac{1}{3}\frac{250}{5} = \frac{50}{3} = 16\frac{2}{3}$$

so SE is equal to

$$SE = 16\frac{2}{3} - 10 = 6\frac{2}{3}$$

and income effect is

$$IE = 10 - 6\frac{2}{3} = 3\frac{1}{3}$$

"The substitution effect is attributed to the pure change in relative price induced by the decrease of nominal price p_1"

"The income effect can be attribute to the pure change in real income induced by the decrease of nominal price p_1"

g) From the functions

$$\begin{aligned} V(x_1, x_2) &= 30\left(x_1^{10} \times x_2^{20}\right) + 3 \\ V(x_1, x_2) &= 10x_1 \times 20x_2 \\ V(x_1, x_2) &= 10\ln x_1 \times 20\ln x_2 + 2 \\ V(x_1, x_2) &= 10\ln x_1 + 20\ln x_2 + 7 \end{aligned}$$

the first and last represent Maggie's preferences - they are monotone transformations of $U()$. The first transformation is

$$f(U) = 30U + 3$$

and the second is

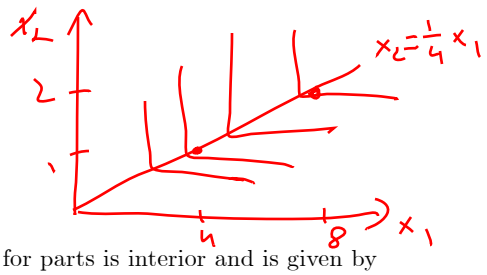
$$f(U) = \ln U + 7$$

Problem 2. .

a) Jimmy's utility function is

$$U(x_1, x_2) = \min(x_1, 4x_2)$$

b) in the commodity space (x_1, x_2), carefully depict Jimmy's indifference curves.



c) Jimmy's demand for parts is interior and is given by

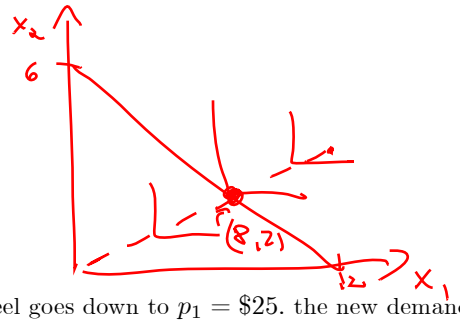
$$\begin{aligned} x_2 &= \frac{1}{4}x_1 \\ 50x_1 + 100x_2 &= 600 \end{aligned}$$

$$50x_1 + 100\frac{1}{4}x_1 = 600$$

$$x_1 = \frac{600}{75} = 8$$

$$x_2 = \frac{1}{4} \times 8 = 2$$

d) geometrically Jimmy's optimal choice is



e) when the price of a wheel goes down to $p_1 = \$25$. the new demand is

$$\begin{aligned} x_2 &= \frac{1}{4}x_1 \\ 25x_1 + 100x_2 &= 600 \end{aligned}$$

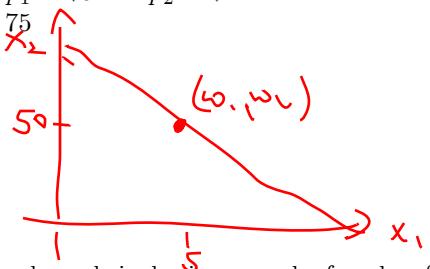
$$\begin{aligned} x_1 &= 12 \\ x_2 &= 3 \end{aligned}$$

Thte substitution effec is zero (perfect complements) and the income effect is 4 wheels.

Problem 3.

a) Ramon's budget set, with $p_1 = \$5$ and $p_2 = \$1$ is

Budget set: $m = 25 + 50 = 75$

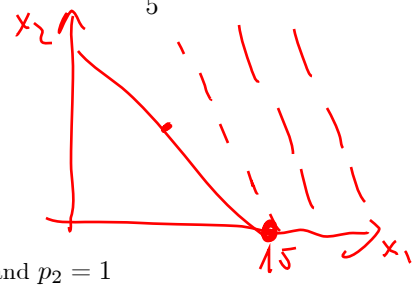


b) The optimal demand for packs and single cigars can be found as follows. Since

$$-MRS = 10 > \frac{p_1}{p_2} = 5$$

therefore the total income will be invested in packs

$$x_1 = \frac{75}{5} = 15 \text{ and } x_2 = 0$$



c) When prices are $p_1 = 20$ and $p_2 = 1$

$$-MRS = 10 < \frac{p_1}{p_2} = 20$$

and hence the total income will be invested in x_2. At such prices demands are

$$x_1 = 0 \text{ and } x_2 = \frac{5 \times 20 + 50}{1} = 150$$

d) The demands are

If $\frac{p_1}{p_2} < 10$ then

$$\begin{aligned} x_1 &= \frac{p_1\omega_1 + p_2\omega_2}{p_1} \\ x_2 &= 0 \end{aligned}$$

if $\frac{p_1}{p_2} > 10$ then

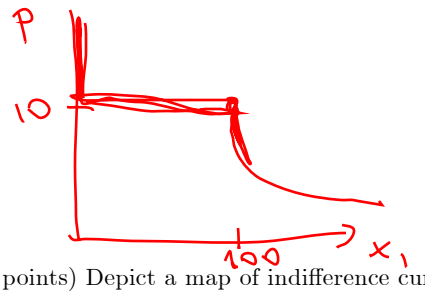
$$\begin{aligned} x_1 &= 0 \\ x_2 &= \frac{p_1\omega_1 + p_2\omega_2}{p_2} \end{aligned}$$

and if $\frac{p_1}{p_2} = 10$

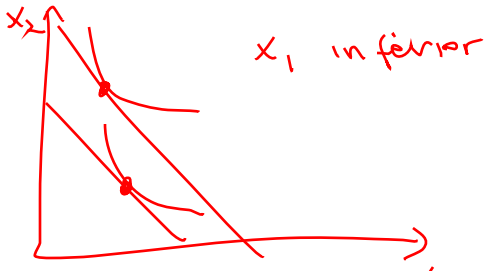
$$\begin{aligned} x_1 &= \alpha\frac{p_1\omega_1 + p_2\omega_2}{p} \\ x_2 &= (1-\alpha)\frac{p_1\omega_1 + p_2\omega_2}{p_2} \end{aligned}$$

for $\alpha \in (0,1)$

The demand curve for x_1 is ($p_2 = 1$, $\omega_1 = 5$ and $\omega_2 = 50$).



Bonus Problem. (extra 10 points) Depict a map of indifference curves that is consistent with
a) inferior goods



b) Giffen goods

(Make sure you explain why these graphs represent the respective preferences.)

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group A)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

Problem 1. (50 points)
Patrick spends his income on books (x_1) and CD (x_2).

a) Suppose the price of a book is $p_1 = \$10$, the price of a CD is $p_2 = \$5$, and Patrick's daily budget is $m = \$40$. Show graphically Patrick's budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 2 CDs (assume that he can always dispose the gift).

b) Patrick's preferences are given by the following utility function

$$U(x_1, x_2) = x_1 + 2\ln x_2.$$

Find Patrick's marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give the formula for MRS).

- What is the value of MRS at consumption bundle $(5, 8)$ (give a number)?

- Suppose Patrick "consumes" 5 books and 8 CDs and one takes away 0.0001 of a book. What is compensation in terms of CDs is sufficient to make Patrick indifferent?

- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle $(5, 8)$.

c) From now on assume no gift. In the commodity space (x_1, x_2), find (geometrically) Patrick's optimal choice. Describe how the two "secrets of happiness" can be seen geometrically in the graph (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).

e) Using the two conditions from d) find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- $p_1 = \$10, p_2 = \$5$ and $m = \$40$ (give two numbers).

and after the price of a book increases:

- for $p_1 = \$30, p_2 = \$5$ and $m = \$40$ (give two numbers).

Is each of the solutions interior? Illustrate the change on the graph.

f) Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

Problem 2. (25 points) Michael *always* consumes three hamburgers x_1 along with one Coke x_2 (this is the only healthy combination of the two products!) .

a) Propose Michael's utility function that represents his preferences over hamburgers and Coke (function $U(x_1, x_2)$).

b) In the commodity space (x_1, x_2), carefully depict Michael's indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters p_1 and p_2 and m). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels's optimal choice of x_1 and x_2 as a function of (p_1, p_2 and m). Is the choice (solution) interior for any price and income? (Give formulas $x_1(p_1, p_2, m)$ and a yes-no answer.)

e) Using $x_1(p_1, p_2, m)$ derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption TCH? (You do not have to give any number. Just relate two effects to TCH.)

Problem 3. (15 points) Adam spends all his income on food (x_1) and clothing x_2).He is a fairly sophisticated fellow and his utility function is quite complicated

$$U(x_1, x_2) = \left[700 \times \sqrt{\ln\left[(2\ln x_1 + \ln x_2)^2\right]} + 10\right]^{800}.$$

a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of x_1 and $x_2$if the prices are $p_1 = 4$ and $p_2 = 4$ and $m = 1200$ (find two numbers x_1 and x_2). Is your solution interior, or corner?

c) Assume $p_2 = 4$ and $m = 1200$. Find analitically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

Problem 4. (10 points) Frank can use his 24h for leisure R or work. The hourly wage rate is $w = 20$. Frank is a committed skier and uses all his income on ski passes in Devil's Head Resort C.

a) Draw Frank's budget set, given that the price of one ski pass is $p_c = \$10$ (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by

$$U(x_1, x_2) = RC^5.$$

b) What is the real wage? (formula + nubmer) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

Problem 1. (50 points)
Patrick spends his income on books (x_1) and CD (x_2).

a) Suppose the price of a book is p_1= \$10, the price of a CD is p_2= \$10, and Patrick's daily budget is m = \$50. Show graphically Patrick's budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 2 CDs (assume that he can always dispose the gift).

b) Patrick's preferences are given by the following utility function

$$U(x_1, x_2) = 2x_1 + 6\ln x_2.$$

Find Patrick's marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give the formula for MRS).

- What is the value of MRS at consumption bundle $(3, 6)$ (give a number)?

- Suppose Patrick "consumes" 3 books and 6 CDs and one takes away 0.0001 of a CD. What is compensation in terms of CDs is sufficient to make Patrick indifferent?

- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle $(3, 6)$.

c) From now on assume no gift. In the commodity space (x_1, x_2), find (geometrically) Patrick's optimal choice. Describe how the two "secrets of happiness" can be seen geometrically in the graph (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).

e) Using the two conditions from d) find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- for p_1= \$10, p_2= \$10 and m = \$50 (give two numbers).

and after the price of a book increases:

- for p_1= \$20, p_2= \$10 and m = \$50 (give two numbers).

Is each of the solutions interior? Illustrate the change on the graph.

f) Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

Problem 2. (25 points) Michael *always* consumes five hamburgers x_1 along with one Coke x_2 (this is the only healthy combination of the two products!) .

a) Propose Michael's utility function that represents his preferences over hamburgers and Coke (function $U(x_1, x_2)$.

b) In the commodity space (x_1, x_2), carefully depict Michael's indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters p_1 and p_2 and m). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels's optimal choice of x_1 and x_2 as a function of (p_1, p_2 and m). Is the choice (solution) interior for any price and income? (Give formulas $x_1(p_1, p_2, m)$ and a yes-no answer.)

e) Using $x_1(p_1, p_2, m)$ derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption TCH? (You do not have to give any number. Just relate two effects to TCH.)

Problem 3. (15 points) Adam spends all his income on food (x_1) and clothing (x_2) .He is a fairly sophisticated fellow and his utility function is quite complicated

$$U(x_1, x_2) = \left[12 \times \sqrt{\ln\left[\left(6\ln x_1 + 2\ln x_2\right)^2\right]} + 3\right]^{300}.$$

a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of x_1 and $x_2$if the prices are p_1= 5 and p_2= 10 and $m = 80$ (find two numbers x_1 and x_2). Is your solution interior, or corner?

c) Assume p_2= 10 and $m = 80$. Find analytically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

Problem 4. (10 points) Frank can use his 24h for leisure R or work. The hourly wage rate is $w = 20$. Frank is a committed skier and uses all his income on ski passes in Devil's Head Resort C.

a) Draw Frank's budget set, given that the price of one ski pass is p_c= \$10 (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by

$$U(x_1, x_2) = R^{17}C^{34}.$$

b) What is the real wage? (formula + number) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group C)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

Problem 1. (50 points)
Patrick spends his income on books (x_1) and CD (x_2).

a) Suppose the price of a book is $p_1 = \$6$, the price of a CD is $p_2 = \$2$, and Patrick's daily budget is $m = \$18$. Show graphically Patrick's budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 2 CDs (assume that he can always dispose the gift).

b) Patrick's preferences are given by the following utility function

$$U(x_1, x_2) = 4x_1 + 4\ln x_2.$$

Find Patrick's marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give the formula for MRS).

- What is the value of MRS at consumption bundle $(3, 8)$ (give a number)?

- Suppose Patrick "consumes" 3 books and 8 CDs and one takes away 0.0001 of a CD. What is compensation in terms of CDs is sufficient to make Patrick indifferent?

- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle $(3, 8)$.

c) From now on assume no gift. In the commodity space (x_1, x_2), find (geometrically) Patrick's optimal choice. Describe how the two "secrets of happiness" can be seen geometrically in the graph (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).

e) Using the two conditions from d) find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- for $p_1 = \$6, p_2 = \$2$ and $m = \$18$ (give two numbers).

and after the price of a book increases:

- for $p_1 = \$20, p_2 = \$2$ and $m = \$18$ (give two numbers).

Is each of the solutions interior? Illustrate the change on the graph.

f) Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

Problem 2. (25 points) Michael *always* consumes one hamburger x_1 along with two Cokes x_2 (this is the only healthy combination of the two products!) .

a) Propose Michael's utility function that represents his preferences over hamburgers and Coke (function $U(x_1, x_2)$.

b) In the commodity space (x_1, x_2), carefully depict Michael's indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters p_1 and p_2 and m). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels's optimal choice of x_1 and x_2 as a function of (p_1, p_2 and m). Is the choice (solution) interior for any price and income? (Give formulas $x_1(p_1, p_2, m)$ and a yes-no answer.)

e) Using $x_1(p_1, p_2, m)$ derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption TCH? (You do not have to give any number. Just relate two effects to TCH.)

Problem 3. (15 points) Adam spends all his income on food (x_1) and clothing (x_2) .He is a fairly sophisticated fellow and his utility function is quite complicated

$$U(x_1, x_2) = \ln\left[0.5\times\sqrt{\left[(10\ln x_1 + 2\ln x_2)^2\right]} + 3\right]^{300}.$$

a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of x_1 and $x_2$if the prices are $p_1 = 2$ and $p_2 = 2$ and $m = 120$ (find two numbers x_1 and x_2). Is your solution interior, or corner?

c) Assume $p_2 = 2$ and $m = 100$. Find analytically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

Problem 4. (10 points) Frank can use his 24h for leisure R or work. The hourly wage rate is $w = \$120$. Frank is a committed skier and uses all his income on ski passes in Devil's Head Resort C.

a) Draw Frank's budget set, given that the price of one ski pass is $p_c = \$30$ (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by

$$U(x_1, x_2) = R^3 C.$$

b) What is the real wage? (formula + number) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+25+15+10=100 points).

Problem 1. (50 points)
Patrick spends his income on books (x_1) and CD (x_2).

a) Suppose the price of a book is p_1= \$10, the price of a CD is p_2= \$2, and Patrick's daily budget is m = \$30. Show graphically Patrick's budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 1 CDs (assume that he can always dispose the gift).

b) Patrick's preferences are given by the following utility function

$$U(x_1, x_2) = 8x_1 + 8\ln x_2.$$

Find Patrick's marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give the formula for MRS).

- What is the value of MRS at consumption bundle $(4, 8)$ (give a number)?

- Suppose Patrick "consumes" 4 books and 8 CDs and one takes away 0.0001 of a CD. What is compensation in terms of CDs is sufficient to make Patrick indifferent?

- Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle $(4, 8)$.

c) From now on assume no gift. In the commodity space (x_1, x_2), find (geometrically) Patrick's optimal choice. Describe how the two "secrets of happiness" can be seen geometrically in the graph (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values). Provide economic intuition behind the two conditions (ca. two sentences for each).

e) Using the two conditions from d) find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- for p_1= \$10, p_2= \$2 and m = \$30 (give two numbers).

and after the price of a book increases:

- for p_1= \$40, p_2= \$2 and m = \$30 (give two numbers).

Is each of the solutions interior? Illustrate the change on the graph.

f) Is the marginal utility of a dollar invested in books and CD equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of a dollar is consistent with optimum.

Problem 2. (25 points) Michael *always* consumes one hamburger x_1 along with four Cokes x_2 (this is the only healthy combination of the two products!) .

a) Propose Michael's utility function that represents his preferences over hamburgers and Coke (function $U(x_1, x_2)$.

b) In the commodity space (x_1, x_2), carefully depict Michael's indifference curves (and mark the optimal proportion line).

c) Write down two secrets of happiness (give two equations) that determine his optimal choice (for parameters p_1 and p_2 and m). Explain economic intuition behind the conditions (one sentence for each secret).

d) Find Michaels's optimal choice of x_1 and x_2 as a function of (p_1, p_2 and m). Is the choice (solution) interior for any price and income? (Give formulas $x_1(p_1, p_2, m)$ and a yes-no answer.)

e) Using $x_1(p_1, p_2, m)$ derived in d), determine whether goods are 1) ordinary or Giffen, 2) normal or inferior and 3) gross substitutes or gross complements (for points 1-3 points chose one option and give one sentence explaining your choice).

f) Compare the substitution and income effects relative to a total change of consumption TCH? (You do not have to give any number. Just relate two effects to TCH.)

Problem 3. (15 points) Adam spends all his income on food (x_1) and clothing (x_2) .He is a fairly sophisticated fellow and his utility function is quite complicated

$$U(x_1, x_2) = \ln\left[0.5\times\sqrt{\left[(12\ln x_1 + 6\ln x_2)^2\right]} + 17 * 21 - 7\right]^{300}.$$

a) Argue that Adam is not really that sophisticated, as his preferences can be represented by a significantly simpler utility function. (one sentence + simpler utility function)

b) What is his optimal choice of x_1 and $x_2$if the prices are $p_1 = 1$ and $p_2 = 1$ and $m = 60$ (find two numbers x_1 and x_2). Is your solution interior, or corner?

c) Assume $p_2 = 1$ and $m = 60$. Find analytically and geometrically the demand curve and the price offer curve.

Hint: In b) and c) you can use the magic formula.

Problem 4. (10 points) Frank can use his 24h for leisure R or work. The hourly wage rate is $w = \$120$. Frank is a committed skier and uses all his income on ski passes in Devil's Head Resort C.

a) Draw Frank's budget set, given that the price of one ski pass is $p_c = \$30$ (mark the endowment point). What is the slope of his budget line? Interpret this slope economically.

Let the utility function be given by

$$U(x_1, x_2) = R^3 C.$$

b) What is the real wage? (formula + number) How can the real wage be seen in the graph of a budget set?

c) What is the optimal choice of leisure, ski passes and labor supply? (Find the optimal choice geometrically and give three numbers).

1 Problem 1

(a) His budget line without the CD gifts is straightforward: $10x_1 + 5x_2 = 40$ and x_1-intercept $\frac{40}{10} = 4$ and x_2-intercept $\frac{40}{5} = 8$ represent his real incomes in terms of books and CDs, respectively. The red line in the figure below exhibits his budget line after those CD gifts.

(b) Since $\text{MU}_1 = \frac{\partial U}{\partial x_1} = 1$ and $\text{MU}_2 = \frac{\partial U}{\partial x_2} = \frac{2}{x_2}$, we can write his MRS as

$$\text{MRS}_{1,2} = -\frac{1}{2/x_2} = -\frac{x_2}{2}. \tag{1.1}$$

At $(5, 8)$, its value is -4(or just its absolute value 4). Note that it does not rely upon x_1, the quantity demanded for books. And he has to be compensated 0.0001×4 CDs in order to preserve his current utility level. The indifference is described below, and the slope at (5,8) should be the same as the value of MRS, 4.

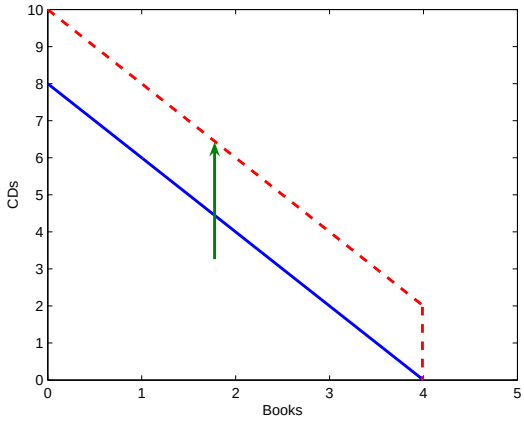


Figure 1: Budget Line

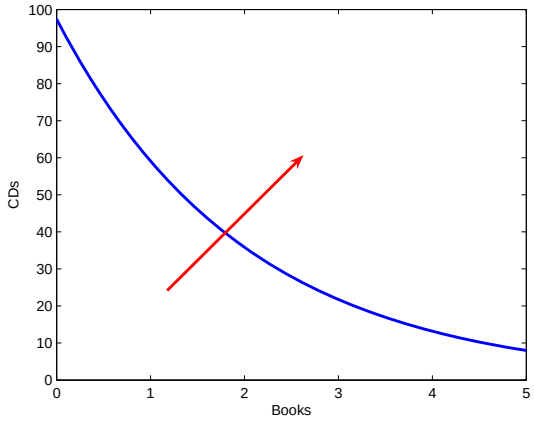


Figure 2: Indifference Curve

(c) Two things are required to be an optimal; first, the tangency condition: the budget line should be a tangent line to the indifference curve. Second, the optimal point should be located on the budget line.

(d) The first Secret of Happiness(SOH) says that the MRS should coincide with (negative) the relative price at equilibrium: $\text{MRS} = -\frac{p_1}{p_2}$. It means that rate of exchange at which the consumer is willing to stay put(MRS) must be equal to the price ratio. The second SOH is $p_1x_1 + p_2x_2 = m$, which simply means that all the money should be exhausted by consuming both of the goods.

(e) Since MRS$= -\frac{x_2}{2}$, $x_2 = 4$ immediately follows from the first SOH. Note that he can afford $x_2 = 4$. The extra money $\$40 - \$5 \times 4 = \$20$ must be spent on good 1 by the second SOH

and thus $x_1 = 2$. Now if p_1 increased to \$30, the first SOH says $\frac{x_2}{2} = \frac{30}{5} = 6$, that is, $x_2 = 12$ but it is not affordable since he needs \$60 for 12 CDs. Plugging $x_2 = 12$ into the budget line

$$30x_1 + 5 \times 12 = 40 \rightarrow x_1 = -\frac{2}{3} < 0. \tag{1.2}$$

Hence $x_1 = 0$ and $x_2 = \frac{m}{p_2} = \frac{40}{5} = 8$. While the first bundle $(x_1, x_2) = (2, 4)$ is interior, the second bundle $(x_1, x_2) = (0, 8)$ is at the corner.

(f) As depicted below, $(2, 4)$ is in the interior. Hence the marginal utilities from a dollar for books and for CDs at $(2, 4)$ must be the same. You can easily verify that

$$\text{when } (p_1, p_2) = (\$10, \$5), \quad \frac{\text{MU}_1}{p_1} = \frac{1}{10} = \frac{2/4}{5} = \frac{2/x_2}{5} = \frac{\text{MU}_2}{p_2}. \tag{1.3}$$

At $(x_1, x_2) = (0, 8)$, on the other hand, their marginal utilities from a dollar would not be the same. Observe that

$$\text{when } (p_1, p_2) = (\$30, \$5), \quad \frac{\text{MU}_1}{p_1} = \frac{1}{30} \text{ but } \frac{\text{MU}_2}{p_2} = \frac{2/x_2}{p_2} = \frac{2/8}{5} = \frac{1}{20} \tag{1.4}$$

2 Problem 2

(a) Since hamburgers and cokes are perfect *complements* for Michael, his utility function over two goods must take a form of

$$U(x_1, x_2) = \min\{x_1, 3x_2\}. \tag{2.1}$$

(b) Recall that the indifference curve has a "L" shape in case of perfect complements. And the optimal preference line should be $x_1 = 3x_2$.

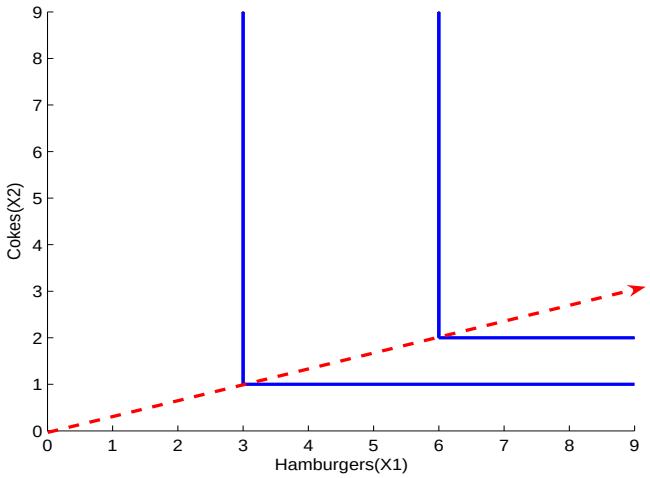


Figure 3: Indifference Curve and Optimal Proportion Line

(c) Two secrets of Happiness in case of perfect complements are as follows:

$$(i) \quad x_1 = 3x_2 \tag{2.2}$$
$$(ii) \quad p_1x_1 + p_2x_2 = m. \tag{2.3}$$

The condition (i) provides us with the economic intuition that he is willing to consume two goods in the same proportion $2 : 1$. The second condition says that his income must be exhausted.

(d) Plugging (2.2) into (2.3) gives us the equation of x_1 only;

$$p_1(3x_2) + p_2x_2 = x_2(3p_1 + p_2) = m. \tag{2.4}$$

Hence $x_2 = \frac{m}{3p_1+p_2}$ and $x_1 = \frac{3m}{3p_1+p_2}$. Since $x_1, x_2 > 0$(without loss of generality, his income $m > 0$.) they are interior.

(e) They are *ordinary* since as x_1 and x_2 are downward sloping. They are *normal* since as m goes up, so do x_1 and x_2. Finally, they are *gross complements* since as the price of one good goes up the quantity of the other good will decrease.

(f) By Slutsky equation we can decompose the total change in demand into two effects; *substitution* and *income* effects. However, its substitution effects would be zero because they are perfect complements. Its main intuition follows from the fact that he is willing to consume in the same proportion so his optimal choice will be determined by the set of kinks in his indifference curve.

3 Problem 3

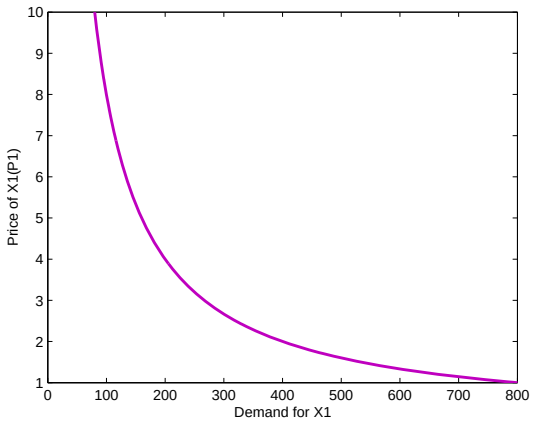


Figure 4: Demand Curve for x_1

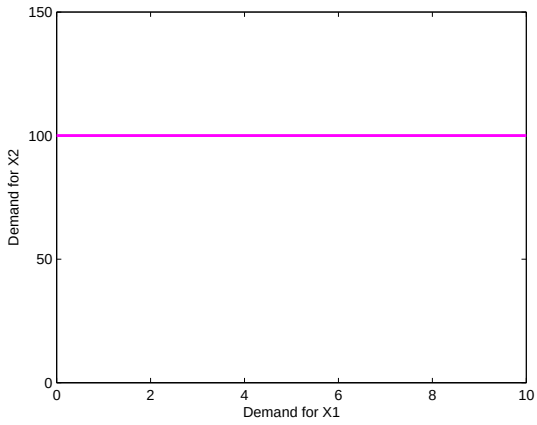


Figure 5: Price(p_1)-Offer Curve

(a) Let a function $V(x_1, x_2) = 2\log x_1 + 1\log x_2$. You can easily see that Adam's complicated utility function is just a monotone transformation of V. In fact, we shall rewrite his original utility function in terms of V as

$$U(x_1, x_2) = \left[700 \times \sqrt{\log V^2} + 10\right]^{800}. \tag{3.1}$$

(b) Part (a) enables us to analyze his consumption behavior using the simple function $V(x_1, x_2)$. Since it is a Cobb-Douglas utility, the magic formulae lead us to his demand function:

$$x_1 = \frac{a}{a+b}\frac{m}{p_1} = \frac{2}{2+1}\frac{1200}{4} = 200$$
$$x_2 = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{2+1}\frac{1200}{4} = 100.$$

(c) Assume that $p_2 = 4$ and $m = 1200$. Now we can think of x_1's demand function which displays how the quantity of x_1 changes as p_1 varies, as well as the p_1-offer curve which displays how the set of optimal bundles (x_1, x_2) changes as p_1 varies. When $p_2 = 4$ and $m = 1200$, the above demand function comes down to

$$x_1 = \frac{a}{a+b}\frac{m}{p_1} = \frac{2}{2+1}\frac{1200}{p_1} = \frac{800}{p_1} \tag{3.2}$$
$$x_2 = \frac{b}{a+b}\frac{m}{p_2} = \frac{1}{2+1}\frac{1200}{4} = 100. \tag{3.3}$$

(3.2) addresses his demand function which is depicted below. With both of them (3.2) and (3.3), you can draw p_1-offer curve which must be flat.

4 Problem 4

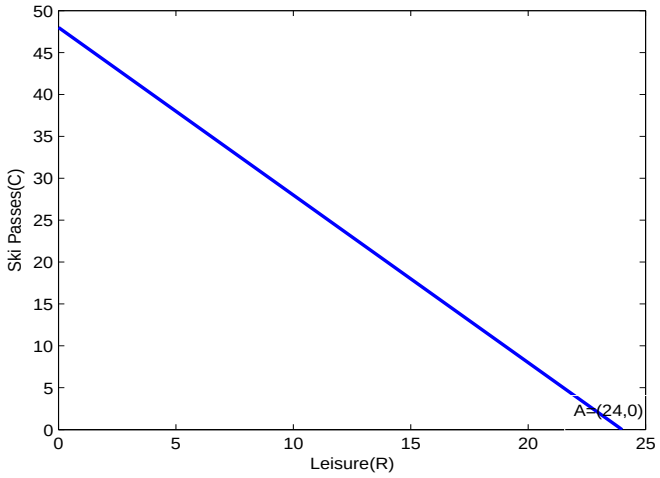


Figure 6: Frank's Budget Line and His Endowments

(a) If Frank enjoys leisure during R hours, his labor income would be $w(24 - R) = 20 \times (24 - R)$. Those money will be totally spent on ski passes(C). Denoting its price by p_C, we can write his budget line as

$$p_C C = w(24 - R). \tag{4.1}$$

Hence you end up with $10C + 20R = 20 \times 24$ when $p_C = 10$ and $w = 20$. Note that he is just endowed with his daily hours, 24 hours. Point $A = (24, 0)$ in the figure indicates his endowment.

(b) The real wage is simply $\frac{w}{p_C} = 2$. It is the slope of his budget line.

(c) Since $U(R, C) = RC^5$ is Cobb-Douglas, his demand for R and C are

$$R = \frac{a}{a+b}\frac{m}{w} = \frac{1}{1+5}\frac{480}{20} = 4$$
$$\text{and} C = \frac{b}{a+b}\frac{m}{p_C} = \frac{5}{1+5}\frac{480}{10} = 40,$$

respectively. From $R = 4$, his labor supply is immediate; $24 - 4 = 20$ hours.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (50+20+15+15=100 points) + bonus (10 extra "e" points). Make sure you answer the first four questions before working on the bonus one!

Problem 1. (50 points) To reenergize for Econ 301 class, in the morning, Tony always drinks Mountain Dew (x_1) and eats Burritos (x_2).

a) Suppose Mountain Dew costs $p_1 = \$2$, burrito costs $p_2 = \$10$, and his daily budget is $m = \$40$. Show graphically the budget constraint in the commodity space. Mark the two extreme consumption bundles (mark concrete values). On the same graph, show how the budget set is affected by inflation of 100% that affects prices of both commodities but does not affect income (so his income stays the same $m = \$40$)?

b) Tony's preferences are given by the following utility function

$$U(x_1, x_2) = x_1^3 x_2^3.$$

Find Tony's marginal rate of substitution (MRS) as a function of x_1, x_2 (give a formula for MRS).

- What is the value of MRS at consumption bundle $(1, 2)$ (give a number)?

- Which of the two goods is more valuable, soda or burrito, if Tony drinks one Mountain Dew and consumes two burritos?

- Depict his indifference curve map in a commodity space. Mark the slope of the indifference curve at the bundle $(1, 2)$.

c) In the commodity space (x_1, x_2), find (geometrically) Tony's optimal choice,.assuming pre-inflation prices $p_1 = \$2$, and $p_2 = \$10$. Describe how the two properties of the optimal bundle, known as two "secrets of happiness" (two short sentences) can be seen in the graph.

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values).

- Provide some economic intuition behind the two conditions (ca. two sentences for each).

- Derive the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation).

- What fraction of income is spent on burritos (give the percentage)?

- Find analytically and geometrically the demand curve for Mountain Dew (given $p_2 = \$10$ and $m = \$40$) and Engel curve (given $p_1 = \$2$, and $p_2 = \$10$)

- Are they Giffen goods? Why? (yes/no answer + one sentence).

e) Using your formula from d) find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- $p_1 = \$2, p_2 = \$10$ and $m = \$40$ (give two numbers).

and after the price of Mountain Dew decreased:

- for $p_1 = \$1, p_2 = \$10$ and $m = \$40$ (give two numbers).

What is the total change in consumption of Mountain Dew? (give a number). Illustrate the change on the graph.

f) decompose the total change in consumption of x_1 from e) into a substitution and income effect. (Calculate the two numbers and show how can you find the effect on the graph.)

Problem 2. (20 points) Bill is a wild-animal lover. From his recent trip to Galapagos Islands he brought a small Iguana. His new pet has only three legs: two left and one right. (Iguanas use magma heated soil to warm their eggs and his favorite pet lost one right leg during the last volcano eruption). To survive the famous Madisonian winter, the iguana has to wear shoes, left (x_1) and right ones (x_2).

a) Write down Bill's utility function representing his preferences over right and left shoe (function $U(x_1, x_2)$).

b) In the commodity space (x_1, x_2), carefully depict Bill's indifference curves.

c) Find analytically Bill's demand for shoes if $p_1 = \$2$ and $p_2 = \$1$ and Bill's budget for iguana shoes is $m = \$15$. Is the solution interior? (give two numbers and a yes/no answer).

d) Illustrate Bill's optimal choice on the graph including the indifference curves and the budget set.

e) Suppose the price of left shoe goes down to $p_1 = \$1$. Find Bill's new demand for shoes. What can you say about the substitution effect? How about the income effect? (Answer the latter question without any calculations, using only a graph).

Problem 3. (15 points) Ramon decides about his new collections of postage stamps. He is interested in two themes: "the birds of the world", x_1, (measures the number of stamps in the subcollection with birds) and "the famous mathematicians", x_2. The utility derived from the collection is given by

$$U(x_1, x_2) = x_1 + 10 \times \ln x_2.$$

a) What is the optimal collection of stamps if the prices are $p_1 = 1$ and $p_2 = 2$ and $m = 15$. (find two numbers x_1 and x_2). Is your solution interior, or corner?

b) Find the optimal collection if the prices are still $p_1 = 1$ and $p_2 = 2$, but the income is only $m = 8$. Depict Ramon's optimal choice in the commodity space.

Problem 4. (15 points) Jacob can use his 24h for leisure R, or work. The hourly wage rate is $w = 5$. Jacob spends all his money on cheese curds C.

a) Draw Jacob's budget set, given the price of cheese is $p_c = \$10$ (mark the endowment point).

Let the utility function be given by

$$U(x_1, x_2) = R + C.$$

b) Are leisure and cheese curds perfect complements, perfect substitutes or none of them?

c) What is the optimal choice of leisure, cheese curds and labor supply? (Find geometrically and give three numbers).

d) Harder: Plot a graph with labor supply (horizontal axis) and wage rate (vertical one) assuming $p_c = \$10$ (hint: for what value of w do we go from one "bang" to the other "bang" solution?)

Bonus Problem. (extra 10 points) Let $U(x_1, x_2) = x_1 x_2^2$, and $V(x_1, x_2) = \ln x_1 + 2 \ln x_2$ be two utility functions.

a) Show that $U(\cdot)$ is a monotone transformation of $V(\cdot)$, and hence they define the same preferences.

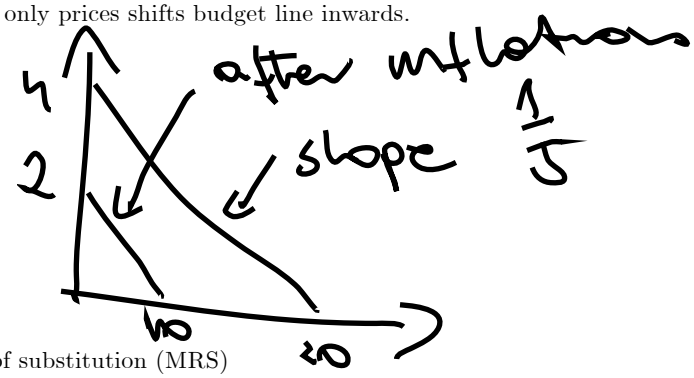
b) Derive MRS for each of the two functions. Using the two formulas for MRS, argue that the functions define the same indifference curve maps.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to midterm 1 (Group B)

Problem 1.

a) With $p_1 = \$2$, $p_2 = \$10$ and $m = \$40$ the budget set is (two extreme consumption bundles are 20 and 4). Inflation that affects only prices shifts budget line inwards.



b) Tony's marginal rate of substitution (MRS)

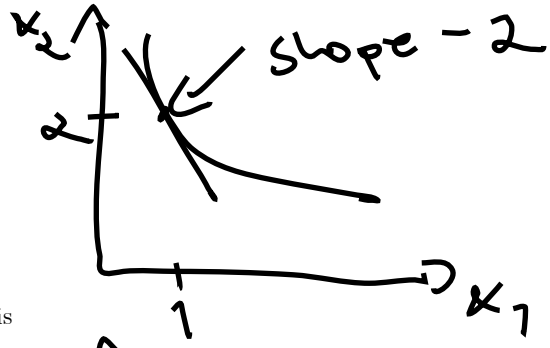
$$MRS = -\frac{MU_1}{MU_2} = -\frac{x_2}{x_1}$$

- The value of MRS at consumption bundle $(1, 2)$ is

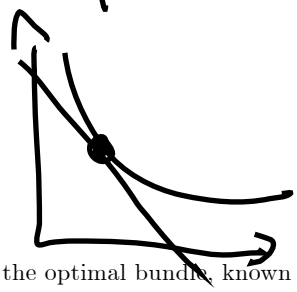
$$|MRS| = |-\frac{2}{1}| = 2$$

- Burrito (x_2) is less valuable than Mountain Dew (x_1)

- Tony's indifference curve map is. (the slope of her indifferent curve that passes through bundle $(1, 2)$ is -2.



c) Tony's optimal choice is



- the two geometric properties of the optimal bundle, known as two "secrets of happiness" are:

1. At the optimal bundle, the indifference curve is tangent to a budget set
2. The optimal bundle is located on budget line

d) mathematically the two secrets of happiness, are

$$\begin{cases} MRS = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

- the economic intuition behind the two conditions is:

The individual value of x_1 in terms of x_2 coincides with the market value

The income of a consumer is exhausted

- the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m can be found as follows

From the MRS condition

$$MRS = -\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

hence

$$x_2 = \frac{p_1}{p_2} x_1$$

plugging in budget constraint

$$p_1 x_1 + p_2 \left(\frac{p_1}{p_2} x_1 \right) = m$$

Solving for x_1 gives

$$x_1 = \frac{1}{2} \frac{m}{p_1}$$

Plugging in

$$x_2 = \frac{p_1}{p_2} \left(\frac{1}{2} \frac{m}{p_1} \right) = \frac{1}{2} \frac{m}{p_2}$$

- the fraction of income spent on burritos is

$$\frac{p_1 x_1}{m} = \frac{1}{2} = 50\%$$

- and the demand curve for burritos book (given $p_2 = \$10$, and $m = \$40$) and Engel curve (given $p_1 = \$2$, and $p_2 = \$10$)

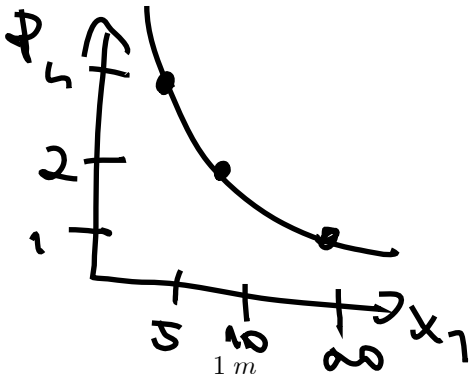
Demand curve

$$x_1 = \frac{1}{2} \frac{m}{p_1} = \frac{1}{2} \frac{40}{p_1} = \frac{20}{p_1}$$

and hence inverse demand is

$$p_1 (x_1) = \frac{20}{x_1}$$

Geometrically



Engel curve: Since

$$x_1 = \frac{1}{2} \frac{m}{p_1}$$

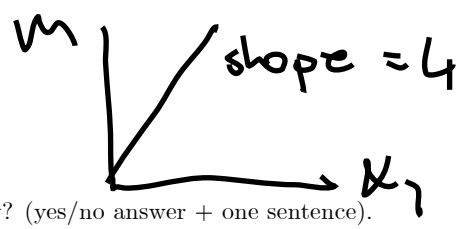
at $p_1 = \$2$

$$x_1 = \frac{1}{2} \frac{m}{2} = \frac{1}{4} m$$

hence

$$m (x_1) = 4 x_1$$

Geometrically



- are they Giffen goods? Why? (yes/no answer + one sentence).

No, because the demand curve is downwardslopping on the whole domain.

e) The optimal consumption levels for (x_1, x_2).

- at $p_1 = \$2, p_2 = \$10$ and $m = \$40$

$$x_1 = \frac{1}{2}\frac{m}{p_1} = \frac{1}{2}\frac{40}{2} = 10$$

and

$$x_2 = \frac{1}{2}\frac{m}{p_2} = \frac{1}{2}\frac{40}{10} = 2$$

and after the price of science-fiction book decreased, for $p_1 = \$1, p_2 = \$10$ and $m = \$40$

$$x_1 = \frac{1}{2}\frac{m}{p_1} = 20$$

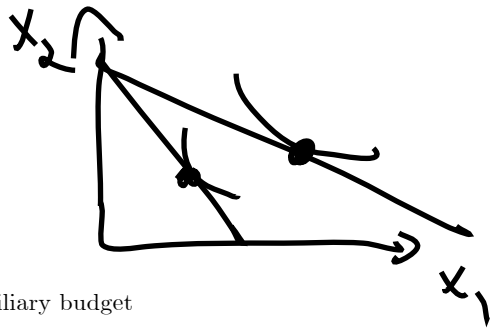
and

$$x_2 = \frac{1}{2}\frac{m}{p_2} = \frac{1}{2}\frac{40}{10} = 2$$

Hence the total change in consumption of x_1 is

$$\Delta x_1 = 20 - 10 = 10$$

Geometrically



f) Substitution effect: auxiliary budget

$$m' = 10 \times 1 + 10 \times 2 = 30$$

and hence

$$x_1 = \frac{1}{2}\frac{30}{1} = 15$$

so SE is equal to

$$SE = 15 - 10 = 5$$

and income effect is

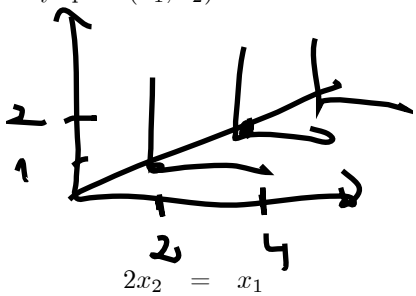
$$IE = 10 - 5 = 5$$

Problem 2. .

a) Bill's utility function is

$$U(x_1, x_2) = \min(x_1, 2x_2)$$

b) indifference curves.in the commodity space (x_1, x_2) are



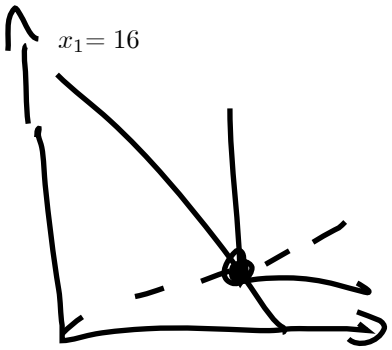
c) Bill's demand for shoes is

$$\begin{aligned} 2x_2 &= x_1 \\ 2x_1 + x_2 &= 40 \end{aligned}$$

$$4x_2 + x_2 = 40$$

$$x_2 = \frac{40}{5} = 8$$

d) geometrically Bills's optimal choice is



e) when the price of a left shoe goes down to $p_1 = \$1$. the new demand is given by the system of equations

$$\begin{aligned} 2x_2 &= x_1 \\ x_1 + x_2 &= 40 \end{aligned}$$

and hence demand is

$$\begin{aligned} x_1 &= \frac{80}{3} = 26\frac{2}{3} \\ x_2 &= \frac{40}{3} = 13\frac{1}{3} \end{aligned}$$

The substitution effect is zero (perfect complements) and the income effect is $4.10\frac{2}{3}$

Problem 3.

a) the two secrets of happiness are

$$\begin{aligned} -\frac{x_2}{10} &= -\frac{1}{2} \\ x_1 + 2x_2 &= 15 \end{aligned}$$

and hence $x_2 = 5$ and $x_1 = 5$. Since both are positive, this is interior solution.

b) the two secrets of happiness are

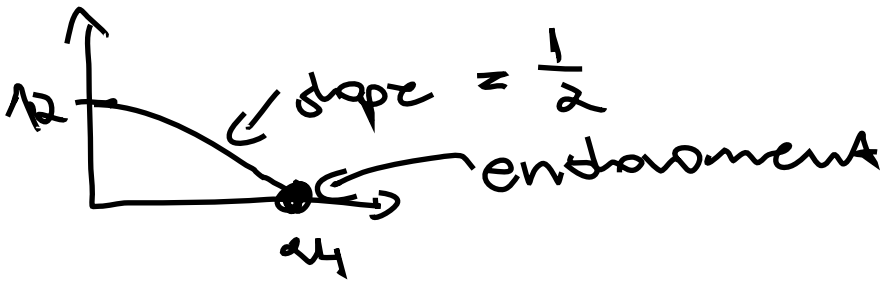
$$\begin{aligned} -\frac{x_2}{10} &= -\frac{1}{2} \\ x_1 + 2x_2 &= 8 \end{aligned}$$

and hence secrets of happiness give $x_2 = 5$ and $x_1 = -5$. Since consumption must be non-negative the optimal consumption is $x_1 = 0$ and $x_2 = 4$, which is a cornet solution.

Problem 4.

a) Jacob's budget set, with $w = \$5$ and $p_c = \$10$ is

Income is $m = 5 \times 24 = \$120$

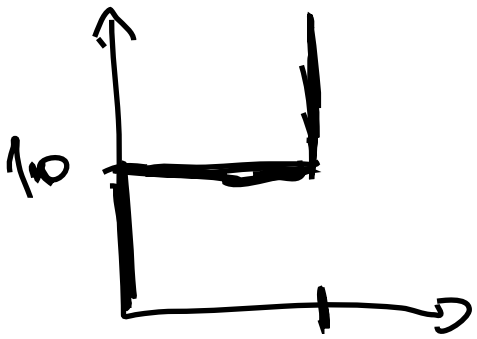


b) They are perfect substitutes

c) $|MRS| = 1 > \frac{w}{p_c} = \frac{1}{2}$ which implies that Jacob cares more about leisure than consumption,.therefore he will spend the whole day at home

$$R = 24, LS = 0 \text{ and } C = 0$$

d)



Bonus Problem. (extra 10 points)

a) Monotone transformation $\ln()$. Take a log of U

$$\ln U() = \ln x_1 x_2^2 = \ln x_1 + \ln x_2^2 = \ln x_1 + 2\ln x_2 = V()$$

where we used two properties of ln function.

b) For $U()$, marginal rate of substitution is

$$MRS = -\frac{MU_1}{MU_2} = -\frac{x_2^2}{2x_1x_2} = -\frac{x_2}{2x_1}$$

and for $V()$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{1/x_1}{2/x_2} = -\frac{x_2}{2x_1}$$

and hence MRS coincides for all (x_1, x_2).It follows that the slopes of indifference curves are the same at any point and hence they must be the same.

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 1 (Group B)

You have 70 minutes to complete the exam. The midterm consists of 3 questions (60+25+15=100 points) + bonus (10 points). Make sure you answer the first three questions before working on the bonus one!

Problem 1. (60 points) Maggie likes to read science fiction (x_1) and romance (x_2) novels. Her preferences over the two types of books are represented by a utility function

$$U(x_1, x_2) = (x_1)^6 (x_2)^3$$

a) Find Maggie's marginal rate of substitution (MRS) as a function of x_1, x_2 (give a formula).

- what is the value of MRS at consumption bundle $(2, 2)$ (give a number).
- complete the sentence: "The Marginal Rate of Substitution is a (marginal) value of a ... in terms of ..."
- how much one must compensate Maggie in terms of romance books, after taking away 0.00001 of a science-fiction book, in order to keep her indifferent? (give a number, assume she consumes bundle $(2, 2)$).
- depict her indifference curve map in a commodity space. Mark the slope of her indifferent curve that passes through bundle $(2, 2)$.

b) Suppose the price of a science-fiction book is $p_1 = \$4$, a romance book costs $p_2 = \$10$ and her total monthly spending on books is $m = \$600$. Show graphically her budget constraint in the commodity space. Mark the two extreme consumption bundles (give values). On the same graph, show how the budget set would be affected by the introduction of an ad valorem tax on science fiction books (x_2) at rate 100%?

c) In the commodity space (x_1, x_2), find (geometrically) Maggie's optimal choice.

- describe the two properties of the optimal bundle, known as two "secrets of happiness" (two short sentences).

d) Write down mathematically two secrets of happiness, assuming that p_1, p_2, m are parameters (and not concrete values).

- provide some economic intuition behind the two conditions (ca. two sentences for each).
- derive the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m (show the derivation).
- what fraction of income is spent on science - fiction novels (give the percentage).
- find analytically and geometrically the demand curve for science finction book (given $p_2 = \$10$, and $m = \$600$) and Engel curve (given $p_1 = \$4$, and $p_2 = \$10$)
- are science-fiction books normal goods? Why? (yes/no answer + one sentence).
- are they Giffen goods? Why? (yes/no answer + one sentence).

e) Using your formula from d) find the optimal consumption levels for both types of books (x_1, x_2).

- for $p_1 = \$4, p_2 = \$10$ and $m = \$600$ (give two numbers).

and after the price of science-fiction book decreased:

- for $p_1 = \$2, p_2 = \$10$ and $m = \$600$ (give two numbers).

What is the total change in consumption of x_1? (give one number). Illustrate this change on the graph.

f) decompose the total change in x_1 from f) into a substitution and income effect. (Calculate the two numbers, show how you found them on the graph.) Complete the two sentences:

"The substitution effect is attributed to the pure change in .... induced by the decrease of nominal price p_1"

"The income effect can be attribute to the pure change in .... induced by the decrease of nominal price p_1"

g) which of the following alternative utility functions represents Maggie's preferences (there are two such

functions)?

$$\begin{aligned} V(x_1, x_2) &= 6x_1 \times 3x_2 + 3 \\ V(x_1, x_2) &= 30\left(x_1^6 \times x_2^3\right) + 3 \\ V(x_1, x_2) &= 6\ln x_1 \times 3\ln x_2 \\ V(x_1, x_2) &= 6\ln x_1 + 3\ln x_2 + 7 \end{aligned}$$

Explain why the utility functions you have selected represent Maggie's preferences (one sentence). Suggest the transformation of $U()$ function that makes the two $V()$ functions equivalent.

Problem 2. (20 points) Jimmy's favorite hobby is slot car racing. He assembles slot trucks from parts, by adding ten wheels (x_1) to an engine (x_2) (these are supertrucks, with five wheels on each side). He purchases the parts on the market.

a) write down Jimmy's utility function representing his preferences over wheels and engines (function $U(x_1, x_2)$).

b) in the commodity space (x_1, x_2), carefully depict Jimmy's indifference curves.

c) find analytically Jimmy's demand for parts if one wheel costs $p_1 = \$2$, an engine is $p_2 = \$10$ and Jimmy's budget for slot cars is $m = \$120$. Is the solution interior (give two numbers and yes/no answer).

d) illustrate Jimmy's optimal choice on the graph including the indifference curves and the budget set.

e) suppose the price of one wheel goes down to $p_1 = \$1$. Find Jimmy's new demand for the parts. What can you say about the substitution effect? How about the income effect? (you can answer the last question without any calculations, using only a graph).

Problem 3. (20 points) Ramon Gonzales M. Panetelas is a specialist in Habanos cigars (famous Cuban cigars). Cuban cigars are sold either in 5 cigar packs (x_1), or in singles (x_2). Ramon has no income. Instead he is initially endowed with $\omega_1 = 10$ packs and $\omega_2 = 50$ cigars.

a) draw Ramon's budget set, given the price of a pack is equal to $p_1 = \$5$ and a single cigar is $p_2 = \$1$ (mark the endowment point).

b) Illustrate geometrically Ramon's optimal demand for packs and single cigars, given his utility function

$$U(x_1, x_2) = 5x_1 + x_2$$

(Give two numbers (x_1, x_2), and mark them on the graph, including budget set and the indifference curves.)

c) What is your answer to b) when prices are $p_1 = \$10$ and $p_2 = \$1$. (Give two numbers (x_1, x_2), and plot the graph.)

d) Harder: Give the formula for the demands x_1, x_2 as a function of p_1, p_2 and endowments ω_1 and ω_2. Show the demand on the graph, assuming $p_2 = \$1$, $\omega_1 = 10$ and $\omega_2 = 50$.

Bonus Problem. (extra 10 points) Depict a map of indifference curves that is consistent with

a) inferior goods

b) Giffen goods

(Make sure you explain why these graphs represent the respective preferences.)

Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Solutions to midterm 1 (Group B)

Problem 1. (60 points)

a) Maggie's marginal rate of substitution (MRS)

$$MRS = -\frac{MU_1}{MU_2} = -\frac{2}{1}\frac{x_2}{x_1}$$

- the value of MRS at consumption bundle $(2,2)$ is

$$MRS = -2$$

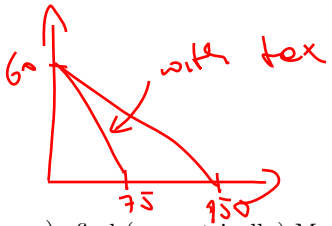
- "The Marginal Rate of Substitution is a (marginal) value of a science finction books in terms of romance novels"

- after taking away 0.00001 of a science-fiction book, to keep her indifferent one must compensate Maggie in terms of romance books

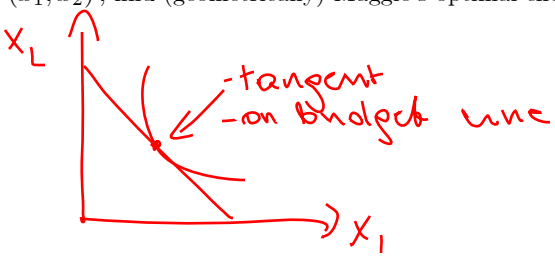
$$0.00001 \times 2 = 0.000002$$

- her indifference curve map is. (the slope of her indifferent curve that passes through bundle $(2,2)$ is -2.

b) With $p_1 = \$4$, $p_2 = \$10$ and $m = \$600$ the budget set is (two extreme consumption bundles are $(150,0)$ and $(0,60)$. The budget set with ad valorem tax shifts inwards.



c) In the commodity space (x_1, x_2), find (geometrically) Maggie's optimal choice.



- the two geometric properties of the optimal bundle, known as two "secrets of happiness" are:

1. At the optimal bundle, the indifference curve is tangent to a budget set
2. The oprimal bundle is locaded on budget line

d) mathematically the two secrets of happiness, are

$$\begin{cases} MRS = -\frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

- the economic intuition behind the two conditions is:

The individual value of x_1 in temrms of x_2 coincides with the market value

The income of a consumer is exchausted

- the optimal consumption of x_1 and x_2 as a function of p_1, p_2, m can be found as follows

From the MRS condition

$$MRS = -2\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

hence

$$x_2 = \frac{1}{2}\frac{p_1}{p_2}x_1$$

plugging in budget constraint

$$p_1 x_1 + p_2\left(\frac{1}{2}\frac{p_1}{p_2}x_1\right) = m$$

Solving for x_1 gives

$$x_1 = \frac{2}{3}\frac{m}{p_1}$$

Pluging in

$$x_2 = \frac{1}{2}\frac{p_1}{p_2}\left(\frac{2}{3}\frac{m}{p_1}\right) = \frac{1}{3}\frac{m}{p_2}$$

- the fraction of income spent on science - fiction novels is

$$\frac{p_1 x_1}{m} = \frac{2}{3} = 66\%$$

- and the demand curve for science finction book (given $p_2 = \$10$, and $m = \$600$) and Engel curve (given $p_1 = \$4$, and $p_2 = \$10$)

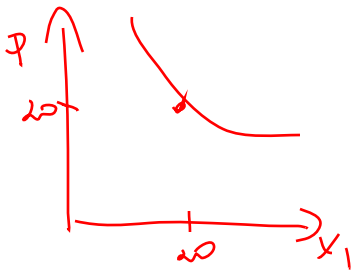
Demand curve

$$x_1 = \frac{2}{3}\frac{m}{p_1} = \frac{400}{p_1}$$

and hence invese demand is

$$p_1(x_1) = \frac{400}{x_1}$$

Geometrically



Engel curve: Since

$$x_1 = \frac{2}{3}\frac{m}{p_1}$$

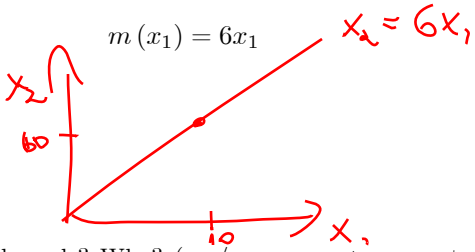
at $p_1 = \$4$

$$x_1 = \frac{2}{3}\frac{m}{4} = \frac{1}{6}m$$

hence

$$m(x_1) = 6x_1$$

Geomettrically



- are science-fiction books normal goods? Why? (yes/no answer + one sentence).

Yes, because their demand increases in income

- are they Giffen goods? Why? (yes/no answer + one sentence).

No, becase the demand curve is downwardslopping on the whole domain.

e) The optimal consumption levels for both types of books (x_1, x_2).

- for $p_1 = \$4, p_2 = \$10$ and $m = \$600$

$$x_1 = \frac{2}{3}\frac{m}{p_1} = \frac{2}{3}\frac{600}{4} = 100$$

and

$$x_2 = \frac{2}{3}\frac{m}{p_2} = \frac{1}{3}\frac{600}{10} = 20$$

and after the price of science-fiction book decreased, for $p_1 = \$2, p_2 = \$10$ and $m = \$600$

$$x_1 = \frac{2}{3}\frac{m}{p_1} = \frac{2}{3}\frac{600}{2} = 200$$

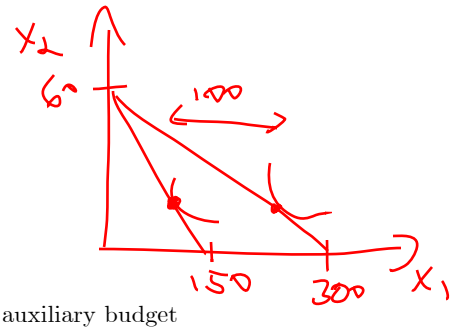
and

$$x_2 = \frac{2}{3}\frac{m}{p_2} = \frac{1}{3}\frac{600}{10} = 20$$

Hence the total change in consumption of x_1 is

$$\Delta x_1 = 200 - 100 = 100$$

Geometrically



f) Substitution effect: auxiliary budget

$$m' = 100 \times 2 + 20 \times 10 = 400$$

and hence

$$x_1 = \frac{2}{3}\frac{400}{2} = 133\frac{1}{3}$$

so SE is equal to

$$SE = 133\frac{1}{3} - 100 = 33\frac{1}{3}$$

and income effect is

$$IE = 100 - 33\frac{1}{3} = 66\frac{2}{3}$$

"The substitution effect is attributed to the pure change in relative price induced by the decrease of nominal price p_1"

"The income effect can be attribute to the pure change in real income induced by the decrease of nominal price p_1"

g) From the functions

$$\begin{aligned} V(x_1, x_2) &= 6x_1 \times 3x_2 + 3 \\ V(x_1, x_2) &= 30\left(x_1^6 \times x_2^3\right) + 3 \\ V(x_1, x_2) &= 6\ln x_1 \times 3\ln x_2 \\ V(x_1, x_2) &= 6\ln x_1 + 3\ln x_2 + 7 \end{aligned}$$

the second and last represent Maggie's preferences - they are monotone transformations of $U(\,)$. The first transformation is

$$f(U) = 30U + 3$$

and the second is

$$f(U) = \ln U + 7$$

Problem 2. .

a) Jimmy's utility function is

$$U(x_1, x_2) = \min(x_1, 10x_2)$$

b) in the commodity space (x_1, x_2), carefully depict Jimmy's indifference curves.

c) Jimmy's demand for parts is interior and is given by

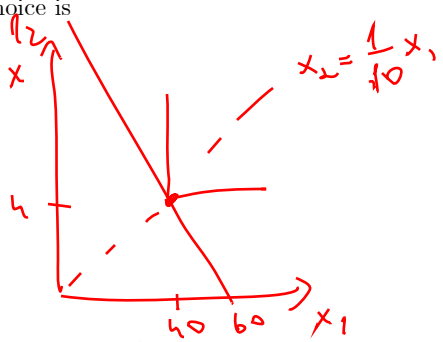
$$\begin{aligned} x_2 &= \frac{1}{10}x_1 \\ 2x_1 + 10x_2 &= 120 \end{aligned}$$

$$2x_1 + 10\frac{1}{10}x_1 = 120$$

$$x_1 = \frac{120}{3} = 40$$

$$x_2 = \frac{1}{10} \times 40 = 4$$

d) geometrically Jimmy's optimal choice is



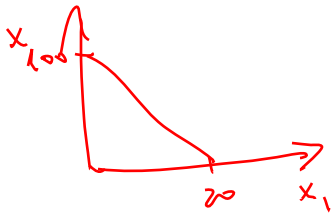
e) when the price of a wheel goes down to $p_1 = \$25$. the new demand is

$$\begin{aligned} x_2 &= \frac{1}{10}x_1 \\ x_1 + 10x_2 &= 120 \end{aligned}$$

$$\begin{aligned} x_1 &= 60 \\ x_2 &= 6 \end{aligned}$$

Thte substitution effec is zero (perfect complements) and the income effect is 20 wheels.

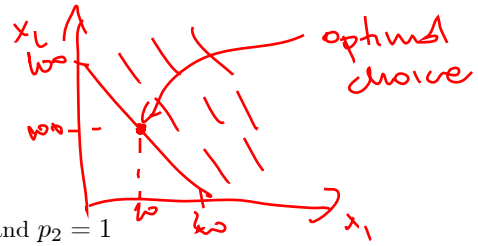
Problem 3.
a) Ramon's budget set, with $p_1 = \$5$ and $p_2 = \$1$ is
Budget set: $m = 10 \times 5 + 50 \times 1 = 100$



b) The optimal demand for packs and single cigars can be found as follows. Since

$$-MRS = 5 = \frac{p_1}{p_2} = 5$$

therefore it does not matter how Ramon allocates his income. For example his endowment point is optimal



c) When prices are $p_1 = 10$ and $p_2 = 1$

$$-MRS = 5 < \frac{p_1}{p_2} = 10$$

and hence the total income will be invested in x_2. At such prices demands are

$$x_1 = 0 \text{ and } x_2 = \frac{10 \times 10 + 50}{1} = 150$$

d) The demands are
If $\frac{p_1}{p_2} < 5$ then

$$\begin{aligned} x_1 &= \frac{p_1\omega_1 + p_2\omega_2}{p_1} \\ x_2 &= 0 \end{aligned}$$

if $\frac{p_1}{p_2} > 5$ then

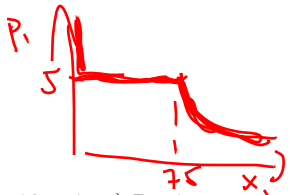
$$\begin{aligned} x_1 &= 0 \\ x_2 &= \frac{p_1\omega_1 + p_2\omega_2}{p_2} \end{aligned}$$

and if $\frac{p_1}{p_2} = 5$

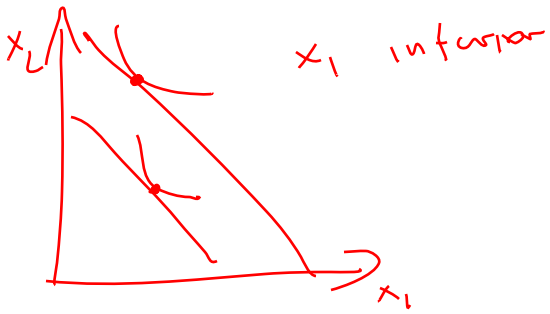
$$\begin{aligned} x_1 &= \alpha\frac{p_1\omega_1 + p_2\omega_2}{p} \\ x_2 &= (1-\alpha)\frac{p_1\omega_1 + p_2\omega_2}{p_2} \end{aligned}$$

for $\alpha \in (0, 1)$

The demand curve for x_1 is ($p_2 = 1$, $\omega_1 = 5$ and $\omega_2 = 50$).



Bonus Problem. (extra 10 points) Depict a map of indifference curves that is consistent with
a) inferior goods



b) Giffen goods

(Make sure you explain why these graphs represent the respective preferences.)